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SIMPLIFIED TRANSMISSION AND GENERATION SYSTEM ANALYSIS PROCEDURES FOR SUBSYNCHRONOUS RESONANCE PROBLEMS

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ABSTRACT

An analysis of subsynchronous resonance problems requires a clear understanding of the physical relationships that produce the phenomenon. This paper presents these relationships and uses them to derive a number of useful formulas for studying the problems. The mathematical basis of these formulas is shown and the approximations required for their derivation are described. These formulas are most useful in planning a series capacitor compensated transmission system to avoid or minimize subsynchronous resonance problems. This application is the subject of a companion paper.

INTRODUCTION

The analysis of subsynchronous resonance problems in series capacitor compensated transmission systems is a unique, but complex technical problem. If the analysis reveals a serious problem, corrective measures may be required. Even with such corrective measures, series capacitors in ac transmission lines are by far the most economical alternative of transmitting power over long distances.

To perform a detailed subsynchronous resonance study requires a method of analysis which can give quantitative results for a wide variety of system operating conditions. The method must be simple enough to visualize all the significant factors involved so that no essential cases are overlooked. This paper presents such a method. It relies on the principles of superposition so that the phenomena can be broken into its essential components and these can be analyzed separately. The analysis is based on a steady state oscillation in the power system and it predicts the corrective measures required to achieve a stable system. The results are compared with more detailed digital computer analysis programs to demonstrate the validity of the approximations that are used.

GENERATOR RELATIONSHIPS INVOLVED IN SELF-EXCITATION

Frequency analysis techniques using the steady state impedances of electrical networks have been traditionally used to analyze system stability. The application of such techniques for the subsynchronous resonance problem requires a circuit representation of the turbine generator. A positive sequence representation of the generator as viewed from its terminals is the most convenient because it can be directly coupled to existing power system network analysis programs. Such a model can also provide a great deal of insight into the phenomena that produces self-excited oscillations and the means of controlling them.

A derivation of the simplified generator model that is being used to indicate the presence of self-excited oscillations is shown in Appendix I. The model shows that for a mechanical oscillation of the generator rotor at a frequency f_m, the transmission system will contain positive sequence oscillating voltages and currents at the frequencies $f_n + f_m$ and $f_n - f_m$ where f_n is the normal system frequency (60 Hz in the U.S.). This can be seen physically by considering the action of an open circuited generator that mechanically contains a steady rotation and an oscillation. The fundamental component of the air gap flux is sinusoidally distributed and it rotates with the machine rotor. An instantaneous voltage is induced in each phase of the stator windings that is proportional to the time derivative of the flux linking the winding. For small amplitude mechanical oscillations the voltage generated is the same as would be generated by having three sinusoidally distributed components of air gap flux rotating with different velocities. The magnitude of the component at synchronous frequency will be unchanged by the oscillation. The magnitude of the other components will be proportional to the amplitude of the oscillation as shown in Figure 1. One will have the frequency $f_n + f_m$ and the other $f_n - f_m$.

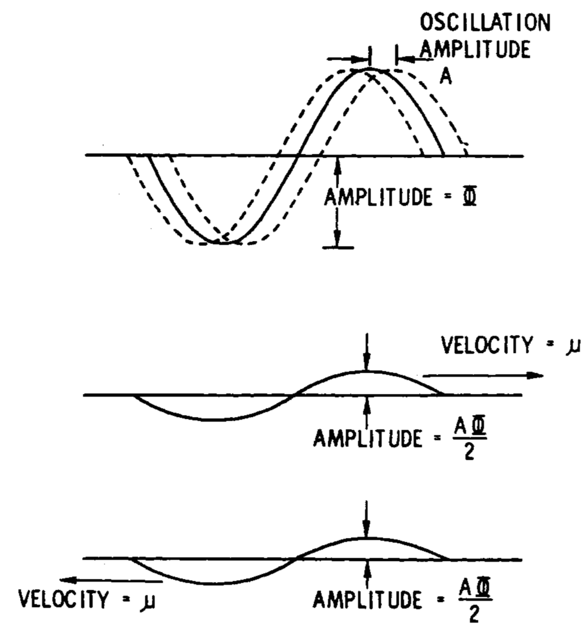

Fig. 1. Fundamental component of air gap flux as viewed from a plane rotating at synchronous speed with an oscillation of the rotor of the form A sin μt. The top view shows the total wave at different points in time. The bottom traces illustrate the fact that the oscillation may be represented by flux waves rotating in opposite directions with respect to the synchronous plane.

An analysis of stator voltage components at two frequencies other than the fundamental can be made by forming two positive sequence networks where the impedances of each network reflect the appropriate frequency. The currents for each network can be calculated separately and the feedback to the rotor circuit determined by the superposition of the two components. When the voltages produced by rotor oscillation are expressed as per unit quantities they have the form

$$V_e = \frac{f_e}{2\,f_m}\,\Phi_o\,\Delta\omega$$

where f_e is electrical frequency ($f_n \pm f_m$) and f_m is one of the torsional resonant frequencies of the turbine generator. For most

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systems Φ_0, the air gap flux, is near 1.0 p.u. so it could be omitted from the expression. $\Delta\omega$ is the per unit magnitude of the mechanical oscillating velocity.

