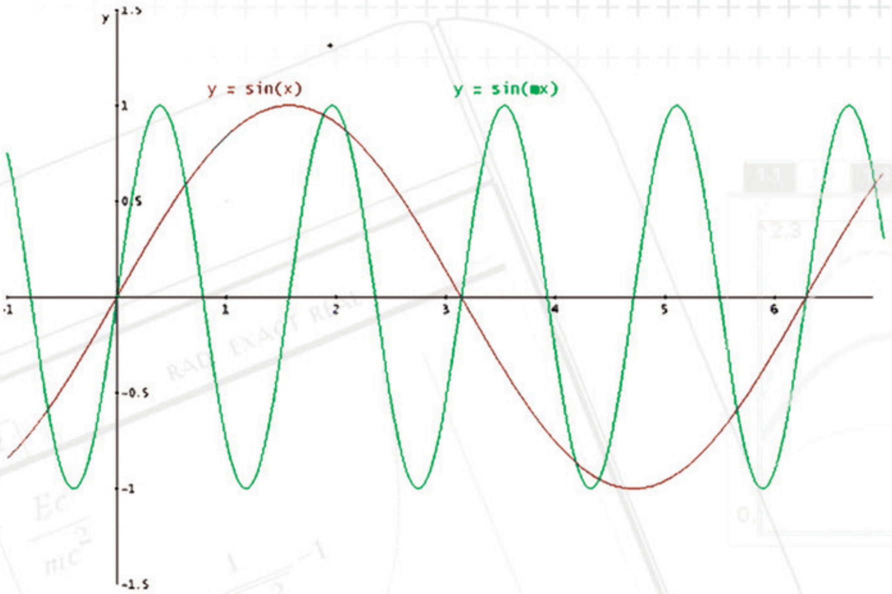
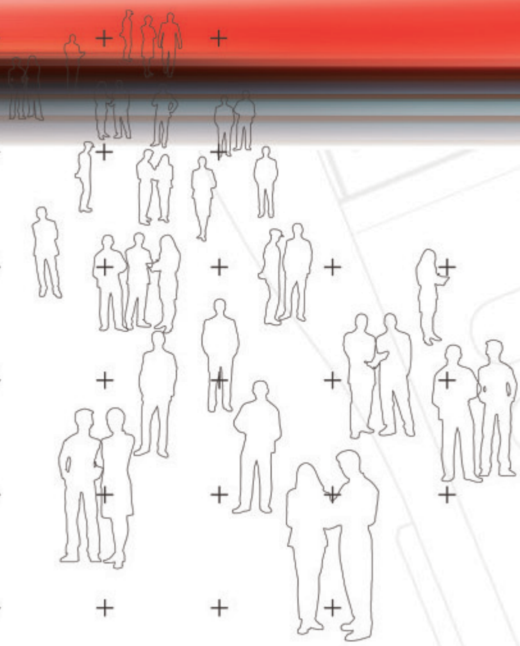
Computer Algebra Systems

in the Mathematics Curriculum





Report of the invitation MEI seminar

supported by Texas Instruments

Held on 27th May 2008

at the London Mathematical Society

$$E_k = m\gamma c^2 - mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

MEI Mathematics in Education and Industry

Innovators in Mathematics Education

Texas Instruments

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Background

The rationale for the seminar

Over recent years technology has progressed to the point where many Computer Algebra Systems (CAS) are widely available. These technologies can perform a wide variety of algebraic manipulation. They include programs such as Mathematica and Maple which can operate on computers and handheld devices such as the TI-Nspire.
In this time the rules governing the use of calculators in school examinations in England have remained relatively unchanged. Although scientific and graphical calculators are allowed in most examinations at GCSE and A level, calculators with the capacity to perform algebraic manipulation are restricted.

During this period many other countries have adopted a different approach to Computer Algebra Systems by embracing them and including them in the school curriculum.

A considerable amount of research has been carried out on the use of Computer Algebra Systems in schools around the world. However, the use of CAS in English schools is very rare. The situation in universities is very different; such technologies are a common feature in many undergraduate courses, especially in Physics and Engineering.

MEI and TI decided to draw together people from a variety of fields to consider whether there was potential to use Computer Algebra Systems effectively in the school mathematics curriculum and if so how best to achieve this.

The aims of the seminar were to:

- explore the potential of Computer Algebra Systems in the mathematics curriculum
- seek a consensus view amongst stakeholders as to a realistic way forward
- recommend a possible framework for curriculum development.

