AN ANALYSIS OF THE RESISTANCE AND

AIRFLOW CHARACTERISTICS OF MINE SHAFTS

BY

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ABSTRACT

During the 1950's there was a flurry of activity on the design of main ventilation shafts for mines. More recently, a new analysis was commissioned in relation to the multi-shaft, nuclear waste isolation facilities currently under design in the United States. This paper presents the major findings of that study and should prove of value as a reference in the design of proposed new shafts.

An overview of mine shaft resistance introduces the effects of the shaft walls, fittings, conveyances and shaft stations. Each of these is analyzed in turn. A completely new technique is described, which facilitates the estimation of wall resistance. The effect of size of a conveyance is quantified and shown to be dominant when the cage occupies more than 30 per cent of the shaft cross-section. The dynamic effects of a moving conveyance are also analyzed.

The study involved a survey of 37 operating shafts encompassing a number of countries. Valuable empirical data were obtained which are presented graphically in the paper.

The paper concludes by giving a step by step procedure for the design of mine shafts that are used for both hoisting and ventilating duties.

INTRODUCTION

In the design of underground repositories for high level nuclear waste in the United States, separate airflow routes must be established for those zones in which emplacement of the nuclear waste is taking place, and those in which mining development is occurring. For this reason, more shafts will be necessary than for a conventional mine. A study was commissioned as part of the Basalt (Nuclear) Waste Isolation Program (BWIP) to investigate those aspects of shaft design that impact upon the ventilation planning for a repository.

This paper summarizes the highlights of that study and is organized into three parts: (1) The determination of shaft resistance; (2) The results of a survey of operating shafts; (3) A procedure for incorporating shaft design into the overall strategic planning of any underground facility.

PART 1: SHAFT RESISTANCE

The resistance to airflow, R, offered by a mine shaft is influenced by four identifiable components:

(a) the shaft walls,
(b) the shaft fittings (buntons, pipes, etc.),
(c) the conveyances (skips or cages) including dynamic effects,
and (d) insets, loading and unloading points.

Each of these four components is examined individually in this part of the paper before combining them to produce a total resistance value.

Shaft Walls

Mine ventilation engineers are familiar with the Atkinson equation for airway resistance:

$$R_w = \frac{k\ L\ per}{A^3} \quad \frac{\rho}{1.2} \quad \frac{Ns^2}{m^8} \qquad (1)$$

where R_w = resistance (Ns^2/m^8) due to wall roughness,
k = friction factor (kg/m^3 or Ns^2/m^4),
L = length of shaft (m),
Per = perimeter of shaft (m),
ρ = actual air density (kg/m^3),
and A = cross-sectional area available for flow (m^2).

Values of k may be found in the literature for various types of airway lining and, also, some that make allowance for combinations of lining and shaft fittings.

A number of authors have, in the past, commented on the fact that the common expression of airway resistance, R, is not a true geometric resistance as it varies with the air density. Furthermore, published lists of the friction factor, k, are also referred to a specified (usually standard) value of air density. Hence the $\rho/1.2$ correction is necessary if equation (1) is to be used.

In order to circumvent the usual annoyances associated with variations in air density and to avoid troublesome corrections, the author has dared, in this paper, to risk the wrath of fellow ventilation engineers by abandoning the traditional concepts of k and R. The former is replaced by the true, original and dimensionless coefficient of friction, f, as used in the well known Chezy Darcy equation:

$$h = \frac{4\ f\ L\ u^2}{2\ g\ D} \quad (m) \qquad (2)$$

where h = frictional head loss (m of fluid),
u = fluid velocity (m/s),
g = gravitational acceleration (m/s^2),
and D = diameter of duct, pipe or airway (m).

(Some authorities use a value of f that is four times the definition used here.) The relationship between f and k is simply:

$$f = 2\frac{k}{\rho} \quad \text{or} \quad \frac{k}{0.6} \qquad (3)$$

when k is referred to standard density of 1.2 kg/m^3.