The remainder of the circuit model required to represent the generator can be visualized by considering the induction motor action of the machine.[2] Positive sequence stator currents at frequencies other than f_n in a generator induce currents in the rotor circuit at the slip frequency $f_m = f_e - f_n$. For a round rotor machine the familiar induction motor circuit can be used to represent these effects. Since two frequencies other than f_n are present in the stator circuit, there will be an induction motor equivalent circuit for each frequency. For the subsynchronous electrical frequency the effective rotor resistance will be negative, corresponding to induction generator action. For the super synchronous electrical frequency the effective rotor resistance will be positive, corresponding to induction motor action.

A positive sequence circuit model which represents the generator for mechanical oscillation is shown in Figure 2. It is simply the combination of the voltage source that represents the oscillation of the main flux and the induction machine equivalent circuit. For practical turbine generators, the induction machine portion of the circuit can be represented by the series circuit which contains the stator resistance, subtransient reactance, and effective rotor resistance divided by slip. In forming this circuit the electrical oscillation frequency f_e is used.

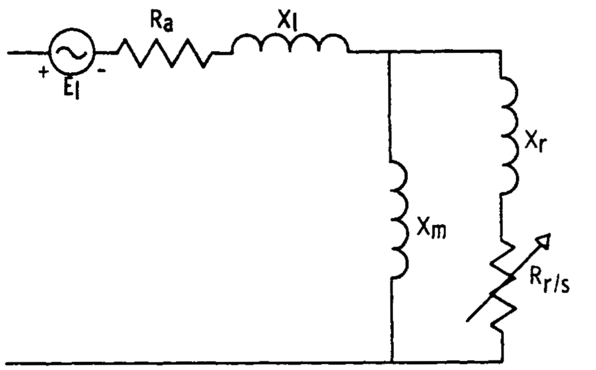

Fig. 2. Equivalent circuit to represent a synchronous machine at a non synchronous frequency. E, is the voltage produced by rotor oscillation. Electrical quantities are in per unit at the non synchronous frequency and for a 60 Hz system s = (fe - 60)/fe.

For analyses where only the motion of one machine is considered, the transmission system, loads, and other machines can be represented by an equivalent impedance at the given stator frequency. This impedance in series with the generator model determines the magnitude and phase of oscillating currents. The formation of the equivalent impedance requires a circuit representation for all loads and machines as well as the transmission system. For most load busses, the short circuit equivalent resistance and reactance is used with the reactance adjusted for the appropriate frequency. The other synchronous machines in the system are represented by their induction machine equivalent circuits based on the assumption that they do not have a mechanical resonance and, hence, no motion at the frequency of interest. More complex representations could be envisioned, but there is no conclusive evidence to date that they are required.

A plot of the equivalent system reactance and resistance including induction generator effects as a function of frequency allows a quick analysis of the possibility of self-excited oscillations due to the induction machine behavior alone. Such a plot for a complex system is shown in Figure 3. The frequencies where the reactance is zero are the electrical resonant frequencies of the system. If the resistance at these frequencies is negative, a self-excited oscillation due to induction generator action is indicated. These plots are often made excluding the resistance and reactance of the generator under study. In that event the generator effects can be added to the plot and the necessity of having pole face amortisseur windings can be evaluated.

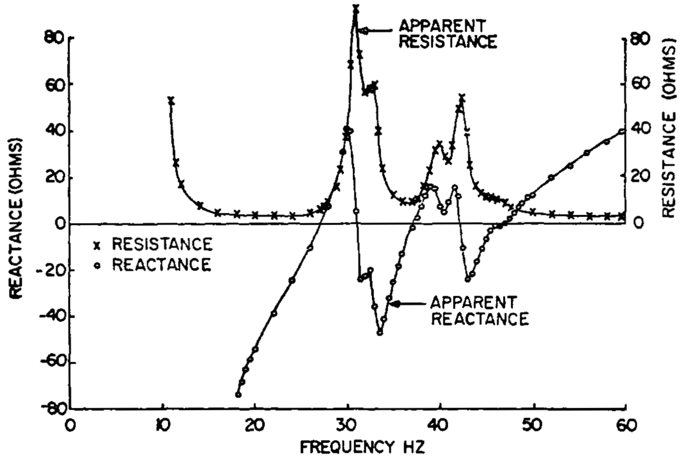

Fig. 3. Transmission system apparent resistance and reactance as a function of frequency. This is the impedance seen from the generator for a series-compensated transmission system.