The seminar programme

Computer Algebra Systems in the Mathematics Curriculum
27 May 2008 at the London Mathematical Society

The programme:

Welcome and introduction

Presentations

- **Professor Celia Hoyles, NCETM**
  My perception of the mathematical needs of the country: the potential & challenge of using ICT
- **Ian Forbes, University of Edinburgh**
  The CASE for CAS?
- **David Leigh-Lancaster, Victoria CAA**
  Practical implications – adoption in Victoria, Australia
- **Conrad Wolfram, Wolfram Research**
  Building a curriculum ground-up with computer maths – a new vision, new challenges

Panel discussion with all keynote speakers

Breakout group discussions

Report back from the break-out groups

Closing plenary

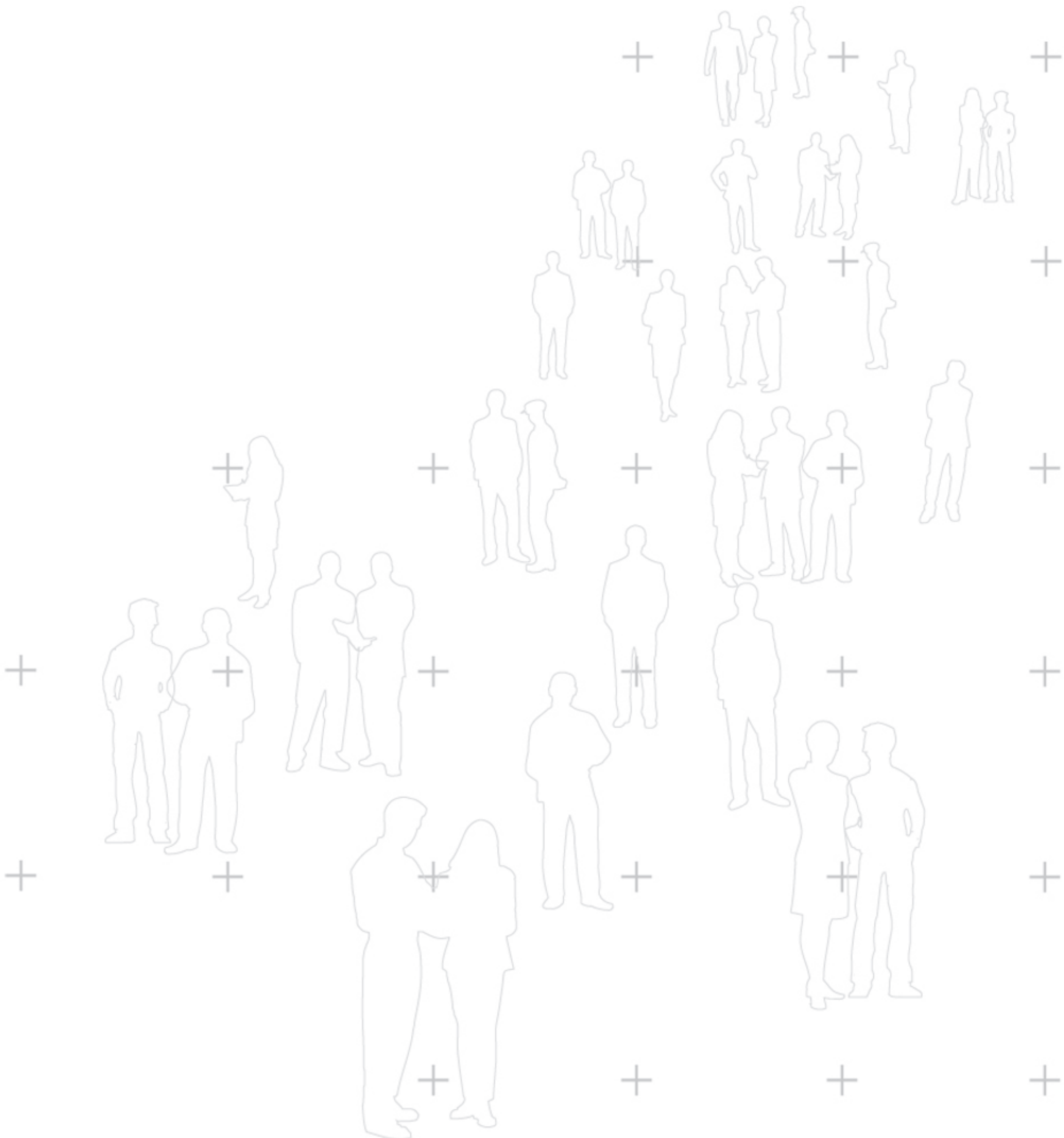
The seminar was chaired by **Neil Sheldon**, OCR Chief Examiner for Pure Mathematics and Numerical Analysis.

How the delegates were identified for invitation

Working together, Texas Instruments (TI) and MEI set out to invite delegates from a variety of fields representing the following:

- Teachers
- University tutors in STEM subjects
- University mathematics educators
- Representatives of QCA and the examination boards
- Mathematics technology producers
- Other delegates with an interest in the school mathematics curriculum

The organisers were very pleased with and grateful to those who attended. The delegates were an interesting and representative group of people who were able to discuss the agenda creatively.



Executive Summary

Introduction

Through a series of presentations and discussions the seminar considered the potential to use CAS in the school mathematics curriculum.

Presentations

- **Professor Celia Hoyles** gave her perception of the mathematical needs of the country and how the potential for using ICT related to this, highlighting the role of NCETM in meeting the professional development needs of mathematics teachers.
- **Ian Forbes** presented the "Case for CAS" based on the book of the same name.
- **Dr David Leigh-Lancaster**, of the Victorian Curriculum and Assessment Authority, described his experiences of integrating CAS into the teaching and assessment of senior secondary mathematics in Victoria, Australia.
- **Conrad Wolfram**, of Wolfram Research, gave his thoughts on how a new mathematics curriculum could be developed if the computer were to be fully exploited in the mathematics classroom.