The resistance, R, is replaced by the rational resistance, r, (McPherson, 1971) where:

$$r = \frac{R}{\rho} \qquad (m^{-4}) \qquad (4)$$

This rational resistance, r, is a true resistance, independent of air density and defined completely by the lining and geometrical configuration of the airway. The corresponding statement of the square law then makes clear the fact that frictional pressure drop, p, varies with the air density as well as resistance and airflow, Q:

$$p = \rho r Q^2 \qquad (= RQ^2)$$

Having introduced f and r, the paper can proceed without any further worries concerning those annoying density corrections. Life would be greatly simplified if all the world's ventilation engineers followed suit.

After making the necessary substitutions from (3) and (4), equation (1) gives shaft wall resistance to be:

$$r_w = \frac{f}{2}\frac{L\ per}{A^3} \qquad (m^{-4}) \qquad (5)$$

In addition to producing simpler and clearer relationships, the employment of f would bring mine ventilation into agreement with other engineering professions who also use the discipline of fluid mechanics. The question now arises, how do we find the coefficient of friction, f, for any given roughness of shaft (or other airway) lining? Well, it is an easy matter to replace published tables of k with the corresponding values of f, or we can still use existing S.I. lists of k, and convert through equation (3), i.e. simply divide by 0.6. There is, however, a more fundamental approach that does not rely on published empirical values and, indeed, can eliminate the need for a coefficient of friction for shaft walls.

The hydraulic roughness of any duct or airway may be quantified as e/D where e is the height of the asperities or projections into an opening of hydraulic mean diameter D. The widely used Moody Chart, found in any basic textbook on fluid mechanics shows that for friction factors, f, of more than 0.005 (k = 0.003 kg/m^3) and Reynold's Numbers exceeding 1,000,000, the friction factor becomes independent of Reynold's Number and varies with roughness, e/D, only. This is the situation in all mine shafts used for ventilation.

The corresponding relationship between f and e/D was shown by the Hungarian fluid mechanician, von Kármán, some fifty years ago, to be:

$$f = \frac{1}{4\ (2\ \log(D/e) + 1.14)^2} \qquad (6)$$

where log is to the base 10.

Strictly speaking, the equation is based on Nikuradse's experiments using uniformly dispersed sand grain roughness whereas, in mine airways, the size of asperities of an unlined opening can vary very considerably. However, experience seems to confirm that the e/D parameter applies reasonably well provided that the walls are not too sinuous and that the shapes of the asperities do not have a directional bias. The latter causes an airway to have a resistance that is dependent upon the direction of airflow. The time may be appropriate for researchers to take a new fundamental approach to the whole question of airway roughness in such circumstances. In the majority of cases, equation (6) works well for mine shafts, even those with tubbed lining. The relationship is shown graphically on Figure 1.

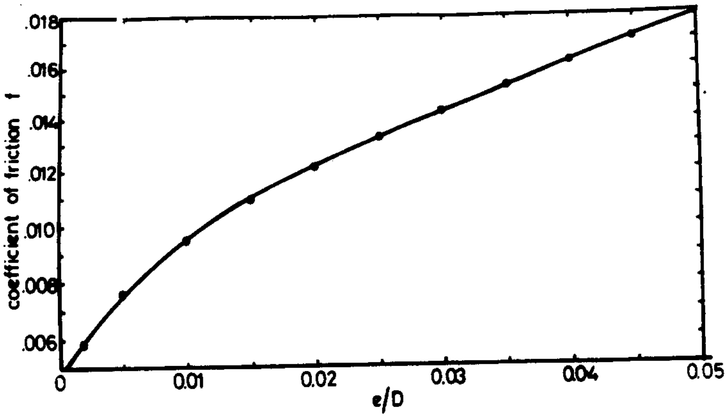

Fig 1. Variation of f with the e/D ratio.

We have now established a straightforward procedure for determining the wall resistance of a shaft.

(a) Determine the ratio e/D and evaluate the coefficient of friction, f, either from equation (6) or Figure 1.

(b) Compute the wall resistance, r_w, from equation (5).

