Convex Hull
(divide and conquer)

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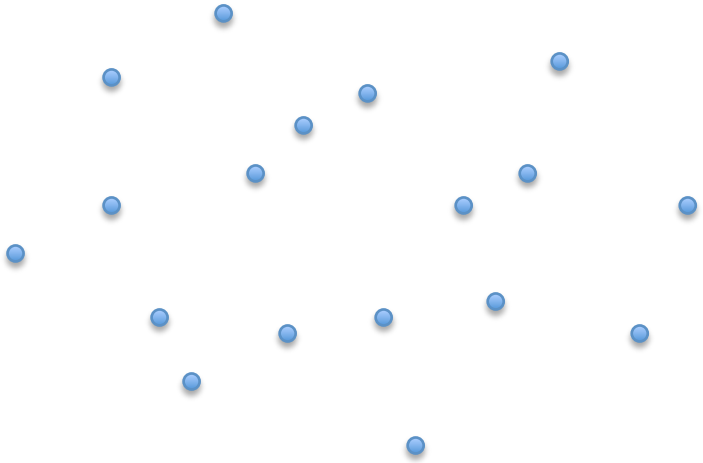
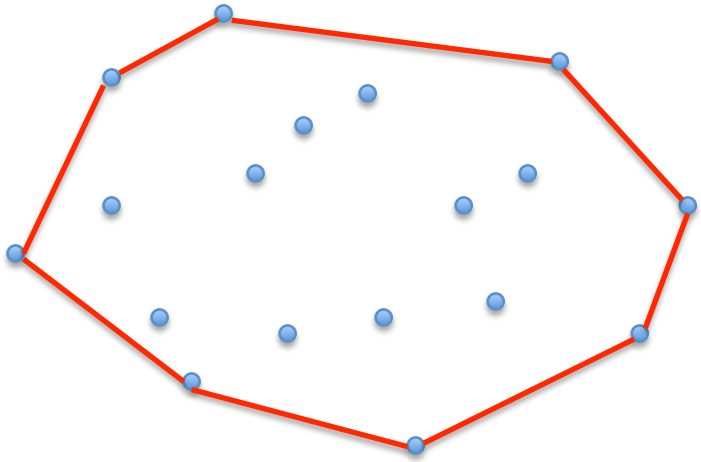
Set of points Q



Convex hull (definition)

