ENGEL CURVES
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Abstract

An Engel curve describes how a consumer's purchases of a good like food varies as the consumer's total resources such as income or total expenditures vary. Engel curves may also depend on demographic variables and other consumer characteristics. A good's Engel curve determines its income elasticity, and hence whether the good is an inferior, normal, or luxury good. Empirical Engel curves are close to linear for some goods, and highly nonlinear for others. Engel curves are used for equivalence scale calculations and related welfare comparisons, and determine properties of demand systems such as aggregability and rank.

An Engel curve is the function describing how a consumer's expenditures on some good or service relates to the consumer's total resources holding prices fixed, so $q_i = g_i(y, z)$, where q_i is the quantity consumed of good i, y is income, wealth, or total expenditures on goods and services, and z is a vector of other characteristics of the consumer, such as age and household composition. Usually y is taken to be total expenditures, to separate the problem of allocating total consumption to various goods from the decision of how much to save or dissave out of current income. Engel curves are frequently expressed in the budget share form $w_i = h_i[\log(y), z]$ where w_i is the fraction of y that is spent buying good i. The goods are typically aggregate commodities such as total food, clothing, or transportation, consumed over some weeks or months, rather than discrete purchases. Engel curves can be defined as Marshallian demand functions holding the prices of all goods fixed.

The term Engel curve is also used to describe the empirical dependence of q_i on y, z in a population of consumers sampled in one time and place. This empirical or statistical Engel curve coincides with the above theoretical Engel curve definition if the law of one price holds (all sampled consumers paying the same prices for all goods), and if all consumers have the same preferences after conditioning on z and possibly on some well behaved error terms. Since these conditions rarely hold, it is important in practice to distinguish between these two definitions.

Using data from Belgian surveys of working class families, Ernst Engel (1857, 1895) studied how households expenditures on food vary with income. He found that food expenditures are an increasing function of income and of family size, but that food budget shares decrease with income. This relationship of food consumption to income, known as Engel's law, has since been found to hold in most economies and time periods, often with the function h_i for food i close to linear in $\log(y)$.

Engel curves can be used to calculate a good's income elasticity, which is roughly the percent change in q_i that results from a one percent change in y, or formally $\partial \log g_i(y, z)/\partial \log(y)$. Goods with income elasticities below zero, between zero and one, and above one are called inferior goods, necessities, and luxuries respectively, so by these definitions what Engel found is that food is a necessity. Elasticities can themselves vary with income, so e.g. a good that is a necessity for the rich can be a luxury for the poor.

Some empirical studies followed Engel (1895), such as Ogburn (1919), but Allen and Bowley (1935) firmly connected their work to utility theory. They estimated linear Engel curves $q_i = a_i + b_i y$ on data sets from a range of countries, and found that the resulting errors in these models were quite large, which they interpreted as indicating considerable heterogeneity in tastes across consumers. Working (1943) proposed the linear budget share specification $w_i = a_i + b_i \log(y)$, which is known as the Working-Leser model, since Leser (1963) found that this functional form fit better than some alternatives. However, Leser obtained still better fits with what would now be called a rank three model, namely, $w_i = a_i + b_i \log(y) + c_i y^{-1}$, and in a similar, earlier, comparative statistical analysis Prais and Houthakker (1955) found $q_i = a_i + b_i \log(y)$ fit best. More recent work documents sometimes considerable nonlinearity in Engel curves. Motivated by this nonlinearity, one of the earlier empirical applications of nonparametric regression methods in econometrics was kernel estimation of Engel curves. Examples include Bierens and Pott-Buter (1990), Lewbel (1991), and Hardle and Jerison (1991). More recent studies that control for complications like measurement error and other covariates z, including Hausman, Newey, and Powell (1995) and Banks, Blundell, and Lewbel (1997), find Engel curves for some goods are close to Working-Leser, while others display considerable curvature, including quadratics or S shapes. Even Allen and Bowley (1935, p. 123) noted "there is a good fit, allowance being made for observation and sampling errors,..., to a linear expenditure relation and occasionally to a parabolic relation."

Other variables z also help explain cross section variation in demand. Commonly used covariates include the number, ages, and gender of family members, location measures, race and ethnicity, seasonal effects, and labor market status. Variables indicating ownership of a home, a car, or other large durables can also have considerable explanatory power, though these are themselves consumption decisions.

Engel's original work showed the relevance of family size, and later studies confirm that larger families typically have larger budget shares of necessities than smaller families at the same income level. Adult equivalence scales model the dependence of utility functions on family size and use this dependence to compare welfare across households, assuming that a large family with a high income is as well off as a smaller family with a lower income if both families have demands that are similar in some way, such as equal food budget shares or equal expenditures on adult goods such as alcohol. The ratio of total expenditures needed to equate food budget shares across households are known as Engel equivalence scales, while the ratio that equates expenditures on adult goods are called Rothbarth (1943) scales.