Discussions

The discussions considered whether there was a place for the use of CAS in the school mathematics curriculum and, if so, how best to implement this. The general consensus was that there is potential to exploit CAS in students' learning of mathematics. It was suggested that introducing CAS into the external assessment of mathematics would ensure its use in the classroom. If CAS is to be used effectively then sufficient professional development opportunities for teachers are needed.

Key points and the way forward

1. ICT-based mathematics qualifications, where students have access to appropriate devices in the classroom and examinations, are an exciting possibility for the future.
2. If CAS were to be introduced hurriedly into the school curriculum, it would be likely to be unsuccessful.
3. A short-term minor adjustment to the current A level would be disruptive and not necessarily produce positive results.
4. Exemplars of good classroom practice are needed and a research programme should be set up to produce these.
5. When good practice is understood it will be possible to move forward, including assessment.
6. The introduction of CAS into schools will require significant professional development for the existing teachers of both mathematics and science.

Contributions from the principal speakers

Celia Hoyles

My perception of the mathematical needs of the country: the potential & challenge of using ICT

Professor Celia Hoyles, Director of the National Centre for Excellence in the Teaching of Mathematics (NCETM), gave a presentation about how she perceived the role of ICT in helping teachers meet the mathematical needs of the country.

The vision of the NCETM is to develop a sustainable national infrastructure for subject-specific professional development of teachers of mathematics that will

- enable the mathematical potential of learners to be fully realised
- raise the status of the profession.

The NCETM is based on the assumption that teaching can be improved through collective effort and ICT invariably serves as an excellent catalyst for this collaborative engagement. This is highlighted by the fact that teachers have expressed the view that time to explore ICT, and design lessons with ICT, is one of their primary mathematics professional development needs. To achieve this NCETM organised a national event: "Potential of ICT in Teaching & learning Mathematics" in March 2008 that was attended by over 350 participants. The event showcased the work of teachers in collaborative groups using ICT in mathematics.

Professor Hoyles expressed the personal view that whether a person(s) can learn and do more mathematics with a computer than with traditional static media is unquestionable. However, we must go beyond generalities and be more explicit about the potential offered to mathematics teaching and learning and also the challenges it brings.

ICT has the potential to offer

- dynamic & visual tools to explore mathematics in shared space
- tools to outsource processing
- new representational infrastructures for mathematics that change what is learned
- visual, dynamic connectivity with real-time interaction
- personal technology & mobility.

Using ICT as a tool to outsource processing can allow students to focus on mathematical concepts by 'setting them free' from technical procedures (such as calculation, manipulation, equations, testing equivalence, graph plotting etc). But the relationship between technical & conceptual is problematic and there may be some costs to outsourcing such as loss in algebraic fluency. There is also an investment of time needed to gain fluency with ICT.

Using ICT can bring with it some challenges. In particular there are barriers relating to ensuring that there is the 'right' technological infrastructure so that students and teachers can access ICT when appropriate. In addition to this ICT use can increase complexity so teachers have to be convinced of the value.

The NCETM has funded projects showcasing the work of teachers using ICT in mathematics and supporting their dissemination. It aims to help other provision and to support groups to design research. In this way it can serve as a catalyst to build and coordinate both supply and demand for professional development opportunities in use of ICT.

Finally, a thought provoking question was posed, 'What can we learn from all the undergraduate students in subjects such as engineering who use CAS as part of their mathematical studies?'

Ian Forbes

The CASE for CAS?

Ian Forbes, the University of Edinburgh, gave a presentation on 'the case for CAS', which is also the name of a book which Ian co-authored.

The book 'The CASE for CAS', **(http://www.t3ww.org/pdf/TheCaseforCAS.pdf** - Boehm, Forbes, Herweyers, Hugelshofer and Schomacker, 2004) argued that

a) teaching of mathematics becomes more interesting with CAS
b) students are more interested and motivated to learn mathematics with CAS
c) students who use CAS are at least as good in 'pencil and paper' skills as their traditional counterparts
d) high stakes assessment and CAS are compatible.

a) Teaching of mathematics becomes more interesting with CAS

The CASE for CAS demonstrates some examples of activities that showed how CAS can be used to teach for understanding.

Here Ian also provided food for thought with an interesting Bert Waits (2000) statement:

"Some Mathematics becomes more important because technology requires it. Some Mathematics becomes less important because technology replaces it. Some Mathematics becomes possible because technology allows it."

b) Students are more interested and motivated to learn mathematics with CAS

Many examples of quotes from students who had learnt using CAS were given including:

- "We all like Maths because it is real and you do not have to remember lots of facts like in History."
- "We don't want to give up when boundaries arise."
- "Technology gives us greater freedom to solve problems by ourselves."
- "You know why you are studying it and it is fun."

c) Students who use CAS are at least as good in 'pencil and paper' skills as their traditional counterparts