The simplified analysis to determine the presence of self-excited oscillations due to the mechanical oscillation of the generator rotor requires an additional assumption. This assumption is that the mechanical oscillation frequency will be one of the torsional resonant frequencies of the turbine generator. The influence of the electrical system will alter these frequencies slightly, but these effects are less than the possible changes caused by inaccuracies in system data. Therefore, this assumption does not limit the usefulness of the analysis. With the mechanical resonant frequency determined, the corresponding electrical frequencies are also known and the oscillating currents can be calculated. These oscillating currents interact with the air gap flux to produce an oscillating torque at the mechanical frequency as shown in Appendix I. The magnitude and phase position of the torque with respect to the oscillating velocity that produced the positive sequence voltages determines whether self-excited oscillations are expected. The apparent mechanical damping produced by the electrical interaction is the in-phase component of oscillating torque divided by oscillating velocity. For the subsynchronous component, Appendix I shows this damping coefficient to be negative and of the magnitude

$$-D = \frac{f_e}{2 f_m} \frac{R}{|Z|^2}$$

In this expression the R is the net resistance of the electrical circuit and Z is the impedance of the electrical circuit at the frequency $f_e = f_n - f_m$. The damping component produced by the supersynchronous component has the same form but opposite sign. If the sum of these two damping components and the mechanical damping of the turbine generator is less than zero, self-excited oscillations are indicated.

Two other ways of expressing the results of this analysis are often useful in determining the amount of corrective action required when self-excited oscillations are indicated. One is to express mechanical interaction as a negative resistance in the stator circuit rather than a voltage source. This expression can be developed from the formulas in Appendix I and it is

$$-R = \frac{f_e}{2 f_m D_m}$$

where D_m is the mechanical damping of the turbine generator. This value of effective resistance, which may exceed 2.0 p.u. on generator base for the first torsional mode, is the resistance that would have to be added in the stator circuit to prevent self-excited oscillations for systems where series electrical resonance could occur. The other expression relates the electrical interaction to a mechanical log decrement. Appendix II shows the development of this expression which becomes

$$\text{Min LOG DEC} = \frac{f_e}{8\, f_m{}^2\, H_m} \frac{R}{|Z|^2}$$

where H_m is the effective generator inertia for the torsional oscillation. The log decrement representation for the damping of oscillations is often used because it is not proportional to the oscillation frequency. Therefore the log decrement would not be expected to show as much variance between mechanical modes as would the time constant. A comparison between the minimum log decrement required to prevent growing oscillations and the modal damping of the turbine generator is a measure of the mechanical damping that would have to be added to prevent growing oscillations.

TRANSIENT TORQUES RESULTING FROM SUBSYNCHRONOUS CURRENTS

In addition to the concern for self-excited oscillations at subsynchronous frequencies, there has been a great amount of effort expended to determine the turbine generator shaft torques resulting from electrical system disturbances. Any disturbance will result in a change in the normal frequency current level of the system and will produce transient currents at the natural frequencies of the system to compensate for the sudden change of normal frequency current. For systems without series capacitors the transient currents are dc currents that decay with time constants determined by the ratios of inductance to resistance in the different current paths of the system.

The simplest system and disturbance to consider in a series capacitor compensated system is a three phase disturbance in a radial system, as shown in Figure 4. Such a system will have a single natural frequency oscillation which will decay with a time constant equal to 2 L/R as long as mechanical interaction effects are neglected. This natural frequency current will interact with the air gap flux of the machine to produce a torque that pulsates at the slip frequency ($f_n - f_e$). As shown in Appendix I, the per unit magnitude of torque and per unit magnitude of current will be the same if the air gap flux is one per unit and the small oscillating flux components are neglected.

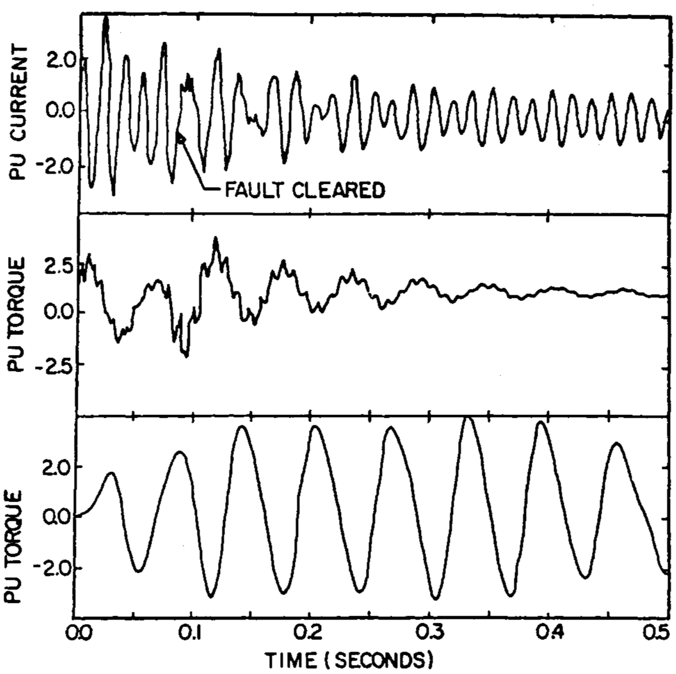

Fig. 4. Plots of generator phase current, electrical torque, and shaft torques between two turbines following a three phase-to-ground fault in a series-compensated transmission system.