However, it is a simple matter to combine equation (5) and (6) in order to determine the wall resistance directly from e/D, and eliminating the need for a coefficient of friction. The combination gives:

$$r_w = \frac{\text{per } L}{8A^3\ [2 \log (D/e) + 1.14]^2} \quad (m^{-4}) \qquad (7)$$

This equation has been used to produce a series of curves for circular shafts. These are shown in Figure 2 and enable the resistance (per meter length) to be read directly for any combination of e and D. This figure gives a rapid manual means of estimating shaft wall resistance. For more precise results, or for non-circular shafts, equation (7) should be employed with D made equal to the hydraulic mean diameter (4A/per).

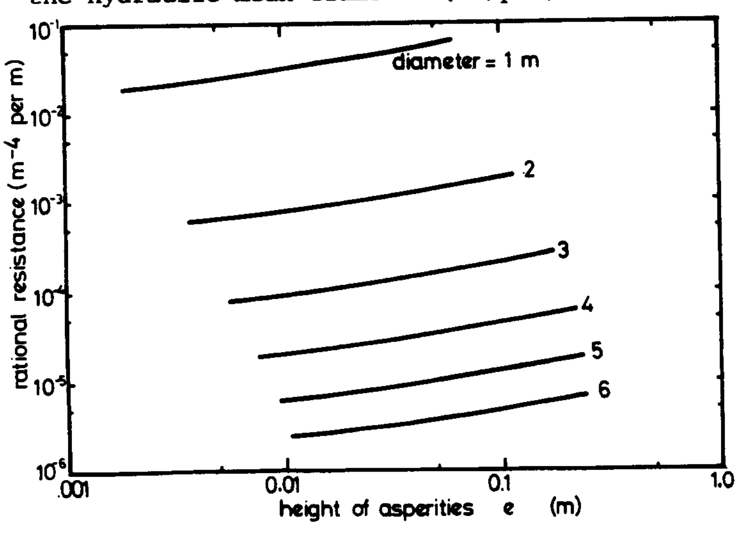

Fig.2. Rational resistance depends on e and D.

Shaft Fittings

Longitudinal fittings such as ropes, guide rails, pipes and cables add to the resistance of a shaft in that they reduce the cross-sectional area available for flow. However, their contribution to skin frictional drag is fairly small compared to the much larger surface area of the shaft walls. Indeed, they may help to reduce swirl. On the other hand, ladderways and associated platforms, often found in metal mines, offer a high resistance and should be compartmentalized behind a smooth wall partition.

Buntons, or other cross-members that are perpendicular to the direction of airflow add greatly to shaft resistance. The drag force on a bunton is given by the expression:

$$\text{Drag} = C_D A_b \frac{\rho u^2}{2} \quad (N) \qquad (8)$$

where C_D = coefficient of drag (dimensionless) depending upon the shape of the bunton,
A_b = frontal or projected area facing into the airstream (m^2),
u = velocity of approaching airstream (m/s),
and ρ = air density (kg/m^3).

The frictional pressure drop, p, caused by a single bunton set is given by the square law and is also equal to the drag/(shaft area).

$$p = \rho r_{1b} Q^2 = \rho r_{1b} u^2 A^2 = C_D \frac{A_b}{A} \frac{\rho u^2}{2} \qquad \frac{N}{m^2}$$

where r_{1b} = rational resistance of a single bunton set.

Solving for r_{1b} gives:

$$r_{1b} = C_D \frac{A_b}{2A^3} \quad (m^{-4}) \qquad (9)$$

This resistance for a single bunton should be multiplied by the total number of bunton sets along the length of the shaft. This may be written as L/S, where L = length of shaft and S = distance between bunton sets. However, unless the buntons are streamlined or far apart, it is probable that turbulent eddies caused by one set will not have died out before reaching the next set. To allow for wake interference and other non-additive effects, Bromilow (1960) developed an empirical "interference factor", F.

$$F = 0.0035\ S/W + 0.44 \quad \text{(dimensionless)} \qquad (10)$$

where W = width of the bunton (m).
This correction applies over the range 10 to 40 for S/W which covers most shafts.