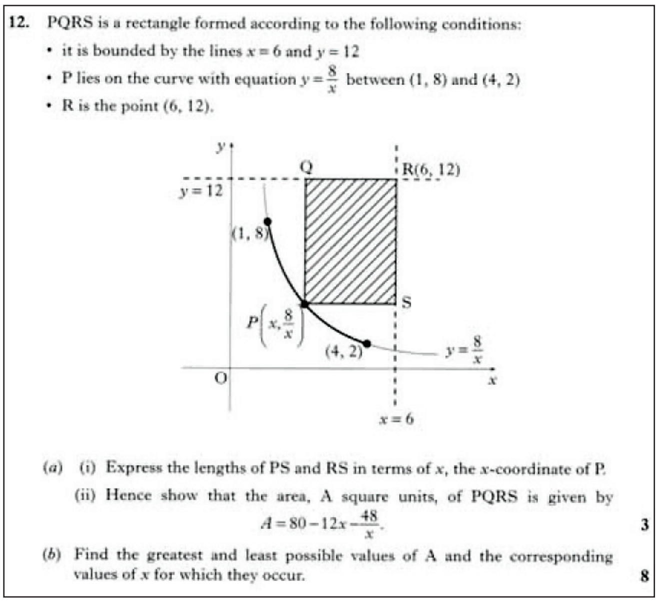
In the International Journal of Computer Algebra in Mathematics Education (2002) Forbes and MacIntyre cited that much research indicates, contrary to speculation, that algebraic skills can actually improve using CAS. For this it is important that CAS is used appropriately and that the assessment does not sabotage this potential.

d) High stakes assessment and CAS are compatible

The perception that CAS cannot be integrated into high stakes assessment has been the main barrier to progress. Indeed, CAS has not been allowed in exams in Scotland since 1996 but this ban is not practical. This is a case of the 'Assessment tail' wagging the 'Curriculum dog'.

Recently, the education minister put pressure on Scottish Qualification Authority, to design a new stand-alone unit exploiting technology which schools could opt-in to. The result was 'Mathematical Modelling with Technology' with the following three outcomes:

- The candidate will be able to model a mathematical problem using Dynamic Geometry software or a hand held device, which embodies such software.
- The candidate will be able to model a mathematical problem using Computer Algebra software or a hand held device, which embodies such software.
- The candidate will be able to demonstrate the ability to model mathematical problems through the use of technology to include Dynamic Geometry and Computer Algebra (CAS) software and, by keeping a folio, communicate the findings to a teacher or group of students.



12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).

(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

Figure 1

In Ian’s view Mathematics in Scotland (and in the UK) is taught to students for them to pass examinations and not for understanding. His talk compared approaches and illustrated how students currently are taught and how they learn. Figure 1 shows an examination question in which the higher order thinking is awarded three marks and the routine procedures given eight marks. This was contrasted with an experiment where students explored mathematics with hand held CAS devices, allowing multiple representations of the problem to be explored as shown in Figure 2.

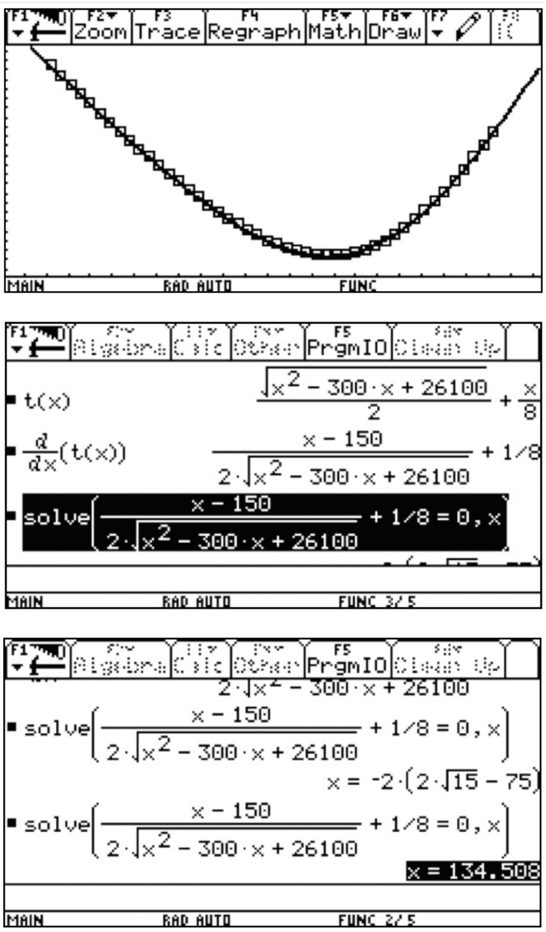


Figure 2

Ian concluded that the use of technology in this way allows us to consider anew Confucius’ saying, “If you give a man a fish, you feed him for a day. If you teach a man how to fish, you feed him for a lifetime.” and perhaps suggest a modification to make it read, “Give your students some mathematics and you feed them for the next exam. Teach your students to fish for mathematics and you feed them for a lifetime.”

Conrad Wolfram

Building a curriculum ground-up with computer maths – a new vision, new challenges

Conrad Wolfram from Wolfram Research gave a presentation about his vision of what a new mathematics curriculum could look like if the full power of the available technology was exploited.

Mathematics is clearly important for an increasing number of technical jobs in a quantitative world. It also develops the ability to think logically. Neither of these aims is supported by excessive focus on the mechanics of calculating. The broader mathematics processes are:

1. translating a problem to a mathematical form (i.e. framing the question)
2. deciding what result is required mathematically
3. doing the calculation (i.e. moving from the mathematical beginning point to the mathematical end point)
4. interpreting and validating the result.