The response of the turbine generator rotor to the applied torque depends on the torsional resonant frequencies of the shaft. Appendix II shows a simple lumped parameter model of the turbine generator that is satisfactory for analysis of the torsional response for the subsynchronous components of the torque. This set of equations will exhibit several natural frequencies or modes of torsional vibration and the shaft system will respond to the applied torque with oscillations in each mode. The technique of modal analysis is quite helpful in visualizing the mechanical response because it transforms the coupled set of differential equations to a set of independent equations. Each of the independent equations will have a single natural frequency so the response to an applied sinusoid is easily determined. Of particular importance is the response when the frequency of the applied torque is one of the modal frequencies of the shaft. If mechanical damping is neglected and the applied torque has the form $\gamma = \gamma_m\, \epsilon^{-\delta t} \sin \omega_m t$ per unit on generator base, the angular response of the generator will be

$$\theta_g = \frac{\gamma_m}{2\, H_m\, \delta\, (\delta^2 + 4\, \omega_m{}^2)^{1/2}} (1 - \epsilon^{-\delta t}) \sin \omega_m t$$

In this equation H_m is the effective modal inertia constant as defined in Appendix II and θ_g is in per unit angle (radians/377). With this result, the mode shape, and the spring constants of the shafts, the peak torques for all the shafts are easily determined. For example, the peak shaft torque for the generator turbine shaft is $K_{gt}\, \theta_g\, (1 - \theta_{mt})$ where θ_{mt} is the angular deflection of the turbine in mode m for a generator deflection of 1.0.

The peak response of the mechanical system in this mode is directly proportional to the magnitude of applied torque (oscillating current magnitude) and very nearly proportional to the electrical time constant ($1/\delta$). Typical numbers for first torsional modes of modern turbine generators are $H_m = 2.5$, $\delta = 5$, $\omega_m = 100$, $\theta_{mt} = 0.3$, and $K_{gt} = 26000$. These constants would give a peak shaft torque of 3.64 per unit for a natural frequency stator current of 1 per unit at 44 Hz.

Alternatively the peak shaft torques can be determined from the inertia constants and the mode shape for the frequency in question. The shaft torque for a lossless mechanical system is simply the sum of the torques applied to the inertias from the shaft to the end of the machine. In practice the term $(\delta^2 + 4w_m{}^2) \approx 4w_m{}^2$. This peak shaft torque can be written

$$\gamma_{peak} = \frac{\gamma_m\, \pi\, f_m}{\delta\, H_m} \Sigma_j\, H_j\, \theta_{mj}$$

where the index j includes only the inertias from the shaft to the end of the machine. For a simple two mass system the peak shaft torque becomes simply

$$\gamma_{peak} = \frac{\gamma_m\, \pi\, f_m}{\delta} \frac{H_1}{H_1 + H_2}$$

This formula shows that the peak shaft torque is related to the number of half cycles of applied torque. For applied torques that are not simple exponentials, the idea of counting the half cycles of applied torque to estimate a shaft torque is very useful.

When the applied torque has several frequencies or is not at one of the modal frequencies, the analysis is more complex. One method to visualize the buildup of torque is as the vector sum of responses to a series of exponentially decreasing half cycle impulses. When the applied torque frequency is not one of the modal frequencies, this series of impulses is alternately in phase and out of phase with the oscillating response of the shaft. If modal analysis is used, the response in each mode must be calculated. For lossless mechanical systems, the steady state component of this response for each mode will be an oscillation at the modal frequency. A pessimistic assumption of the peak torque for each shaft would be the sum of the peak torques from each mode. The assumption would be that all of the modes were at peak value at the same time, before the mechanical damping could dissipate the energy. Usually these types of transient disturbances are analyzed using a digital computer program and the results are compared to the simplified formulas to insure that the digital programs are working correctly. Figure 4 shows the applied torque

