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Chapter 5

A GAME THEORETIC APPROACH TO THE THEORY OF MONEY AND FINANCIAL INSTITUTIONS

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1. Process or equilibrium?

This is a sketch of a game theoretic and gaming approach to the development of an appropriate microeconomic theory of money and financial institutions.

The phrase "money and financial institutions" is used to stress that a theory of money alone cannot be fruitfully constructed in an institutional vacuum. The monetary and financial system of an economy are part of the socio-politico-economic control mechanism used by every state to connect the economy with the polity and society. This neural network provides the administrative means to collect taxes, direct investment, provide public goods, finance wars and facilitate international and intertemporal trade.

The money measures provide a crude but serviceable basis for the accounting system which, in turn, along with the codification of commercial law and financial regulation, are the basis for economic evaluation and the measurement of trust and fiduciary responsibility among the economic agents.

A central feature of a control mechanism is that it is designed to influence process. Dynamics is its natural domain. Equilibrium is not the prime concern, the ability to control the direction of motion is what counts.

Bagehot (1962) noted that a financial instrument originally designed for one purpose may take on a life of its own and serve a different purpose. In particular, most of the instruments may have been invented to facilitate trade but they provided a means for control. Money and financial institutions provide the command and control system of a modern society. The study of the mechanisms, how they are formed, how they are controlled and manipulated and how their influence is measured in terms of social, political and economic purpose pose questions not in pure economies, not even in a narrow political economy, but in the broad compass of a political economy set in the context of society.

A basic purpose of the approach adopted here is to show the minimal conditions which require that financial institutions and instruments emerge as necessary carriers of process. The thrust is for the development of a mathematical institutional economics.

2. One theme, many problems

The macroeconomist deals with here and now. Keynes (1973, p. 296) noted that: "Economics is a science of thinking in terms of models joined to the art of choosing models which are relevant to the contemporary world." The very nature of economic advice is such that it has to be based, at its best, upon a blend of perceptive ad hoc assumptions combined together to provide a sufficient socio-political context for the economic argument presented to justify the advice. The niceties of tight logical checks, completeness and consistency analyses and broad sensitivity analysis can only be afforded as they are called forth in the hurried battle of the adversarial process in which economic advice and policy advocacy is embedded.

There are many microeconomic theorists who use their skills to address problems of policy in the small, such as pricing of utilities, regulation of banks or evaluation of special subsidies. Yet the development and study of the microeconomic foundations of the price system, other means of exchange [see Shubik (1970)] and the politico-economic institutions of economic guidance calls for a level of detachment, scope and abstraction that are all difficult to justify when any form of direct economic advice is being given.

Often apparently complex and ill-defined problems are genuinely complicated. The historian is well aware that there may be thousands of interacting factors which at one time or another come to the front center of the stage for a brief moment in the limelight and are then replaced by other factors. Thus, historians of the American Civil War can still debate the relative importance of the northern railway and logistic system in contrast with the death of Stonewall Jackson prior to Gettysburg. Whole theories may be based on the economic value of slaves; the importance of central land masses; the strategic value of control of the seas; the linkage between tyranny and the control of irrigation systems; freedom and mountain valleys and so forth.

In a lesser, but fortunately more precise, way, the economist concerned with the understanding of economic institutions faces a complex multivariate system. But hopefully there is enough special structure and stability of structure that he can break up his general investigations into more specialized and well-defined segments prior to assembling a general theory.

It is argued here that there are several basic subtopics which can be investigated separately, yet which are needed jointly in the eventual construction of an underlying theory of money and financial institutions. Before we begin to discuss the different, but necessary, partial approaches, several general assertions concerning the construction of any satisfactory theory of money and financial institutions are made.

2.1. Some assertions

Assertion 1

An adequate theory of money and financial institutions must be able to account for segments of the economic system where the economic agents are few in number and large.

Comment. Virtually all of the efficient market price system and information conditions results depend upon there being a continuum of small agents. Yet the evidence is that this is not so for the major tax authorities, banks of issue and other government agencies; this is also a poor approximation for banking, insurance, investment banking, utilities and major manufacturing in general.

Assertion 2

The existence of money and financial institutions would be needed in a world with no exogenous uncertainty whatsoever. The presence of transactions costs caused by the technology of exchange is sufficient to cause aggregation and hence uncertainty.

Comment. Decisions and contracts, even in a world without exogenous uncertainty, utilize time and other economic resources. Lawyers, accountants, bookkeepers and other administrative workers in the information, communication and control structure of the economy of any society organized as a state are not and never have been free goods, even as a first approximation for those interested in answering most of the basic questions in political economy.

Assertion 3

The counterfactural assumption of complete markets (often defended as an adequate first approximation for some problems) may obviate the need for money, contracts and other financial institutions in a world with time and perfect trust. However, the need to write the rules of the game to enforce contracts in a world with less than perfect trust is sufficient to call for financial instruments and institutions even with zero transactions technology costs.

Comment. Transactions costs, whether generated by technology or by strategic considerations of trust and enforcement, are more central to the presence of money and financial institutions than is the presence of exogenous uncertainty. The presence of transactions costs creates many new problems when exogenous uncertainty is present, but without any costs to a transactions technology or any cost to trust, the presence of complete markets wipes out the problems of exogenous uncertainty.

Assertion 4

The theory of the general equilibrium system provides a highly useful benchmark and starting point from the development of a theory of money and financial institutions. But the next steps involve recognizing that game theoretic modeling and methods are more general. The basic results of general equilibrium theory may be reinterpreted as results obtained from a class of strategic market games with a continuum of economic agents with no agent of significant size, or, in some instances, from games with a finite number of players.

Assertion 5

Although game theoretic methods are advocated here as appropriate to helping to reconcile micro- and macroeconomics, two warnings are called for. Many of the basic difficulties involve the modeling of carriers of process. The techniques of cooperative game theory appear to be of only limited worth [but see Gale (1982)]. In particular, for problems involving economic dynamics, the core and value solutions do not seem to be fruitful.

Although the extensive and strategic forms for representing games appear to be of considerable value in modeling,[1] there are many difficulties with the noncooperative equilibrium solution concept which is most frequently used. We do not have a universally accepted prescription or description for how rational economic agents should or do behave in a multiperson game of strategy. The macroeconomists have "solved" their problem in an ad hoc manner by making behavioral assumptions and claiming that their approximations, such as a propensity to consume or save, are "good enough". The microeconomic model of individualistic rational, maximizing man does not extend to provide a unique prescription for behavior. Although noncooperative solution concepts such as *perfect equilibrium* are frequently utilized, the justification for doing so is by no means always compelling.

2.2. A division of tasks

The four major factors which must be considered are (1) topics, (2) models, (3) solution concepts and (4) numbers.

The complexity of the monetary and financial system of our societies considered as a whole must be divided into bite-sized pieces so that we can isolate and analyze the many different phenomena to avoid being overwhelmed by their interactions.

A natural starting point is one period models without exogeneous uncertainty. Figure 5.1 suggests a structure for a sequence of problems starting with the

[1]For some problems such as the study of random pairing in a barter market the extensive and strategic forms may offer *too much* structure The coalitional form may be regarded as more flexible

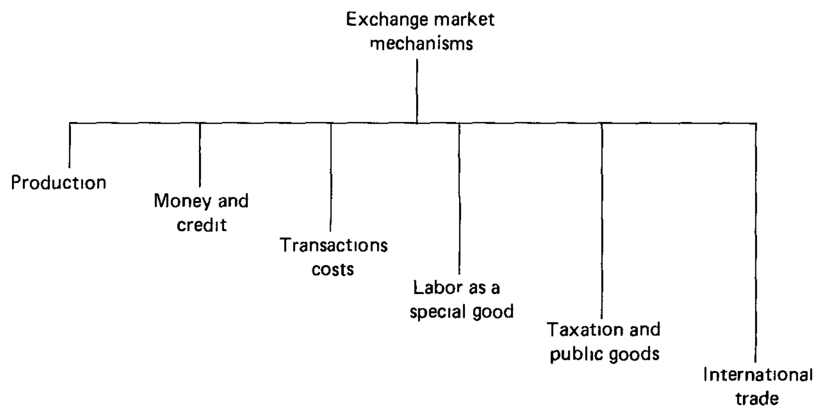

Figure 5 1

simplest; that being of an exchange economy. Complications may then be added individually or jointly. At the top of Figure 5.1 is an exchange market technology with no frills. We may then add, one at a time or in combinations, production, money and credit, transactions costs; the treatment of labor as a special commodity or set of commodities; the addition of mechanisms for the collection of taxes and the distribution of public goods; and international trade.

Associated with every one of these topics there are four classes of treatment which must be considered. They are:

(1) utility and preference assumptions;
(2) exogenous uncertainty and information conditions;
(3) multistage models and full dynamics; and
(4) number of participants: few, many, countable, other.

Prior to discussing the specific work and its relevance, we turn to comments on general equilibrium and macroeconomics.

3. The debt to general equilibrium

The theory of general equilibrium has provided an elegant tight mathematical description for the existence of an efficient price system. In order to be able to produce a precise and analyzable mathematical structure for the intermixture of the insightful, verbal, but loosely mathematical description of Walras (1954), Arrow and Debreu (1954), McKenzie (1959), Debreu (1959) and others refined and presented a sparse noninstitutional set of conditions on preferences and production technology for the existence of a price system.[2]

[2]Koopmans (1977, p 264) has referred to the work on the static price system as "preinstitutional"

Paradoxically the quest for precision provides the key to the understanding of the importance of institutional detail. By starting with the simplest most stripped-down noninstitutional description of production and exchange it becomes easy to start to construct mechanisms and add minimal institutional factors one at a time. The simplicity provides clarity. In particular, the ease with which it is possible to see both what the general equilibrium model does and does not do contributes to its usefulness.

The general equilibrium theory has provided a basis for understanding the efficiency properties of a price system. But up to and including the Debreu version it did not establish any distinction between a competitive or a centralized price system. The existence theorems are independent of the number of agents. The concept of complete markets is made clear. No matter how "unrealistic" we may find a time dated contingent commodity, it is well defined and complete markets provide a meaningful economic upper bound. But implicit, if not explicit, in the mathematics is that time, chance and trust do not matter when markets are complete. Implicit, but not explicit in the first vigorous mathematical treatments of general equilibrium, is that somehow either information, perception and understanding are perfect or the differentials among individuals do not matter.

Clearly abstracted away are transactions costs, government, public goods, oligopolistic segments, the need for money, or any special role for labor. Implicit in the many time period model is the meaning of a real rate of interest and the need for highly restrictive assumptions before this scalar index can be regarded as more than a crude approximation for the generally uneven change in the growth of resources.

The clear mathematical formulation of general equilibrium answered some important questions concerning the existence of an efficient price system; but its abstract unreality provided a sound basis for posing in a precise manner many more questions than it has answered. The attempts to define and answer these new questions amount to the construction of a mathematical institutional economics where the emphasis is upon the description and analysis of mechanisms which are the carriers of process. But the carriers of process are implicit in the rules of the game when the economy is viewed as a game of strategy. Thus, features of the economy such as money and financial institutions should emerge as any attempt is made to enlarge a static mathematical description into a process oriented description. Even the work on computation and algorithms for the calculation of general equilibrium prices of Scarf with Hansen (1973) and Smale (1976), or the more recent work of Scarf (1986) on prices with indivisibilities, can be given a process or institutional interpretation. Who is meant to be producing the prices (the Gosplan, or are they meant to emerge from markets?)? How much information is required by whom?

