Research Article

MULTIDIMENSIONAL SCALING OF LARGE CHROMATIC DIFFERENCES BY NORMAL AND COLOR-DEFICIENT SUBJECTS

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The following two articles offer a glimpse at current experimental psychology in the Soviet Union, normally screened from the English-speaking world by all but impenetrable language barriers. The accompanying Commentary by Tarow Indow helps place these contributions in the framework of contemporary work on the psychophysics of color perception.

Abstract—*Fifteen normal trichromatic subjects, two protanopes, and two deuteranopes judged pairs of successively presented foveal color stimuli. Multidimensional scaling of the data yielded estimates of a three-dimensional space with axes interpreted as red–green, blue–yellow, and white–black. For color-deficient subjects, the average radius of the space differed from that of normals, being smaller for the protanopes and larger for the deuteranopes. For both types of color deficiency, the blue–yellow axis was stretched relative to the red–green, more strongly in the protanopes. The findings are taken to support the generality of a "spherical" model.*

A uniform color space is a representation of perceived colors as points in an n-dimensional space, in which the distances between pairs of colors correspond with their perceptual differences, and each dimension coincides with an independent neuronal channel in the color-coding system.

The multidimensional scaling of large chromatic differences has shown that colors of equal luminance for normal trichromats are represented as points on the spherical surface in the three-dimensional Euclidean space, with white color on the pole and monochromatic colors on the curve located along the equator. The dimensions of the color space are interpreted as red–green, blue–yellow, and white–black opponent neuronal channels. The subjective color attributes are characterized by two spherical coordinates: a horizontal angle referred to hue and a vertical angle referred to saturation, respectively (Izmailov, 1982; Sokolov & Izmailov, 1983).

The present study is designed to provide information concerning the color space of color-deficient subjects.

METHOD

The experimental apparatus was constructed from a color television set "Raduga-701," operated by a minicomputer "Setun'-70," which controlled the luminance, duration, and chromaticity of the stimuli exposed on the screen of the TV set. The stimuli were 13 equiluminant light flashes (25 ± 2 cd/m^2) differing in hue and saturation. The CIE coordinates of the stimuli are given in Table 1.

The foveal color stimuli (2.0 ± 0.5° in diameter and 0.5 s of duration) were successively presented in pairs after 5 min dark adaptation. The interstimulus interval was 1 s; and the interval between stimulus pairs was 3 s.

Seventy-eight stimulus pairs were exposed during one session, each pair 10 times, in quasirandom order. The subject's estimation of perceived differences between members of each pair was registered by pushing one of 10 terminal keys numbered from 0 (no difference) to 9 (maximal difference).

SUBJECTS

Subjects were 15 normal trichromats, two protans, and two deutans. The four color-deficient subjects had different degrees of abnormalities, as tested by Rabkin's pseudoisochromatic plates (Rabkin, 1971) and Rautian's anomaloscope (Rautian, 1957).

RESULTS

Multidimensional Scaling (MDS)

The averaged triangular matrix of perceived color differences for the 15 normal trichromats, and two individual matri-

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Table 1. *CIE coordinates of color stimuli and color-points coordinates in the three-dimensional Euclidean space*