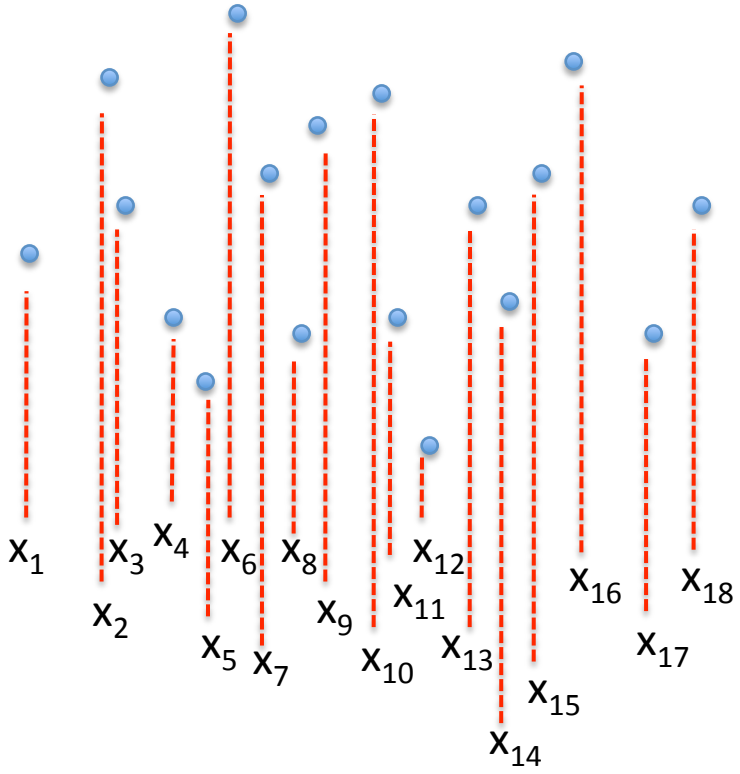


H is the smallest convex polygon that contains all the points of Q

Principle

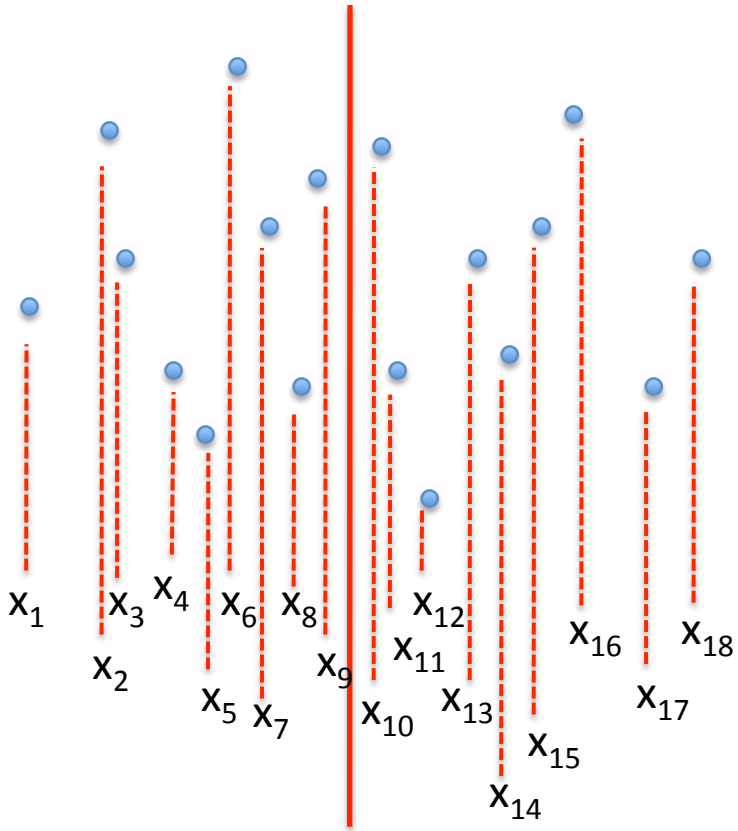
Decompose the set of points in equal parts (Qleft and Qright)
Solve the sub-problems respectively on Qleft and Qright
Merge both convex hulls Hleft and Hright

