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Multireservoir system optimization in the Han River basin using multi-objective genetic algorithms

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Abstract:

In this study, NSGA-II is applied to multireservoir system optimization. Here, a four-dimensional multireservoir system in the Han River basin was formulated. Two objective functions and three cases having different constraint conditions are used to achieve nondominated solutions. NSGA-II effectively determines these solutions without being subject to any user-defined penalty function, as it is applied to a multireservoir system optimization having a number of constraints (here, 246), multi-objectives, and infeasible initial solutions. Most research by multi-objective genetic algorithms only reveals a trade-off in the objective function space present, and thus the decision maker must reanalyse this trade-off relationship in order to obtain information on the decision variable. Contrastingly, this study suggests a method for identifying the best solutions among the nondominated ones by analysing the relation between objective function values and decision variables. Our conclusions demonstrated that NSGA-II performs well in multireservoir system optimization having multi-objectives. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS multireservoir system optimization; multi-objectives genetic algorithms; NSGA-II; Han River basin

INTRODUCTION

Multireservoir system optimization has been quite extensively studied over the last few decades (Yeh, 1985; Labadie, 2004). However, there has not been consensus on the best method to achieve a global optimal solution. This is because: (1) considering the relation among reservoirs that consist of a multireservoir system is difficult; (2) a decision variable which is a release in multireservoir system optimization varies within a wide range; thus, the search space, the set of all possible solutions to a problem, is also correspondingly wide; (3) the objectives of multireservoir system optimization are usually multiple in nature. In addition to these difficulties, making a correct prediction of inflow is almost impossible to do because inflow results from precipitation, one of the most unpredictable of natural phenomena.

Multireservoir system optimization can be categorized as both stochastic and deterministic according to the method of computing inflow. The first approach treats inflow as an unknown parameter, but the second computes inflow as a known parameter. The main objective of the stochastic approach focuses on both evaluating a multireservoir system through changes of inflow and developing a reservoir operating rule. The deterministic approach, however, mainly takes account of how well the optimization technique performs. Therefore, the deterministic approach has been applied in this study to evaluate the performance of multi-objective genetic algorithms (MOGAs).

Even though the deterministic approach is used and inflow is a known parameter, multireservoir system optimization remains difficult to attain because of the three difficulties mentioned above. In this study, MOGAs, which are types of genetic algorithm (GA), are applied to overcome these obstacles. MOGAs can efficiently

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explore and exploit the search space by doing more than just identifying local optimal solutions, and a variable in MOGAs can be easily encoded into an integer, a real number, or a user-defined symbol. Above all, MOGAs represent the proper method for optimizing multi-objective problems because they use a set of solutions, called a population. MOGAs are competent optimization techniques and, therefore, make it relatively easy to establish a trade-off curve. In contrast, most traditional optimization techniques can achieve only one solution set in a single run; thus, many simulations have to be performed to obtain the trade-off curve. Goldberg and Kuo (1987) used the 'population-by-population approach' for GAs and the 'point-by-point method' for traditional optimization techniques. This population-by-population approach is the most important characteristic of MOGAs compared with traditional optimization techniques.

The objective of this study is to demonstrate the performance of the NSGA-II, developed by Deb *et al.* (2002), as it is applied to multireservoir system optimization. Three cases are applied, each having different constraints regarding storage, water supply demand, and a specified water level by the end of the month in question. Through these cases, we suggest a method of analysing both a trade-off curve and decision variables.

PAST RESEARCH ON GAS AND MOGAS IN WATER RESOURCES ENGINEERING

GAs are applied to many water resource optimization problems, such as groundwater management problems (Smalley *et al.*, 2000; Prasad and Rastogi, 2001), water distribution and irrigation problems (Savic and Walters, 1997; Montesinos *et al.*, 1999), the calibration of rainfall-runoff models (Liong *et al.*, 1995), and the short-term scheduling of hydrothermal systems (Wu *et al.*, 2000).

In reservoir operating optimization, only a few studies have been published using GAs. Esat and Hall (1994) demonstrated the utility of GAs through their application to the 'four-reservoir problem'. They compared GAs with discrete differential dynamic programming (DDDP) to show that the requirements for both the computer time and memory of GAs increased linearly, whereas those of DDDP increased exponentially, and they concluded that GAs were the preferred method when the dimensionality of a given system exceeded four. Oliveira and Loucks (1997) applied GAs to derive multireservoir operating policies and used real-coded chromosomes, elitism, arithmetic crossover, mutation, and modified GAs to make all the new candidate solutions feasible. They concluded that GAs might be a practical and efficient method of estimating operating policies for multireservoir systems. Wardlaw and Sharif (1999) extended the four-reservoir problem to show that real-coded GAs incorporating tournament selection, uniform crossover, and modified uniform mutation produced the best results among several alternative GA formulations for reservoir systems, and they considered a more complex 10-reservoir problem.

The applications of MOGAs to water resources problems have increased recently. Ritzel *et al.* (1994) applied MOGAs to a multi-objective groundwater pollution containment problem. They used simple GAs, a vector-evaluated GA (VEGA), and a Pareto GA in order to compare the results with those attained by mixed integer chance constrained programming; here, the Pareto GA seemed to achieve results superior to those of the VEGA. Cieniawski *et al.* (1995) investigated MOGAs to solve a multi-objective groundwater monitoring problem. They used a VEGA, a Pareto GA, and the combination of a VEGA and a Pareto GA; of these, the VEGA–Pareto combination outperformed the other two MOGAs. Burn and Yulianti (2001) applied MOGAs to examine different waste-load allocation problems. Their work showed both a trade-off curve and a decision space representation of selected points based on this curve. Prasad and Park (2004) presented a MOGA applied to the design of a water distribution network, and they suggested a constraint handling technique that does not require a penalty coefficient. Reed and Minsker (2004) demonstrated the use of high-order Pareto optimization on a long-term monitoring application and demonstrated that high-order Pareto optimization can be used in a balanced design of water resource systems. Recently, Pareto-based selection approaches have become the most popular of the MOGA solution techniques (Van Veldhuizen and Lamont, 2000). Of the Pareto-based selection approaches in use, the NSGA-II method is used in this study.

MOGAS

Multi-objective optimization problems (MOPs) are defined as follows (Van Veldhuizen and Lamont, 2000).

Definition 1 (General MOP). *In general, an MOP minimizes* $F(\vec{x}) = (f_1(\vec{x}), \cdots, f_k(\vec{x}))$ *subject to* $g_i(\vec{x}) \leq 0,\ i = 1, \cdots, m,\ \vec{x} \in \Omega$. *An MOP solution minimizes the components of a vector* $F(\vec{x})$, *where* $\vec{x}$ *is an n-dimensional decision variable vector* ($\vec{x} = x_1, \cdots, x_n$) *from some universe* Ω.

In addition to Definition 1, the Pareto dominance and the Pareto optimality are defined as follows.

Definition 2 (Pareto Dominance). *A vector* $\vec{u} = (u_1, \cdots, u_k)$ *is said to dominate* $\vec{v} = (v_1, \cdots, v_k)$ *(denoted by* $\vec{u} \preceq \vec{v}$*) if and only if u is partially less than v, i.e.* $\forall i \in \{1, \cdots, k\},\ u_i \leq v_i \wedge \exists i \in \{1, \cdots, k\} : u_i < v_i$.

Definition 3 (Pareto Optimality). *A solution* $x \in \Omega$ *is said to be Pareto optimal with respect to* Ω *if and only if there is no* $x' \in \Omega$ *for which* $\vec{v} = F(x') = (f_1(x'), \cdots, f_k(x'))$ *dominates* $\vec{u} = F(x) = (f_1(x), \cdots, f_k(x))$. *The term 'Pareto optimal' is taken to mean with respect to the entire decision variable space.*

The key definition of MOGAs is a Pareto-dominance concept used to discriminate nondominated solutions from a search space. The nondominated solution to multi-objective optimization cannot simply be a single one, as is found in single-objective optimization, because in the former at least two objectives compete to achieve better performance simultaneously. Therefore, that one solution is superior to all the other solutions for the given objectives is implausible. Consequently, MOGAs should assign a rank to each solution using Pareto dominance and attempt to ascertain the nondominated solutions (known also as Pareto-optimal solutions or a Pareto front) that have a rank of one and are not dominated by the other solutions.