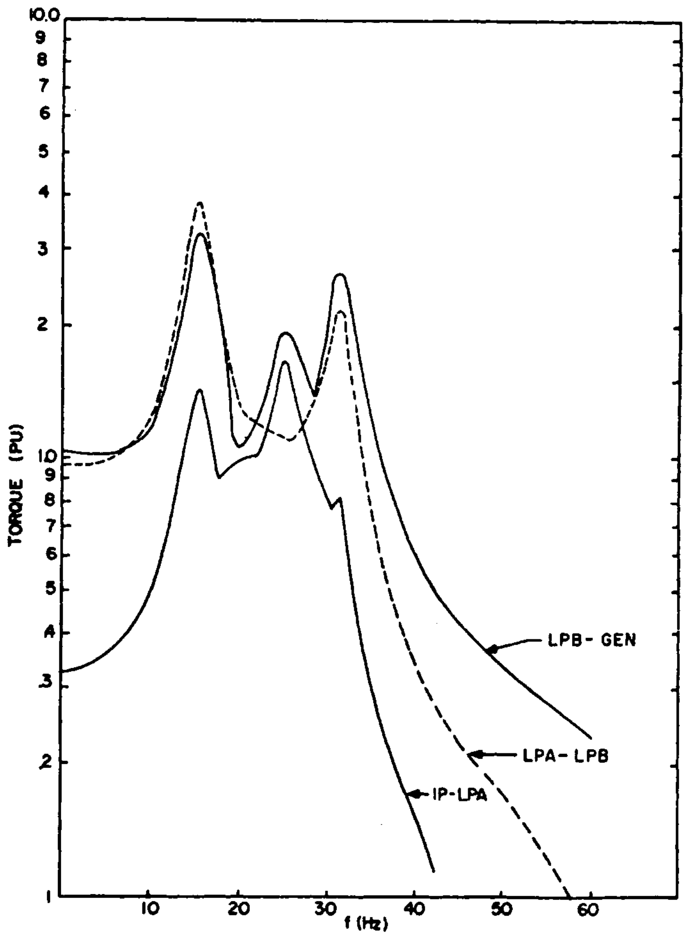

Fig. 5. Peak transient shaft torque for an applied torque on a 900 MVA generator of the form $\gamma = 1.0e^{-\delta t}\cos \omega t$. IP-LPA is the shaft torque between the intermediate and low pressure turbines; LPA-LPB is the shaft torque between the two low pressure turbines; and LPB-GEN is the shaft torque between the low pressure turbine and the generator.

and shaft torque from such a program and Figure 5 shows a plot of peak shaft torques as a function of frequency for applied torques of the form described above. The first mode parameters were very nearly those listed above and the results show very good agreement with the predicted value.

Most transmission system disturbances are more complex than the simple exponential described above. A three phase-to-ground fault that is successfully cleared will produce two separate shocks on the machine, one when the fault is initiated and one when it is cleared. Each shock will have natural frequency torque oscillations superimposed on the step change in steady torque that accompanies a short circuit condition. The frequencies of the oscillating torques will often not be the same during the fault as after it is cleared because they are determined by the changed transmission system configuration. The transient torque oscillations resulting from the fault clearing operation may either add to or cancel those produced by the fault application, depending on the clearing time. Digital computer simulation is presently used to determine the transient wave form resulting from these sequential switching operations. Peak torque values are compared either with those obtained from short circuits on the terminals of the machine or with recommended limits supplied by the manufacturer.

A complete analysis of the effects of a transient disturbance on the turbine generator rotor is quite complex. It requires a consideration not only of the peak torque but also of the cumulative damage effects referred to as low cycle fatigue. These cumulative effects are influenced by the total stress history of the machine. At present there are no complete records of stress histories of turbine generators to compare with calculated results, so the accuracy of the calculation methods is uncertain. Therefore, at the present time, these long term effects of shock excitation of the machine are being actively studied, but the results of the studies and simplified calculation methods are not available.

CONCLUSIONS

The relationship derived show the most significant factors in analyzing subsynchronous resonance problems. For determining the likelihood of having self-excited oscillations due to torsional interaction, these factors are the net electrical resistance and reactance at the frequency $f_n - f_m$, the mechanical damping, and the effective modal inertia. For simple electrical networks, the electrical quantities can be calculated by hand. However, for more complex networks digital computer programs are used. The results of this simplified analysis are shown in the companion paper to be quite accurate when compared to more exact methods.[1] Improved accuracy can be obtained by including electrical effects at the frequency $f_n + f_m$ if it is required.

The analysis of transient shaft torques on synchronous machines is most accurately performed with computers if the disturbances are complex. However, the fundamental equations presented show the buildup in resonance that can lead to dangerous torque levels if the applied torque has a component at the resonant frequency. These equations also show that the electrical time constant, mechanical frequency, modal inertia and mode shape are important factors in determining peak shaft torques. For transient applied torques with a component at a number of frequencies, an accurate calculation of peak shaft torque requires a great deal of computation. The simple approximation that this peak is the sum of the torques produced by each mode equivalent is usually quite pessimistic.

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SYMBOLS

V_e - voltage at frequency f_e resulting from rotor oscillation.

f_n - normal system electrical frequency (60 Hz for U.S. systems).

f_e - electrical oscillation frequency (base frequency $\pm f_m$).

f_m - torsional resonant frequency.

Φ_o - generator steady state air gap flux.

$\Delta\omega$ - per unit magnitude of velocity deviation.

D_m - damping coefficient for the modal frequency.

R - per unit resistance.

Z - per unit impedance.

H_m - effective modal inertia constant (as derived in Appendix 2).

θ_{mt} - relative deflection of a turbine for mode m.

ω_m - mechanical frequency in radians per second.

K_{gt} - spring constant for generator-turbine shaft.

γ - per unit torque.

p - differential operator d/dt.

