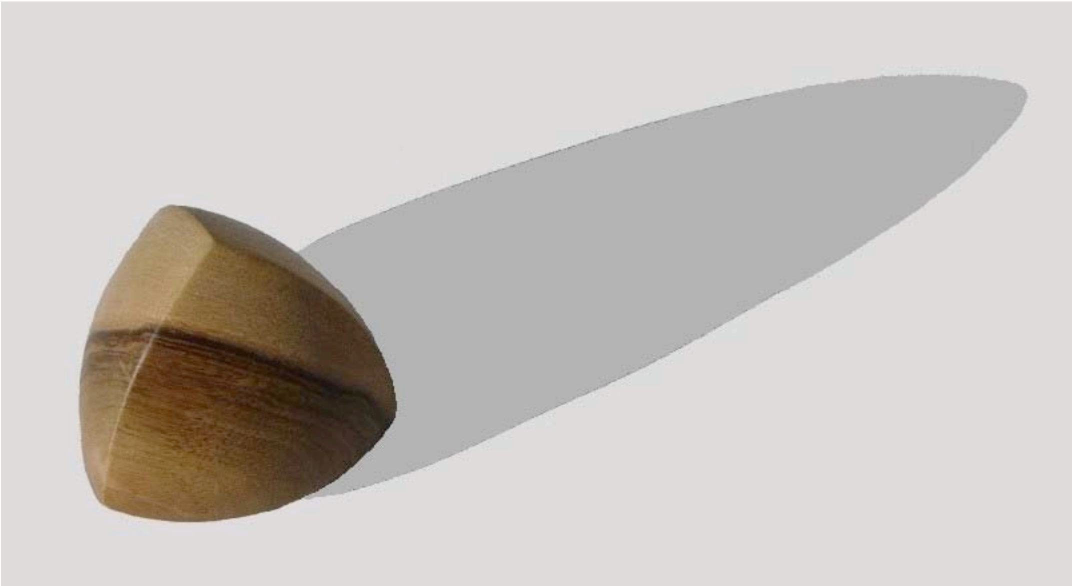
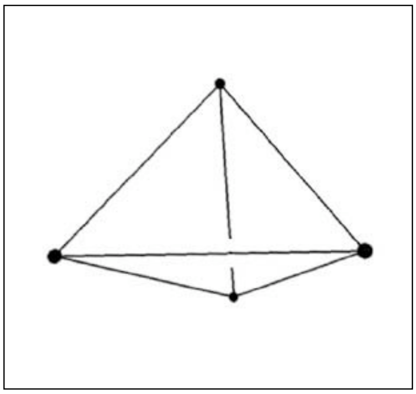
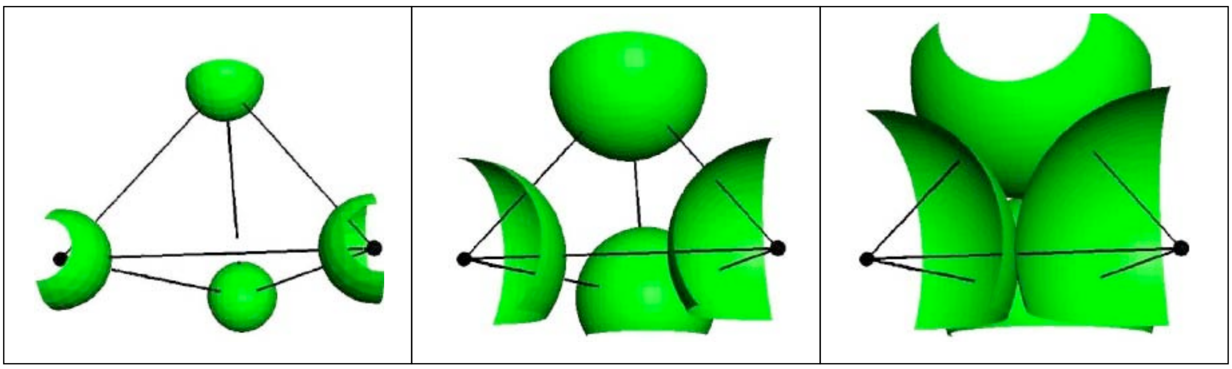
What does this solid have to do with a ball?