For the reasons mentioned above, a specialized optimization technique for MOPs must create an effective set of nondominated solutions having ranks. In this regard, NSGA-II has been applied successfully to many optimization problems. This method uses tournament selection (Goldberg and Deb, 1991), simulated binary crossover (SBX; Deb and Agrawal, 1995), mutation operator (Deb, 2000), and crowding distance for diversity preservation (Deb, 2001). The detailed process of NSGA-II is explained in Deb *et al.* (2002).

CASE STUDY AREA (HAN RIVER BASIN)

The Han River basin is located in the middle of the Korean Peninsula (Figure 1). The catchment area is 23 292 km^2, and the stream length is 471·16 km with an annual discharge of around 18×10^9 m^3. The average annual precipitation in the basin is 1294 mm, with 65% of the total precipitation occurring between July and September. The water use in the basin mainly satisfies municipal water supply demand in the Seoul metropolitan area, which has a population of 17 million. Its municipal water supply demand is around $3{\cdot}6 \times 10^6$ m^3 day^{-1}, as determined by the Korea Water Resources Corporation (KOWACO) in 2001.

There are four major reservoirs with a hydropower plant in the Han River basin. Their physical characteristics are given in Table I. The Hwacheon and the Soyanggang reservoirs are located in parallel areas upstream in the North Han River, and the Choongju reservoir is located upstream in the South Han River. The Paldang reservoir is located at the confluence of the North and South Han Rivers. The main functions of the Soyanggang and the Choongju reservoirs are to mitigate flooding in the Seoul metropolitan area and to supply municipal water. The Hwacheon reservoir can also provide a degree of flood control capacity, but its primary function is to generate hydroelectric power. The Hwacheon, Soyanggang, and Choongju reservoirs also supply agricultural and industrial water downstream. The Paldang reservoir is charged with the role of control point in supplying municipal water, since over 95% of municipal water is taken from this reservoir.

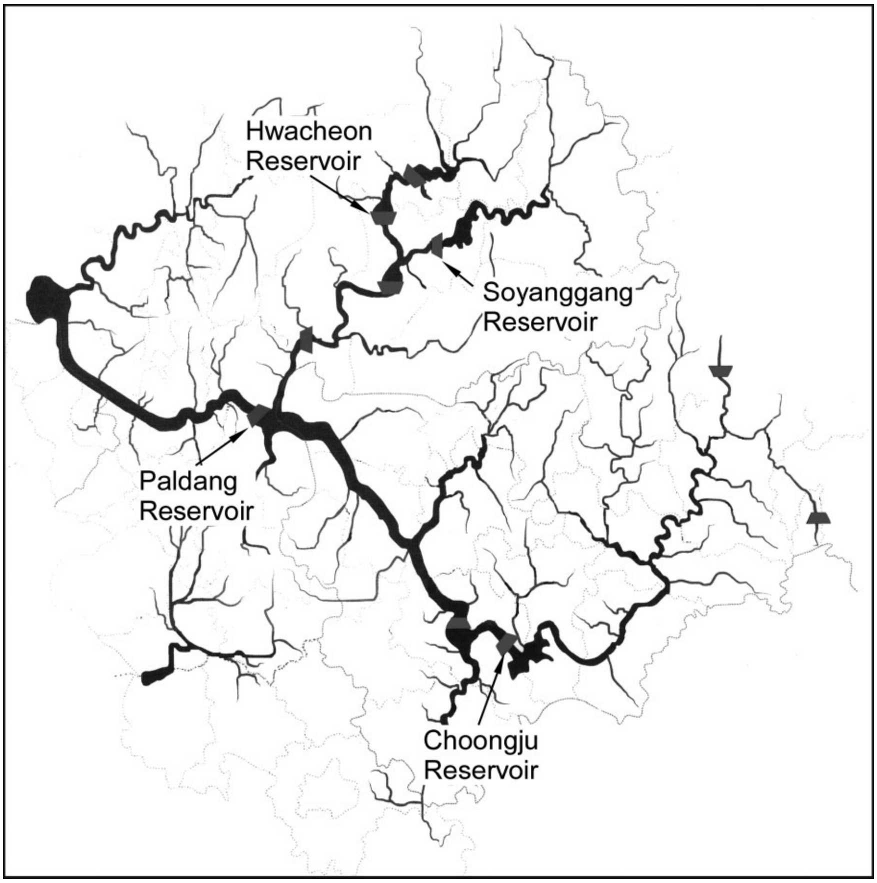

Figure 1. The four major reservoirs in the Han River basin

Table I. Physical characteristics of the four major reservoirs

Reservoir[a]	Basin area (km^2)	Storage (10^6 m^3)			Flood control capacity (10^6 m^3)	Maximum release (m^3 s^{-1})	
		Normal pool level	Regulated water level	Low water level		Spillway	Turbine
HC	3901	1022	809	364	213	5428	185
SY	2703	2543	2346	687	500	5500	251
CJ	6648	2385	2134	596	616	14 200	828
PD	23 800	244	244	226	29	26 000	800

[a] HC, SY, CJ, PD represent Hwacheon, Soyanggang, Choongju, and Paldang reservoirs respectively.

Supplying sufficient water resources downstream while avoiding possible shortages of water is not easy, because precipitation is usually concentrated in only a 3-month span, from July to September. The inflow created by this intense precipitation is usually released through a spillway, so it is, in effect, a useless water resource. During the flood season (from 21 June to 20 September), the water levels of the three upstream reservoirs should be lowered to the regulated water level in order to secure supplementary storage and to prevent possible flooding. As a result, the intense precipitation and lowering water level can result in insufficient water resources downstream.

To overcome inadequate water resources in the multireservoir system, increasing flow downstream is the best method, but this necessarily creates a concomitant shortage of storage. These two objectives, maximizing release and storage, conflict with each other, so multi-objective optimization techniques must be introduced to reconcile these two objectives. Therefore, multireservoir system optimization in the Han River basin needs to satisfy the following requirements:

1. The main objective of reservoir operation must be minimizing water shortage, because the water taken from the Paldang reservoir is the only water resource serving the Seoul metropolitan area.
2. Another important objective must be maximizing the storage of each reservoir; if the following years' precipitation is insufficient, then the water in the reservoir is the only available water resource in the basin.

MODEL FORMULATION

Coding scheme, crossover, and mutation

Decision variables in reservoir operating optimization are usually represented by release from the reservoirs considered, and the range of this release is wide. For example, the release from the Choongju reservoir in July is between 0 and $40\,251 \times 10^6$ m^3; thus, if binary coding is used and the release is divided by 1×10^6 m^3 size ranges, 16 bits ($32\,768 = 2^{15} < 40\,251 < 2^{16} = 65\,536$) are required to represent one gene, and approximately 411 bits to represent one chromosome in this study. Of course, binary-coded GAs can handle lengthy bits, but they require more chromosomes and more time in order to attain nondominated solutions than do real-coded GAs. Therefore, a real-coded chromosome consisting of the release from each reservoir is used in this study. Real-coded GAs have, in fact, been used successfully in various water resources fields (Chang and Chen, 1998; Wardlaw and Sharif, 1999; Yoon and Shoemaker, 2001).

In NSGA-II, the SBX operator is used. SBX, which has search power similar to that of a single-point binary crossover, creates offspring from a given pair of parent solutions. The procedure for calculating offspring (c_1, c_2) from parent solutions (p_1, p_2) is as follows.

A uniform random number u between zero and one is generated, and then the spread factor β is computed using

$$\beta = \begin{cases} (2u)^{1/(n+1)} & \text{if } u \le 0.5 \\ \left[\dfrac{1}{2(1-u)}\right]^{1/(n+1)} & \text{otherwise} \end{cases} \tag{1}$$

in which n is the distribution index of SBX and can be any nonnegative real number. Then, offspring are calculated as follows:

$$\begin{aligned} c_1 &= 0{\cdot}5[(1+\beta)p_1 + (1-\beta)p_2] \\ c_2 &= 0{\cdot}5[(1-\beta)p_1 + (1+\beta)p_2] \end{aligned} \tag{2}$$

These offspring are symmetric with respect to parent solutions. A larger value of the distribution index n allows offspring to be closer to parent solutions. A smaller value of n results in a more uniform distribution in the range $0 \le \beta \le 1$; and if $n = 0$, then it creates an exact, uniform distribution in the same range. In this study, n is set to 3·0 and the probability of crossover is 0·9 (Kim and Heo, 2004).