Mathematics education often currently focuses on the third of these stages even though this is the only one that computers can do better! Many examples were displayed that demonstrated how using technology to perform the calculations, manipulations, or to demonstrate results graphically could free students to concentrate on the other three processes. There is also the added advantage that real, often complex, datasets can be easily used.

The following features were stated to be desirable for a new curriculum:

- The reason for teaching a topic must be
    - a) It is useful for a technical job
    - b) It is useful for every day living
    - c) It is culturally highly significant...

    … or it is leading to these.

- Manual computation should only be taught when
    - a) It explains a concept
    - b) It is important to be able to perform it without a computer.
- Exercises and examination questions should solve real-world problems.
- The emphasis should be on problem set-up and interpretation, not the mechanics of calculating.
- Validation of results is important based on judgements made by approximating.
- Computers must be available for teaching, homework, examinations (which should be open-book).
- Collaborative problem-solving is built in.

However, there are some potential pit-falls to a technology-based curriculum that would need to be avoided:

- 'Mindless button-pushing': this is no better that mindless manual computation. This can be avoided with appropriate open-ended tasks
- Education-only technology: this will quickly become outmoded and not be relevant afterwards
- Push for computers to replace teachers: good teachers are essential to implement any new curriculum.

Mathematics is both more important than ever and more derided than ever. A computer-aided mathematics curriculum is the silver bullet that can bridge this gap. This would allow us to:

- Teach much more applicable mathematics skills, relevant outside education
- Create problems sorted by conceptual difficulty, not calculating mechanics
- Make maths fun!

David Leigh-Lancaster

Practical implications – adoption in Victoria, Australia

Dr David Leigh-Lancaster, Mathematics Curriculum Manager, Victorian Curriculum and Assessment Authority (VCAA) gave his perspective on the implementation of the use of CAS in senior secondary mathematics. What follows is a short technical article which was written by David.

Background

While CAS have been around for over 20 years, and are commonly used tools in these areas, their use in early undergraduate and senior secondary mathematics courses is a relatively recent phenomenon. In the senior secondary school context, CAS were initially used as pedagogical tools, and subsequently incorporated to varying degrees in curriculum design. Their permitted use in formal examinations (or some components of formal examinations) has developed slowly and selectively, and in quite specific contexts, from the mid to late 1990s. While most systems permit (if not actively require) the use of graphics calculators in some component final examinations, the use of CAS is not widely permitted or expected in such examinations. However, there are an increasing number of systems around the world that do permit the use of CAS (eg France, the College Board, Queensland Australia - has an entirely school based assessment regime) or require the use of CAS (eg Denmark, Victoria Australia, Western Australia). Other systems such as Switzerland and Austria leave the choice to teachers, while New Zealand has a modularised standards approach and provides for technology free units and equivalent technology active units for graphics calculators and for CAS calculators.

In Australia, over the past decade, there has generally been strong bi-partisan support for technology as a positive driver of economic growth and prosperity. The clear directions and strong support of the VCAA Board have provided a robust framework from which the developments in Victoria have proceeded. The practical motivations for such developments included (as they also do for new technologies such as CAS), freeing up time to work on other matters; to increase the speed of computation; and to increase the reliability of computations.

The senior secondary Victorian Certificate of Education (VCE) Mathematics curriculum in Victoria is based on paired semester long sequences of units, with Units 1 and 2 corresponding to a Year 11 (penultimate year) level of study and Units 3 and 4 corresponding to a Year 12 (final year) level of study. The Unit 3 and 4 sequences are assessed by a combination of two examinations, worth 66% of the final score, and VCAA prescribed but teacher devised and school assessed (coursework) tasks, worth 34% of the final score, with coursework statistically moderated against the examinations. The Unit 3 and 4 VCE Mathematics studies are shown in Table 1.

The pilot

Table 1: Unit 3 and 4 VCE Mathematics studies

Study	Nature	Examinations and technology 2006 - 2009
Further Mathematics	A practical applications and discrete mathematics based study with a data analysis core and a selection of three from six available modules	Two 1.5 hour examinations, approved graphics calculator or CAS (calculator or software*) assumed
Mathematical Methods	Mainstream function, algebra, calculus and probability study	One 1 hour technology free examination (common with the CAS study) One 2 hour examination, approved graphics calculator assumed
Mathematical Methods (CAS)	Mainstream function, algebra, calculus and probability study	One 1 hour technology free examination (common with the non - CAS study) One 2 hour examination, approved CAS (calculator or software*) assumed
Specialist Mathematics	Assumes concurrent or previous study of Mathematical Methods / Mathematical Methods(CAS) Advanced pure and applied mathematics eg, calculus and differential equations, complex numbers, vectors, mechanics	One 1 hour technology free examination One 2 hour examination, approved graphics calculator or CAS (calculator or software*) assumed

* subject to separate VCAA approval process on a school by school basis - only a small number of schools have used this option

The mechanism the Board of Studies, and its successor, the VCAA chose to was to implement a three stage pilot from January 2001 to 31 December 2005. These units were developed from the then current Mathematical Methods study through the work of the Mathematical Methods (CAS) accreditation panel, informed by the early work of the computer algebra (CAS-CAT) research project. Thus, the pilot Mathematical Methods (CAS) Units 1 – 4 were a natural evolution of Mathematical Methods Units 1 – 4.