1. decomposition



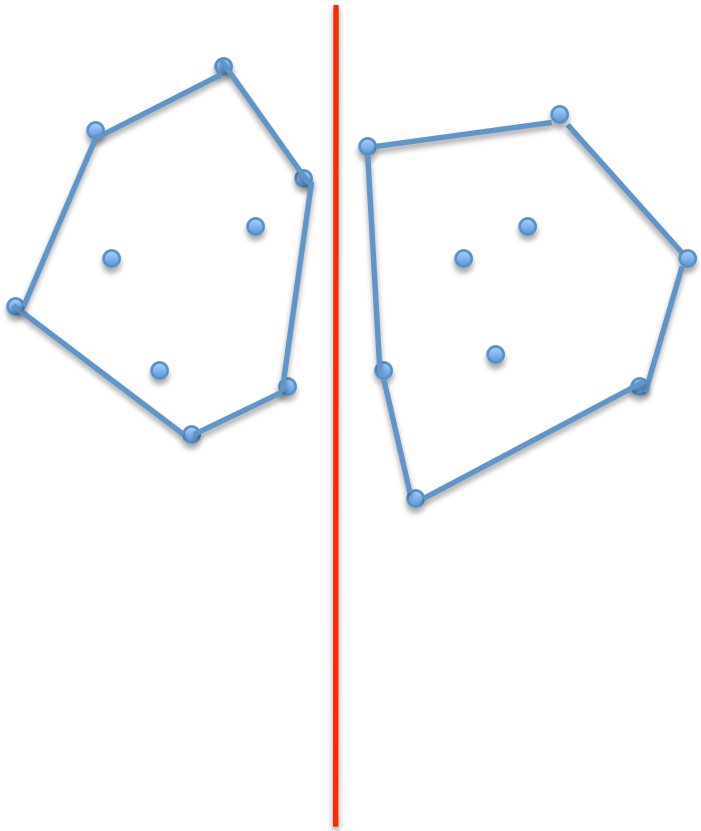
Sort the points by increasing abscissa

1. decomposition



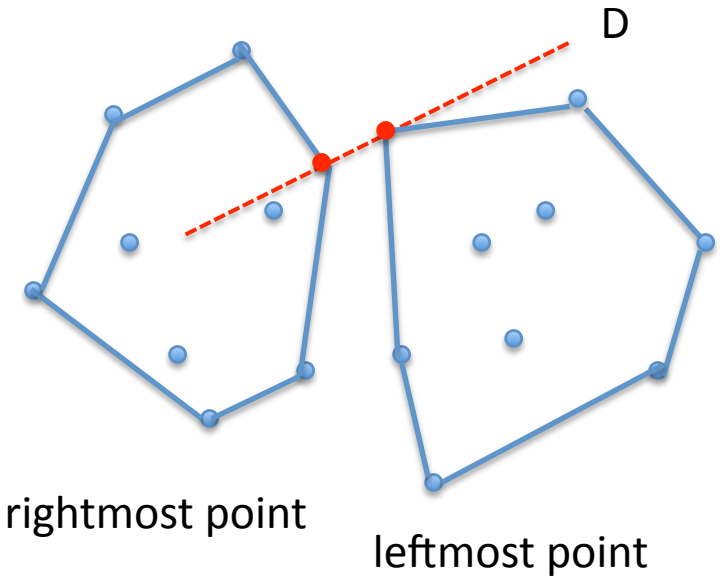
Split the points into two sets of equal size