	CIE			15 Normal Subjects				Protanope AA				Deuteranope HE			
Color	x	y	λ (mn)	X_1	X_2	X_3	R	X_1	X_2	X_3	R	X_1	X_2	X_3	R
Indigo	.12	.15	480	−14.6	−37.5	10.9	41.7	−7.0	−35.0	15.0	38.7	−9.5	−39.1	87.9	96.6
Blue	.14	.22	484	−15.2	−34.0	13.2	39.5	−4.0	−33.0	9.0	34.4	−8.5	−29.2	86.7	91.9
Bl-Gr	.20	.34	495	−19.6	−11.6	32.7	39.9	−13.0	−6.0	28.0	31.4	−7.6	−5.3	92.3	92.8
Green	.21	.59	520	−32.7	14.5	29.1	46.1	−2.0	30.0	5.0	30.5	−21.9	18.2	82.6	87.4
Gr-Yel	.25	.56	535	−29.1	14.5	33.8	46.9	−1.0	31.0	10.0	32.6	−14.3	20.7	67.5	72.0
Yel-Gr	.35	.52	560	−15.2	16.7	35.6	42.2	−3.0	30.0	17.0	34.6	−4.1	25.3	80.3	84.3
Yellow	.50	.40	588	16.1	14.0	32.6	38.9	0.0	28.0	23.0	36.2	7.9	38.0	86.8	95.1
Orange	.56	.32	615	30.8	15.6	23.4	41.7	5.0	30.0	8.0	31.4	14.6	45.0	74.5	88.3
Red	.59	.31	620	35.1	16.8	21.6	44.5	5.0	29.0	21.0	36.2	24.4	39.8	69.0	84.9
Pur-Rd	.29	.17	558*	26.1	−16.8	32.2	44.8	3.0	−24.0	17.0	29.6	16.6	−17.8	92.7	95.9
Pur-Bl	.24	.14	567*	17.1	−20.2	35.3	44.1	3.0	−28.0	12.0	30.6	9.1	−23.2	83.5	87.2
Purple	.25	.16	565*	22.7	−21.7	32.9	45.5	0.0	−30.0	15.0	33.5	6.9	−25.0	84.2	88.0
White	.31	.32	—	−1.8	−1.3	44.4	44.5	−15.0	−3.0	28.0	31.9	1.7	−5.4	82.2	82.4
Averaged radius (R)							43.1				33.2				88.2
Standard deviation (SD)							2.6				2.7				6.7
Variance coefficient, % (R)							6.1				8.1				7.6
Correlation coefficient (r)							0.98				0.99				0.97

ces for the protanope and deuteranope, are presented in Tables 2 and 3. Each element of every matrix is the sum of 10 values given by observers to a stimulus pair. The matrices were analyzed by an MDS program based on the Young-Torgerson procedure (Young & Torgerson, 1967).The results of MDS of each triangular matrix are represented by n coordinates of each color point in the Euclidean space and by n characteristic roots—one for each axis (Tables 1, 4). The dimensionality of the subjective space was estimated from the number of large characteristic roots (as a formal criterion) and by the number of interpretable axes (as a substantive criterion). Both criteria indicated a three-dimensional space (see Discussion). The axes of the space were rotated to facilitate interpretation as red–green, blue–yellow, and white–black color-opponent functions (Hurvich & Jameson, 1955; Sokolov & Izmailov, 1983). The resulting Euclidean coordinates of color points for each type of data are presented in Table 1.

A Test of Color Sphericity

The initial hypothesis concerning the spherical configuration of color points was tested by finding a center equidistant from all color points. The test requires fulfillment of two conditions: (1) maintenance of high linearity between interpoint distances and perceived color differences as evaluated by a correlation coefficient (the main condition of color space uniformity); and (2) existence of a geometrical center for the configuration of color points, such that all points are nearly equidistant from that center. The constancy of these radial distances was evaluated by a variance coefficient (the ratio of a standard deviation to a mean radius). The optimal solution was considered to be achieved when the maximal correlation coefficient was associated with the minimal variance coefficient. For the data obtained, the correlation coefficients between Euclidean distances and perceived differences varied from 0.97 to 0.99, and the

Table 2. *Subjective color differences matrix averaged for 15 normal trichromats*

Color	No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Indigo	1		11	42	60	60	61	68	73	76	57	47	52	62
Blue	2			34	58	60	57	61	72	75	56	50	50	55
Bl-Grn	3				36	41	42	50	60	66	56	47	49	33
Green	4					18	29	55	68	68	67	63	68	46
Gr-Yel	5						19	54	63	69	67	60	62	43
Yel-Gr	6							37	52	56	54	51	58	29
Yellow	7								24	32	43	44	44	36
Orange	8									8	45	45	48	53
Red	9										43	49	44	53
Pur-Rd	10											22	12	48
Pur-Bl	11												7	33
Purple	12													44
White	13													

Table 3. *Subjective color differences matrices for protanope AA (upper triangle) and for deuteranope HE (lower triangle)*