At the request of the VCAA, the Victorian Qualifications Authority (VQA) approved a two year extension of the accreditation period for the current VCE Mathematics study until December 2005, which enabled the review process for Mathematical Methods and Mathematical Methods (CAS) to proceed concurrently. The pilot examinations for Mathematical Methods (CAS) 2002 - 2005 had the same structure and format as the Mathematical Methods examinations and were held at the same time. Examination 1: Facts, skills and standard applications, was a one and a half hour multiple choice and short answer paper while Examination 2: Analysis task, was a one and a half hour extended response paper. As was the case at that time with graphics calculators for Mathematical Methods, both Mathematical Methods (CAS) examinations assumed student access to an approved CAS.

Following an excellent response from schools to an invitation for expressions of interest in the first half of 2001, the VCAA implemented the second and third stages of the expanded VCAA pilot program 2002 – 2005. This included the original three pilot schools, and an additional 16 schools, staggered with respect to when students will first undertake Units 3 and 4, and corresponding examinations, from 2003 and from 2004 respectively.

Factors which were important to the pilot's smooth implementation included:

- the accreditation of a well designed CAS study that was evolutionary from the accredited Mathematical Methods Units 1 – 4 study, and which clearly and explicitly specified required mathematics and important mental, by hand and CAS concepts, skills and processes through both its areas of study and its outcomes;
- the goodwill, interest and enthusiasm of teachers, students and schools involved in the pilot.

Information about the pilot study and related research was disseminated through a variety of mechanisms such as the CAS pilot section of the VCAA website, the CAS-CAT project, the Mathematical Association of Victoria website, the VCAA Bulletin, conferences, professional development workshops and various papers and articles published in newspapers, research and teacher professional journals and through professional dialogue by teachers from pilot schools.

In November 2003, the VCAA Board approved the Mathematics Expert Studies Committee recommendation that Mathematical Methods (CAS) be fully accredited for the next accreditation period, as a parallel and alternative study to Mathematical Methods, available for all schools from 2006. The Board also approved, in principle, the recommendation that the calculus based studies have a technology free and technology assumed examination structure, (with a common technology free paper for Mathematical and Mathematical Methods (CAS)), subject to consultation on models for these examinations with key stakeholders.

After several months of consultation, the current examination structure was put in place for the 2006 - 2009 accreditation period. The examinations for these and technology allowed are displayed in table 1. The VCAA also foreshadowed in 2005 and 2006, that the parallel studies would be merged to a single CAS enabled Mathematical Methods study for 2010, and that the advanced mathematics study would also be CAS enabled.

In November 2008, the VCAA extended the accreditation period of all VCE Mathematics studies, with the exception of Mathematical Methods, to 2010. Thus all students undertaking this type of study (mainstream function, algebra, calculus and probability) will do Mathematical Methods (CAS) Units 1 and 2 in 2009 and Mathematical Methods (CAS) Units 3 and 4 in 2010. From 2010, students of Further Mathematics will continue to be able to use either an approved graphics calculator or an approved CAS as their assumed technology, while students of Specialist Mathematics will use an approved CAS. Schools have been able to plan for this transition over several years as best suits their circumstances.

Table 2: assumed technologies for end of schooling mathematics examinations in Victoria from 2000

Year	Technology
2000 -	assumed access for graphics calculators in all mathematics examinations, for revised VCE Mathematics study 2000 – 5 incorporating questions that require the use graphics calculator functionality
2001 -	2001 – Mathematical Methods (CAS) pilot study, assumed access for approved CAS in pilot examinations
2006 - 9	assumed access for approved graphics calculators or CAS, as applicable, for the relevant course
2010 -	assumed access for approved graphics calculators or CAS for Further Mathematics; assumed access for approved CAS for Mathematical Methods and Specialist Mathematics

Practical issues relating to implementation

Students in mathematics are expected to develop and apply mathematical knowledge, capacities and dispositions whereby mental, by hand and technology assisted approaches to working mathematically are used. Thus, the use of technology in a component of VCE examination assessment is not only permitted (technology neutral position) but assumed (technology active position). All students are expected to work towards achieving standards that relate to the appropriate selection and effective use of technology. What is worthwhile applies for all students. The corresponding assessment structure needs to be congruent with curriculum expectations - it should include opportunity for explicit assessment of what is valued and also opportunity for students to be rewarded by judicious and effective choices in their approach to working mathematically.

There are three substantial practical issues for systems with respect to preparation for widespread implementation of a newly-accredited CAS study:

- availability and affordability of CAS;
- teacher training and professional development and;
- availability of suitable teaching and learning resources.

Ensuring the availability and affordability of CAS and related teachers and student support and resources requires considered and timely dialogue with technology companies. In particular, the VCAA indicated clearly to technology companies that equity and access were critical issues of significant social and political sensitivity – it would not be tenable to proceed with wider implementation if the cost of a hand-held CAS device was not comparable to that of a corresponding model graphics calculator. Notwithstanding the fact that this has now been the case for several years it is still not uncommon to have various commentators with an interest in the field claim otherwise.