In Section 5 it is argued that the most promising way to go forward from the basis provided by general equilibrium theory is by means of game theoretic methods, in particular strategic market games. But enormous simplifications are available when it is possible to make the assumption that individuals are all price-takers because they are individually of a size that is insignificant with respect to the markets as a whole. This assumption can be made rigorous by considering a continuum of economic agents and was first done formally by Aumann (1964) for cooperative games, and Dubey and Shapley (1977), Schmeidler (1980) and Jaynes, Okuno and Schmeidler (1978) for games in strategic form. These provide a linkage in the reinterpretation of models with price-taking as a special class of the broader game theoretic models.

4. From general equilibrium to macroeconomics: The challenge

The hope to unify through a theory of money the insights and advocacy of a Keynes with the general equilibrium theory is somewhat misplaced. The assumptions made about economic institutions, politico-economic power and socio-economic behavior, whether of entrepreneurs, investors, consumers or government agencies, are necessary to the socio-politico-economic applied orientation of virtually all of macroeconomic theory. These assumptions and observations are highly specific to the socio-political and institutional structure of the society being examined.

A full general theory of macroeconimic dynamics does not lie solely within the domain of economics. The monetary and financial structure of the society provides a command and control system through which the socio-political forces exert guidance and control on the economy. But a control system is not the control.

A theory of money and financial institutions at the level of abstraction of general equilibrium theory is a theory of mechanisms and how they might work given the combination of economic forces and more or less exogenous socio-political forces.

At best the microeconomic theorist, for some time to come, cannot bridge the gap between the parsimonious description of rational economic man and the socio-political behavioral actors of the macroeconomic advisors.

A view adopted here is that staying with the parsimonious and counter-factual model of economic man, an extensive and strategic form game theoretic view of the structure of financial control enables us to go beyond general equilibrium. This can be done at a level of abstraction at which the logic of economic process enables us to examine the nature and need for minimal financial instruments and institutions. Thus, the questions which must be asked and answered are, for example: What are the essential properties of money and credit? What are the essences of the commercial, merchant and central banking functions? What are the essential functions of insurance? At what level of economic complexity will bankruptcy laws, seniority conditions, futures contracts, bonds or other instruments come into existence? Is there a natural structure and upper bound to the number of essentially different financial instruments? I suggest that there is [Shubik (1975)].

But even with an exploration of minimal financial institutions and functions, the micro foundations of macroeconomics will have only begun to have been erected. The institutions and instruments appear as a logical necessity in any attempt to well define the rules of a strategic market game. Yet the dynamics of process models depend delicately upon the behavioral assumptions concerning the actors and it is here that the gap between the viewpoints of the microeconomic theorists and macroeconomists is large.

5. Strategic market games

5.1. Game theory models and playable games

Good modeling requires a judicious balance between detail and abstraction, between "realism and relevance", and simplification and tractability. Models should not only be sufficiently well defined to be mathematically analyzable but they should be playable as games (and possibly used as experimental games). This additional gaming criterion provides a "debugging device" and serves as a check on the complexity and ease or difficulty with which the mechanism is run. The need for clarity concerning details of the rules of the game is such that clearing houses, small change, warehouse receipts or bills of lading appear as necessities. An instrument may have many institutionally different forms but its function will be necessary to all systems. Thus, clearing houses and bankruptcy laws have many manifestations in different societies but are necessary whenever mass credit operations exist.

The actual playing of a game also calls attention to the limitations caused by transactions costs and time to many mechanisms. Thus, for example, there is nothing logically wrong in giving one's stockbroker a continuous function expressing one's demand for IBM shares as a function of the prices of wheat and gold; but any attempt to play this as a game will suggest the use of simpler messages.

In our attempt to stress mechanisms and to find the minimal conditions which require the invention of financial instruments and institutions, the risk is run that one set of critics will feel that the simplifications are gross distortions of "the real world", while the mathematically oriented theorist may feel that the models are too cluttered up with unnecessary detail. It is possibly helpful to look at the models as playable games which can be analyzed rather than stress immediate realism.

5.2. Market mechanisms

How is price formed? There are undoubtedly many ways. We can imagine pairs meeting randomly and contracting or recontracting; or an auctioneer calling price or individuals haggling, matching with partners elsewhere and returning to bargain if not satisfied. More prevalent in a mass retail economy is where one walks into a store and accepts the posted price or walks out without buying that item, although other purchases may be made. In a stock exchange a double auction mechanism may be used. Retail price formation is often different from wholesale sales to the trade and these may differ from intrafirm producer sales. The facts of inventory costs, transportation and delivery time lags modify the price-setting process. In complicated deals price may appear fixed to the untutored eye, but the lawyers are adjusting price via conditions on the deal. In socialist economies there may be a feedback system involving market, political and bureaucratic pressure; in capitalist economies oligopolistic power plays a role in some sectors.

Short of performing detailed ad hoc industrial studies, "realism" is not that easy to achieve in describing price mechanisms. Rather than propose realism as the criterion, the gaming test is used – mechanisms that are simple and easy to use are constructed. Gale (1986a, 1986b), Rubinstein and Wolinsky (1984), Binmore and Herrero (1984) and others have considered random pairing mechanisms. These are discussed elsewhere. Here the emphasis is on one-move mechanisms. An eventual program is to classify classes of mechanisms by axiomatic properties. Dubey, Mas-Colell and Shubik (1980) present five axioms which can be stated nontechnically and intuitively as follows:

(i) *Convexity*: traders have available a convex set of strategies.

(ii) *Anonymity*: in the market only the message sent by the trader matters.

(iii) *Continuity*: the outcomes vary continuously with the strategies.

(iv) *Aggregation*: the trading opportunities for any player are influenced by all others only through the mean of the messages of all others.

(v) *Nondegeneracy*: it must be possible for individual players to influence to a substantial extent their trading possibilities in the market.

With these five axioms they established inefficiency of all interior boundary noncooperative equilibria (N.E.) with a finite number of traders and efficiency of interior N.E. with a continuum of traders.

Axiom (iii) rules out the Bertrand–Edgeworth class of models.

An interesting distinction between the games satisfying the Dubey, Mas-Colell and Shubik axioms and the Bertrand–Edgeworth model is that in the former there is continuity in the variation of payoffs to variation in an individual's strategy, but the efficiency of equilibria requires a continuum of players, while in the latter there is a discontinuity in the variation of payoffs as strategies are varied but the efficiency of equilibrium can be achieved with a finite number of traders. [See Benassy (1986) for further analysis, and Dubey, Sahi and Shubik (1989) for axioms covering both Cournot and price mechanisms.]

The concept of strategic market game is formally related to Hurwicz's (1960, 1973) approach to the design of resource allocation methods.

Shubik (1973), Shapley (1976), Shapley and Shubik (1977), Dubey and Shubik (1978a, 1980b), Okuno and Schmeidler (1986) and Dubey (1982) constructed and established the noncooperative equilibrium properties of three models which can be described as a one-sided Cournot type of model, a two-sided Cournot and a double auction or two-sided Bertrand–Edgeworth model.

Consider an exchange economy with n players and $m+1$ commodities where the $m+1$st commodity is used as a money and there are m markets. Let the utility function of individual i be $\varphi_i(x_1^i, x_2^i, \ldots, x_m^i, x_{m+1}^i)$, where x_j^i is individual i's final holding (in R_+^{m+1}) of good j $(j=1, \ldots, m+1)$. Let the initial endowment of individual i be $(a_1^i, a_2^i, \ldots, a_{m+1}^i)$.

In model 1 (the sell-all model) all individuals are required to put up for sale all their resources except their money. They use the money to buy resources and after trade are paid for what they have sold. A strategy for individual i is a set of m numbers $(b_1^i, b_2^i, \ldots, b_m^i)$, where $b_j^i \geq 0$ and $\sum_{j=1}^m b_j^i \leq a_{m+1}^i$:

$$p_{m+1} = 1\,, \qquad p_j = \sum_{i=1}^n b_j^i \Big/ \sum_{i=1}^n a_j^i\,, \qquad x_j^i = b_j^i / p_j\,, \quad \text{for } j = 1, \ldots, m\,, \tag{1}$$

and

$$x_{m+1}^i = a_{m+1}^i - \sum_{j=1}^m b_j^i + \sum_{j=1}^m a_j^i p_j\,, \tag{2}$$

where p_j = the price of good j.

This model is exposited in detail in Shapley and Shubik (1977). It should be noted that a strategy is a physical act not a bid or a verbal statement. The strategy of an individual has the "unrealistic" feature that an individual must sell all of his goods rather than consume directly from his resources, buying only for consumption above his initial endowment. The second or two-sided Cournot model corrects for this.

The strategy of an individual i is a set of $2m$ numbers $(b_1^i, q_1^i; b_2^i, q_2^i; \ldots; b_m^i, q_m^i)$, where $0 \leq b_j^i$ and $\sum_{j=1}^m b_j^i \leq a_{m+1}^i$ and $0 \leq q_j^i \leq a_j^i$, for $j = 1, \ldots, m$. Here $p_{m+1} = 1$ and

$$p_j = \sum_{i=1}^{n} b^i_j \Big/ \sum_{j=1}^{n} q^i_j\,, \quad \text{if } \sum_{i=1}^{n} q^i_j > 0\,,$$

$$= 0\,, \qquad\qquad \text{if } \sum_{i=1}^{n} q^i_j = 0\,, \tag{3}$$

$$x^i_j = a^i_j - q^i_j + b^i_j / p_j\,, \quad \text{for } j = 1, \ldots, n\,, \tag{4}$$

and

$$x^i_{m+1} = a^i_{m+1} + \sum_{j=1}^{m} q^i_j p_j - \sum_{j=1}^{m} b^i_j\,. \tag{5}$$

As individuals can buy and sell simultaneously in the same market the possibility for "wash sales" appears[3] and for finite numbers of players a continuum of equilibria may be encountered. An example of this possibility is given by Shubik (1984a, pp. 434–438). The proof of convergence of N.E. to the competitive equilibria (C.E.) as the players are replicated is given in Dubey and Shubik (1978a).[4]

The third model is in the style of Bertrand–Edgeworth in the sense that price rather than quantity is the prime strategic variable. A strategy is no longer a physical act of sending goods to the market but a contingent statement of size $4m$ of the form $(p^i_1, q^i_1, \tilde{p}^i_1, \tilde{q}^i_1; \ldots; p^i_m, q^i_m, \tilde{p}^i_m, \tilde{q}^i_m)$, where p^i_j and q^i_j are the price and amount of good j an individual i is willing to buy, and $\tilde{p}^i_j$ and $\tilde{q}^i_j$ are the price and amount an individual i is willing to sell of good j. Theoretically we would like to be able to enforce:

$$\sum_{j=1}^{m} p^i_j q^i_j \le a^i_{m+1}\,. \tag{6}$$

This states that an individual's buying commitment can never be larger than his cash on hand. But ex ante there is no way to enforce this unless an inspection

[3]A wash sale occurs when an individual both buys and sells simultaneously in the same market thereby thickening it. For example if an individual has 100 units of a commodity and wants 100 more he could buy 100 or combine the sale of 100 with the purchase of 200. This would create a wash sale of 100. The wash sale provides a useful enlargement of strategies to establish the existence of N.E. with active trade [see Shapley (1976), Peck and Shell (1985, 1986) and Yao (1987)].

[4]Okuno and Schmeidler (1986) construct a strategic market game where the players use as strategies linear excess demand functions. They obtain approximate efficiency of the N.E. They observe that their main motivation "is to eliminate myopic assumptions inherent in the Walrasian model on the one hand and in the Shubik or Cournot type models on the other". I have no argument with their logic, but I suggest that playability is an important criterion.