After new chromosomes are combined by SBX, a mutation operator is used, and is given by

$$c_1' = c_1 + \delta_1 \Delta_{\max} \tag{3}$$

in which Δ_{max} is the maximum perturbance, and δ_1 is defined as

$$\delta_1 = \begin{cases} (2u)^{1/(n_m+1)} - 1 & \text{if } u < 0{\cdot}5 \\ 1 - [2(1-u)]^{1/(n_m+1)} & \text{if } u \geq 0{\cdot}5 \end{cases} \quad (4)$$

in which n_m is the distribution index of mutation and is set to 50 in this study, and the probability of mutation is set to 1/number of genes.

The multireservoir optimization problem

In this study, a chromosome is composed of 36 genes, three sets of 12 monthly releases in each reservoir, and both a 1 month time span and a 1 year reservoir operating plan are considered. The decision variables are the releases from each reservoir and the state variables are the storages. Using the decision variables and other parameters, the state variables are computed using

$$S_i^{t+1} = S_i^t + I_i^t - R_i^t \quad i = \text{HC,SY,CJ}; t = 1, \cdots, 12 \quad (5)$$

$$S_{PD}^{t+1} = S_{PD}^t - R_{PD}^t + R_{HC}^t + R_{SY}^t + R_{CJ}^t + \text{Local Inflow}^t - \text{Intake}_{PD}^t \quad t = 1, \cdots, 12 \quad (6)$$

in which S_i^t, I_i^t, and R_i^t are the month t average storage, inflow, and release at reservoir i respectively. Local Inflow^t is the sum of discharges from tributaries, and Intake$_{PD}^t$ means the water supply taken from the Paldang reservoir. HC, SY, CJ, and PD indicate the Hwacheon, Soyanggang, Choongju and Paldang reservoirs respectively.

The objective functions are defined thus:

$$\text{Minimize } f_1 = -\sum_t \sum_i S_i^t \quad (7)$$

$$\text{Minimize } f_2 = -\sum_t \sum_i R_i^t \quad (8)$$

subject to

$$R_L \leq R_i^t \leq R_U \quad (9)$$

$$S_L \leq S_i^t \leq S_U \quad (10)$$

$$R_{DUTY} \leq R_i^t \quad (11)$$

$$S_L^{LAST} \leq S_i^{LAST} \leq S_U^{LAST} \quad (12)$$

in which R_L, R_U and S_L, S_U are the original lower and upper limits of release and storage respectively, and S_i^{LAST} means the storage by the end of the last month in simulation. S_L^{LAST} and S_U^{LAST} are the changed lower and upper storage limits having a smaller range than do the original lower and upper limits, and R_{DUTY} is the designated release for satisfying water supply demand. Equations (9) and (10) each have 96 constraints, since four reservoirs and 12 time spans are considered; Equation (11) has 48 constraints, and Equation (12) has six constraints. The constraints in Equations (9)–(11) are applied to the entire time steps of simulation, but the last constraint, that of Equation (12), is satisfied only by the last month. This condition makes Equation (12) a very strict constraint, because the water level by the end of the month is calculated by the storages and the releases occurring in all the previous 11 months; thus, when these storages and releases are calculated, the feasibility of maintaining an appropriate water level up to the month's end should be rechecked. In this study, the number of generations is increased to 2500 to satisfy this strict constraint.

COMPUTATIONAL RESULTS

In this study, three cases are analysed to evaluate the performance of NSGA-II as it is applied to multireservoir system optimization having multi-objectives. Each case has a different constraint condition, and the number of generations in Case 3 is increased to 2500 because it exhibits a more complicated constraint condition than do the other two cases mentioned above (Table II). More detailed explanations of each of these cases are as follows.

Case 1. This is the simplest case in this study and indicates whether the nondominated solutions obtained by NSGA-II are appropriate for multireservoir system optimization. The only constraints in Case 1 concern the storage and release limits for each reservoir in the Han River basin; no constraints regarding supplying water downstream or maintaining the water level existing at the end of last month in a particular range affect this case. Consequently, water shortages or impractical storage values in the last month can occur in Case 1.

Case 2. This is meant to permit an evaluation of the NSGA-II constraint handling technique as additional constraints are added to Case 1. Two additional constraints, Equations (11) and (12), are used in this study. The first satisfies the amount of water supply demand downstream for each reservoir, and the second achieves the proper water level by the end of the last month. In Case 2, only the first additional constraint, that regarding water supply demand, is added, whereas in Case 3 both the first and second additional constraints are added.

Case 3. This has the largest number of constraints (246) and concerns storage and release limits (192), water supply demand (48), and the proper water level by the end of the last month (six). In order to satisfy the constraint regarding water level, the number of generations is increased to 2500, in contrast to Cases 1 and 2, in which the number of generations is 500. Most applications by MOGAs only show a trade-off relation among multi-objectives. They do not suggest how a decision maker might choose an acceptable alternative solution among the Pareto-optimal solutions; thus, the trade-off relation must be analysed once again by a decision maker. Contrastingly, the method of how to analyse a decision variable (as release is analysed in this study) and how to choose a suitable alternative solution are suggested in this study.

Random seed impacts and population sizing

The search process of GAs has stochastic features, so, even if the same parameter settings are used in simulation, the decision variables or the objective function values are changed between each optimization run (Bayer and Finkel, 2004). To overcome this drawback of GAs and get reliable results for the analysis, many optimization runs with each different random seed number are performed and fully investigated. However, it has been generally acknowledged that multireservoir system optimization is computationally too complex to achieve a proper parameter setting, even though many optimization runs are employed. Hence, we confine our attention to achieving a proper parameter setting and results for a single random seed number.

Another key factor in solution quality and search reliability of GAs is population size. Reed *et al*. (2003) suggested the three-step design methodology for solving MOPs automatically with only a few simple user

Table II. Initial conditions and constraints in each case

	Number of chromosomes	Number of generations	Constraints
Case 1	1000	500	Maximum and minimum limits of storage (96) + Maximum and minimum limits of release (96)
Case 2	1000	500	Maximum and minimum limits of storage (96) + Maximum and minimum limits of release (96) + Water supply demand (48)
Case 3	1000	2500	Maximum and minimum limits of storage (96) + Maximum and minimum limits of release (96) + Water supply demand (48) + Storage limit reached by the end of the last month (6)

Table III. Number of Pareto-optimal solutions in each case

Population size	Case 1 Generation number: 500	Case 2 Generation number: 500	Case 3 Generation number: 1000
250	250	250	125
500	500	500	231
750	750	750	417
1000	1000	1000	178
1250	1250	1250	523
1500	1500	1500	231

inputs. The first user-defined parameter is the initial goal for the number of nondominated solutions. It is really difficult or almost impossible to obtain the true trade-off relation of objective functions in multireservoir system optimization, since the entire decision space is too large to be able to calculate the whole solutions with enumeration. Thus, the maximum number of nondominated solutions cannot be estimated, and the initial goal for the number of nondominated solutions is set to a reasonably small value, i.e. 250.

The second parameter is for the stopping criteria of the MOGA. Reed *et al.* (2003) used the minimum percentage change in the number of nondominated solutions for two successive runs to be considered identical. Table III shows the number of Pareto-optimal solutions in each case. In both Cases 1 and 2, NSGA-II can find the nondominated solutions as many as the population size shown in Table III. On the contrary, the Pareto-optimal solutions in Case 3 show no definite trend even though the generation number is increased up to 1000. This might be because the added constraint to Case 3, Equation (12), is the hardest constraint to be satisfied. To satisfy this constraint, as mentioned above, all the releases or the storages of the preceding months should be appropriate values without constraint violation, and this means that the search power of the GAs is very limited.

In order to decide the proper population size, a different kind of criteria with regard to the constraint violation is used in this study. Figure 2 shows the violation values for Case 3 when the population sizes vary from 250 to 1500. The violation values are generally smallest when the population size is 1000, and even when the population sizes are increased (up to 1500) there are no improvements in the sense of violation values. Note that the violation values are getting larger for a population size of 1250, and are slightly damped for a population size of 1500. As a result, the population size is determined to be 1000, and the generation number in Case 3 is set to 2500 to give NSGA-II sufficient objective function evaluations.