Color	No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Indigo	1		3	30	67	65	61	65	66	65	6	4	4	35
Blue	2	4		34	61	63	61	63	57	63	6	0	2	35
Bl-Grn	3	20	14		40	43	41	38	35	36	22	27	29	0
Green	4	52	52	28		1	2	3	3	12	54	54	63	40
Gr-Yel	5	51	54	31	7		8	2	4	8	55	59	63	39
Yel-Gr	6	66	49	36	15	13		3	8	2	57	62	60	36
Yellow	7	80	69	36	17	27	6		3	6	52	55	49	37
Orange	8	88	73	45	31	33	30	7		4	56	58	63	36
Red	9	86	82	63	60	35	34	14	3		49	59	59	34
Pur-Rd	10	28	23	30	45	56	55	53	64	60		4	5	26
Pur-Bl	11	11	12	11	53	52	45	50	75	65	15		3	33
Purple	12	12	17	10	50	52	54	64	70	67	12	2		34
White	13	30	17	6	31	32	28	44	55	41	8	19	14	

variance coefficient fluctuated in a range of 6–11% (see Table 1). These data show good agreement between the spherical representation and the data.

After determining the center and rotating the configuration, the coordinates were normalized along their radial lines so that they all fell exactly on an ideal unitary sphere. The resulting configurations are represented by Figure 1 as they project on the equatorial (constant luminance) plane of the sphere.

The Color Space of Normal Trichromats

The configuration of color points projected on the equatorial plane of the sphere with radius equal to 43.1 is represented in Figure 1a. All color points are spaced in such an order that the horizontal angle corresponds to a hue, and white is located close to the pole of the sphere.

The area covered by the normal trichromatic configuration, as achievable by TV-set colors, is smaller than that covered by the monochromatic stimulus configuration for normal trichromats (Izmailov, 1982). This difference is explained by the smaller saturation of TV-set colors.

The normal color space was used as the reference for different color deficiencies.

The Color Space of Color-Deficient Subjects

The protanopic color space has smaller average radius (33.2) and larger curvature of bounding surface than that of normal observers. The deuteranopic color space, in turn, has larger average radius and smaller curvature of bounding surface. This means that the color surface of a deuteranope approximates a plane.

The protanopic configuration is compressed along the red–green axis and is stretched along the blue–yellow one. At the same time, it lies much higher on the white–black axis than the configuration for normal observers. From additional data, not reported here, the degree of shrinkage on the red–green axis corresponds to the degree of color defect determined by traditional methods.

The deuteranopic configuration demonstrates the simultaneous rapprochement of points along both chromatic axes with a rising along the white–black axis to the white pole. The same

Table 4. *Characteristic roots of subjective matrices, listed in the order of magnitude*

No.	15 Normals	AA, Protanope	SA, Protanomal	HE, Deuteranope	PR, Deuteranomal
1	7,092	9,962	13,997	10,717	8,781
2	5,322	752	3,701	2,256	5,553
3	1,687	500	2,690	995	1,516
4	940	369	1,179	858	1,412
5	488	237	614	554	896
6	296	159	386	401	506
7	137	114	65	0	0
8	55	0	0	−59	0
9	0	−48	−150	−241	−22
10	−164	−187	−280	−587	−375
11	−202	−328	−612	−755	−840
12	−202	−463	−885	−1,026	−1,041
13	−469	−771	−1,194	−1,372	−1,489