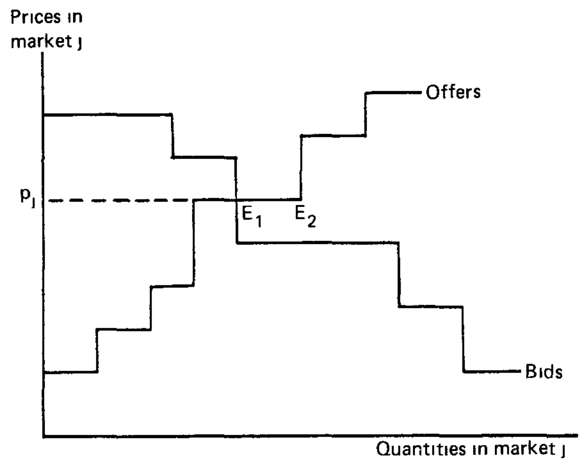

Figure 5 2

procedure is instituted. The condition (6) when p_j^i and q_j^i are bids can be extremely strong as the probability that one can purchase everything may be low.

In this market mechanism a bid and an offer histogram is constructed for each market. Bids are displayed as summed in descending order and offers in ascending order. Market price is established at the intersection of the two histograms. This is shown in Figure 5.2. There are several details concerning market price information and residual excess supply or demand (in Figure 5.2 the amount E_1E_2 is excess offers at the market price p_j) and these, together with the study of the N.E. and the relation to the C.E. are given in Dubey (1982) and Dubey and Shubik (1980b). An important distinction between the previous models and this is that there are noncooperative equilibria (N.E.) which are competitive equilibria (C.E) with as few as two individuals active on each side of a market.

5.2.1. On the number of simple mechanisms

One can envision a special language for trade involving only names of goods, quantities, bids, offers and prices. A move by a trader is a message concerning bids and offers for quantities of goods at various prices. The mass market is a device which aggregates the messages of traders and determines final market prices and trades. It is suggested elsewhere [Shubik (1979)] that if each trader has only a single move and if the market mechanism is limited in complexity, there are only a few simple market mechanisms.

5.2.2. Enough money

All three games described above have one move per player and all moves are made simultaneously; thus the velocity of money is at most one. In order for it to be feasible to achieve a C.E. of the exchange economy as an N.E. of the strategic market game, for the sell-all model it is required that at equilibrium:

$$\sum p_j x^i_j \leq a^i_{m+1} . \tag{7}$$

As all nonmonetary assets are sold and all purchases paid for in money, the amount of money needed is at least the market value of all assets other than money.

For the two models, where the individual can consume directly from his assets the cash requirements are smaller, they are:

$$\sum p_j \max[(x^i_j - a^i_j), 0] \leq a^i_{m+1} . \tag{8}$$

Given the condition that all transactions must involve the use of a specific commodity and given the transactions technology as fixed, the feasible set of trades for any trader will be constrained by his holding of cash. The worst case is where he must buy everything. This is shown in (7), the best case is shown in (8). If the exchange economy associated with a strategic market game (i.e. with the same preferences and endowments) has one or more C.E. such that the cash requirements at some C.E. cannot be met, this C.E. will not be a feasible outcome of the strategic market game, but a boundary N.E. will exist where the relative price of the commodity money will reflect not only its value in consumption, but also its shadow price as a capacity constraint on trade.

Even with a continuum of traders a N.E. of a strategic market game cannot approach a C.E. unless that outcome is feasible, but this requires enough cash. Without enough cash N.E. exist but are not efficient.

It is possible that the inequalities may not be satisfied by selecting good $m+1$ as the means of payment, but by changing to good j they could be satisfied. A discussion of alternative choices of a means of payment is given by Shubik (1986b).

Suppose that there were not enough money, in the sense that there were no money that could satisfy the inequalities. A natural question to ask is: If we can change the game (and the exchange economy) by increasing the monetary endowments of the players, can we guarantee that we can eventually satisfy the enough money conditions at a C.E. of the new exchange economy?

As the amount of money[5] is varied the number of C.E. can change and the

[5] The word "money" is used often as a broad phrase covering an item used for the means of payment, but also having properties such as a store of value, serving as numeraire and possibly having other features. Here possibly "means of payment" is more precise and restrictive than "money", but this should be clear enough from the context.

relative prices can change radically. In particular, the marginal relative worth of the money could drop with increasing quantities so that the enough money condition is never satisfied. A sufficient condition to eventually achieve enough money is that:

$$\frac{\partial \varphi_i / \partial x^i_{m+1}}{\partial \varphi_i / \partial x^i_j} \geq \Delta \,, \quad \text{for all } j, \text{ for all endowments} \,. \tag{9}$$

Intuitively this says that there will always be some lower bound to the purchasing power of money for any commodity. Dubey and Shapley (1977c) discuss this further.

If we assume that utility functions are of the form:

$$F(\varphi_i(x^i_1, \ldots, x^i_m) + x^i_{m+1}) \,, \tag{10}$$

or more simply:

$$\varphi_i(x^i_1, x^i_2, \ldots, x^i_m) + f_i(x^i_{m+1}) \,, \quad \text{with } f' > 0 \,, f'' < 0 \,, \tag{11}$$

then all exchange economies associated with the class of games $\Gamma(n, k)$, defined by having n traders with preferences as in (10) and (11) and initial endowments of the form $(a^i_1, a^i_2, \ldots, a^i_m, a^i_{m+1} + k)$, will all have the same number of C.E. In particular, if the game $\Gamma(n, 0)$ has no interior C.E. it becomes meaningful to consider the minimum amount of money required to be able to achieve one C.E. as a N.E., and the amount required to attain all C.E.[6]

For this class of games it is meaningful to identify the C.E. which require the minimal and maximal amounts of money to finance trade. An immediate objection which can be raised against this observation is that condition (10) is unreasonable as it implies a no income effect. This is, to some extent, a difficult empirical question; as a first-order approximation it may be reasonable to assume that as individuals become richer the income effect of prices becomes weaker. Thus, for example, it might be that as a first approximation we have:

$$\begin{aligned} \phi_i(x^i_1, \ldots, x^i_{m+1}) &= \varphi_i(x^i_1, \ldots, x^i_{m+1}) \,, & \text{for } x^i_{m+1} \leq k_1 \,, \\ &= \varphi_i(x^i_1, \ldots, x^i_m, k_1) + f_i(x^i_{m+1} - k_1) \,, & \text{for } k_1 \leq x^i_{m+1} \,. \end{aligned} \tag{12}$$

For the extremely rich it might even be reasonable to assume that beyond some k_2 the marginal utility of further wealth becomes approximately constant.

[6]Although with continuum of agents it may be somewhat evident to establish that with enough money any C E is a N.E., the establishing of strict equivalence is more problematical.

5.3. Complete markets and enough money

We may adopt the definition of commodity money as a commodity which has intrinsic value to a consumer (i.e. it appears as an argument of the utility function) and has complete markets, or is directly exchangeable into every other good. Figure 5.3 illustrates this. There are four commodities represented by the numbered points. Each line connecting two points signifies a simple market, i.e. a market (with some unspecified mechanism) where i can be exchanged directly for j. In Figure 5.3(a) there is no money; in 5.3(b) good 1 is a money; in 5.3(c) goods 1 and 3 are monies; and in 5.3(d) all goods are monies.

With complete markets all goods are money; there is always enough money to finance every individual's trade without credit. Using an extension of the Dubey–Shubik (1978a) model a strategy of an individual trader becomes of dimension $m(m-1)$ if there are m goods; as each can be directly exchanged for any other there are $m(m-1)/2$ markets, but in any market (say apples for oranges) an individual can be on either side (or both simultaneously). Amir, Sahi, Shubik and Yao (1987) have been able to prove the existence of pure strategy N.E.[7]

A natural question to ask is: Are there N.E. in the game with complete markets which are not C.E.? The answer is yes and is established by the example below where there are four types of traders, a continuum of each type and three commodities.

Let the utility functions be given:

$$u_0(x_1, x_2, x_3) = x_1^{1/3} x_2^{1/3} x_3^{1/3} ,$$
$$u_1(y_1, y_2, y_3) = y_1^{1/3} y_2^{2/3} ,$$
$$u_2(z_1, z_2, z_3) = z_2^{1/3} z_3^{2/3} ,$$
$$u_3(w_1, w_2, w_3) = w_1^{2/3} w_3^{1/3} .$$

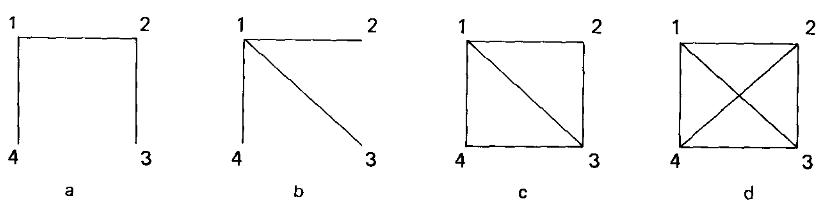

Figure 5 3

[7]A somewhat different model proposed by Shapley is that after all bids and offers are received they are all aggregated in a central agency which calculates and announces the clearing prices. Sahi and Yao (1987) have analyzed this model and proved existence.

The initial endowments are:

$$(a_1^0, a_2^0, a_3^0) = (1, 1, 1),$$
$$(a_1^1, a_2^1, a_3^1) = (0, 3, 0),$$
$$(a_1^2, a_2^2, a_3^2) = (0, 0, 3),$$
$$(a_1^3, a_2^3, a_3^3) = (3, 0, 0).$$

The final N.E. allocations are:

$$(x_1, x_2, x_3) = (2, 2, 2),$$
$$(y_1^1, y_2^1, y_3^1) = (1, 1, 1),$$
$$(z_1^2, z_2^2, z_3^2) = (0, 1, 1),$$
$$(w_1^3, w_2^3, w_3^3) = (1, 0, 1),$$

with prices

$$p_{12} = 2 = p_{23} = p_{21}$$

and

$$p_{12} p_{23} p_{31} = 8.$$

However, the unique C.E. is easily seen to consist of the price vector $(1, 1, 1)$ and gives rise to the following final allocations:

$$(1, 1, 1), \quad (2, 1, 0), \quad (0, 2, 1) \quad \text{and} \quad (1, 0, 2).$$

This N.E. violates the no-arbitrage condition that $p_{ij} p_{jk} p_{ki} = 1$. It would be destroyed as a N.E. if credit were available. It depends upon there being only one round of trade so that no individual can take advantage of the full arbitrage circuit for $p_{ij} p_{jk} p_{ki} > 1$.

This model cannot be generalized to many periods without specifically introducing instruments for futures trading. Although logically we can imagine the trade of today's eggs for wheat to be delivered six years from now, if they are traded at all, commodities for lengthy future delivery are traded via futures contracts which are paper instruments which take on a life of their own until they are extinguished when the contract is filled.

When there are only a finite number of traders the N.E. are not efficient. The approach to efficiency depends on the thickness of markets. But two phenomena influence the thickness of markets when numbers are finite. They are the number of markets used (the more, the thinner the markets for the same volume of trade) and the absence or presence of wash sales (as already noted).

The presence of transactions costs[8] combined with the thinness of markets both contrive to provide reasons for some markets to fail to materialize.

Remark on the selection of a numeraire

When there is only a single money and m markets it is easy and natural to select money as a numeraire and set $p_{m+1} = 1$ without any reference to equilibrium conditions. The game only determines m prices, thus prices emerge in terms of the money. With complete markets prior to the emergence of equilibrium $m(m-1)/2$, prices must be determined and the numeraire cannot be neutrally meaningfully selected without imposing a no-arbitrage requirement.

5.4. *Not enough money and the need for credit*

When there is a restriction to one money and markets exist only between it and all other goods three cases must be considered. Both the absolute quantity and the distribution of the monetary good count, thus three possibilities are that:

(1) there is enough money well distributed;
(2) there is enough money but badly distributed; and
(3) there is not enough money.