Case 1

The objectives in Case 1 show (1) how well the nondominated solutions are distributed in a Pareto front, (2) to what degree constraint violations can decrease, and (3) the basic method of analysing Pareto-optimal solutions when considering multi-objectives. The first objective is important, because if nondominated solutions are not well spread in a Pareto front, then this means that the objective functions have not properly formed or that the initial parameters, like crossover and mutation probabilities, have not been well chosen. In such cases, new objective functions or different initial parameters would become necessary. Figure 3 shows the nondominated solutions (called the Pareto front) in the last generation in Case 1. The shape of the Pareto front is very typical of Min–Min optimization problems, and the nondominated solutions are very well spread in the Pareto front without any concentration in a narrow range of objective function values. This indicates that the two objective functions and the initial parameters in this study are appropriate.

It is also apparent in Figure 4 that the number of violations (NOV) and the violation values of constraint (VioVAL) decreased rapidly. The NOV is close to 1000 for the first 10 generations, and all solutions are infeasible solutions with a large violation. However, over half of the solutions become feasible after the

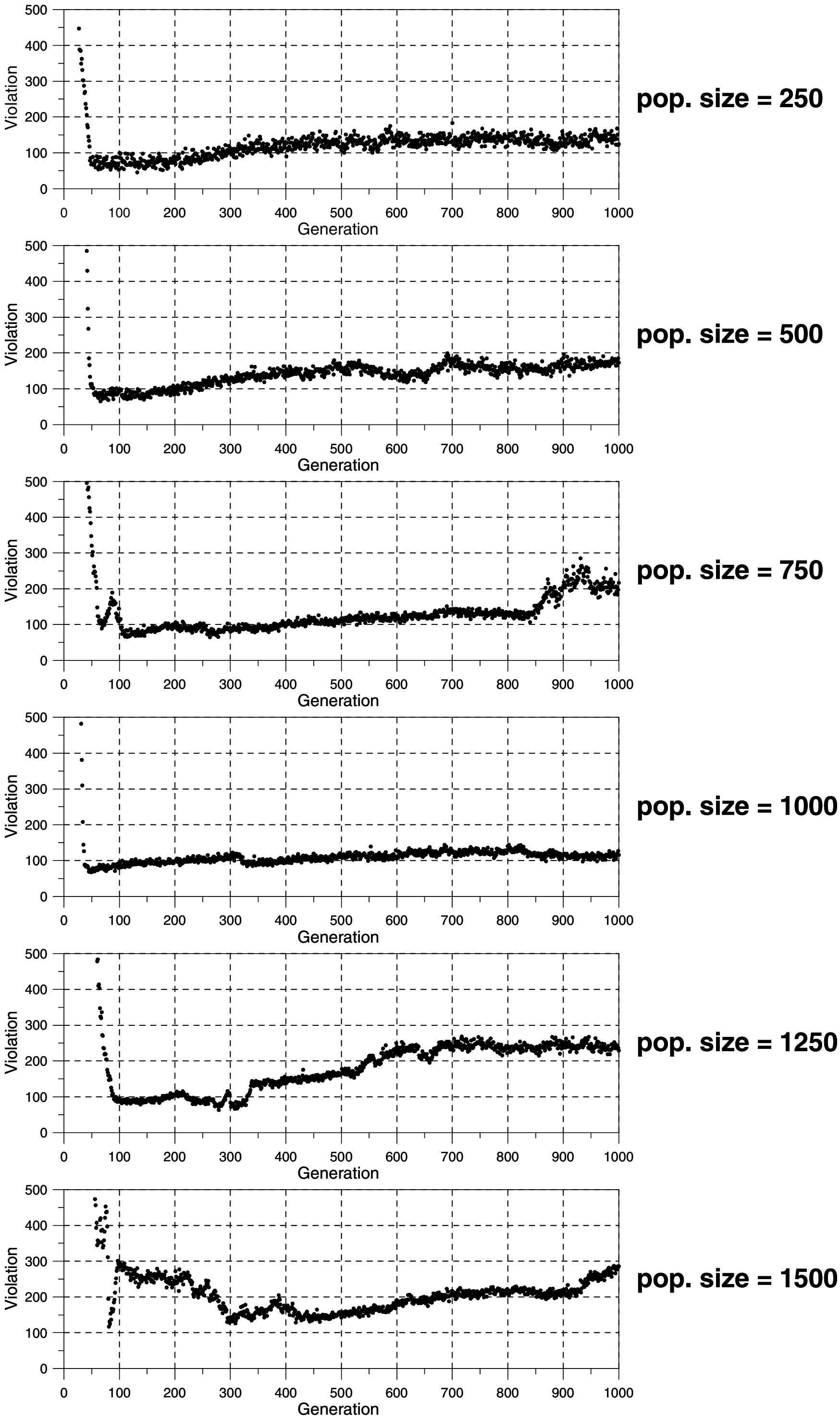

Figure 2. The violation values of constraint in Case 3 with different population sizes

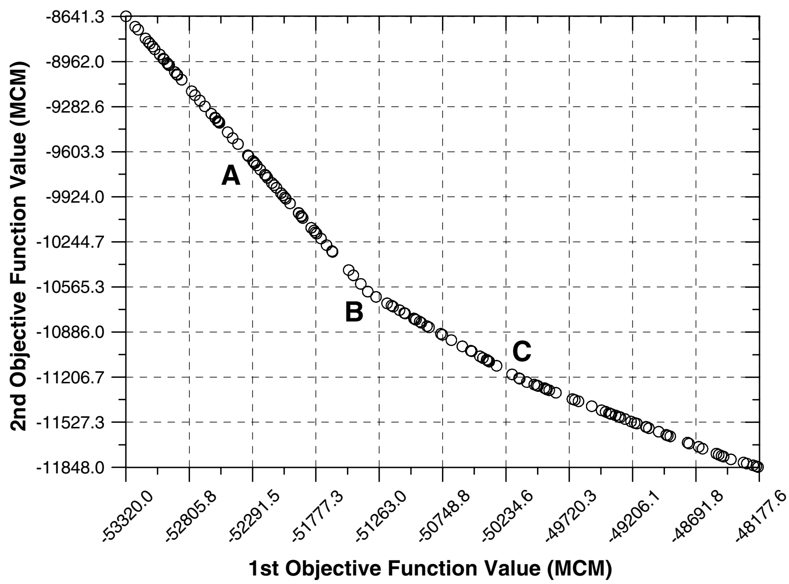

Figure 3. The nondominated solutions in Case 1

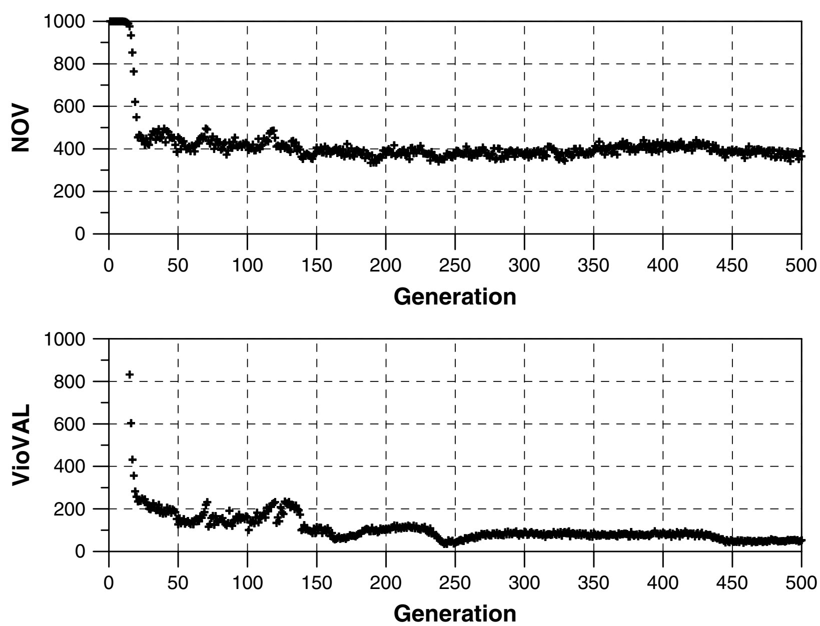

Figure 4. The number of violations (NOV) and the violation values of constraint (VioVAL) in Case 1