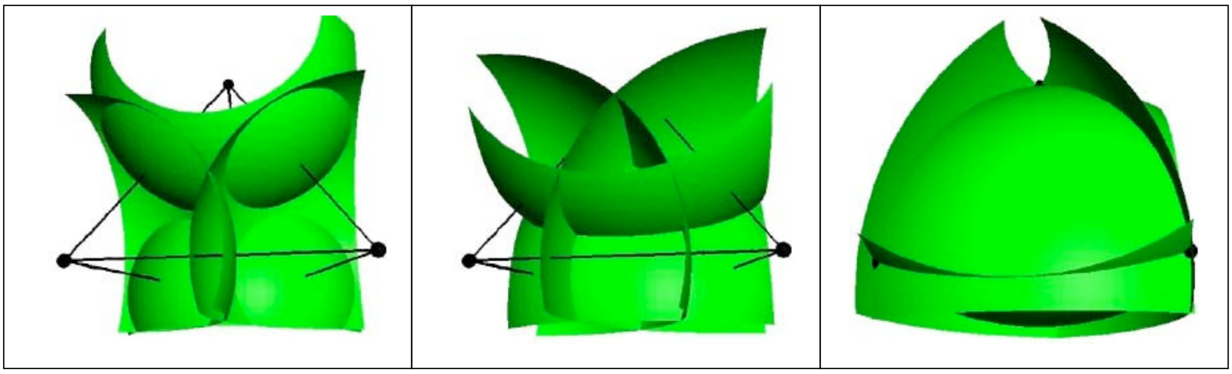
When four points in space are arranged such that they are equidistant from each other, they form the vertices of a *tetrahedron*:



Spheres of equal size are placed at each of the four tetrahedron vertices. They expand at the same rate until they touch each other:

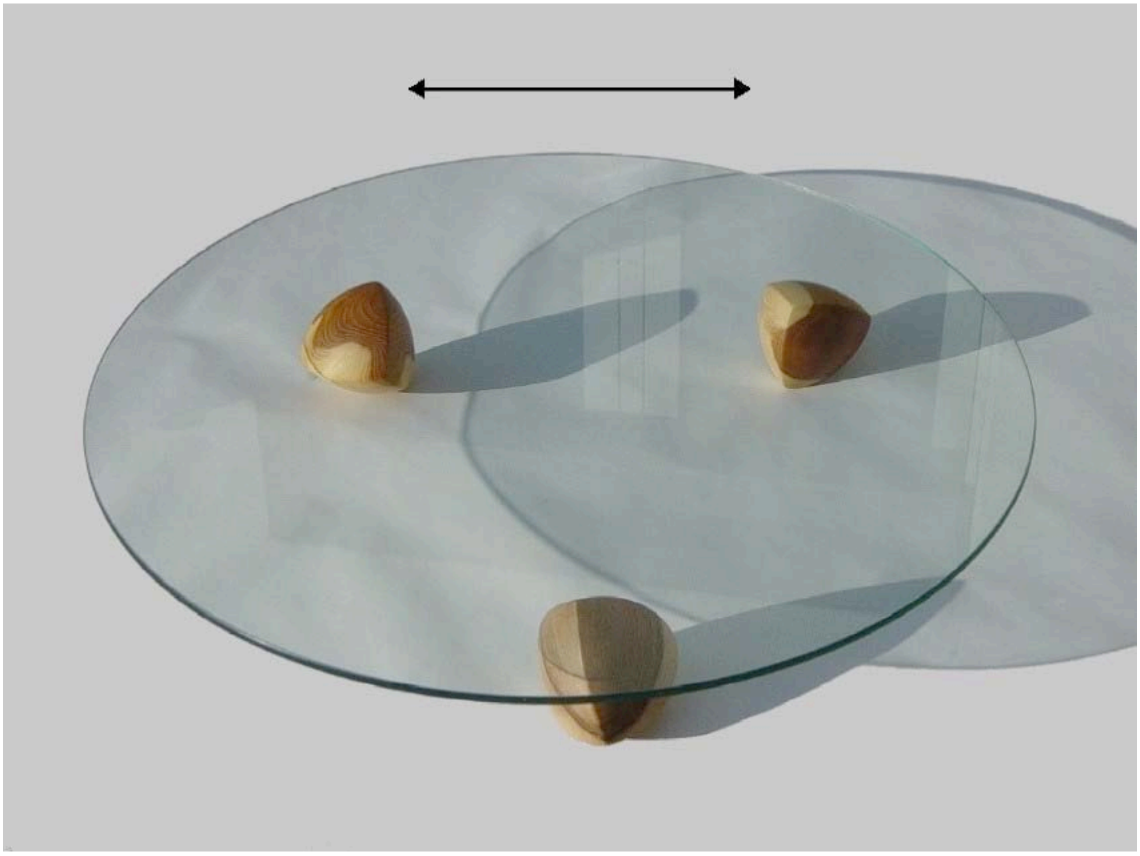


To continue expanding, the surfaces of the spheres must penetrate each other:



In the illustration on the right, the surface of each sphere intersects the three tetrahedron vertices opposite the centre of the sphere.

The body bounded in this way looks like a bulging tetrahedron (see the top illustration on page 1). It is called a *Reuleaux tetrahedron* and has the remarkable property of being of almost *constant width*:



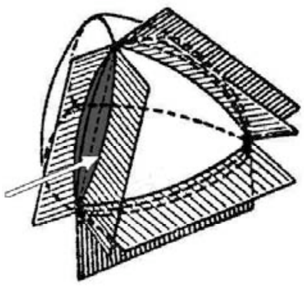
If a flat plate is placed on three equal Reuleaux tetrahedrons and pushed back and forth, it moves almost parallel to the surface of the table without 'wobbling'—as if the glass plate were rolling on three *spherical balls* of equal size!

Details

· A three-dimensional convex body is of *constant width* if it has the following property: Whichever way it is clamped between two parallel plates, both plates are always at exactly the same distance from each other. Bodies of this kind are therefore usually called *bodies of constant width* (cf. [2], [4], [5]), or sometimes also bodies of *constant breadth* ([1], [7]) or *spheroforms* ([1]).

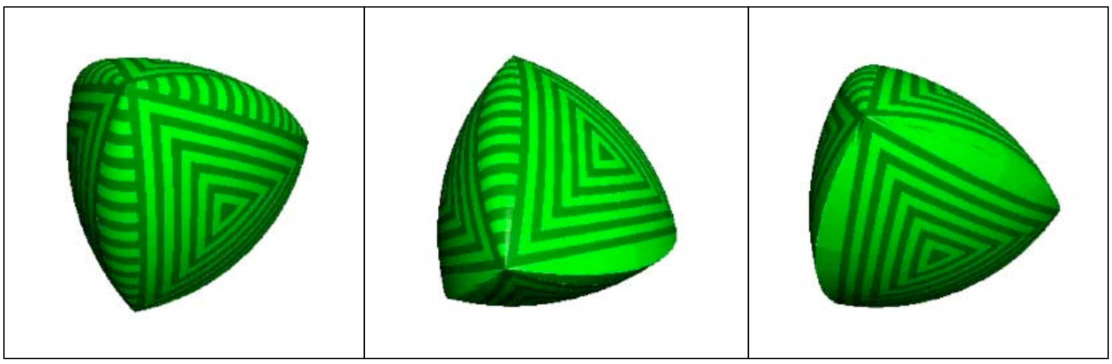
· A Reuleaux tetrahedron is only of *almost* constant width. Mostly the body touches two plates between which it is clamped with one of its vertices and one point on the opposite surface of the solid. In such cases, the two plates are at the same distance from each other due to construction. However, the two possible points of contact may be on two opposite edges of the solid. Here the width is greater: For two edge midpoints it is maximally $\sqrt{3}-\frac{\sqrt{2}}{2}\approx 1.0249$ times larger than the smallest width (cf. [11] and [12]).