The equation for the resistance due to buntons then becomes:

$$r_b = \frac{L}{S} C_D \frac{A_b}{2A^3}\left[0.0035\frac{S}{W} + 0.44\right] \quad (m^{-4}) \quad (11)$$

Values of the coefficient of drag, C_D, for buntons in shafts are higher than those for free standing bodies of the same shape because of the containment of turbulence within the boundaries of the shaft. Table 1 indicates values of C_D for shaft buntons.

DIRECTION OF AIRFLOW

Section	C_D	*Authority*
I-girder	2·75	SKOCHINSKY.
I-girder	2·05	HOERNER
Rectangle	2·05	,,
Tee	2·00	,,
Triangle	2·00	,,
Plate	1·98	,,
Angle	1·98	,,
Angle	1·82	,,
Triangle	1·55	,,
Square	1·55	,,
Angle	1·45	,,
Capped rectangle	1·40	Estimated from results by SKOCHINSKY BARCZA and HOUBERECHTS
Rounded square	1·35	Approx. value. Value depends on ratio: radius of corner/side of square
Dumb-bell	1·30	Calculated from MARTINSON's results
Cylinder	1·20	HOERNER
Streamlined I-girder	1·06	SKOCHINSKY

Note: The drag coefficients given are approximate values for a Reynolds number of about that for a bunton in a full-scale mine shaft.

Table 1. Drag coefficients of elongated bodies of infinite span [1]. Reproduced by kind permission of the Institution of Mining Engineers (U.K)

Conveyances

The resistance of a cage or skip may be considered in two parts; (a) that due to the obstruction to airflow by a stationary conveyance, and (b) the transient effects caused by motion of the conveyance.

(a) Resistance of a stationary conveyance.

The shock (pressure) loss incurred when an airstream is caused to change direction is usually quoted as a multiple of velocity heads, i.e.

$$p_c = X\rho\frac{u_a^2}{2} \quad (N/m^2) \quad (12)$$

where p_c = frictional pressure drop due to the stationary conveyance (N/m^2)
u_a = velocity of approaching airstream (m/s)
and X = shock loss factor

By combining equation (12) with the Square Law, the effective resistance of the stationary cage, r_c, can be expressed in terms of X.

$$p_c = \rho r_c Q^2 = \rho r_c u_a^2 A^2 = X\rho u_a^2/2$$

$$\text{or } r_c = \frac{X}{2A^2} \quad (13)$$

where A = free shaft area (m^2).

The problem now becomes one of evaluating X. One of the most comprehensive laboratory studies on cage resistance was carried out by A. Stevenson in 1956 using physical models. A new analysis of his observation data led to the production of Figure 3. This shows the variation of X with respect to coefficient of fill, C_f, the latter being defined as the fraction of the shaft cross sectional area, A, that is filled by the plan area of the conveyance, a, or

$$C_f = a/A \text{ (dimensionless)} \quad (14)$$

The curves on Figure 3 encompass the cage height/ width (H/W) range of 1.5 to 6.0 and refer to a length/width (L/W) ratio of 1.5. For other values of L/W, the multiplying correction factor included in Figure 3 may be employed. Furthermore, the curves refer to cages with open ends. Totally enclosed skips give a lower resistance and the X factor should be reduced by 15 per cent (Bromilow, 1960).

A major feature of Figure 3 is the dominant effect of coefficient of fill, the shock loss increasing rapidly after C_f exceeds 0.3 or 30 per cent. The significance of the vertical height of conveyance lies in the length and restricted area of shaft through which the flow is constricted around the cage, rather than increased skin friction. Hence, the H/W ratio has little effect at low C_f but is of increasing influence at higher values of C_f.

Each of the curves on Figure 3 has the form:

$$X = a + b\, C_f^{\,c} \qquad (14)$$

where a, b and c are constants for that curve. However, to generalize all of the information given on the figure, and to facilitate incorporation into computer programs, a trend surface analysis was carried out on Stevenson's experimental data. This led to a useful algorithm which is given in the appendix.