There are four basic ways in which a money shortage or maldistribution can be overcome, they are by:

(1) introducing more markets;
(2) producing more money and distributing it appropriately;
(3) increasing velocity of circulation; or
(4) introducing credit.

As transactions costs decrease, communications improve and population increases, new markets may appear, but even to casual empiricists there are futures contracts which are not available and risks which cannot be hedged efficiently.

In the description of macroeconomic disequilibrium and inflation, stories abound about governments resorting to the printing press, to debasing coinage or to plundering gold and silver mines elsewhere. Even with the entry of new gold, it is necessary to specify how the gold is introduced to the economy. Does the government sell it in return for labor (pay the army, for instance) or other resources? Does it enter via gold miners selling the extra resources? Purely

[8]See Chapter 1 by Ostroy and Starr in this Handbook for a detailed discussion of transactions costs.

formally it is straightforward to regard the supply of a commodity as parametrically given and study the variations. One can also model the production of gold, but unless the production and preference conditions are appropriate even with production there is no guarantee of enough money. A more prevalent way in which the lack of money is ameliorated is by the invention of credit instruments.

Before turning to credit, some comments on the velocity of money are in order. There are technological bounds on many timing aspects of economic life. The quickening or slowing of payments strategically without resorting to credit instruments has only limited possibilities. Individuals can change the frequency of their purchases of some inventories and modify or select faster production processes. However, although changes in the velocity of money may be an important problem in economic policy and control, except for adding considerable complication to the description of a strategic market game to provide strategic choice over timing, no central conceptual problems in a theory of money appear to depend on the changes in velocity. For this reason it is suggested that in spite of Tobin's criticism [see Karaken and Wallace (1980, p. 90)] fixed move models probably provide a useful simplification in the development of a theory of money. Variation in velocity is an important applied problem but a red herring in the basic understanding of money and financial institutions.

The most essential way in which a monetary economy copes with shortage of money is with credit and other financial instruments.

ı. Strategic market games with credit

n an attempt to construct games with credit arrangements the following factors ıust be taken into account, and how they are dealt with must be specified ithin the model:

(a) the nature of the credit instrument;
(b) the nature of the issuer and borrower;
(c) default conditions on the borrower;
(d) default conditions on the lender;
(e) a one-period or multiperiod model;
(f) the presence or absence of exogenous uncertainty; and
g) ıntergenerational transfer of assets and liabilities.

Instruments and issuers

re are three divisions of economic agents as issuers and borrowers which it consideration. They are (1) individuals, (2) private institutions and (3) public institutions. The fundamental distinctions among them are reflected, in general, in large differences in economic size, life and special rules differentiating and delimiting their strategic possibilities. These differences will be manifested in the construction of a game with distinguished classes of players.

In a world with only private individuals there are two sorts of loan arrangements we may consider. The first is where there is bilateral personal credit, as is often found among friends, and the second is where an anonymous money market exists in which all I.O.U. notes are accepted and treated as fungible. These are discussed further in Subsection 6.5.

Although both large merchants and manufacturers have provided and often do provide banking and insurance functions, we consider just the bank and its issue of banknotes and checks.[9] The models are noted in Subsection 6.8.

Possibly one of the more important ways the money supply has been supplemented has been by the bill of exchange involving originally merchants and bill specialists and now usually involving two merchants and two banks. No attempt has been made yet to build a formal model isolating the factors which contribute to giving it an independent life.

The last special agent is the government or government agencies. At a level of some detail there are important distinctions. In the United States much government agency financing is done with the "the full faith and credit" of the U.S. government, but the level of guarantee may vary with the agency involved. At the level of abstraction and aggregation employed here "the government" is considered as a single financial agency. As a modeling choice we may wish to consider it outside of the private economy – as the referee or controller of the game. Alternatively, it may be considered as within the game in which case as it is such a specialized player, a careful specification of its motivation and payoff function is required. The government may issue coin, banknotes and checks. This is discussed further in Subsection 7.6.

6.2. *Default conditions, single- and multiperiod models*

As soon as credit is introduced into a game, unless it is limited so that even under the worst scenario the debtor can pay back fully, it becomes necessary to add default conditions as part of the rules of the game. In all modern enterprise economies there is a body of law dealing with insolvency, bankruptcy, corporate liquidation and reorganization, the garnishing of income and other factors which come into play when there is a failure to repay debt at the appointed time.

[9]It is of interest to ask at what level of detail in modeling and game design one would expect the need for coin as contrasted with banknotes to appear – most probably where there is a need for low value, and high velocity, and high durability – an easy natural source for a central government to provide a public service and take a seignorage profit

Insolvency may involve only a timing problem. A debt is not met at a due date, but available assets are more than adequate to cover it (and any accrual) given enough time for orderly liquidation. In a bankruptcy the firm or individual, even fully liquidated, may not be able to cover the debt outstanding. At the level of abstraction here rather than use the legal language with its many special meanings, perhaps a more neutral terminology such as "repayment failure rules" are called for. Shubik and Wilson (1977) and Dubey and Shubik (1979) unfortunately did not follow this advice.

When the rules are needed to take care of a settlement after default, a considerable difference emerges between the single-period and multiperiod game. In the single-period game the settlement takes place at the end of play. When there is more than one period a decision must be made whether to settle during the game or roll over the loan hoping to recoup before the end of play. When a settlement is made, the procedure has to reflect the purposes of the design of the rule. These may include deterrence to discourage debtors from failure, rehabilitation to improve their chances of being economically valuable to the society, restitution to help optimize the return to creditors, and administrative speed and low cost. But in making restitution, if borrowing and lending have involved aggregations of individuals we must specify seniority conditions. There is also the danger that a single failure may set off a chain of failures. The question of "what are the optimum repayment failure rules" is complex and has not yet been fully answered. This is also true of the role of limited liability.

6.3. A digression on the cashless society, transactions costs and Pareto optimality

Implicit in much of the discussion on exchange and exchange and production is the proposition that the technology of exchange is costless. Pesek (1976) has noted that much of the discussion on banking is as though it is a costless occupation, yet an examination of the actual costs involved indicates that it is not unlike the telephone companies or manufacturing.

Even a casual examination of transactions costs suggests many operational reasons for different financial instruments which, from some point of view, would be regarded as the same. Thus, for some sets of questions there are no interesting differences among economies run with coins, fiat paper, banknotes, checks or a computerized accounting credit system. Given the technological breakthrough in the last few decades in communication and computation, it might appear that transactions costs are falling so precipitously that the cashless society where only accounting money exists is becoming a real possibility [see Black (1970) and Fama (1980)]. As a start in theorizing about the properties of the means of payment it is possibly worth not making any distinction. But there are indications that the transactions costs, accounting and anonymity features are such that coins and paper fiat money are here to stay while the use of checks may be eroded by accounting transfers. Mayer (1974, p. 168) in a popular book on banking suggests that the cost of processing a check might have been around $0.20 in the early 1970s. In a second book [Mayer (1984, pp. 71–74)] the estimates appear to be higher. The conceptual and accounting difficulties in attributing individual cost to this joint production operation are at least as bad as in evaluating the cost of the individual telephone call, but the rough message is that the total expense of the payments system of the banks has a nontrivial average transactions cost. Paper money, in contrast, may last for a few years[10] and at the cost of a few cents for the note, serves for many anonymous transactions. Coins may last for several decades,[11] each costs a fraction of the face value of the coin and serves for even more transactions than notes. There are different ways in which we might try to attach significance to these difficulties, such as weighting the different instruments by both the dollar size and frequency of use of the instrument. There are conceptual difficulties, the empirical information is shaky, but it is suggested here that the prospects for the cashless society are not high.

Once the importance of transactions costs are acknowledged the concept of Pareto optimality of competitive exchange needs adjustment. As noted by Arrow (1981) the feasible set of outcomes with transactions costs depends on the initial distribution of resources. Rogawski and Shubik (1986) have studied a strategic market game with transactions costs and shown the existence of pure strategy N.E. and for large numbers their approach to efficiency in the transactions cost constrained set. But although results may be obtained with production technologies represented by cones or convex sets, many of the more interesting problems with transactions costs involve set-up costs, indivisibilities and complicated joint production, none of which has yet been treated game theoretically in the context of a theory of money and financial institutions.[12]

6.4. *A digression on two types of anonymity*

For the major purposes of the economics of payment, the check, credit card or accounting entry are as anonymous as cash. From the point of view of Mrs.

[10]The average life of a Federal Reserve Note (from an estimate of the Department of the Treasury, Bureau of Engraving and Printing in 1986, personal correspondence, 19 September 1986) is 18 months for the $1 bill; 2 years for $5, 3 years for $10; 5 years for $20; 9 years for $50, and 23 years for $100. The cost of production is $26 per 1000 notes regardless of denomination.

[11]The U.S Mint produces the coinage A casual estimate based on sampling pocket change is that a quarter may remain in circulation for more than 25 years

[12]But see Gale and Hellwig (1984) for a treatment of nonconvex transactions costs.

Jones who tells her husband that she had to stay late in the office while she had a night on the town, there is a considerable difference. The anonymity of the clearinghouse dealing with records is not the anonymity of cash. The presence of vast quantities of $100 bills used by the underground economy and others attests to the differences between cash payment and a system of clearance that leaves a paper trail for the tax man, suspicious spouse or other control group.

The difference between the two types of anonymity is sufficient to provide a reason strong enough by itself to be a barrier to the introduction of a cashless society.

6.5. The money market

Everyone a banker

A popular thought in the casual discussion of competitive finance is that anyone should be able to become a banker. A way to try to operationalize this as a playable game is as follows.

There are n individuals and m goods, and for simplicity in minimizing the need to discuss the need for future markets, we limit the game to one period. Each individual i has a utility function of the form:

$$\varphi_i(x^i_1, \ldots, x^i_m) + \mu^i \min\left[0, \sum_{j=1}^{m} p_j(x^i_j - a^i_j)\right],$$

where $(a^i_1, \ldots, a^i_m)$ is the initial endowment of i and the p_j are the prices formed. We must specify how prices are formed and justify the structure of the extra term on the utility function.

If every individual is able to issue I.O.U. notes which are accepted as a means of payment by all others, then we may use the same sort of market mechanism as was shown for the two-sided Cournot market with the following changes. here, as in that market, a bid by an individual is of the form $(b^i_1, q^i_1, b^i_2, q^i_2, \ldots, b^i_m, q^i_m)$, but the sum $\sum_{j=1}^{m} b^i_j$ is no longer bounded by a quantity of commodity money a^i_{m+1}; each b^i_j is a personal I.O.U. note sent to the market denominated in a unit of account, say dollars.

In order for this game to become playable we have to have rules concerning the redemption of I.O.U. notes and what happens if there is a failure to redeem. If there were no punishment for default, the incentive to issue I.O.U. notes would be unbounded and would destroy any proposed active equilibrium. By introducing a bankruptcy penalty which can be extraeconomic, such as going to prison or losing a hand, where the severity of the penalty increases with the level of debt, we bound the level of borrowing of any individual who is maximizing his payoff given the actions of all others. Without loss of generality the specific functional form suggested in (13) serves as well as the form:

$$\varphi_i(x_1^i, x_2^i, \ldots, x_m^i; x_{m+1}^i) \tag{14}$$

defined on $R_m^+ \times R$, where $x_{m+1}^i = \sum_{j=1}^m p_j(x_j^i - a_j^i)$. This is illustrated in Figure 5.4 where the key controlling feature of the severity of the penalty as a deterrent to failing to redeem I.O.U. notes is the slope of the contours as they proceed into the negative orthant. As long as the penalty for issuing one dollar of unredeemed debt is greater than the worth of an extra dollar of income, the penalty is sufficiently severe to discourage strategic default. If there is no strategic default at equilibrium, the specifics of the severity of the penalty for varying levels of final debt will not matter. "Harsh enough" can be operationalized. This is not so with incomplete markets and exogenous uncertainty where an optimal bankruptcy penalty may involve a number of bankruptcies.