APPENDIX I
GENERATOR MODEL FOR SUBSYNCHRONOUS RESONANCE STUDIES

The basic equations that are used to represent the machine make use of the familiar D-Q transform originally defined by R. H. Park. For simplicity a two phase machine is considered and a per unit system is defined for steady state equations in terms of positive sequence stator variables. The definition of variables and sign conventions are those used by Kimbark for generator action.[3] The basic equations required to define the instantaneous stator voltages and currents are the transformation from D-Q variables to stator variables, a set of equations representing the rotor circuit, and a torque equation. The transformation from D-Q to two phase variables is

$$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

The equations to represent the rotor voltages and currents are:

$$E_d = -R_a I_d + p\Phi_d - \omega\Phi_q$$

$$E_q = -R_a I_q + p\Phi_q + \omega\Phi_d \tag{2}$$

Now assume that each variable contains a main component and a variation Δ. For example $E_d = E_{do} + \Delta E_d$. For small amplitude variations all ΔΔ products are ignored. In addition, the trigonometric relations are written:

$$\cos(\theta_o + \Delta\theta) = \cos\theta_o - \Delta\theta \sin\theta_o$$

$$\sin(\theta_o + \Delta\theta) = \sin\theta_o + \Delta\theta \cos\theta_o$$

These steps will allow a linearized set of pertubation equations to be written around the initial operating point. For example

$$\Delta E_d = -R_a\Delta I_d + p\Delta\Phi_d - \omega_o\Delta\Phi_q - \Delta\omega\Phi_{qo}$$

Using this expansion the variation in phase voltages becomes

$$\begin{bmatrix} \Delta E_a \\ \Delta E_b \end{bmatrix} = \begin{bmatrix} \Delta E_{a1} \\ \Delta E_{b1} \end{bmatrix} + \begin{bmatrix} \Delta E_{a2} \\ \Delta E_{b2} \end{bmatrix} \tag{3}$$

where, the variables are separated to represent a voltage due to oscillatory motion of the rotor and the induction machine action. If θ is equal to $\omega_o t + \Delta\theta$ then

$$\begin{bmatrix} \Delta E_{a1} \\ \Delta E_{b1} \end{bmatrix} = \Delta\omega \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} -\Phi_{qo} \\ \Phi_{do} \end{bmatrix} + \omega_o\Delta\theta \begin{bmatrix} -\sin\theta_o & -\cos\theta_o \\ -\cos\theta_o & \sin\theta_o \end{bmatrix} \begin{bmatrix} -\Phi_{qo} \\ \Phi_{do} \end{bmatrix} \tag{4}$$

For a sinusoidal pertubation at frequency μ rad/sec, $\Delta\theta = A\sin\mu t$ and $\Delta\omega = A\mu\cos\mu t$. In these expressions A is the amplitude of the oscillation in radians. Using the relation $\cos\delta = j\sin\delta$, the voltage ΔE_{al} becomes

$$\Delta E_{a1} = \frac{A(\omega_o - \mu)}{2}(\Phi_{do} + j\Phi_{qo})\sin(\omega_o - \mu)t$$

$$- \frac{A(\omega_o + \mu)}{2}(\Phi_{do} + j\Phi_{qo})\sin(\omega_o + \mu)t \tag{5}$$

A positive sequence per unit system can be defined where $E_+ = E_a \angle\delta_o$ in per unit. A consistent set of per unit variables requires that $V_{base} = \omega_o\ \Phi_{base}$ so that in per unit the positive sequence subsynchronous oscillating voltage is

$$\Delta E_1 = \frac{A(\omega_o - \mu)}{2}(\Phi_{do} + j\Phi_{qo}) \tag{6}$$

In this per unit system $\Delta\omega = A\mu/\omega_o$. If this expression is substituted into equation 6 and $\Phi_{do} + j\Phi_{qo}$ is assumed to be 1.0 pu, the per unit subsynchronous voltage is:

$$\Delta E_1 = \frac{(\omega_o - \mu)}{2\mu}\Delta\omega = \frac{f_e}{2f_m}\Delta\omega \tag{7}$$

A similar expression is obtained for the per unit supersynchronous voltage.

The other components of the voltage variation are:

$$\begin{bmatrix} \Delta E_{a2} \\ \Delta E_{b2} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} -R_a\Delta I_d + p\Delta\Phi_d - \omega_o\Delta\Phi_d \\ -R_a\Delta I_q + p\Delta\Phi_q + \omega_o\Delta\Phi_d \end{bmatrix} \tag{8}$$

For a round rotor machine with one rotor circuit on each axis, this equation leads to the familiar induction machine model for the generator. The flux linkage terms can be written in terms of current using the relation

$$\Delta\Phi_d = \left[-L_\ell - \frac{L_m(R_r + pL_r)}{R_r + p(L_m + L_r)} \right] \Delta I_d = -K\Delta I_d \tag{9}$$

By using the appropriate trigonometric relations, the equation for ΔE_{a2} becomes

$$\Delta E_{a2} = [-R_a + (p + j\omega_o)K]\ [\Delta I_d + j\Delta I_q]\ \cos\omega_o t \tag{10}$$