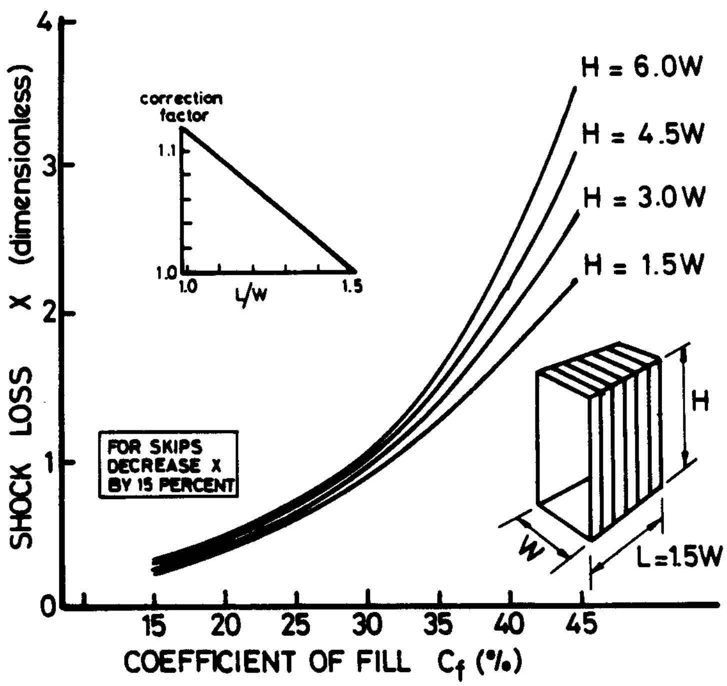

Fig.3. Shock loss due to a conveyance varies with the coefficient of fill.

(b) Dynamic resistance of a moving conveyance.

The motion of a conveyance will influence the effective pressure drop and resistance of a mine shaft. Consider a conveyance moving at its maximum velocity, u_c, against an airflow of velocity u_a (Figure 4). If the resistance of the cage, when stationary, is r_c (determined from the methodology of the previous section), then the corresponding pressure drop caused by the standing cage will be:

$$p_c = r_c\,\rho\,Q^2 = r_c\,\rho\,u_a^2 A^2 \quad (N/m^2) \qquad (15)$$

Now suppose that the cage were stationary in the shaft when the free air velocity was u_c. The corresponding pressure drop would be.

$$p_c' = r_c\,\rho\,u_c^2 A^2 \quad (N/m^2) \qquad (16)$$

This expression approximates the additional pressure drop caused by movement of the conveyance. The total pressure drop, p_{max}, across the moving cage is then given by summing equations (15) and (16).

$$p_{max} = p_c + p_c' = r_c\,\rho(u_a^2 + u_c^2)A^2 \quad (N/m^2) \qquad (17)$$

However, if the effective resistance of the moving conveyance is r_{cmax} then the square law gives:

$$p_{max} = r_{cmax}\,\rho\,Q^2 = r_{cmax}\,\rho\,u_a^2 A^2 \quad (N/m^2) \qquad (18)$$

Comparing equations (17) and (18) shows that

$$r_{cmax} = r_c \left[1 + \frac{u_c^2}{u_a^2} \right] \quad (m^{-4}) \qquad (19)$$

Similarly, when the cage is moving at its maximum velocity <u>with</u> the airflow, its effective resistance becomes

$$r_{cmin} = r_c \left[1 - \frac{u_c^2}{u_a^2} \right] \quad (m^{-4}) \qquad (20)$$

Hence, the effective resistance of a conveyance in a hoisting shaft will oscillate between r_{cmax} and r_{cmin}. If the velocity of the cage, u_c, exceeds that of the airflow, u_a, then r_{cmin} (and the corresponding pressure drop) become negative. In this case, the motion of the cage assists rather than impedes the airflow. The cyclic variation in pressure may be measurable throughout the ventilation network and particularly at locations close to the shaft. This is most likely to occur if the conveyance contributes a large proportion of the overall shaft resistance, i.e. if the coefficient of fill is high. A further effect may be the imposition of a fluctuating load on main fans.