In Figure 5.4 the basic distinction between a utility function of forms (13) and (14) is that (13) gives an indifference curve of the form $A'CE'$ which implies that in this one-shot game left over ownership of I.O.U. notes are of no value, but negative amounts are of negative value. The curve ACE arising from (14) looks like a more conventional indifference curve, somewhat transposed, so that part appears in the negative orthant. This indicates that the creditor has

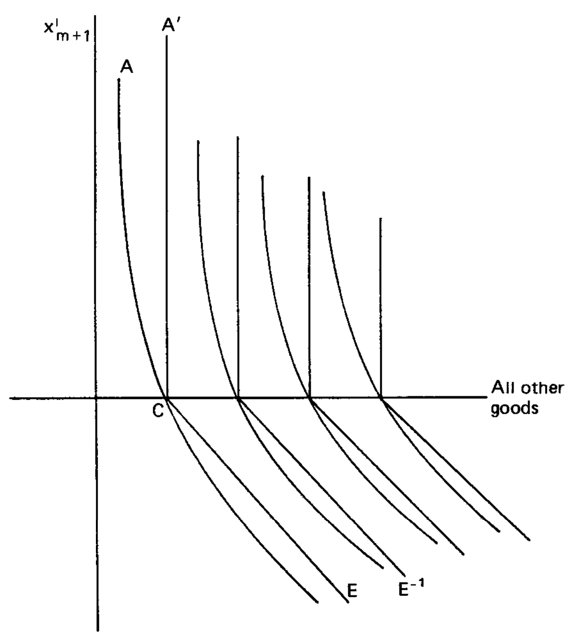

Figure 5 4

value for outstanding debt, a situation which is usually true if the game continues and there is some hope of repayment.

For a continuum of traders all of the C.E. of the associated exchange economy will be N.E., but in each instance the default penalty establishes only a lower bound on price. Beyond that lower bound the multiplication of all prices by $k > 1$ gives an equilibrium associated in its real good distribution with the equilibrium for $k = 1$.

After all players have selected their strategies and the m prices are formed by adding together all goods, the goods are shipped and the net balances of each trader are reported from each market to a clearinghouse. The balances bear only the name of the individuals. The clearinghouse sets up accounts for all individuals and cancels debits and credits to achieve a net balance. In this one-period model, as final positive balances are worthless no one wishes to end with positive balances. Those with negative balances suffer the penalty. In the one-period game there is no need to consider seniority or credit failure domino effects. Implicit in this formulation is an avoidance of having to deal with others than those who fail to redeem their paper at an outside bank.

Mathematically this model can be regarded as identical with there being a shadow bank clearinghouse which exchanges personal I.O.U. notes for bank notes. As long as there is only one trading period and a nonzero default penalty, with a continuum of traders the coincidence of the exchange economy's C.E. with N.E. of the strategic market game can be established. One can make the default penalty infinite [see Schmeidler (1980)] but there is no need to. The harsher the penalty, the lower the bound on equilibrium prices.[13]

There is no particular logical or technological reason to rule out letting all individuals directly monetize their I.O.U.s. A playable one-period game has been described. Even at this level of simplicity a clearinghouse is needed as well as a device or referee to report failure to redeem and to enforce a penalty on defaulters. In general, individual I.O.U.s are not accepted as money because of lack of information and trust. Even in a highly computerized world the cost of a universal trustworthy credit evaluation system is sufficiently substantial to rule out "everyone a bank" as a viable arrangement.

6.6. *The money market, gold and liquidity*

Above it has been suggested that although there is nothing logically wrong with all individuals using their own I.O.U.s as money, for reasons of trust,

[13]As the penalty is increased the disutility of going bankrupt for a dollar increases. Any price level where the disutility of going bankrupt exceeds the marginal value of extra purchase can be supported. With a high penalty the lower bound on the price level approaches zero The distribution of real resources is not influenced.

administration and identification this is not a good approximation of the way things are or even will be. A closer approximation is that there is an identifiable trusted known institution which serves as an intermediary. This institution evaluates and judges creditworthiness. It may accept the debt of otherwise unknown individuals and exchange this unknown debt for its known debt which is accepted in payment by all. In doing this the institution either becomes a specialized differentiated player in the game or should be regarded as a controller out of the game. In the next two sections this institution is considered in the form of public or private banks.

Before considering banks, an economy is examined which already has gold as a commodity money, but might nevertheless benefit from the existence of a market in which the gold can be borrowed.

In examining an exchange economy which uses a commodity money for exchange there are three possibilities concerning the amount of money available and the needs of trade. They are:

(a) All have enough to finance efficient trade:

$$\sum_{j=1}^{m} p_j \max[(x^i_j - a^i_j), 0] \leq a^i_{m+1}, \quad \forall i \in N, \tag{15}$$

where p_j are prices at a C.E.[14]

(b) There is enough money in the economy as a whole to finance efficient trade, but it is badly distributed:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} p_j \max[(x^i_j - a^i_j), 0] \leq \sum_{i=1}^{n} a^i_{m+1}, \tag{16}$$

but for some i $\sum_{j=1}^{m} p_j \max[(x^i_j, a^i_j), 0] > a^i_{m+1}$.

(c) There is not enough money for efficient trade:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} p_j \max[(x^i_j - a^i_j), 0] > \sum_{i=1}^{n} a^i_{m+1}. \tag{17}$$

These observations are for a one-period (or normalized form) model. New problems appear when a multiperiod model is considered and futures markets, short sales and other trades involving timing are considered.

In the first of the three cases, as there is both enough money and it is well distributed, no credit or money market facilities are needed. In the third case (relative to the C.E. under consideration) there is not enough money in the system to finance liquidity. In the second case, by introducing a money market

[14]We have discussed the possibility that there may only be enough money to attain one C E. and not the others, in Subsection 5 2

it is possible to correct the maldistribution of money and achieve the C.E. trade in the associated strategic market game. The money market is a simple quantity mechanism in which those who wish to lend money offer it to the market and those who wish to borrow bid I.O.U. notes. Figure 5.5 shows the two-stage game. The amount an individual i offers to lend is z^i. The amount that an individual j offers to pay for his loan at the end of play is u^j. Hence, an endogenous rate of interest ρ emerges as:

$$1+\rho = \sum_{j=1}^{n} u^j \Big/ \sum_{k=1}^{n} z^k$$
$$= 1\,, \quad \text{if either sum is zero}\,. \tag{18}$$

After the money market has cleared, exchange takes place with the conditions that:

$$\sum_{j=1}^{m} b_j^i \le a_{m+1}^i - z^i + u^i/(1+\rho)\,. \tag{19}$$

The final payoff is given by:

$$\varphi_i(x_1^i, \ldots, x_m^i)$$
$$+ \mu_i \min\left[0, \left(a_{m+1}^i - \sum_{j=1}^{m} b_j^i + \sum_{j=1}^{m} p_j q_j^i - \rho u^i/(1+\rho) + \rho z^i\right)\right], \tag{20}$$

where $x_j^i = a_j^i - q_j^i + b_j^i/p_j$, for $j = 1, \ldots, m$. For a sufficiently strong default penalty there will be a pure strategy N.E. with $\rho > 0$ for finite numbers[15] and $\rho = 0$ for a continuum of traders.

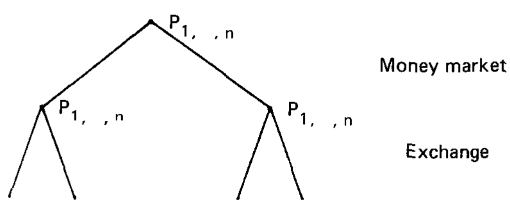

Figure 5 5

[15]As this is a multistage game, as soon as we consider a finite number of players we need extra conditions (such as perfect equilibrium) to cut down on the possible proliferation of equilibria The justification of the extra conditions is not obvious

Gold and liquidity

The consideration of borrowing and lending over more than one period poses new problems concerning default penalties and the meaning of a commodity money and how it enters the utility function. It also poses problems concerning the need for short sales or futures markets in order to achieve efficiency.

A fully satisfactory model of period-by-period default penalties and reorganization has not yet been achieved. For a finite horizon the problem can be avoided by refinancing interim defaults until the end.

The cost of liquidity is modeled as follows. Consider a game which is played for k periods. In each period there are m nondurable commodities and a durable commodity money, say gold. Each individual i wishes to maximize

$$\sum_{t=1}^{k} \beta^{t-1}\varphi_t(x_{1t}^i, \ldots, x_{mt}^i, x_{m+1,t}^i)\,, \tag{21}$$

where the component $x_{m+1,t}^i$ which enters the utility function is to be interpreted as the consumption value of gold as a consumer good, say as jewelry. We may consider a game with three moves in each period. At the start of the period each individual divides the gold he possesses into two piles, one for jewelry and one for money. For simplicity, the production processes which take gold money to jewelry and vice versa are regarded as costless and immediate. There is no wear and tear on the gold. Each individual i starts with an amount of gold $a_{m+1,1}^i$. At each period t he divides it so that:

$$a_{m+1,t}^i = y_t^i + w_{m+1,t}^i\,, \tag{22}$$

where y_t^i is the amount of monetary gold at the start of period t and $w_{m+1,t}^i$ is the amount of jewelry. Jewelry can be sold for money but yields no consumption value[16] if sold and the money is not available until the end of the period. Without production costs and time, rather than define jewelry separately we could just give consumption value to untraded gold.

The amount of monetary gold at the start of $t+1$ is:

$$a_{m+1,t+1}^i = y_t^i - \sum_{j=1}^{m+1} b_{jt}^i + \sum_{j=1}^{m+1} p_{jt}q_{jt}^i - \rho_t u_t^i/(1+\rho) + \rho_t z_t^i + x_{m+1,t}^i\,, \tag{23}$$

where the u_t^i and z_t^i are interpreted as in (18), i.e. the net returns from

[16]The modeling here is somewhat arbitrary, we could split the consumption value of jewelry between buyer and seller in any proportion.

borrowing and lending gold. The $x^i_{t,m+1}$ is the ending amount of jewelry:

$$x^i_{m+1,t} = w^i_{m+1,t} - q^i_{m+1,t} + b^i_{m+1,t}/p_{m+1,t} . \tag{24}$$

The existence of active pure strategy N.E. has been proved by Shubik and Yao (1989). The cost or value of liquidity is the consumer value foregone by utilizing a unit of gold for money.

As long as payments are actually made in gold a real cost is paid for liquidity and hence if trade is required at a C.E. outcome this cannot be attained as a N.E. of the market game.

Even with separable utilities intertemporal trade may be required for optimality if endowments vary sufficiently from period to period. A trivial one-nondurable commodity, three-period trading model is useful to focus attention on the modeling and definitional problems concerning credit, futures markets and short sales.

