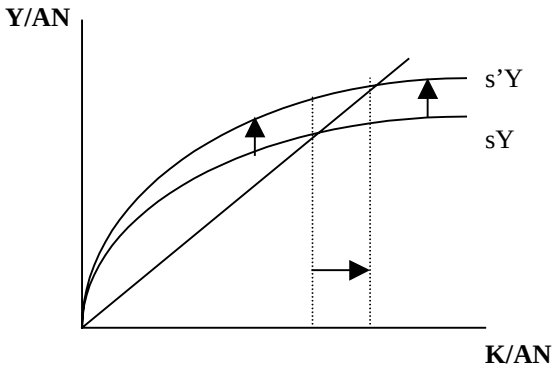
PROBLEM SET SEVEN SOLUTIONS

PROBLEM 1

a. Solow Growth model where the savings rate increases.

Conclusion: increase in steady-state output and capital (per effective worker), but has no effect on the long-run growth rate of the economy. This is really right out of the book. Graphics shown below.



b. New production function.

$Y = AK^{0.5} = K^{a+0.5}$

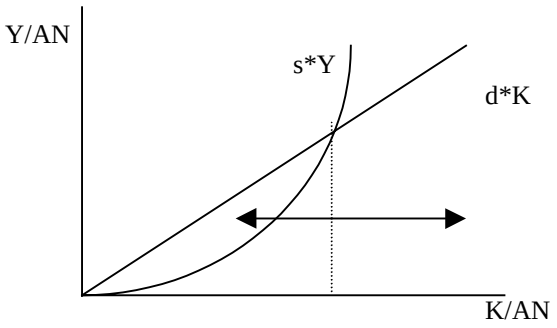
$dK/dt = sY-dK$

Steady state is where $dK/dt = 0$ or where $K^* = (d/s)^{1/(a-0.5)}$ and $Y^* = (K^*)^{a+0.5} = (d/s)^{(a+0.5)/(a-0.5)}$

Check the convexity/concavity of the savings function (sY) with respect to K.

$d(sY)/dK = s(a+0.5)K^{a-0,5}$
$d^2(sY)/dK^2 = s(a+0.5)(a-0.5)K^{a-1.5} > 0$ iff $a > 0.5$ (here function is convex)
$d^2(sY)/dK^2 = s(a+0.5)(a-0.5)K^{a-1.5} < 0$ iff $a < 0.5$ (here function is concave)

When $a > 0.5$ the savings function is convex and the steady-state is locally unstable. If the initial endowment of capital is above the steady-state, the economy will grow forever. Looking at the capital accumulation equation, capital accumulates whenever saving (sY) is larger than depreciation (dK), or whenever the saving curve lies above the depreciation curve. This is illustrated below.



On the other hand, if initial capital is below the steady-state, the economy will shrink to nothing. Dynamics are illustrated by the arrows.

c. Convergence

The idea of convergence is motivated by a concave savings function. The dynamics of the steady-state are stable, so countries with similar savings rates, technology growth rates, and depreciation and rates will converge to the same level of per capita income over time.

When the production function is convex, no country with an initial endowment above the steady-state will ever converge to any level of output per worker, and grow forever. In this sense there will not be convergence in the levels of per capital income over time.

d. Capital and Technology

New computers (physical capital) may make manufacturing processes more efficient, but consultants could do the same thing by better managing the flow of materials to reduce inventory and down time. The consultants here are really human capital.

Problem 2.

a. Social Security is currently financed as a "pay-as-you-go system". In other words, the government takes tax revenues from current workers and uses them to pay benefits for currently retired workers. Most private pension plans take contributions from employees (and employers) and invest those funds in stocks and bonds until the employee retires. Social Security is not fully funded in the sense that current retirees are receiving benefits out of the contributions of younger people. The quick aging of the population implies that the ratio of retirees/young worker is increasing over time, meaning either taxes will have to increase or benefits will have to be cut in order for the program to continue its existence. Doing nothing means the program will be bankrupt in the near future.

b. Raising benefits implies lifetime wealth has increased for everyone. PIH says consumption depends on lifetime wealth, so people increase consumption (reducing their current saving rate). Essentially, the government is now saving for them, implying there is less need to save for retirement. The IS curve shifts to the right, and in the short-run output and interest rates increase. As this is a demand shock, we know as long as output is above its natural level, inflation will accelerate. The budget deficit will increase, though by less in the short-run as tax revenues increase with output, and the savings rate of young people has decreased.

c. Higher taxes offset the increase in lifetime wealth. If there is no change in overall wealth, there is no change in consumption, and savings falls one for one with the increase in taxes, simply substituting taxes for private saving. Note for young people, the tax effect will be stronger than the benefit effect if the older generation is larger (as is the case here), and lifetime resources

actually decrease. The opposite is true for the older generation. It is plausible that the IS curve does not shift at all (if the taxes are simply a transfer from young to old), so there are no dynamics on output, inflation, or interest rates. Savings rates are still lower, however, as private saving has been displaced by taxes.