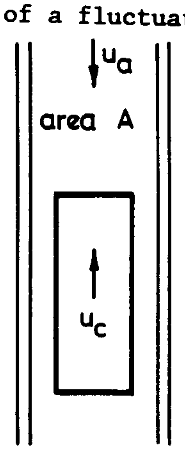

Fig.4. Moving conveyance in a shaft.

In the case of two conveyances of equal dimensions and travelling in synchronization, but in opposite directions, these dynamic effects cancel out. There is, however, a short-lived and rapidly damped pressure pulse when the cages pass. Aerodynamic effects also cause lateral forces and vibration on passing conveyances (McPherson, 1987).

Entry and Exit Losses

The shock losses at insets, loading and unloading points or any other point of air entry or exit may, again, be converted into a resistance

$$r_{sh} = \frac{X}{2A^2} \quad (m^{-4}) \qquad \text{(see eqn}^n \text{ 13)} \quad (21)$$

The literature gives some assistance in estimating X factors for simple geometries (e.g. Hartman, 1982). Conditions are seldom ideal, particularly at loading/unloading stations and measurements of actual installations indicate wide variations in X values. As far as the author is aware, there is no method currently available for the analytical determination of such shock losses, although numerical simulations of flow patterns can be helpful. Physical models still appear to be the best means of investigating proposed designs of insets.

Entry and exit losses can be reduced by aerodynamic design of the shaft geometry at those locations. At loading/unloading points, widening of the shaft, increasing the height of the station so that the stationary conveyance is clear of the shaft, or air bypass drifts all assist in reducing shock losses.

Overall Shaft Resistance

Although there will be some interaction between the four major components of shaft resistance, it errs on the side of conservative design to assume that they are additive. Equations (7), (11), (19), (20), and (21) then give the total resistance of the shaft as:

$$r_{tot}=r_w+r_b+r_c\left[1\pm\frac{u_c^2}{u_a^2}\right]+r_{sh} \quad (m^{-4}) \qquad (22)$$

PART 2: SURVEY OF OPERATING SHAFTS

When designing a main shaft, engineers will wish to compare their proposed configurations with those of similar existing shafts elsewhere. Such information is not always readily available to private companies. As part of this study, a survey of sixteen major mining organizations, encompassing several countries, produced detailed data for 37 shafts used both for hoisting and as main airflow routes. This section of the paper indicates the highlight findings of that survey. Although 37 shafts are a very small sample from the world of mining, they represented coal, metal and other non-metal mines owned by large operating companies. Detailed results are given by Wallace and Rogers, 1987.

The depths of the shafts varied from 354 to 2264 m. Table 2 gives some of the shaft configuration data. It is clear that in this sample, concrete lining and rigid guides are favored; a reflection, perhaps, of the fact that the survey was directed specifically to shafts used jointly for hoisting and ventilation. This may also explain the dominant number of downcast shafts reported.

	Number of Shafts
Lining	
Concrete	30
Concrete/tubbing	1
Concrete/steel	1
Unlined	5
Guides	
Rope	6
Rigid	28
Combined rope/rigid	3
Airflow direction	
Downcast	31
Upcast	6
Hoisting function	
Mineral	12
Men and material	11
Combined mineral, men and material	14
Number of Conveyances in a Single Shaft	
One	8
Two	13
Three	7
Four	4
Five	1
Six	3
Seven	1

Table 2. Summary of 37 shafts included in the survey.

Figure 5 shows that 26 out of the 37 coefficients of fill were less than 30 per cent (above which the conveyance shock loss increases rapidly (Figure 3)). The eight shafts reporting coefficients of fill of more than 50 per cent were the single conveyance shafts included Table 2. These were all men and materials shafts. The largest reported coefficient of fill was 57.7 per cent.