For subsynchronous currents in the stator circuit, the direct and quadrature axis currents will be sinusoids at the mechanical frequency μ. They will rotate in the reverse direction to that established by the stator to rotor transformation. Therefore for steady state equations as viewed from the rotor, $p = -j\mu$, $\Delta I_d = \Delta I\cos\mu t$, and $\Delta I_q = -\Delta I\sin\mu t$. If slip is defined as $s = -\mu/(\omega_o - \mu)$, the equation for positive sequence voltage becomes

$$\Delta E_2 = \left[-R_a - jX_L - \frac{jX_m(R_r/s + jX_r)}{R_r/s + j(X_m + X_r)} \right] \Delta I \tag{11}$$

In this expression the reactances are defined for the stator frequency $(\omega_o - \mu)$. This equation is the standard equation used to represent an induction machine. An approximation that is often used for synchronous machines is to neglect X_m and replace $X_L + X_r$ by X". The equation becomes

$$\Delta E_2 = [-R_a - jX'' - R_r/s]\ \Delta I \tag{12}$$

The complete model for the subsynchronous frequency is shown in Figure 2. A similar model can be derived for the supersynchronous frequency.

The representation of mechanical interaction requires the use of the relationship between electrical current and torque. An expression for the torque applied to the generator from the electrical system is

$$\gamma = \Phi_q I_d - \Phi_d I_q \tag{13}$$

In deriving a simplified model for small amplitude pertubations, it is useful to represent only the largest of the pertubations. If the stator circuit is near resonance, this quantity will be the oscillating phase current. The subsynchronous component of the phase A current in phase with the oscillating voltage E_{al} will have the form

$$\Delta I_a = \Delta I\sin[(\omega_o - \mu)t + \delta]$$

$$\text{where } \tan\delta = \Phi_{qo}/\Phi_{do} \tag{14}$$

The phase B subsynchronous current has the same form, but it is delayed by 90 degrees. When these currents are transformed to D-Q coordinates with the transformation T^{-1} and the torque calculated, the result is

$$\Delta\gamma = \Delta I \cos \mu t \tag{15}$$

In this expression it is assumed that the magnitude of $\Phi_o = 1.0$ pu and that the $\Delta\Phi$ terms could be neglected. Note that this torque has the same phase position as velocity. Therefore it would appear to be a negative damping in the mechanical circuit.

The component of current that is in phase with the subsynchronous voltage can be written from the positive sequence circuit as:

$$\Delta I = \Delta E_1 R/(R^2 + X^2) \tag{16}$$

In this expression R is the net resistance of circuit and X is the net reactance as viewed from the subsynchronous voltage source. If equations 7 and 16 are substituted into equation 15, the result is:

$$\frac{\Delta\gamma}{\Delta\omega} = D_m = \frac{f_e}{2 f_m} \; \frac{R}{R^2 + X^2} \tag{17}$$

The model for the supersynchronous frequency yields the same expression for damping except the sign is reversed and resistances and reactances are calculated at the appropriate frequency. The net mechanical damping resulting from torsional oscillation is the sum of these two components. Since this torque is applied to the machine, a positive torque represents a negative damping coefficient.

APPENDIX II

MECHANICAL EQUATIONS TO REPRESENT TURBINE GENERATORS FOR SUBSYNCHRONOUS RESONANCE STUDIES

The mechanical equations used to represent the turbine generator for subsynchronous resonance studies are those required to describe the torsional motion of the unit. The model normally used is a lumped parameter spring-mass model with each major element (e.g., a turbine or the generator) represented by a mass and the shafts between elements represented by torsional springs. Damping is usually represented by viscous damping coefficients associated with each mass and spring although it could just as readily be represented as a viscous damping coefficient associated with each mode of oscillation.

The development of the transformations from the coupled differential equations of motion to a system of orthogonal equations associated with each mode assumes that there is no mechanical damping. This assumption does not cause appreciable error because the quality factor (Q) of these systems is nearly 400 for most modes. The development of the transformation is most easily done using matrix arithmetic and it follows the methods outlined by Hildebrand.[4]

The initial equations of motion are

$$2[H]\,[\ddot{\theta}_i] + [K]\,[\theta_i] = [F_i] \tag{1}$$

where [H] is a diagonal matrix of the inertia constants associated with each mass and [K] is the tridiagonal matrix of spring constants connecting the masses. The vector $[\theta_i]$ is the angular position of each mass and the $[F_i]$ vector is the applied torque for each mass. For a consistent set of variables with the torques in per unit on generator KVA base, the inertia constants are in kw-sec/KVA and the spring constants are in per unit power per radian of deflection. For analyzing the electromechanical interactions of the generator, all applied torques will be zero except that associated with the generator.

To form the transformation matrix, first define a matrix [D] such that 2[H] = [D] [D]. For a diagonal matrix [H], the matrix [D] will also be diagonal and $d_{ii} = \sqrt{2h_{ii}}$. Other properties of the D matrix that will be used are that it is its own transpose and its inverse is also diagonal with elements $d_{ii}^{-1} = 1/d_{ii}$.