Summary of discussions

During the afternoon the delegates were split into four groups and Chairs were asked to ensure that their groups addressed the following questions:

1. Is there a place for CAS in the mathematics curriculum?
2. Is A level the best place to start introducing CAS to mathematics students?
3. Is it possible to exploit CAS without it being used in assessment?
4. Should CAS be used in assessment?
5. If so, how should the assessment be designed:
    a. alternative assessments (CAS/non-CAS)?
    b. CAS-only assessments that assume the use of CAS?
    c. CAS-neutral assessments?
6. How could teachers be encouraged to exploit CAS in teaching and learning mathematics?
7. What are the professional development implications of CAS?
8. How would the use of CAS alter our view of what is important in mathematics – would it influence the content of the curriculum?

A summary of comments from the four groups for each of the questions can be seen below.

Q1. Is there a place for CAS in the mathematics curriculum?

There was widespread support for a place for CAS in the mathematics curriculum. If implemented well, it has the potential to give a broader conceptual view rather than simply computation. This view is likely to be improved if the use of CAS is implemented as part of wider move to exploit the potential of computers, including dynamic geometry software and graph plotters.

CAS is used extensively at university level by STEM subjects, especially engineering and physics, and in industry. Students of STEM subjects will be better prepared for university if they have used CAS whilst studying mathematics in school. However, not all university lecturers are positive about CAS, especially those in mathematics departments. I.B. has tried to run a pilot course for 3 years world-wide but take up has been low. One factor that seems to influence this has been a problem with universities accepting this for entrance. I.B. contacted 100 universities and half of them reacted negatively to the introduction of CAS in schools/colleges. Victoria and New Zealand have dealt with this negative attitude and we can learn from their experiences.

Q2. Is A level the best place to start introducing CAS?

The use of technology within mathematics should be encouraged with students of all ages. CAS should be incorporated into school mathematics prior to A level, possibly as early as Key Stage 3. It has great potential for teaching modelling and problem solving and for helping develop mathematical understanding. Consequently it allows students to engage with problems that are more realistic and more interesting and this is of benefit to students of all ages. This is reflected in the use of CAS in many European countries where it is introduced from age 12-14.

Although there is a desire to use CAS with younger students it may be easiest initially to introduce it with post-16 students. There may be transitional problems and therefore introducing CAS with A level students could be a more manageable first step as it will impact on fewer students. Introducing it as an optional component of Further Mathematics may be more acceptable; however, this could result in very few students using it.

The issue of where to introduce CAS is also affected by whether it is implemented as a change to the current curriculum or as part of a new curriculum. Implementing it as part of a new curriculum is likely to be more successful than as an amendment to a current curriculum either at KS3/4 or A level.

Q3. Is it possible to use CAS without it being used in assessment?

As with other forms of technology it would be possible to encourage the use of CAS in the classroom without allowing its use in examinations. It would be possible to use it solely to enrich students' mathematical education; however, it is likely that such use would be both limited and variable. Learning to use it will take time and so should be rewarded if we want teachers and students to invest time in it. CAS is useful regardless of assessment, but assessment really is important to motivate its use.

A secondary issue that affects the issue of using CAS in assessment is the availability of technology during examinations. It would be desirable for students to have access to an appropriate CAS-enabled device, such as a graphical calculator or small laptop computer, which could be used in current examination environments. The availability of such devices was perceived as essential to using CAS in assessment.

Q4 and Q5. Should CAS be used in assessment and how?

There is a distinction between the skill of being able to recall information and the skill of being able to solve problems. The skill of knowing that there is a solution technique, not necessarily recalling it, is more valuable and this should be rewarded. Using CAS in assessment, so students are rewarded for identifying methods to solve problems and applying them, would more accurately reflect happens in real life. There is a danger though that excessively rewarding the ability to get the answer correctly by using the technology could result in a perception that mathematics education is about training students to use technology.

Various suggestions for how to use CAS in assessment were made, including:

- Two separate papers at A level, both compulsory.
- As part of a coursework element at A level.
- As part of an optional A Level module.
- A new module/qualification in mathematics modelling in computational methods or mathematics modelling in computer aided environments (not just limited to CAS).
- In Use of Mathematics AS level.
- As an element of the forthcoming changes to GCSE Mathematics.
- As part of the functional skills element of the new GCSEs in Mathematics.

Experience from other countries suggests that there are issues we should be aware of. CAS-neutral assessments are difficult to write and students who have access to CAS could be advantaged by using it for checking. If alternative CAS/non-CAS assessments are used then there is the problem of comparability: it has to be seen to be fair. If CAS were only a small part of the assessment, then there is a danger that students and teachers would not invest the necessary time for teaching and learning.

It is likely that there would be resistance to including CAS in assessment. The pressure on schools/colleges to ensure their students perform well in examinations is great. If CAS is perceived as a threat to this then it is not likely to be well-received.

There was support for running a pilot in the first instance. This would involve persuading QCA of the value of it and getting one of the awarding bodies interested in administering it. There would need to be funding for such a pilot.