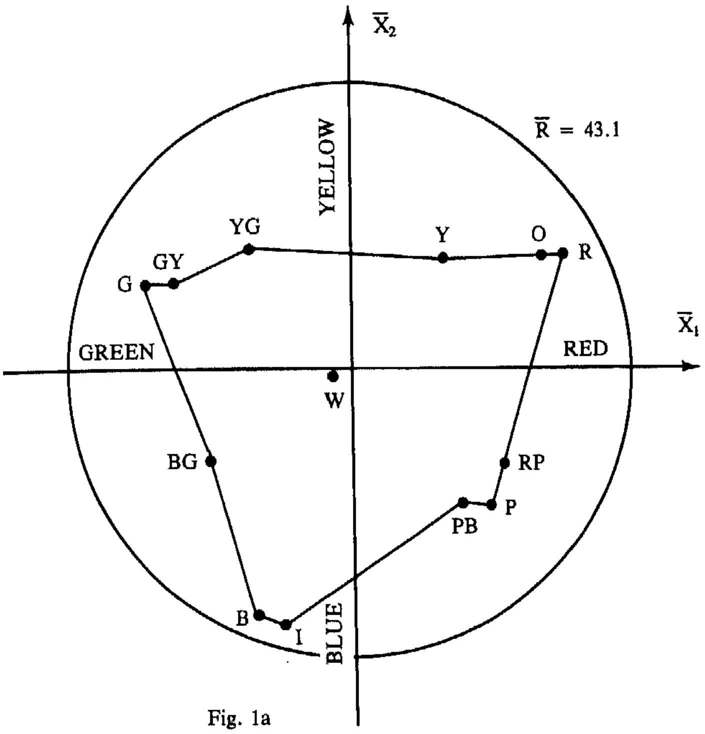

Fig. 1a

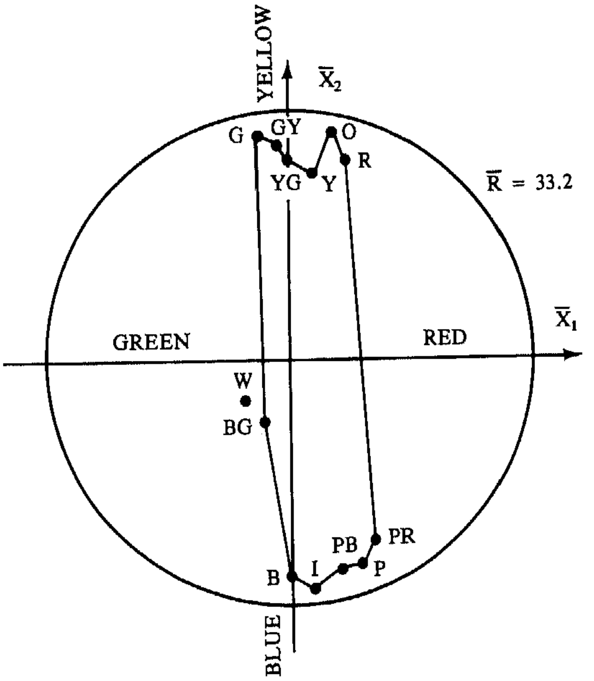

Fig. 1b

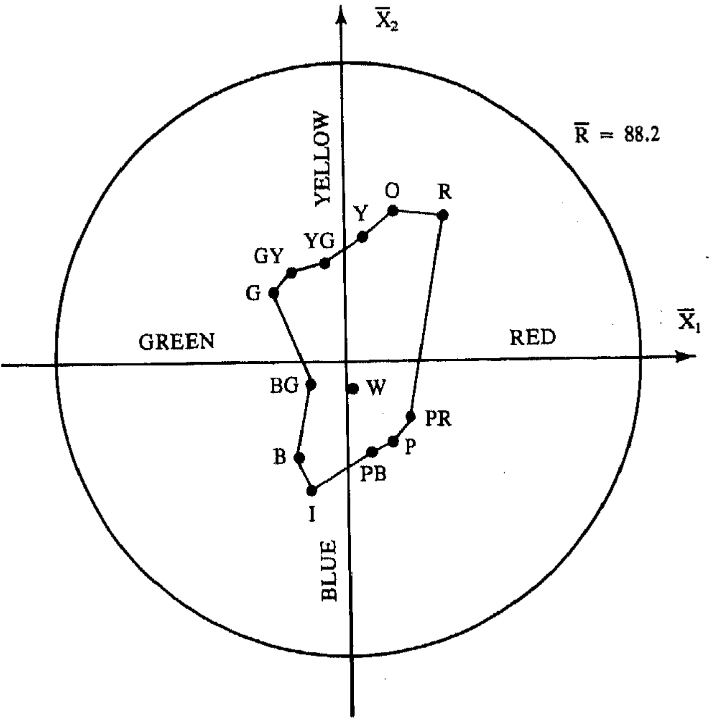

Fig. 1c

Fig. 1. Spherical model of color discrimination as projected on the equatorial plane X_1X_2: (a) averaged data for normal trichromats; (b) protanopic configuration; (c) deuteranopic configuration.

tendency can be seen here as was seen in the protanopic case: The more color defect, the more condensed is the stimulus configuration.