Consider two types of traders with utility functions for three perishable goods:

$$U_1 = 5x_1 + 5x_2 + x_3 \quad \text{and} \quad U_2 = y_1 + y_2 + y_3 , \tag{25}$$

with endowments of $(0, 0, 4)$ and $(1, 3, 0)$, respectively. By inspection a (boundary) C.E. solution is $p_1 = p_2 = p_3 = 1$, final endowments are $(1, 3, 0)$ and $(0, 0, 4)$. The general equilibrium model gives no information about the payment sequence. All that is required is that $\sum_{j=1}^{3} p_j(x^i_j - a^i_j) = 0$, for $i = 1, 2$. But for a game to be able to achieve the equilibrium outcome the trading markets must be specified. If there are m new goods each period and they are durable over all k periods, then there will be in total mk time-dated goods by the last period or, in total, for all periods:

$$A = \sum_{t=1}^{k} mt = mk(mk - 1)/2 \quad \text{goods} . \tag{26}$$

Thus, complete markets call for $A(A - 1)/2$ markets.

If no goods are durable, then the number of markets for a m goods per period is:

$$mk(mk - 1)/2 . \tag{27}$$

The number of spot markets in (27) is $km(m - 1)/2$. Thus, in the simple example with $m = 1$ and $k = 3$ we have three futures markets and no spot markets.

In a complete market model we may ask what is meant precisely by a futures market. Is an exchange of beans now for carrots in six years time a futures trade? Does such an exchange require legal documents and are these documents negotiable and fungible with documents with the same delivery conditions?

It is not useful to be trapped in institutional detail and terminological niceties, but some operational distinctions must be made when there are strategic differences in the market structure.

When we think of trying to play a multiperiod complete market model we are forced to invent financial paper. In the price formation mechanism of (3), where individuals submit quantities, these are physical acts. But you cannot bid now with carrots to be grown six years from now. Carrots are represented by a paper promise to deliver. Furthermore, as soon as a future promise becomes a part of strategy its fulfillment must be guaranteed. But there may be a stage of the game where this is not feasible. A failure to deliver penalty must be specified. As soon as an individual is permitted to sell any item he is not *absolutely* guaranteed to own when delivery is due, a penalty is required.

The distinction between naked and covered futures contracts and short sales can be made on the basis of feasibility. In the exchange model an individual, who by the rules will own 100 tons of wheat in two years time, can sell two-year contracts now and one-year contracts next year to sum to 100 tons, but no more, if it is not certain that he can buy more before the delivery date.

A short sale is a current sale of shares for money together with a promise to deliver the shares at a future, possibly unspecified date. A naked short involves selling shares that are not currently owned. A covered short sale is a sale where the seller already owns the shares to be delivered. The feasibility conditions were aptly phrased by Daniel Drew to whom the following verse is attributed:

He who sells what isn't his'n
Must pay up or go to prison.

If an individual is permitted to sell shares he does not own, he has to either have made a contract to borrow them for current delivery from a third party or the system is permitted to violate conservation. More shares can be sold than exist. If the short contract has an open date, then the calling of the short for delivery becomes part of the lender's strategy set.

The existence of active pure strategy N.E. in a k-period exchange model with complete futures markets (where a future is an exchange of a good now for a contract to deliver a good in the future) follows formally from Amir, Sahi, Shubik and Yao (1987). Existence where there are full futures traded in money follows from Dubey and Shubik (1978b). The more interesting open problem is to be able to characterize the relationship between the amount of a commodity money or credit needed for optimal financing and the configuration and number of futures markets. In business life comments are made about "minimizing or conserving the use of cash". The challenge is to give a precise operational meaning to this statement in the context of a multiperiod strategic market game.

6.7. An outside or central bank

In this and the next section the discussion is once more restricted to the single-period game. A way of running an exchange economy efficiently is by the use of fiat or outside money. Heuristically we may believe that in a mass economy the government's I.O.U. notes can be more generally acceptable than those of a stranger.

Three questions arise. How is the money introduced into the economy? How does it get out? What are the goals and role of the government?

We dispense with the last question first. The government may be regarded as an outside player or referee. Its purpose is to supply money to enable the traders to achieve optimal trade in the markets. If the government actually earns spending power through supplying money, then a convention for spending or disposing of this money must be given and in some sense justified. For example, it might wish to control the direction of growth in the economy.

Two models are noted and motivated. The first can be regarded as apparently nothing more than a reinterpretation of the same mathematical model discussed in Subsection 5.3 for every man his own banker. But the actual play of the game would be different.

6.7.1. The unlimited credit model

The government acts as referee, the single banker, clearinghouse and credit evaluator. Each trader is given a checkbook with bank checks. They can bid any amount they want, but at the end of the game must repay to the government any debt outstanding.[17] Failure to repay triggers a penalty.

In this model there is no limit to the volume of government credit that is issued. There is no charge for the government credit. But we can think of a slightly more complicated game where exogenously the government fixes a money rate of interest and policy as to how it will use its profits.

The game is as follows. The referee fixes $\rho, \mu^1, \mu^2, \dots, \mu^n$ and

[17]To play this as a game it is possibly easiest to computerize the credit system and clearinghouse. An individual enters his name and amounts bid in each market The system calculates prices and informs all players of their final balances. The government enforces the penalty The fundamental modeling distinction between this and every man his own banker involves the concept of trust. But the intuitive idea that we trust the government paper more than individual paper is not modeled at this level of simplicity

$p_1^*, p_2^*, \ldots, p_n^*$. ρ is the rate of interest charged, μ^i is the penalty for a unit of unpaid debt levied against trader i, and p_j^* is the price the referee is willing to pay any player for goods he wishes to give to the referee as payment of debt. A player i attempts to maximize

$$\varphi_i(x_1^i, x_2^i, \ldots, x_m^i) + \mu^i \min\left[0, -(1+\rho)\sum_{j=1}^{m} b_j^i + \sum_{j=1}^{m} p_j q_j^i + \sum_{j=1}^{m} p_j^* s_j^i\right]. \tag{28}$$

A strategy by a trader is as in the two-sided Cournot market model. It is of the form $(b_1^i, q_1^i, \ldots, b_m^i, q_m^i)$, where $0 \le q_j^i \le a_j^i$ for $j = 1, \ldots, m$ and $0 \le b_j^i$, but as he has an open checkbook there is no bound on the sum of b_j^i. After the market prices have been formed and trade has taken place, the remaining part of the strategy of each individual i is that he decides upon a set of goods $(s_1^i, s_2^i, \ldots, s_m^i)$ to sell to the referee. Dubey and Shubik (1983) establish that for $\mu^i > 0$, for n types of traders, it is possible to select ρ and $p^* = (p_1^*, \ldots, p_m^*)$ to point the vector of leftover resources in any direction.

The interpretation of this model is that the referee can use its power to supply the means of payment, as a tax by charging a money rate of interest. The proceeds of the tax can be utilized to remove real resources at the end of the game. Thus, here the rate of interest and disposal prices are exogenous, and are used to control the direction of growth.

6.7.2. *The fixed credit or fiat money model*

A two-stage model with the government supplying a fixed amount of fiat money or credit has been given by Shubik and Wilson (1977) and Dubey and Shubik (1979). The distinction between this game and the previous one is that the relationship between the specific amount of money M in the economy and the severity of the default penalty $\mu^1, \mu^2, \ldots, \mu^n$ is critical in determining the possibility that some players may elect to go bankrupt for strategic reasons. The limit in the quantity of money places a bound on prices, but the bankruptcy penalty connects the shortfall in money to disutility, hence the strategic choice between the utility of an increased purchase versus the disutility of the penalty must be considered.

Figures 5.6(a) and 5.6(b) show the two-stage process. A player i first bids an amount of I.O.U. notes u^i in order to obtain his share of the fiat money M. For the game with a finite number of players one can consider a strategy where as information about the others is available after each player i has bid u^i the bids in the second stage $(b_1^i, \ldots, b_m^i)$ are all functions of the moves in the first stage. A simpler version is where a strategy by i is a vector $(u^i; b_1^i, \ldots, b_m^i)$, where it is assumed that i is uninformed of the actions of others, as Figure

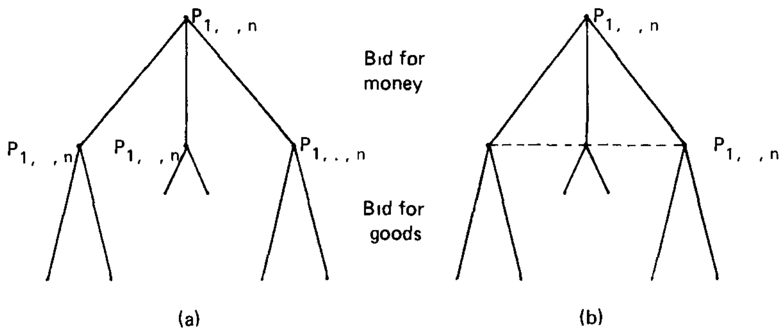

Figure 5 6

5.6(b), but he can bid fractions so that:

$$\sum_{j=1}^{m} b_j^i = s^i = \frac{u^i M}{\sum_{k=1}^{n} u^k} . \tag{29}$$

For simplicity it is assumed that all goods are put up for sale; thus, if the initial endowment of i is $(a_1^i, \ldots, a_m^i)$, then the jth market has $\sum_{i=1}^{n} a_j^i$. The Dubey–Shubik analysis is carried out in terms of a continuum of traders and it is shown that any C.E. can be obtained as a N.E. with an appropriate selection of the M and the default penalty. But if the default penalty is too low for some trader types, they will elect strategic bankruptcy and the money rate of interest, which is defined (in the continuum model) as

$$1 + \rho = \int u/M , \tag{30}$$

will be positive.

6.8. Inside banks

A common way to obtain credit is via a privately owned banking system, but the existence of a private banking system raises a host of new features. A partial list is suggested.

(1) Who owns the banks?
(2) If the banks have shares how are they paid for?
(3) How many banks are needed for efficiency?
(4) What is the goal of the bank?

(5) What determines the lending limit, if any?

(6) What is the strategy set of the bank?

The two fairly natural answers to Who owns the banks? are that either they are individually owned, as was the case with many merchant banks and banking families, or that they are owned by private stockholders. With the former one can argue for utility maximization by a special player. With the latter profit maximization for the stockholders needs to be justified as a goal.

If we adopt the view (confirmed even by casual empiricism) that the important and prevalent manifestation of bank ownership is via shares, then we need to introduce shares into the model and explain how they are distributed.

An underlying theme in banking and the extension of credit is the minimization of the need for individual trust. To a great extent a commodity money such as gold is a hostage which can be held as a substitute for trust. In a society with not enough gold to be an efficient commodity money we can economize on its use by creating a bank whose capital must be paid up in gold (or in fiat money if it exists and is trusted). If the individuals of the society are willing to abide by the rules and if the failure penalties are sufficiently harsh, then the money supply can be expanded by permitting a bank to lend some multiple ($k > 1$) of its capital. This can be stated as a formal rule, but the justification of such a rule appears to lie in some form of risk assessment concerning the trust of banking.

The question of how many banks are needed for sufficient competition is linked to the nature of the strategic variables of the banks and the solution concept considered. It is argued here that the two main candidates for a model of bank strategy are the Cournot model, where the banks use the quantity of credit offered as the strategic variable, or the Bertrand–Edgeworth model, where the rate of interest is the strategic variable. For the first to attain efficiency necessarily requires a large number of banks; for the second it requires only two for an efficient noncooperative equilibrium solution.

With only two banks, even though a noncooperative equilibrium solution might include the competitive equilibria, it is possible that there might be a temptation to communicate directly and to collude.

Shubik (1976) somewhat nonrigorously has considered both the Cournot and Bertrand–Edgeworth models of banking. Figure 5.7 shows a Cournot model for the sale of shares and an interest rate setting model for bank competition. $P_{1,2,c}$ stands for both banks and a continuum of traders moving simultaneously. P_c and $P_{1,2}$ indicate the traders and the banks moving alone, respectively.