· The Reuleaux tetrahedron can be made into a body of constant width by *rounding three of its edges*: To do this, first remove the part of the surface of this body which is located between the extensions of two adjacent lateral surfaces of the tetrahedron [shaded grey]. Then, in the space thus created, insert a portion of new, spindle-shaped surface generated by rotating the circular arc in which the Reuleaux tetrahedron intersects the extensions of the lateral surfaces of the tetrahedron [marked with an arrow] around the corresponding edge of the tetrahedron [broken line] (illustration from [5]).



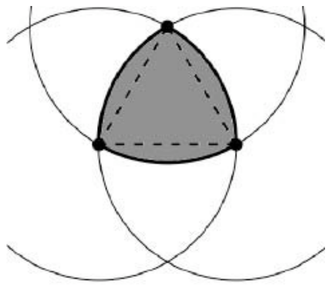
When three edges of a Reuleaux tetrahedron that meet at a *common vertex* are rounded as described, the resultant solid is a body of constant width. This is also known as a *Meissner body*.

The following frames show this Meissner body from above, from the side and from below:

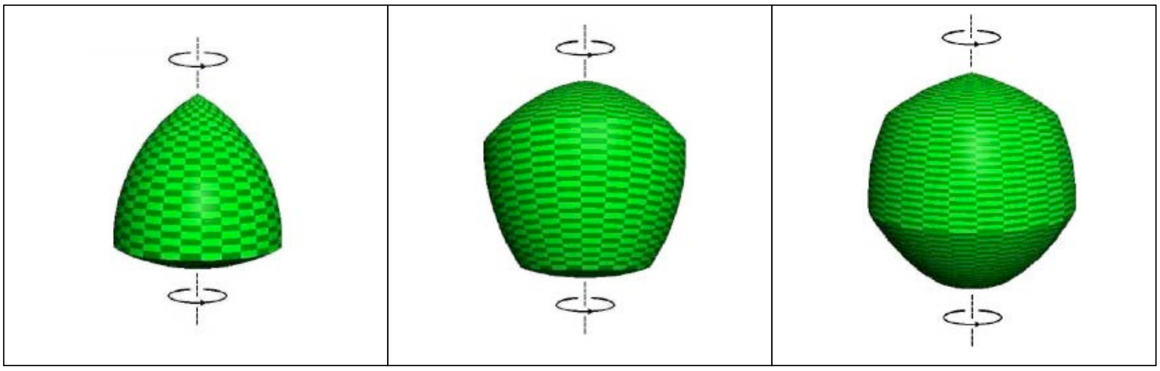


If instead the edges surrounding *one lateral surface* of a Reuleaux tetrahedron are rounded as described, a second type of a Meissner body results (cf. [1], [2], [3], [4], [6], [9] and [12]).

· Franz Reuleaux (1829–1905) was a German engineer and developed a school of mechanical engineering based on mathematical methodology. He was a professor at the ETH Zurich (Swiss Federal Institute of Technology) from 1856 to 1864 before serving at the Berlin Trade Academy until 1896. In his book *The Kinematics* he demonstrated that convex planar figures other than circles can also be of constant width. The most well-known of these figures is the equilateral triangle in which each vertex is the centre of an arc described between the two opposite vertices. The *Reuleaux triangle* is used in engineering due to its constant width, for instance in the Wankel engine. (cf. [3], [4], [7], [8] and [10])



· Convex bodies of constant width can also be *axisymmetric*. Thus a Reuleaux triangle rotated around one of the axes of symmetry is of constant width, as are all Reuleaux pentagons, heptagons, etc. There is therefore an *infinite number* of convex bodies with the same constant width (cf. [2], [3], [4] and [9]):



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