Figure 6 shows the distribution of free area air velocity. The most popular range was 5.0 to 7.5 m/s. A rule of thumb often adopted by ventilation planners is that the free area velocity of airflow in a hoisting

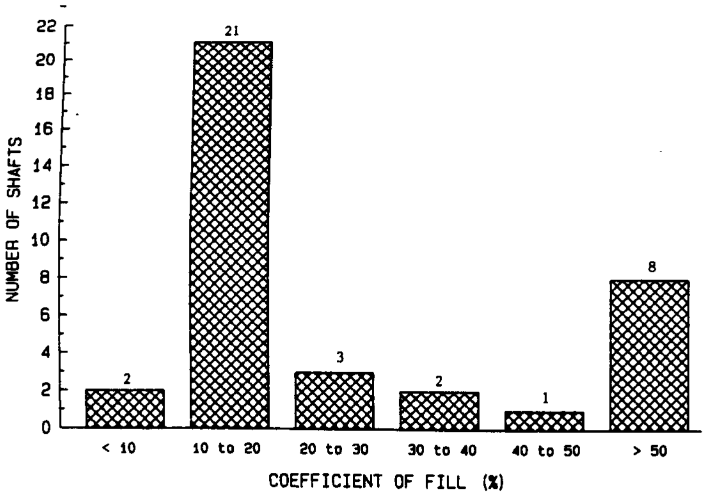

Fig.5. Distribution of coefficient of fill.

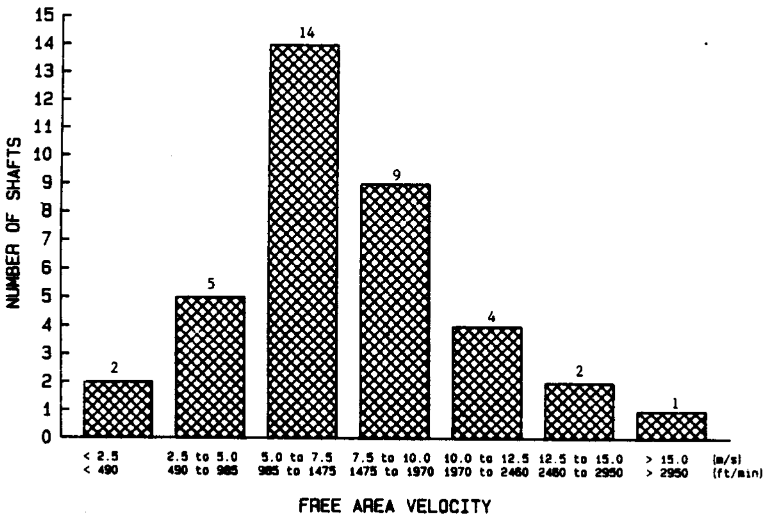

Fig.6. Distribution of free area air velocity.

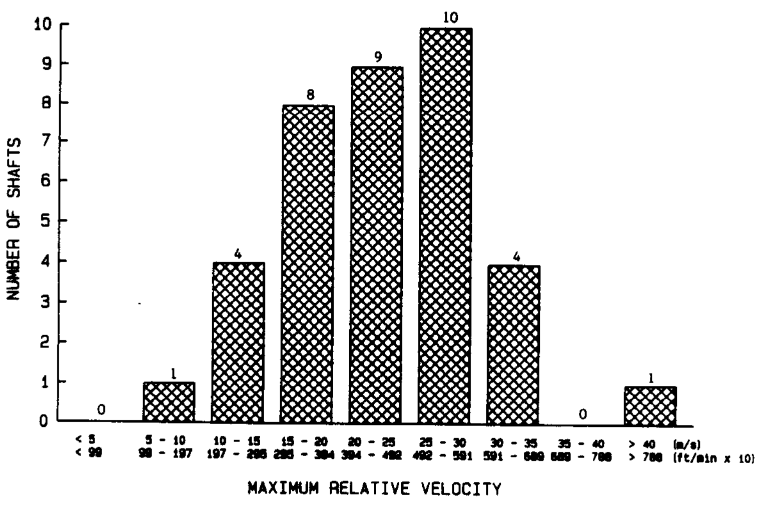

Fig.7. Distribution of maximum relative velocity between the airflow and the conveyance.

shaft should not exceed 10 m/s. This is fairly well reflected in the survey with only seven shafts reporting higher velocities. The highest air velocity was 17.85 m/s. For comparison, ventilation shafts that are not used for hoisting have velocities limited primarily by economics. A maximum design velocity of 20 m/s is often adopted in these cases, with the range 7 to 12 m/s precluded in wet upcast shafts in order to avoid water blanketing.