The dichromatic cases demonstrate a spacing out of the configuration in three loci: near yellow, blue, and white. This finding conforms to phenomenological data on the reduction of all visible colors by dichromats at these three loci (Jameson & Hurvich, 1956).

DISCUSSION

In constructing the color discrimination space for color-deficient subjects, we were interested in the number of spatial dimensions needed to interpret the configuration of points in color space.

The second column of Table 4 gives the characteristic roots for 15 normals combined. The contrast between three large and 10 small roots is apparent. To test the meaningfulness of the axes, the values of the coordinates were given on the ordinate, and the wavelengths of corresponding nonspectral colors used were given on the abscissa. Only the first three coordinates gave systematic dependencies. These resemble very much the curves known as opponent functions by Hurvich and Jameson (1955). In accordance with the latter we interpreted the basic dimensions as red–green (X_1), blue–yellow (X_2), and white–black (X_3) opponent systems (Izmailov, 1982).

For anomalous cases, the systematic character of the curves was regarded as the more reliable criterion for determination of three basic factors (see Romeskie, 1978). In color-deficient subjects, three coordinates were also chosen. In this respect, we depart from Helm (1964), who investigated the color space for color-deficient subjects. He found only two dimensions, which were interpreted for normal and deuteranomalous cases as blue–yellow (I) and red–green (II) systems. The most likely explanation of the two-dimensionality of Helm's color space might be that the saturations of the stimuli he used were equal. Apparently, this circumstance made the third (achromatic) axis insignificant.

Except for our mild deuteranomal subject, PR, the blue–

yellow axis was the most significant in all our color-deficient subjects, and this coincides with Helm's results. Moreover, dimension II, which was not labeled by Helm for protanomal JN, is apparently interpreted for our protanope AA as an achromatic axis—the second, in order of size, of the characteristic roots. On the X_1X_2 plane (Fig. 1b) our result agrees with the protanomalous configuration in dimensions I and II in Figure 5 in the paper by Helm.

The dimension considered insignificant for normal and deuteranomalous cases became very important for the protanomal. This fact was ignored, however, by Wish and Carroll (1974), who reanalyzed Helm's data using the INDSCAL method. Those authors did not note the role of this dimension and constructed a total space for red–green-blind subjects with an elliptical configuration stretched along the blue–yellow axis and shortened along the red–green one. Such a solution permits determination of degrees of color deficiency but fails to identify the type of deficiency. The identification of type of color deficiency is possible in the framework of a three-dimensional solution that, added to the change of color axes, emphasizes also the role of achromatic axis for diagnostic purposes (Sokolov, Izmailov, & Paramei, 1980; Sokolov et al., 1984).

It was shown that the colors in normal subjects can be represented as points on a sphere in three-dimensional space. The protan and deutan color points are also represented on a sphere in the three-dimensional space having different degrees of transformation of each coordinate. This causes modification of configuration of color points in the space. Different distortions of the normal configuration of color points in protan and deutan color vision are due to different mechanisms in these two forms of red–green blindness. The three-dimensionality of the color-deficient space indicates that higher levels of the neuronal color-coding system operate properly. At the same time the deficiency of color vision is attributable not just to the receptor level. A retention of the sphericity of color-deficient space may be interpreted as the retention of a normalization process in the color vision system, in which a reduction of any opponent function is compensated by an amplification of the others. The reduction of a color-opponent function causes an exactly compensating gain in a contribution of achromatic function.

The basic result of the present study is support for the universal validity of the spherical model of the three-dimensional color space, both for normal and color-deficient color vision. The construction of individual color maps on the surface of the sphere offers a unique opportunity for precise quantification of color-deficient vision.

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