21st generation, and the VioVAL also decreases rapidly. In Case 1, the VioVAL is computed by considering constraint with respect to the minimum and the maximum storage capacities of each reservoir. In the first generation, the VioVAL is calculated as 280 592. However, this decreases to 90 177 (32% of the initial value) in the fifth generation and to 11 144 (4% of the initial value) in the 10th generation. The average of the VioVALs from the 21st to the last generation is 101, only 0·03% of the initial value. In addition,

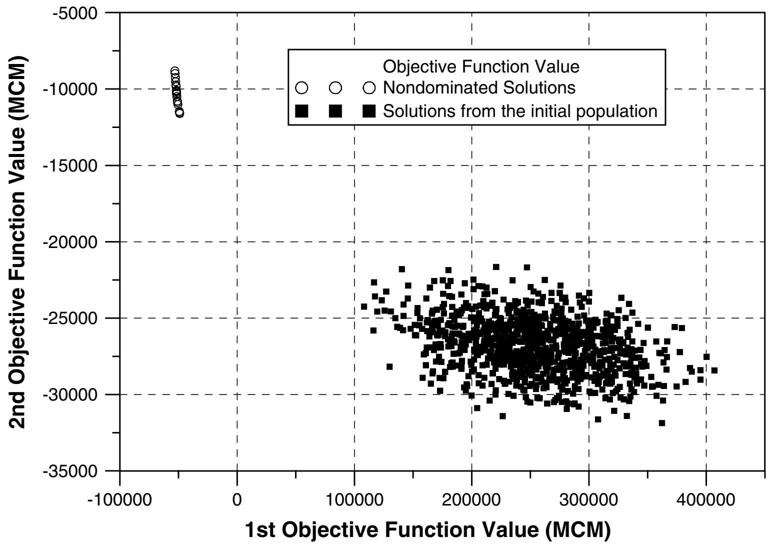

Figure 5. Objective function values from the initial population and the nondominated solutions in Case 1

Figure 5 shows the objective function values calculated from the initial population as filled squares and the nondominated solutions from the last generation as open circles. All the initial solutions are infeasible solutions with a large violation, as shown in Figure 4, and are set in a very wide range far away from the Pareto front. However, the nondominated solutions in the last generation exhibit no violation of constraint and form a shape typical in Min–Min optimization problems. This means that, even though the initial random solutions are located in an area in which the optimization technique can hardly lead to nondominated solutions contained in the search space, NSGA-II can ascertain the Pareto-optimal solutions very efficiently.

Figure 3 shows the nondominated solutions in Case 1. The range of each objective function is divided into 10 parts and the release average (the decision variables) within each part is determined in order to analyse the nondominated solutions. Table IV provides these separate release averages (R1A, R1B, R1C, R2A, R2B, and R2C), standard deviation (SD), and the coefficient of variation (VAR) near points A, B, and C in Figure 3. If the SD and VAR are larger in a particular month, then this indicates that the decision variable (release) in that month can change the objective function value into negative or positive values. Using this information, the decision maker is thus capable of discriminating between an important and an unimportant month. A more detailed explanation of decision variables is as follows:

1. If the decision maker wants to operate a reservoir satisfying two objectives simultaneously, the solutions near point B in Figure 3 may be the best ones, and the averaged releases from column R1B or R2B will be used in Table IV.
2. After the decision maker selects the averaged releases from column R1B, and if a greater amount of storage is necessary to operate the reservoir properly, a new solution near point A can be chosen and the averaged releases easily changed into those from column R1A in Table IV. For example, the releases from the Hwacheon, Soyanggang, and Choongju reservoirs in the last month will change from $406{\cdot}4 \times 10^6$ m^3, $597{\cdot}0 \times 10^6$ m^3, and $2085{\cdot}6 \times 10^6$ m^3 respectively to $344{\cdot}8 \times 10^6$ m^3, $490{\cdot}5 \times 10^6$ m^3, and $1250{\cdot}3 \times 10^6$ m^3 respectively. The other releases do not need to be changed, since they display similar values between R1A and R1B.
3. When the decision maker wants a greater water supply (release), the right part of Table IV is useful, since these releases are calculated by using the second objective function, the sum of the releases. For example,

Table IV. Decision variables near points A, B, and C

Dam	Month	First objective function[a]					Second objective function[a]				
		R1A	R1B	R1C	SD[b]	VAR[c]	R2A	R2B	R2C	SD[b]	VAR[c]
HC	10	72·6	72·6	72·6	0·01	0·00	72·6	72·6	72·6	0·00	0·00
	11	56·2	56·2	56·2	0·00	0·00	56·3	56·3	56·3	0·00	0·00
	12	69·3	69·3	69·3	0·01	0·00	69·3	69·3	69·3	0·01	0·00
	1	66·1	66·1	66·1	0·01	0·00	66·1	66·1	66·1	0·00	0·00
	2	56·0	56·0	56·0	0·01	0·00	56·0	56·0	56·1	0·01	0·00
	3	55·8	55·8	55·8	0·02	0·00	55·8	55·8	55·8	0·02	0·00
	4	86·5	86·5	86·6	0·07	0·00	86·5	86·6	86·6	0·07	0·00
	5	134·9	135·0	135·3	0·41	0·00	135·0	135·1	135·2	0·30	0·00
	6	421·8	423·3	455·8	17·52	0·04	421·8	448·0	454·3	16·85	0·04
	7	495·3	495·3	495·5	0·11	0·00	495·3	495·4	495·4	0·09	0·00
	8	718·5	717·1	684·7	17·27	0·02	718·6	692·6	686·4	16·64	0·02
	9	344·8	406·4	479·2	64·05	0·15	350·9	477·8	478·9	66·84	0·17
SY	10	113·7	113·7	113·7	0·00	0·00	113·8	113·8	113·8	0·00	0·00
	11	109·7	109·7	109·7	0·00	0·00	109·8	109·8	109·8	0·00	0·00
	12	113·7	113·7	113·7	0·00	0·00	113·7	113·7	113·7	0·00	0·00
	1	113·7	113·7	113·7	0·00	0·00	113·7	113·7	113·7	0·00	0·00
	2	101·7	101·7	101·7	0·00	0·00	101·8	101·8	101·8	0·00	0·00
	3	113·7	113·7	113·7	0·00	0·00	113·7	113·7	113·7	0·00	0·00
	4	109·7	109·7	109·7	0·01	0·00	109·8	109·8	109·8	0·01	0·00
	5	113·7	113·7	113·7	0·01	0·00	113·8	113·8	113·8	0·01	0·00
	6	109·9	109·9	110·2	4·45	0·04	109·9	110·0	110·1	2·36	0·02
	7[d]	114·1	114·2	162·7	195·16	0·84	114·2	114·5	114·6	148·69	0·85
	8	277·6	277·6	665·7	190·81	0·41	277·6	326·5	535·7	169·69	0·44
	9	490·5	597·0	649·9	87·83	0·15	482·9	649·5	649·8	94·54	0·17
CJ	10	271·7	271·7	271·7	0·00	0·00	271·8	271·8	271·8	0·00	0·00
	11	241·9	241·9	241·9	0·00	0·00	241·9	241·9	241·9	0·00	0·00
	12	250·3	250·3	250·3	0·00	0·00	250·3	250·3	250·3	0·00	0·00
	1	250·3	250·3	250·3	0·02	0·00	250·3	250·4	250·4	0·01	0·00
	2	225·1	225·1	225·1	0·00	0·00	225·1	225·1	225·1	0·00	0·00
	3	250·3	250·3	250·3	0·02	0·00	250·3	250·4	250·4	0·02	0·00
	4	265·5	265·5	265·5	0·00	0·00	265·5	265·5	265·5	0·00	0·00
	5	308·7	308·7	308·7	0·01	0·00	308·8	308·8	308·8	0·00	0·00
	6	314·5	314·5	314·5	0·02	0·00	314·6	314·6	314·6	0·02	0·00
	7	298·6	298·6	298·8	0·12	0·00	298·6	298·7	298·7	0·10	0·00
	8	1251·7	1253·0	1437·3	89·65	0·07	1251·7	1305·0	1396·4	82·63	0·06
	9	1250·3	2085·6	2146·2	469·24	0·25	1596·9	2144·9	2146·1	537·23	0·31

[a] R1A, R1B, R1C and R2A, R2B, R2C indicate the releases computed by the nondominated solution near A, B, and C in Figure 3 in terms of the first objective function and the second function respectively ($\times 10^6$ m^3).
[b] SD means the standard deviation of all releases ($\times 10^6$ m^3).
[c] VAR means the coefficient of variation which is defined as VAR = SD/Average.
[d] The SD for Soyanggang reservoir in July has significant values because the optimal solutions located on the right side of point C have a much larger release.

more water can be supplied if releases are set to the values in column R2C from those in column R2B in Table IV.