Consider an economy where a representative trader of type i has an initial endowment of $(a_1^i, a_2^i, \ldots, a_{m+1}^i)$, where the $m+1$st commodity is gold which serves as a commodity money which is in short supply. Both banks put up all of their shares for sale. These can both be normalized to one. There are at least

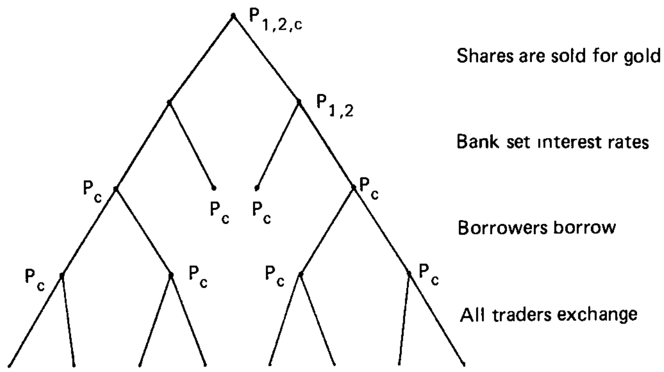

Figure 5.7

two individuals with gold. The referee or government has set two parameters k, the capital to lending ratio or reserve requirements. If capital is u, a bank can lend up to ku. The other parameter is μ, the failure to repay penalty; this could be made individual, $\mu^1, \mu^2, \ldots, \mu^n$, but this refinement does not appear to be needed for the one-stage game. (Although it does influence the range of the price system in equilibrium.) Furthermore, individual penalties, like purely individual taxes, appear to be administratively unwieldy. Historically there has been a split in the bankruptcy law into two sets: personal and corporate. In the model here neither dividends nor bank failure pose a problem as earnings (positive or negative) are flowed through to the stockholders without limited liability.

Let u_k^i be the amount of gold bid by a representative trader of type i for the shares of bank k, where $k = 1, 2$. Thus, traders of type i will obtain:[18]

$$w_k^i = u_k^i \Big/ \sum_{j=1}^{n} u_k^j \,. \tag{31}$$

The banks select rates of interest ρ_1 and ρ_2 at which they are prepared to lend up to ku_1 and ku_2, where u_j is the total capital of bank j.

The second move of a trader i is to ask for a loan of size g^i with the servicing rule that his bid goes to the bank with the lowest rate. If banks have the same rate the bid is split. If bank j has a higher rate than k, its demand is a decreasing function of $\rho_j - \rho_k$.

[18]The notation given is for a finite number of traders n which shows the structure. This can be replaced by n finite types with a continuum of traders.

Loan demand for bank j is:

$$\begin{aligned} &\sum_{i=1}^{n} g^{i}, && \text{for } \rho_j < \rho_k, \\ &\sum_{i=1}^{n} g^{i}/2, && \text{for } \rho_j = \rho_k, \qquad (32) \\ &\max\left\{0, f\left[\left(\sum_{i=1}^{n} g^{i} - ku_k\right), (\rho_j - \rho_k)\right]\right\}, && \text{for } \rho_j > \rho_k. \end{aligned}$$

There are many rationing mechanisms for oligopolistic demand [see Shubik (1959)], but the key driving force in obtaining a pure strategy equilibrium is the requirement that if there are t banks (here there are two), then at equilibrium:

$$k \sum_{i \neq j}^{t} u_i \geq \sum_{k=1}^{n} g^{k}, \quad \text{for } j = 1, \ldots, t.$$

This states that the loan capacity for any $t - 1$ banks is enough to satisfy all demand.

An individual i obtains the amount of money:

$$v^{i} = a^{i}_{m+1} - \sum_{j=1}^{2} u^{i}_{j} + g^{i}, \quad \text{if (33) holds}. \qquad (34)$$

In the third move of all traders they each bid and offer in all markets. Trader i bids $(b^i_1, q^i_1, \ldots, b^i_m, q^i_m)$, where

$$\sum_{j=1}^{m} b^{i}_{j} \leq v^{i} \quad \text{and} \quad b^{i}_{j} \geq 0. \qquad (35)$$

At the end of the game all accounts are settled. If there is no bankruptcy, then the banks will liquidate, pay out profits and return capital to the investors. If the bank reserve money were fiat, that is enough to construct a game with a large enough reserve rate k to have any C.E. attainable as a N.E. If the reserve money is a commodity which enters the utility function, then one can approach efficiency as k becomes large, but not quite attain it. This has not yet been proved rigorously. The heuristic argument is if the reserve money has consumption value (say as jewelry, for example), then there is a utility loss while it is sterilized as bank reserves. This is the cost of liquidity as discussed in Subsection 6.6 in the model with monetary gold and jewelry.

6.8.1. *Comment on the uses of a central bank with private banks*

In the one-period model it is easy to build either an inside or outside banking system to finance trade with a zero money rate of interest. At the level of abstraction of general equilibrium theory there is no need to discuss the finer distinctions among the existence of markets, short sales, futures contracts, fiat money or bank money. In a multistage economy with differentiated strategic players a government will use its central bank for at least two purposes: control of the economy and to aid the efficient financing of trade. In general, even with control there will be a need to vary the money supply. This can be done by a central planning agency or via a set of private banks controlled by a few special weapons in control of the central bank. Two natural candidates are a reserve ratio and a reserve money rate of interest.

6.8.2. *Comment on the costs of banking*

An economic reason for having private banking is that the administrative, informational and other costs are such that the private system is more efficient than the central agency. Political and bureaucratic reasons concern the dilution of power.

The role of the nature of the actual costs of banking is central to these considerations. Part of conventional wisdom might argue for considerable increasing returns to scale in banking, like in a telephone network. Another temptation is to assume, as was done implicitly above, that banking is virtually costless. It has already been noted that the literature and annual reports indicate otherwise.[19]

7. Further directions

The discussion above has concentrated on the game theoretic and gaming applications applied primarily to the one-period exchange economy with and without the need for credit. This is only a start in the investigation of financial instruments and institutions designed to deal with long- and short-term lending, futures, insurance and other factors depending explicitly on the length of production, incomplete markets, transactions costs and exogenous uncertainty.

[19]See, for example, the proceedings of the annual conference on Bank Structure and Competition of the Federal Reserve Bank of Chicago started in 1963 Also see the annual reports of any major bank for a cost breakdown.

There are only a few papers on game theoretic applications of strategic market games to these broader problems. A brief commentary, together with some observations on some open problems, closes this survey.

7.1. Production

Dubey and Shubik (1978b) have constructed a strategic market game with a finite number of traders and firms with three stages: trade by all, production, and then trade by all. The existence of the N.E. is proved together with the convergence of the type symmetric noncooperative equilibria to the C.E. Problems with financing are avoided, shares are not actively traded, and it is assumed that firms maximize profits. In a separate paper Dubey and Shubik (1980a) examine the conditions required to have managers maximize profits as a sequence of finite economies approach that with a continuum. Intuitively a manager who owns a substantial part of a firm and uses its product could have three influences on his payoffs if he has market power, they are the substitution effect, the consumer income effect and the owner income effect. All three must attenuate.

In spite of these formal results and especially for the development of a theory of money and financial institutions there are two gaps of importance. There is no satisfactory model of stock voting and stock trading. Unless there is adequate stockholders protection [Shubik (1984a, pp. 574–577)] with voting stock there may easily be no equilibrium price system.[20] The premium paid for control in a takeover, leveraged buyout or tender offer may appear as disequilibrium behavior [see Herman and Lowenstein (1986)] but may be premia in a market where, because of inadequate rules, no pure strategy price equilibrium exists.

As was even indicated in Boehm-Bawerk (1923) and Wicksell (1935) in the development of a theory of capital, the length of production is of considerable importance. It is the length of production which causes much of the need for long-term finance and the length of the time of trade or exchange that causes much of the need for short-term finance. Given a structure of production through time one may wish to ask the analogous question: What is enough money to finance exchange in the one-period, one-stage model? That is, what constitutes enough money to finance multistage, multiperiod exchange and production? This question is also linked to concepts of liquidity and is not yet defined in a satisfactory manner.

[20]This can be seen by trying to model the cooperative game and showing that it has no core, unless the appropriate minority protection exists

7.2. *Transactions costs*

7.2.1. *A comment on exogenous and endogenous uncertainty*

It is suggested that if there were absolutely no transactions costs, then there could be complete markets costlessly constructed; hence, exogenous uncertainty could be wiped out by completing markets in contingent trades. If, on the other hand, there is no exogenous uncertainty present but there are transactions costs, if the costs are high enough we may expect aggregate mechanisms to appear and markets to be endogenously incomplete. The uncertainty will be created. This suggests that even without exogenous uncertainty a theory of money and financial institutions is called for to account for the influence of strategic behavior and transactions costs.

An open question is how to characterize conditions where limited markets are superior to complete markets when transactions costs are present. A possible approach is to start with complete markets [Amir, Sahi, Shubik and Yao (1987)], add transactions costs [Rogawski and Shubik (1986)] and then attempt to examine the class of alternative incomplete market games.

7.2.2. *On bankers*

One simple manifestation of transactions costs is the emergence of brokers and other middlemen. These can be modeled directly. Nti and Shubik (1984) have considered exchange where the exchange must be carried through broker–dealers. A dealer is an individual who buys to resell, a broker is not a principal agent. He acts as agent or middleman serving both buyer and seller for a fee or share of the sale. The existence of equilibria and the convergence to zero profit of competitive dealers is established.

A natural extension of this model, which has not yet been carried out, is indicated as in Figure 5.8. Here the buyers and sellers have the option of

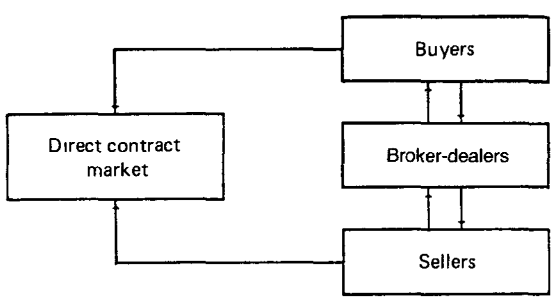

Figure 5.8

trading via brokers and dealers and electing a search mechanism [like that of Gale (1986a, 1986b), for instance], where, however, each encounter has a cost.

7.3. *Bankruptcy and insolvency: Complete and incomplete markets*

There are four major problems in the modeling of bankruptcy. They can be characterized by (1) the bankruptcy rule for a single-period game with a money and no exogenous uncertainty; (2) the rule for a single-period game with exogenous uncertainty; (3) the rule for bankruptcy, reorganization and insolvency in finite multiperiod game; and (4) the modeling of bankruptcy in infinite horizon models.

The first instance which arises from the need to bound strategic behavior enables us to define an optimal penalty as one which is sufficient to act as a disincentive to strategic default. It does not pay to elect to default. The discussion and analysis in Shubik and Wilson (1977) and Dubey and Shubik (1979) covers this.

When there is exogenous uncertainty without a market for every contingency new problems appear, when there is a random element to the payoffs an extremely harsh penalty may be sufficient to stop all borrowing, and a low enough penalty might stop all lending. A contemplation of inventory theory suggests that under virtually any criterion without infinite penalties an optimal policy should have a positive number of expected bankruptcies much of the time, and a game with an intermediate penalty should have equilibria which dominate the equilibria in games with extreme penalties. Dubey, Geanakoplos and Shubik (1989) have been able to establish this.

The multistage model introduces completely new considerations. At this point the economic welfare of the creditors as well as the fate of the debtors becomes relevant. In an ongoing economy an insolvent debtor may be given time to liquidate enough assets to pay his debts. A bankrupt may be reorganized or totally liquidated. An individual may have assets seized or his income garnished. In modeling these phenomena there is a temptation to become lost in detail, or alternatively to oversimplify by devices such as having simple extremely harsh penalties. No satisfactory treatment of this appears to have been given.