Q6 and Q7. How could teachers be encouraged to exploit CAS in teaching and learning mathematics? What are the professional development implications?

Most teachers want to improve their skills and do a good job; however, many do not have confidence in using technology and some do not have a strong mathematical background. There would need to be a substantial initial investment in training teachers not just to use the technology itself, but to have the subject knowledge and confidence to know where it is appropriate to use CAS in their own mathematics. Only then would they really have the confidence to use it with students.

When students have access to technology they will do the unexpected and problems expand as a result. Teachers need to be comfortable with the consequences of setting students problems that may have hidden depths. The teacher's role is then no longer a deliverer of information but a facilitator of real learning.

The following features of a professional development programme were suggested.

- Training is better done for a department than for individuals.
- Time needs to be devoted to improve teachers' subject knowledge so they can judge when the use of technology is appropriate.
- Different systems may have different syntaxes and there should be an opportunity to explore these.
- An online reference for teachers would be preferable to a paper-based textbook.

Q8. How would the use of CAS alter our view of what is important in mathematics – would it influence the content of the curriculum?

CAS allows the time to go deeper into more interesting questions; it allows more modelling and is one of the tools of a mathematician's trade. It has the potential to allow students to engage with more realistic problems and this is more valuable than considering whether individual topics should form part of the content of the curriculum.

Some topics, such as algebra and calculus, are particularly applicable to CAS. It is important than in studying these areas students are presented with an approach that blends the use of CAS with non-CAS methods so that they are able to understand the concepts. The general perception of mathematics by society is that mathematics is procedural; this view needs to be changed so that mathematics is perceived as conceptual.

Key points and the way forward

As is evident from the feedback from the discussion groups, there was energetic discussion among seminar participants of the wide implications of CAS in the mathematics curriculum. There was a general consensus that there is significant potential to exploit technology in the school mathematics curriculum and that this has not been fully realised.

As many additional questions were raised as were initially posed for discussion; some key themes and suggestions which emerged included are the following.

1. An ICT-based qualification where students have access to appropriate devices in the classroom and examinations would be useful. It could be a much more realistic qualification that allowed them to be better problem-solvers and mathematicians. The possibility of such a qualification should be investigated. Such a qualification would need a suitable pilot.
2. If CAS were to be introduced hurriedly into the school curriculum, considerable difficulties could be expected and it would be likely to be unsuccessful.
3. A short-term minor adjustment to the current A level would be disruptive and not necessarily produce positive results.
4. Exemplars of good classroom practice are needed and a research programme should be set up to produce these.
5. When good practice is understood it will be possible to move forward, including assessment.
6. The introduction of CAS into schools will require significant professional development for the existing teachers of both mathematics and science.

APPENDICES

Conference delegates

Name	Institution
Bobbie Baird	VP TI Europe & Asia
Miggy Biller	York College
Richard Browne	MEI
Douglas Butler	Autograph
Tom Button	MEI
Carlos Coelho	Product Development TI
Graham Cumming	Edexcel
Sue de Pomerai	MEI
Stella Dudzic	MEI
Bryan Dye	Hewett School, Norwich
Andrea Forbes	TI UK & Ireland
Ian Forbes	University of Edinburgh
Bob Francis	Exeter College
Gerald Goodall	RSS
Ted Graham	University of Plymouth
Martin Greenhow	Brunel University
Jonny Griffiths	Paston College, Norfolk
David Holland	MEI
Celia Hoyles	NCETM
Sue Johnston-Wilder	University of Warwick
Keith Jones	Further Mathematics Network
Stephen Kean	Casio
Andy Kemp	Warwick School
Zsolt Lavicza	Geogebra
Stephen Lee	MEI
David Leigh-Lancaster	Victorian CAA
Bernard Murphy	MEI
Chris Olley	King's College, London
Jenny Orton	TI UK & Ireland
Roger Porkess	MEI
Phil Ramsden	Imperial College, London
Chris Rath	TI UK & Ireland
Carol Robinson	University of Loughborough
Chris Saker	Further Mathematics Network
Amanda Sandland	International Baccalaureate Organization
Neil Sheldon	Manchester Grammar School
Charlie Stripp	MEI
Peter Thomas	ACME
James Thompson	AQA
Nick Thorpe	Long Road Sixth Form College
Spencer Williams	TI UK & Ireland
Conrad Wolfram	Wolfram Research

Acronyms and other abbreviations used in the report

AS/A	Level Advanced Subsidiary/Advanced Level
CAS	Computer Algebra Systems
CPD	Continuing Professional Development
FE	Further Education
FMN	Further Mathematics Network
FM	Further Mathematics
GCSE	General Certificate of Secondary Education
HE	Higher Education
IB	International Baccalaureate
ICT	Information and Communications Technology
IJCAME	International Journal of Computer Algebra in Mathematics Education
KS	Key Stage
MEI	Mathematics in Education and Industry
NCETM	The National Centre for Excellence in the Teaching of Mathematics
OCR	Oxford, Cambridge and RSA Examinations
STEM	Science, Technology, Engineering and Mathematics
TI	Texas Instruments
VCAA	Victorian Curriculum and Assessment Authority
VCE	Victorian Certificate of Education
VQA	Victorian Qualifications Authority

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