It is important that two conflicting objectives are simultaneously considered in this case. Traditional multi-objective optimization techniques generally use one objective function formulated by the weighted sum of multi-objective functions; moreover, these techniques require many runs because they can only arrive at a single solution from one run. However, NSGA-II uses two objective functions simultaneously without employing a weighting function or any user-defined function, and it can compute nondominated solutions with simply a single run despite the various constraints.

Case 2

Three kinds of constraint are used in this study. The basic constraint includes the storage and the release limits of each reservoir, and the other two constraints centre on supplying water downstream and maintaining the water level up to the end of the last month within a proper range. When only the basic constraint is used, the simulation results sometimes become impractical, as a much larger release in a given month can cause a serious water shortage in the next month. This is because storage and release are connected as a mass balance equation of water in a reservoir. Thus, two additional constraints are essential to achieve acceptable results through NSGA-II. The first additional constraint regarding water supply demand is used together with the basic constraint in Case 2. The second additional constraint connected to the water level in the last month is added to Case 3, in which the number of generations is increased to 2500.

When the performance of the constraint handling technique in NSGA-II is examined, it is evident that finding feasible solutions through this technique would not be seriously hampered in the face of additional constraints. In this study, the number of basic constraints is 192, and the number of first additional constraints is 48, a quarter of the basic constraints and, therefore, not an insignificant proportion. However, the NOV and the VioVAL in Case 2 are almost the same as those in Case 1, except for the slight increase in NOV between approximately 50 and 150 generations (Figures 4 and 6). In addition, NOV and VioVAL decrease rapidly after the 21st generation, as in Case 1.

As additional constraints with respect to water supply demand are added, Case 2 shows a lesser shortage of water than does Case 1. Table V shows the shortages of water in Case 1 and Case 2. The average of releases (Avg) is calculated based on releases from the nondominated solutions in the last generation, and the deficit is computed by subtracting water supply demand from the average of releases. The deficits in Case 1 (marked with A and C in Table V) indicate negative numbers between October and the following May at Hwacheon, between October and the following June at Soyanggang, and between October and the following

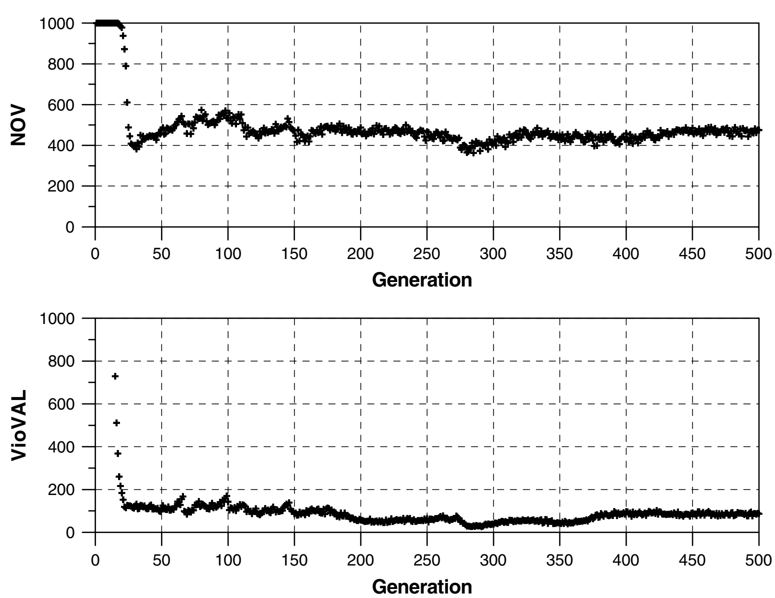

Figure 6. The number of violations (NOV) and the violation values of constraint (VioVAL) in Case 2

Table V. The shortages of water in Cases 1 and 2

Dam	Month	Water demand (10^6 m^3)	First objective function				Second objective function			
			Case 1		Case 2		Case 1		Case 2	
			Avg (10^6 m^3)	Deficit (A) (10^6 m^3)	Avg (10^6 m^3)	Deficit (B) (10^6 m^3)	Avg (10^6 m^3)	Deficit (C) (10^6 m^3)	Avg (10^6 m^3)	Deficit (D) (10^6 m^3)
HC	10	82·6	72·6	−10·0	82·7	0·1	72·6	−10·0	82·7	0·1
	11	66·3	56·3	−10·0	66·4	0·1	56·3	−10·0	66·4	0·1
	12	79·3	69·3	−10·0	79·3	0·0	69·3	−10·0	79·3	0·0
	1	76·1	66·1	−10·0	76·3	0·2	66·1	−10·0	76·3	0·2
	2	66·0	56·0	−10·0	66·0	0·0	56·0	−10·0	66·0	0·0
	3	65·6	55·8	−9·8	65·8	0·2	55·8	−9·8	65·8	0·2
	4	96·4	86·6	−9·8	96·6	0·2	86·6	−9·8	96·6	0·2
	5	144·3	135·3	−9·0	145·3	1·0	135·1	−9·2	145·3	1·0
	6	145·0	442·5	297·5	393·7	248·7	434·8	289·8	369·1	224·1
	7	181·5	495·4	313·9	494·7	313·2	495·4	313·9	494·5	313·0
	8	238·4	698·2	459·8	667·4	429·0	705·8	467·4	692·0	453·6
	9	178·9	431·6	252·7	477·9	299·0	403·4	224·5	477·2	298·3
SY	10	123·7	113·8	−9·9	123·8	0·1	113·8	−9·9	123·8	0·1
	11	119·8	109·8	−10·0	119·8	0·0	109·8	−10·0	119·8	0·0
	12	123·7	113·7	−10·0	123·8	0·1	113·7	−10·0	123·8	0·1
	1	123·7	113·7	−10·0	123·8	0·1	113·7	−10·0	123·8	0·1
	2	111·8	101·8	−10·0	111·8	0·0	101·8	−10·0	111·8	0·0
	3	123·7	113·7	−10·0	123·8	0·1	113·7	−10·0	123·8	0·1
	4	119·8	109·8	−10·0	119·8	0·0	109·8	−10·0	119·8	0·0
	5	123·7	113·8	−9·9	123·8	0·1	113·8	−9·9	123·8	0·1
	6	119·8	111·5	−8·3	120·4	0·6	110·7	−9·1	120·3	0·5
	7	123·7	232·6	108·9	244·2	120·5	175·2	51·5	196·7	73·0
	8	123·7	466·8	343·1	479·4	355·7	387·1	263·4	393·3	269·6
	9	119·8	589·8	470·0	644·5	524·7	553·8	434·0	640·1	520·3
CJ	10	281·8	271·8	−10·0	281·8	0·0	271·8	−10·0	281·8	0·0
	11	251·9	241·9	−10·0	252·0	0·1	241·9	−10·0	252·0	0·1
	12	260·3	250·3	−10·0	260·4	0·1	250·3	−10·0	260·4	0·1
	1	260·3	250·4	−9·9	260·4	0·1	250·4	−9·9	260·4	0·1
	2	235·1	225·2	−9·9	235·2	0·1	225·2	−9·9	235·2	0·1
	3	260·3	250·4	−9·9	260·4	0·1	250·4	−9·9	260·4	0·1
	4	275·5	265·5	−10·0	275·6	0·1	265·5	−10·0	275·6	0·1
	5	318·7	308·8	−9·9	318·8	0·1	308·8	−9·9	318·8	0·1
	6	324·5	314·6	−9·9	324·9	0·4	314·6	−9·9	324·9	0·4
	7	308·6	298·7	−9·9	309·6	1·0	298·7	−9·9	309·4	0·8
	8	323·8	1348·2	1024·4	1261·7	937·9	1309·1	985·3	1229·4	905·6
	9	279·4	1879·9	1600·5	2048·3	1768·9	1717·8	1438·4	1963·5	1684·1

July at Choongju. However, the deficits in Case 2 (marked with B and D in Table V) show no water shortages in any of the reservoirs.