The eigenvalue problem that is formed from this system of equations is expressed by the equation

$$[P]\,[\delta_j] = \omega_j^2\,[\delta_j] \tag{2}$$

where ωj is one of the natural frequencies, $[\delta_j]$ is the corresponding eigenvector, and

$$[P] = [D]^{-1}\,[K]\,[D]^{-1} \tag{3}$$

The eigenvector is related to one of the desired mode shapes by the relation

$$[\theta_j] = [D]^{-1}\,[\delta_j] \tag{4}$$

The elements of the eigenvector can be multiplied by any constant without altering their usefulness as a transformation. For this problem with the applied torque on the generator mass, they should be adjusted to make θ_{gj}, the modal displacement of the generator in the jth mode equal to one radian.

The transformation matrix from angles associated with masses to those associated with modes is the matrix whose columns are the mode shapes defined above. This matrix is denoted by the symbol [T]. Applying this transformation to equation 1 gives

$$2[H]\,[T]\,[\ddot{\theta}_m] + [K]\,[T]\,[\theta_m] = [F_i] \tag{5}$$

If this equation is multiplied by the transpose of the transformation $\{[T]^* = [\delta]^*\,[D]^{-1}\}$, and the result is simplified, it will be

$$[M]\,[\ddot{\theta}_m] + [\omega^2]\,[M]\,[\theta_m] = [T]^*\,[F_i] \tag{6}$$

With the mode shape defined above and only an applied torque at the generator mass $[T]^*\,[F_i] = [F_i]$. The matrix $[\omega^2]$ is a diagonal matrix of modal frequencies and the matrix [M] is a diagonal matrix of modal masses of the form $m_{ii} = 2\,\Sigma\, h_j\, \theta^2{}_{ji} = 2\,H_m$.

Therefore the coupled set of n differential equations become a set of n equations of the form

$$\ddot{\theta}_m + \omega_m^2\,\theta_m = f_g/2\,H_m \tag{7}$$

A damping term is often added to equations to include the effects of modal damping. With this addition, the equations become

$$\ddot{\theta}_m + (D_m/2H_m)\,\dot{\theta}_m + \omega_m^2\,\theta_m = f_g/(2H_m) \tag{8}$$

These equations are identical to those for R-L-C electrical circuits and their natural frequency response to a disturbance is a sinusoid with an exponential decay. The frequency is ω_m and the decay time constant is $4H_m/D_m$. One common form of expressing mechanical damping is the logarithmic decrement (Log Dec). It is defined as LOG DEC = $(1/n)\ \mathrm{LOG}_e\ (A_o/A_n)$ where A_o is the amplitude of the first measured oscillation cylce and A_n is the amplitude n cycles later.

For a simple exponential decay the log decrement is related to D_m by the expression

$$D_m = 4H_m\, f_m \log \text{dec.}$$

Discussion

Edward W. Kimbark (Bonneville Power Administration, Portland, Oregon): The paper states that a single-frequency angular oscillation of very small amplitude, superposed on the constant rotational speed, produces an upper side frequency (supersynchronous frequency) and a lower side frequency (subsynchronous frequency) in the stator voltages. But what about an angular oscillation of large amplitude, which could result from a disturbance in the external circuits connected to the stator or to a series of such disturbances?

Since the stator frequency is proportional to the rotational speed, a variation of that speed produces frequency modulation. Such modulation, applied to communication circuits, has been thoroughly investigated. [1, 2] A single signal frequency modulating a carrier wave produces infinite sets of upper and lower side frequencies all of which differ from the carrier frequency by whole multiples of the signal frequency. Each side band differs from the carrier frequency by a Fourier series. As the degree of modulation approaches zero, the lower side frequencies predominate, giving the result obtained in the paper by small-perturbation analysis.

I wonder whether this spectrum of side frequencies would not increase the probability of a near-coincidence of one of them with the complement of one of the mechanical frequencies.

Manuscript received February 22, 1977.

REFERENCE

[1] Stanford Goldman, *Frequency Analysis, Modulation and Noise,* McGraw-Hill Book Co., 1948.

[2] P. F. Panter, *Modulation, Noise and Spectral Analysis,* McGraw-Hill Book Co., 1965.

L. A. Kilgore, D. G. Ramey, and M. C. Hall: The authors wish to thank Dr. Kimbark for the interest shown in our paper. His point that the oscillation of the generator rotor produces sets of upper and lower side frequencies in the output voltage is correct. However, it seems unlikely that analysis of any frequencies other than the first sidebands (synchronous frequency plus and minus the mechanical oscillation frequency) will result in other than very secondary modifications to the results. The magnitude of oscillatory voltage at the first sidebands rarely exceeds 3 percent of rated voltage for oscillatory shaft stresses at the material yield point. Voltage magnitudes for higher sidebands decrease from that for the first sideband in proportion to $1/N!$ for the Nth sideband.

If any frequency other than the first side frequency were considered, the bilateral coupling described in the paper (rotor oscillation at frequency f_m produces voltages at frequency $f_e = f_n - f_m$; current oscillations at frequency f_e produce rotor torques at frequency f_m) would not exist. Only in the extremely rare case where either a machine has resonant frequencies at both f_m and $2f_m$, or the electrical system has resonant frequencies at $f_n - f_m$ and $f_n - 2f_m$, would there be any response.

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