An important parameter in governing the stability of a moving conveyance is the maximum relative velocity between the conveyance and the air passing it. The maximum relative velocity can be shown to be (Wallace and Rogers, 1987):

$$u_{rel} = \frac{u_a + u_c}{1 - C_f} \quad \text{(m/s)} \tag{23}$$

Figure 7 shows that the majority of shafts reported relative velocities of less than 30 m/s. In one exceptional case, the relative velocity was 50 m/s. This was a two-skip shaft with a high free air velocity, the greatest reported skip speed and a coefficient of fill (for each skip) of 35 per cent. The skip in this shaft has been fitted with angled lips to counteract a "sideways drag" that occurred when a skip was moving against the airflow.

PART 3: SUMMARY OF DESIGN PROCEDURE

The demands made upon a mine shaft may include access for personnel and equipment, the hoisting of planned tonnages of rock and the passing of specified airflows. This paper has concentrated on those features of shaft design that pertain to ventilation. However, the design of a shaft must meet all of the demands made upon that shaft. Based on the findings of this study, a set of guidelines has been compiled to assist in the management of a shaft design exercise:

1. Assess the duties required for rock hoisting (tons/hour), number of personnel to be transported and time allowed at shift changes, and the size, weight and frequency of hoisting materials and equipment.
2. Determine alternative combinations of conveyance sizes and hoisting speeds.
3. Conduct ventilation network analyses, initially on the basis of an estimated shaft resistance, in order to establish required airflows in the shaft.
4. Assess the dimensions of proposed shaft fittings, including pipes, cables, guides and buntons.

5. Conduct an optimization exercise (computer programs are now available for this) to find the optimum size of shaft that will pass the required airflow at the minimum combination of operating costs and capital expense of shaft construction
6. Check the free area velocity. If this exceeds 10 m/s in a hoisting shaft or 20 m/s in a pure ventilation shaft, then the cost of enlarging the shaft should be reviewed -the total cost vs. diameter curve is often fairly flat above its minimum (optimum) point. Furthermore, sizing a shaft slightly above optimum gives additional flexibility. Hard won experience has show the wisdom of such foresight.
7. Determine the coefficient of fill for the largest of the proposed conveyances. Should this exceed 30 per cent for two or more conveyances, or 50 per cent for a single conveyance shaft, then the dimensions of the skip or cage should be reviewed and/or the size of the shaft, again, re-examined.
8. Determine the maximum relative velocity between the airflow and the largest conveyance, $(u_a+u_c)/(1-C_f)$. If this exceeds 30 m/s then additional precautions should be taken to ensure stability of the conveyances. In any event, the relative air velocity should not exceed 50 m/s.
9. Examine the air velocities at all loading/unloading points and, if necessary, redesign the excavations to include air bypasses or enlarged shaft stations.
10. Determine the total resistance of the shafts, including the transient effects of moving conveyances. Examine all feasible means of reducing the resistance including the reduction and streamlining of buntons and aerodynamic design of intersections. For shafts of major importance, construct and test physical models of lengths of shaft and main intersections.
11. Rerun ventilation network analyses with the established value(s) of shaft resistance in order to determine the final fan pressures required.

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APPENDIX

Algorithm to compute the shock loss factor, X, for a stationary cage.

Required data: (i) Height (H), Width (W), Length (L) of conveyance as defined on Figure 3; (ii) Coefficient of fill, C_f, expressed as a fraction

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a = (3.056 H/W + 5.378)**0.3986
b = 11.254 + 2.206 ((H/W)**1.659)
c = 0.0387 H/W + 0.0641
X = c + b (Cf ** a)
X = X (1.36 - 0.24 L/W)
X = 0.85X (for skips only)
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Notes: (i) ** denotes "Raised to the power of"
(ii) The algorithm is applicable for the ranges 1 to 1.5 for L/W; 1.5 to 6.0 for H/W; and 0.1 to 0.45 for C_f
(iii) a, b and c are local variables for the algorithm
(iv) The units of H, W and L must be consistent to give a dimensionless X

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