In ascertaining nondominated solutions with constraint, the remaining feasible search space should be minimized, and thereby the violation redundancy too should be minimized. In Table V, where the water shortages show negative values in column A, most of these values are very close to -10^7 m^3 (the smallest violation redundancy), because the minimum release is set to a value of 10^7 m^3 subtracted from the water supply demand. Therefore, this means that NSGA-II can use the most feasible solution space. The other shortages of water in columns B, C, and D show the same results.

Case 3

There are 246 constraints in Case 3, as displayed in Table II. The storage constraint of the last month must be satisfied for all three upstream reservoirs. The upper and lower storage limits reached by the end of the last month are defined by the initial storage $\pm 10^7$ m^3. NSGA-II is run with a population size of 500, the same as in Cases 1 and 2, and the number of generations is increased to 2500 because Case 3 has the largest number of constraints. The average time to complete a run on a 3·06 GHz Pentium 4 computer is 17 min.

If the variables of optimization problems have no relation and are independent of each other, then computing feasible solutions is an easy task, as only the limits of each variable need to be considered. However, it is more complex in multireservoir system optimization, because the release of the current month strongly affects the storage for the following months. Consequently, even if the release of the current month is a feasible solution, this can cause an infeasible storage value in one of the following months. Therefore, how efficiently the optimization technique determines the feasible and nondominated solutions in the search space is an important criterion when evaluating optimization technique performance.

Figure 7 shows the 1000 initial solutions from the first generation as crosses and 657 Pareto-optimal solutions from the last generation as solid squares in the inset. The first objective function values of the Pareto-optimal solutions vary between $-49\,993{\cdot}8 \times 10^6$ m^3 and $-49\,301{\cdot}5 \times 10^6$ m^3, compared with the $108\,169{\cdot}1 \times 10^6$ m^3 to $407\,005{\cdot}8 \times 10^6$ m^3 range obtained from the initial solutions, so the extent of the Pareto-optimal solutions is only 0·23% of the initial solutions. Furthermore, a high degree of violation does occur in the initial solutions, but all the Pareto-optimal solutions are in fact feasible. These results demonstrate that NSGA-II can effectively arrive at Pareto-optimal and feasible solutions in the search space, even though initial solutions are far from the Pareto-optimal solutions and do contain many violations.

The Pareto-optimal solutions show only objective function values, so decision makers cannot ascertain how they might change decision variables in order to achieve a specific objective function value. However, the information concerning the decision variable is more important to decision makers than the objective function values are. Therefore, a method of discriminating critical decision variables, which can set an objective function value to a particular range, is suggested in this study.

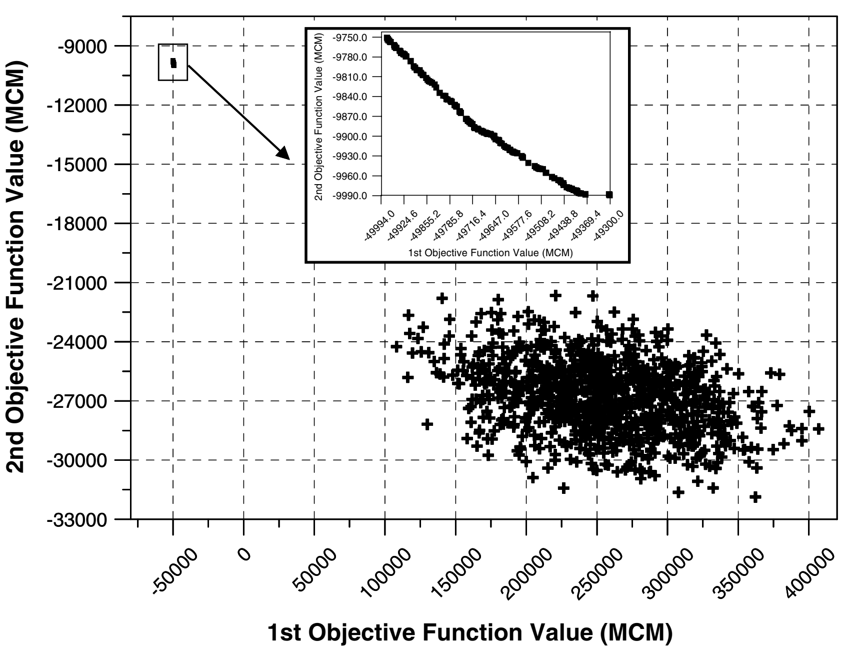

Figure 7. The initial solutions and the nondominated solutions in Case 3

Table VI. The statistical values of decision variables in Case 3

Month	Hwacheon reservoir			Soyanggang reservoir			Choongju reservoir		
	Avg (10^6 m^3)	SD (10^6 m^3)	VAR	Avg (10^6 m^3)	SD (10^6 m^3)	VAR	Avg (10^6 m^3)	SD (10^6 m^3)	VAR
10	82·65	1·66E−02	2·01E−04	123·75	4·56E−03	3·69E−05	281·78	3·73E−03	1·33E−05
11	66·31	1·71E−02	2·58E−04	119·75	3·65E−04	3·05E-06	251·95	1·31E−03	5·21E-06
12	79·32	3·67E−03	4·62E−05	123·78	6·77E−03	5·47E−05	260·36	2·95E−03	1·13E−05
1	76·09	1·70E−03	2·24E−05	123·75	3·81E−03	3·08E−05	260·36	3·86E−03	1·48E−05
2	66·04	1·89E−03	2·86E−05	111·78	8·93E−03	7·99E−05	235·16	9·44E−03	4·02E−05
3	65·70	1·52E−02	2·31E−04	123·76	6·18E−03	5·00E−05	260·36	1·80E−02	6·91E−05
4	96·47	1·60E−02	1·66E−04	119·81	3·42E−02	2·86E−04	275·57	1·97E−02	7·14E−05
5	172·5	3·66E+01	2·12E−01	123·78	1·62E−02	1·31E−04	318·80	7·53E−03	2·36E−05
6	450·73	4·31E+01	9·56E−02	119·82	5·50E−03	4·59E−05	324·59	3·60E−02	1·11E−04
7	495·58	2·60E−02	5·24E−05	125·18	2·50E+00	2·00E−02	515·60	1·09E+01	2·12E−02
8	626·7	6·84E+01	1·09E−01	285·71	5·92E+00	2·07E−02	1314·86	2·17E+00	1·65E−03
9	479·61	2·60E−02	5·41E−05	650·14	5·66E−02	8·71E−05	807·66	7·21E+00	8·93E−03

At first, those decision variables that exhibit significant change are observed because some decision variables are fixed at almost the same values as those occurring in Pareto-optimal solutions, and seem to have no influence in determining how to find Pareto-optimal solutions. Thus, the decision variables having a large SD value or VAR are investigated first. Table VI shows the Avg, SD, and the VAR of the decision variables calculated from the Pareto-optimal solutions in Case 3. The standard deviations of the decision variables are relatively large in May, June, and August at Hwacheon reservoir, in July and August at Soyanggang reservoir, and in July, August, and September at Choongju reservoir. The other decision variables display minor SDs, which are $5{\cdot}66 \times 10^4$ m^3 at most, and the VARs are almost zero. Moreover, the variation of the second objective function values is $246{\cdot}2 \times 10^6$ m^3, and the sum of the changes in these eight decision variables for the Hwacheon, Soyanggang, and Choongju reservoirs is $249{\cdot}2 \times 10^6$ m^3. These results demonstrate that the change in second objective function values is mainly caused by changes in these eight decision variables. These variables, therefore, are referred to as the critical decision variables.

Figure 8 shows the sum of the critical decision variables. From top to bottom, the critical decision variables for the Hwacheon, Soyanggang and Choongju reservoirs are shown. The three Y-axes in each graph exhibit the same difference (250×10^6 m^3) between the maximum and minimum indices. In these graphs, the critical decision variables for Hwacheon show a monotonic increase and change within $209{\cdot}7 \times 10^6$ m^3. However, the critical decision variables for Soyanggang and Choongju reservoirs vary within only $19{\cdot}6 \times 10^6$ m^3 and $19{\cdot}9 \times 10^6$ m^3, figures much smaller than that of the Hwacheon reservoir, and they do not indicate any clear increase or decrease. As a result, the releases in May, June, and August at Hwacheon reservoir are the major critical decision variables, and the remaining critical variables are minor ones.