Shape invariance assumes that budget share Engel curves for one type of consumer, such as a household with children, is a linear transformation of the budget share Engel curves for other types of consumers, such as households without children. Shape invariance is necessary for constructing what are known as Exact or Independent of Base equivalence scales, and has been found to at least approximately hold in some data sets. See Lewbel (1989), Blackorby and Donaldson (1991), Gozalo (1997), Pendakur (1999), and Blundell, Browning and Crawford (2003).

The level of aggregation across goods affects Engel curve estimates. Demand for a narrowly defined good like apples varies erratically across consumers and over time, while Engel curves based on broad aggregates like food are affected by variation in the mix of goods purchased. The aggregate necessity food could include inferior goods like cabbage and luxuries like caviar, which may have very different Engel curve shapes.

Other empirical Engel curve complications include unobserved variations in the quality of goods purchased, and violations of the law of one price. When price or quality variation is unobserved, their effects may correlate with, and so be erroneously attributed to, y or z. Examples of such correlations could include the wealthy systematically favoring higher quality goods, and the poor facing higher prices than other consumers because they cannot afford to travel to discount stores.

Assume a consumer (household) h determines demands q_{hi} facing prices p_i for each good i by maximizing a well behaved utility function over goods (which could depend on z_h), subject to a budget constraint $\sum_i p_i q_{hi} \leq y_h$. This yields Marshallian demand functions $q_{hi} = G_{hi}(p, y_h, z_h)$, with Engel curves given by these functions holding the price vector p fixed. Utility functions that yield Engel curves of the form $q_{hi} = b_i(z) y_h$ are called homothetic, and $q_{hi} = a_i(z) + b_i(z) y_h$ are quasihomothetic. Many theoretical results regarding two stage budgeting and aggregation across goods require homotheticity or quasihomotheticity, most notably Gorman (1953).

The shape of Engel curves plays an important role in the determination of macroeconomic demand relationships. For example, ignoring z for now, suppose individual consumers h each have Engel curves of the quasihomothetic form $q_{hi} = a_{hi} + b_i y_h$. Then, letting Q_i and Y be aggregate per capita quantities and total expenditures in the population, we get $Q_i = A_i + b_i Y$ by averaging q_{hi} across consumers h. This is a representative consumer model, in the sense that the distribution of y only affects aggregate demand Q_i through its mean $E(y) = Y$. Gorman (1953) showed that only linear Engel curves have this property, though linear Engel curve aggregation dates back at least to Antonelli (1886). Gorman's linearity requirement, which does not usually hold empirically, can be relaxed given restrictions on the distribution of y, e.g., Lewbel (1991) shows that $E(y \log y)/Y - \log(Y)$ is very close to constant in United States data, and if it is constant then Working-Leser household Engel curves yield Working-Leser aggregate, representative consumer demands.

Exactly aggregable demands are defined by $q_i = \sum_{j=1}^{J} A_{ji}(p) c_j(y, z)$, and so have Engel curves $q_i = \sum_{j=1}^{J} a_{ji} c_j(y, z)$ that are linear in the functions $c_j(y, z)$. These models have the property that appregate demands Q_i depend only on the means of $c_j(y, z)$. Utility theory imposes constraints on the functional forms of $c_j(y, z)$. Properties of exactly aggregable demands and associated Engel curves are derived in Muellbauer (1975), Jorgenson, Lau, and Stoker (1982), and Lewbel (1990), but primarily by Gorman (1981), who proved the surprising result that utility maximization forces the matrix of Engel curve coefficients a_{ji} to have rank three or less.

Lewbel (1991) extends Gorman's rank idea to arbitrary demands, not just those in the exactly aggregable class, by defining the rank of a demand system as the dimension of the space spanned by its Engel curves. Engel curve rank can be nonparametrically tested, and has implications for utility function separability, welfare comparisons, and for aggregation across goods and across consumers. Many empirical studies find demands have rank three.

One area of current research concerns the observable implications of collective models, that is, households that determine expenditures based on bargaining among members. For example, the Engel curves of such households could violate Gorman's rank theorem, even if each member had exactly aggregable preferences. Another topic attracting current attention is the role of errors in demand models, particularly their interpretation as unobserved preference heterogeneity, random utility model parameters. This matters in part because another of Allen and Bowley's (1935) findings remains true today, namely, Engel curve and demand function models still fail to explain most of the observed variation in individual consumption behavior.

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