Another open problem is the treatment of bankruptcy in the infinite horizon model. The lack of a specific end together with the possibility for the rollover of loans opens up the opportunities for Ponzi games[21] and attempts to cover up inability to repay by increased borrowing.

[21]Named in honor of a great Boston swindler, Mr Ponzi, who specialized in borrowing from B to pay A

7.4. Incomplete markets and information

Dubey and Shubik (1977a, 1977b) considered games with an exogenous random move by Nature. Before all individuals move Nature selects one of k states. The traders may have any structure of information concerning the state selected by Nature.

If there are m commodities (other than money) and k states of Nature, then there may be as few as m or as many as mk trading posts depending upon the availability of contingent good markets.

A strategy by a trader consists of a set of moves (one in each market) contingent upon the information he has available prior to having to move.

It is proved that an active N.E. always exists for this game. Furthermore, if all are uninformed and all mk markets exist, then for many traders the C.E. of the Arrow–Debreu market with uncertainty will be approached by N.E.[22]

Regardless of the information conditions a price system always exists which reflects the lack of symmetry of information in the markets. It is also shown by example that more refined futures markets are not necessarily beneficial to all when traders have different information. There are instances where those who are less well informed prefer crude futures markets as it becomes more difficult to sell them worthless paper when there are more markets.

The strategic market game approach to nonsymmetric information is related to the work of Radner (1968, 1979).

A particularly difficult problem to consider is the sale of information and the meaning of the sale of information. A key difficulty in constructing market models for the sale of information comes in the description of the ownership and reproducibility of information viewed as an item of commerce. Shubik (1984b) has constructed a game where one set of players can sell refinements of information sets to others. Equilibrium may be destroyed by the possible sale of information. All may buy or none may buy. An example shows that the results depend on costs. Two problems emerge. The first concerns the rules of dissemination and appropriation and these are dealt with. The second is more fundamental: Is it information or interpretation and analysis that is sold? This has not been dealt with.

Dubey, Geanakoplos and Shubik (1987) have reconsidered the concept of rational expectations in the context of a strategic market game. The type of paradox suggested by Grossman and Stiglitz (1980) is not encountered. There are three basic problems which are encountered with the concept of rational expectations which can be misleading. (1) The idea that the uninformed can figure out what the price will be before they have moved. (2) The idea that the

[22]A key element is that all individuals must move simultaneously before price is formed as it is their moves which form price.

informed cannot profit from their information. And (3) the implicit assumption that there are enough participants that the informed will be unable to conceal usefully their knowledge.

It is the actions of individuals which form price; hence, in a simultaneous move game the price which ex post reveals to the uninformed what the informed know is *not* known ex ante. As soon as price formation is modeled as a well-defined mechanism of the game, no paradoxes appear. When there is a continuum of traders ex post the uninformed may now be informed but it is too late, the informed have made their profit. If there are situations where the continuum of agents hypothesis is unreasonable (such as assuming a continuum of central banks, commercial banks and insurance companies), then it is shown by example with the presence of large players their behavior at a N.E. need not reveal their information.

Strategic market games have also been employed by Peck and Shell (1985, 1986) [see also Cass and Shell (1983)] to investigate "sunspot equilibria" where it can be established that an event essentially exogenous to the economy can be used as a correlating device for a set of N.E. strategies.

An aside on insurance and competition

An important open problem in the study of risk, competition and insurance which is naturally formulated as a strategic market game is the tradeoff between the law of large numbers helpful in the providing of insurance and the number of insurance companies where the larger the number of firms, the more likely there will be competition. There is a tradeoff between the law of large numbers and the oligopoly effect which should produce an optimum number of insurance firms. This leaves aside the cooperative possibilities of reinsurance.

7.5. Dynamics

The general attitude adopted here is that except for one-person nonconstant sum games and two-person antagonistic games played repeatedly, the selection of the solution concept and the interpretation of the solution to a dynamic game should be regarded with great circumspection. Even when the markets are thick (modeled by a continuum of agents) the extreme simplicity of the models calls for care in the interpretation of results.

Shubik and Thompson (1959) in a dynamic one-person model have analyzed a *game of economic survival* which illustrates the potential differences in motivation which can arise when a firm must trade off the possibilities of bankruptcy against its ability to pay dividends in an uncertain environment. In essence the firm is regarded as having two accounts; a corporate account, which if depleted it is forced into bankruptcy, and a bank account paid to the owner. The latter represents the accrual of dividends. The maximization of the discounted expected value of the stream of dividends can be contrasted with the minimization of the probability of being ruined in a specified time. There is a close relationship between the policy for the maximization of expected discounted income and the avoidance of bankruptcy and an optimal inventory policy avoiding stockout penalties.[23]

An attempt to extend the analysis to two firms immediately runs into conceptual difficulties concerning the role of threat. When there are only few large players neither normative nor behavioral criteria currently suggested appear to be totally satisfying.

Shubik and Whitt (1973) have examined the simplest of strategic market games with an infinite horizon. Suppose that two individuals initially have $1-\alpha-\gamma$ and $\alpha+\gamma$ units of paper money. Each period one unit of the (sole) consumer good is put up for sale. In period t the individuals bid x_t and y_t, respectively, and obtain $x_t/(x_t+y_t)$ and $y_t/(x_t+y_t)$, or zero if $x_t=y_t=0$.

The ownership claims of the players to the income received by the market are $1-\alpha$ and α, respectively, where $0<\alpha<1$, thus:

$$w_{t+1}=w_t-x_t+(1-\alpha)(x_t+y_t)\,,\quad \text{where } w_1=1-\alpha-\gamma\,, \tag{36}$$

and $0\le x_t\le w_t$.

Each player i wishes to maximize

$$U_i=\sum_{t=1}^{\infty}\beta_i^{t-1}u_{it}\,,\quad \text{where } u_{1t}=x_t/(x_t+y_t)\,. \tag{37}$$

It is shown that for $\beta_1=\beta_2<1$ a stationary state[24] is eventually reached with $u_1=1-\alpha$ and $u_2=\alpha$.

Define

$$c=\frac{1-\alpha-\gamma}{1-\alpha}\,.$$

Let $V_n(x,\gamma)$ represent the excess utility obtained by an individual starting with γ more money than his real good income in a game lasting n periods. It

[23]Further work on games of economic survival has been done by Miyasawa (1962) and Borch (1966). More recently Radner and Duffie have considered related problems and were able to extend the limited model of Shubik and Thompson considering continuous time.

[24]With $\beta_1>\beta_2$ one may lose uniqueness [see Shubik and Whitt (1973)]

was shown that if $\beta_1 = 1$, then

$$V_n(p, \gamma) = n(1-p)(1-c^{1/n})$$

and

$$\lim V_n(x, \alpha) = -(1-\alpha)\log\left(1 - \frac{\gamma}{1-\alpha}\right).$$

The equilibrium analysis immediately extends to concave utility functions φ_i and to a continuum of traders. In this instance the money advantage ends after one period as all immediately spend all.

In his Ph.D. thesis, Housman (1983) has extended this model to include inside borrowing and lending in a money market. An endogenous rate of interest is obtained. There is an open credit market before bids are made. The market is the same type as the commodity market. I.O.U. notes are traded for money. In the Shubik–Whitt model with a finite number of players, hoarding could take place. With a continuum the advantage of extra money is cashed in immediately. With a loan market it is converted into an income stream via a rate of interest.

A major challenge is to extend these models to include not merely borrowing and lending, but also exogenous uncertainty. An important conceptual problem emerges concerning the relationship among the uses of markets and fiat money, banks and insurance companies.

One can ask specifically: How close a substitute for insurance is the use of fiat money and markets? Then, how much closer is lending for a specific number of periods? These problems are related to the work of Bewley (1980) and Friedman (1969). In a recent paper Shubik (1986a) has formulated this problem as a set of stochastic parallel dynamic programs, generalizing the Shubik–Whitt model. The basic model and observations are as follows. Suppose that there are many individuals who every period will have a claim to some of the goods being sold in the market. The claim of each individual is determined by the draw from an independent random variable. Each individual begins with an amount of fiat money and bids before he knows his income. Price is determined as in (1) and each obtains his income. The adding together of all of the goods offered for sale enables the market on the supply side to take advantage of the law of large numbers. No protection, however, is offered for the variability in the individual's income. Because the game is repeated, if there were no banking the individuals could achieve an extra level of self-insurance by utilizing the opportunities for a serial law of large numbers on their income. It is conjectured that as the number of individuals becomes large, each will be able to follow a stationary policy described by the solution to a set of one-person dynamic programs. Much of the money will be in hoard providing insurance. A distribution of wealth which over time maps into itself is expected.

Insurance would immediately supply the use of the cross-sectional law of large numbers for income and would be more efficient than hoarding. This assumes that insurance is a mechanism for public service and not a monopoly.

The introduction of bank loans poses several basic problems in modeling. A well-defined but not totally satisfactory model is that there is a single bank mechanism which accepts deposits and lends. It has an upper bound on debt to any individual and once that level is reached it garnishes all future income until fully paid back. The ability of the bank to provide insurance via deposits and loans can then be studied as a function of two parameters: ρ, the money rate of interest, and D, the upper bound on the amount of debt any individual can attain.

8. On minimal financial institutions

It is suggested here that as money and financial institutions serve both to facilitate trade and provide a control mechanism over the economy, a game theoretic approach is natural to many of the basic problems.

Much of microeconomic theory, including general equilibrium theory, is essentially oriented towards the study of short-term conscious optimization. Macroeconomics and development theory are aimed at a lower horizon and with a viewpoint that is more behavioral rather than strict optimization.

Money and financial institutions provide an important part of the mechanisms and carriers of process which tie in and help embed the economy, with its everyday short-term optimizing forces, into the polity with its concerns for control and the society which provides considerable special structure through custom, law and habit which delineates and constrains economic behavior.

As much as policymakers might like an all encompassing theory of economic dynamics, we do not have one. Furthermore [including structures such as that of Keynes (1936)] no grand theory of economic dynamics that has validity for more than a few years exists, or is likely to exist. Any theory of dynamics which purports to cover more than a few years is socio-economic and politico-economic. It depends heavily on its cultural and political environment. There is, however, a middle ground between the preinstitutional statics of general equilibrium and the grand dynamics of a Marx or Keynes. That middle ground involves the study of the logical and technological properties of the mechanisms required to connect economic life into its political and social environment. Because I believe that it is extremely important to avoid being prematurely trapped in institutional detail, the stress here is upon "unrealistic games", where the mechanisms for price formation, lending, borrowing, banking, insurance, brokering and dealing are minimal.

The advantages of simple playable games are even more than previously stated. There are at least four. The playability criterion provides a simple check for completeness and consistency and comprehensibility. The fact that the game is playable provides an opportunity for experimentation. The stress on simplicity maximizes the chances for analysis. Last, but definitely not least, is the feature that the simplicity of the mechanisms and the games is such that they are not easily mistaken for institutional reality. They avoid the air of apparent relevance to policy of the day.

The economic theorist qua theorist is not a policy advisor. His role is to further basic understanding and to be able to sort out the essence of economic problems and institutions from the relatively ephemeral trappings of current economic problems in a particular polity in a particular society. It is for this reason that the approach to the development of the theory of money and financial institutions is regarded as an exercise in mathematical institutional economics. Furthermore, the role of game theory is suggested as the natural tool to stress the strategic structure of any financial control system. Finally, strategic market games appear to offer the appropriate modeling methods for the development and the exploration of minimal financial institutions using some of the results of general equilibrium analysis as a reference point and a base from which one advances to a more structured understanding of process.

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