After discriminating the major critical decision variables, the relation between these decision variables and objective function values is examined. In this study, two objective functions are used. The first objective function is the sum of storages in each reservoir, and the second is the sum of releases. The first objective function is not directly connected to the decision variables, but the second one is the sum of release decision variables; thus, it is the second objective function that is used in our examination. In reservoir operation, the decision maker needs to know both how to change release and what the expected storage is. Although release is acceptable, it is only useful if storage is also suitable; and the opposite is also true. Therefore, both release and storage must be appropriate to operate a reservoir efficiently.

Table VII shows the major critical decision variables at Hwacheon reservoir according to the second objective function values in Case 3. In order to achieve the second objective function values in rows A–J, the three releases in May, June, and August at Hwacheon reservoir should be set to those values appearing in the

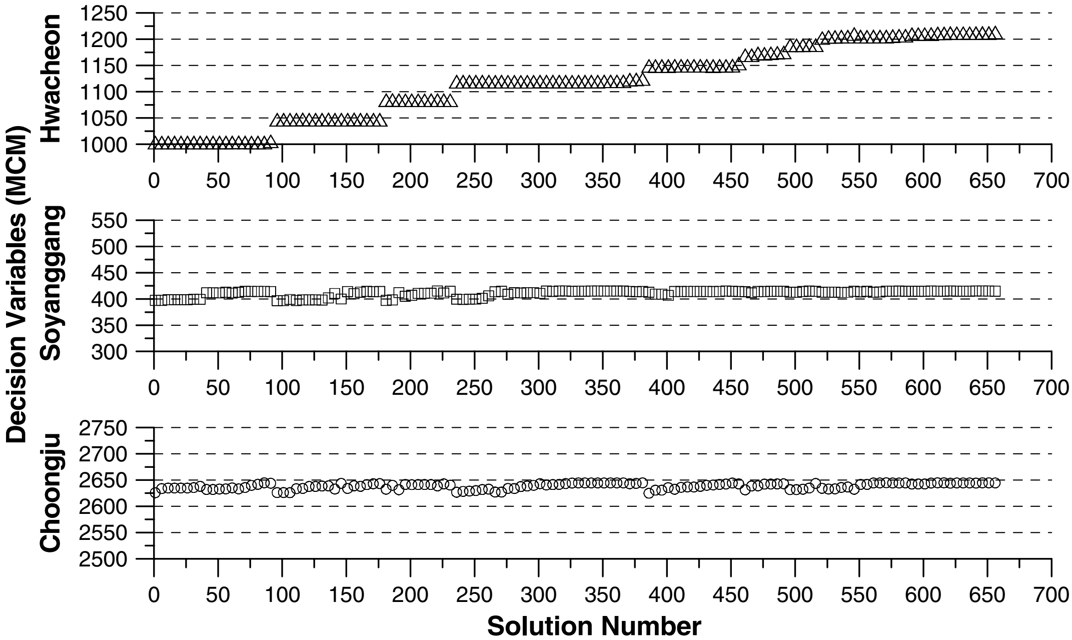

Figure 8. The sum of the critical decision variables in Case 3

Table VII. Major critical decision variables at Hwacheon reservoir in Case 3

	Second objective function value (10^6 m^3)	Releases (10^6 m^3)		
		May	June	August
A	−9743·1 to −9781·2	144·6	363·6	741·0
B	−9786·0 to −9821·8	144·6	407·1	697·8
C	−9826·3 to −9855·5	144·5	444·2	660·2
D	−9859·6 to −9899·8	145·1	479·4	625·1
E	−9901·6 to −9927·5	174·9	479·3	595·1
F	−9928·6 to −9945·4	196·0	479·1	574·6
G	−9945·4 to −9949·5	199·9	479·2	571·5
H	−9950·0 to −9965·2	214·7	479·3	555·9
I	−9966·2 to −9982·8	230·6	479·2	540·1
J	−9983·5 to −9989·3	236·6	479·3	538·1

same rows in Table VII, and the other releases should be set to the average values in Table VI. The releases in Table VII may be appropriate releases at Hwacheon reservoir because these values are not biased compared with the average values in Table VI, which are $172{\cdot}50 \times 10^6$ m^3, $450{\cdot}73 \times 10^6$ m^3, and $626{\cdot}70 \times 10^6$ m^3 respectively. Moreover, these increase in May and decrease in August without exhibiting any sudden change.

Figure 9 shows the patterns of the storages at Hwacheon reservoir as the releases in rows A–J in Table VII are applied. The storages in June, July, and August change because the releases in May and June also vary, but the storages in September concentrate at nearly the same values even though the release in August changes. This is because the water level reached by the end of the last month should be set to a specific value defined by constraint. Consequently, the decision maker can obtain the desired objective function value and determine how best to adjust the decision variables using Tables VI and VII and Figure 9.

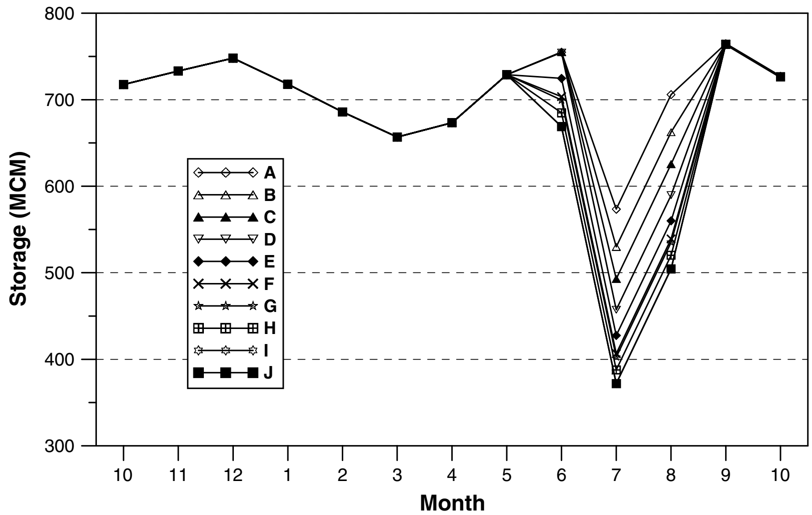

Figure 9. Storage patterns at Hwacheon reservoir in Case 3

CONCLUSIONS

Classical optimization provides only one optimal solution in a single run, and employs a preference vector that requires very careful adjustments to obtain optimal solutions. However, MOGAs can achieve nondominated solutions to multi-objective problems in a single simulation using the population-by-population approach. In addition, their effectiveness does not depend on a user-defined function or a preference vector, so the objective functions can be used directly without necessitating any changes.

In this study, NSGA-II, one of the most widely used MOGA solution techniques, is applied to find the Pareto-optimal solutions to a multireservoir system optimization problem with multi-objectives. Starting from the simplest model, Case 1 (having only storage and release constraints), additional constraints concerning water supply demand or the water level reached by the end of the last month are added in Cases 2 and 3 respectively. In multireservoir system optimization, determining feasible solutions in the search space is a complex task because of the serial connection of storages and releases. Therefore, when the optimization technique is applied, the effectiveness of the method used to ascertain feasible solutions become crucial. All three of the cases examined in this study demonstrate that NSGA-II performs well when Pareto-optimal solutions need to be ascertained.

Most applications by MOGAs contain Pareto-optimal solutions in the objective function space, but they do not suggest how the decision maker might choose the best solution. However, in this study, a method of analysing the Pareto-optimal solutions and the decision variables is proposed in Case 3 from the point of view of reservoir operation, and the relation between the objective function values and the decision variables is also given. The decision maker can easily select which Pareto-optimal solution best achieves the desired objective function value using the method proposed here.

Future areas of study are identified in the ranking procedure of NSGA-II. If the number of objective functions is more than two, then the ranking procedure in NSGA-II creates many nondominated solutions having a rank of one in a very early generation, and the objective function value improves only a little from that point on. Although the ε-dominance concept has been introduced recently to overcome this ‘premature problem’, more research is necessary to achieve better NSGA-II performance.

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