2. Solve sub-problems

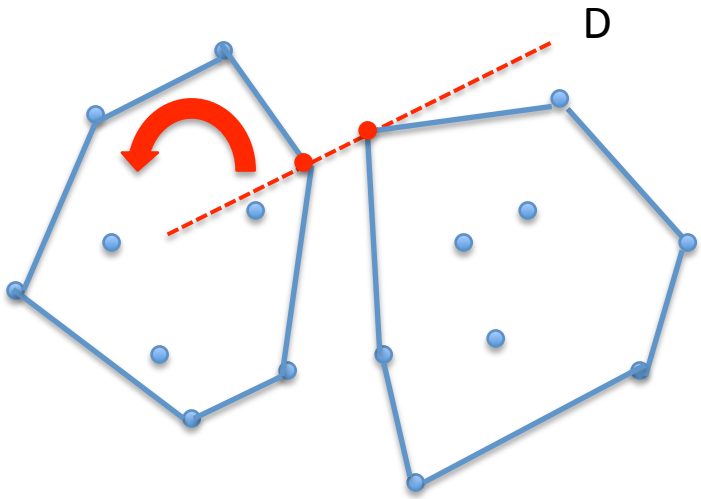


Compute the convex hulls on Qleft and Qright

3. Merge both solutions

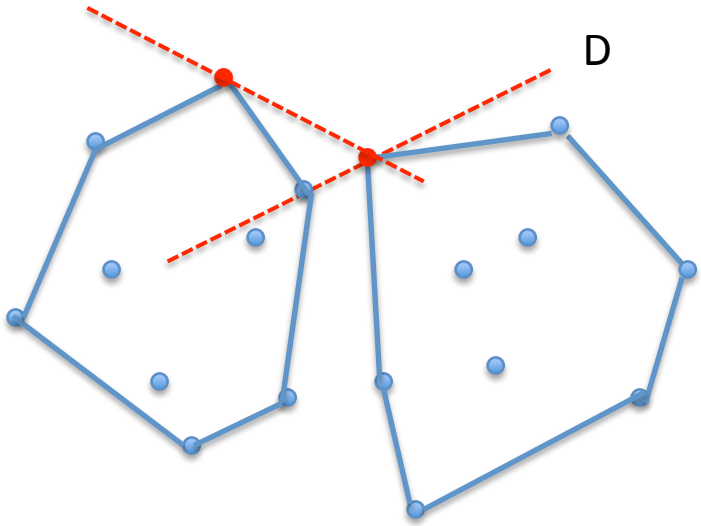


3. Merge both solutions



Determine the upper tangente

3. Merge both solutions



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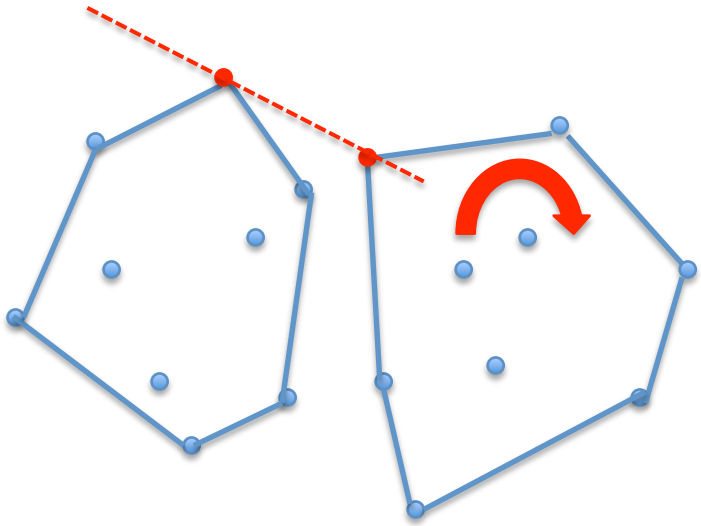

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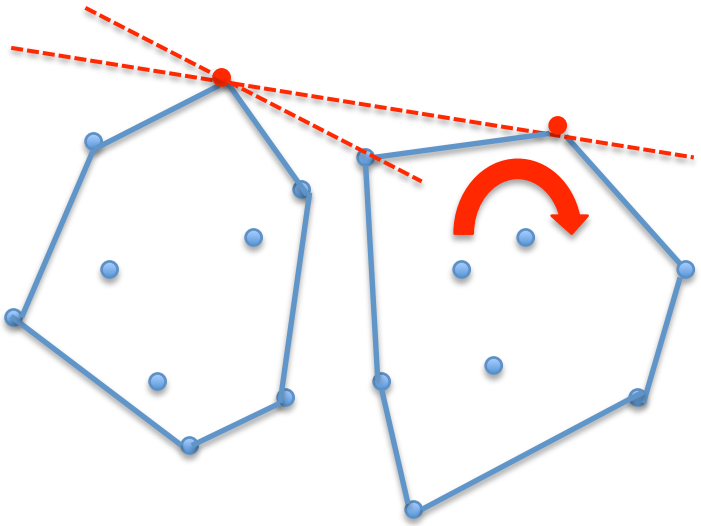

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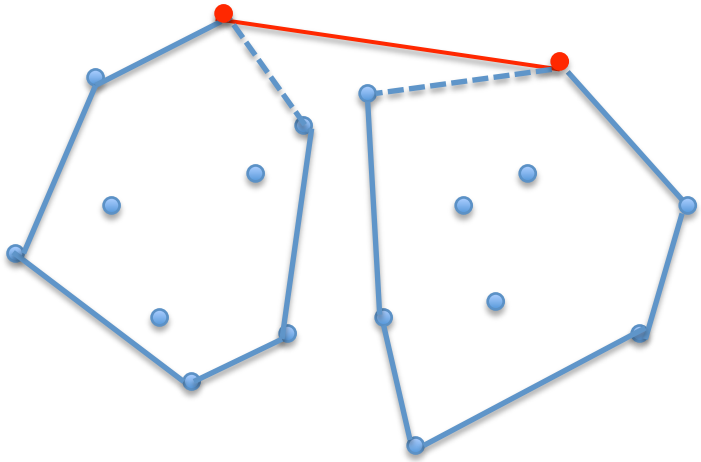

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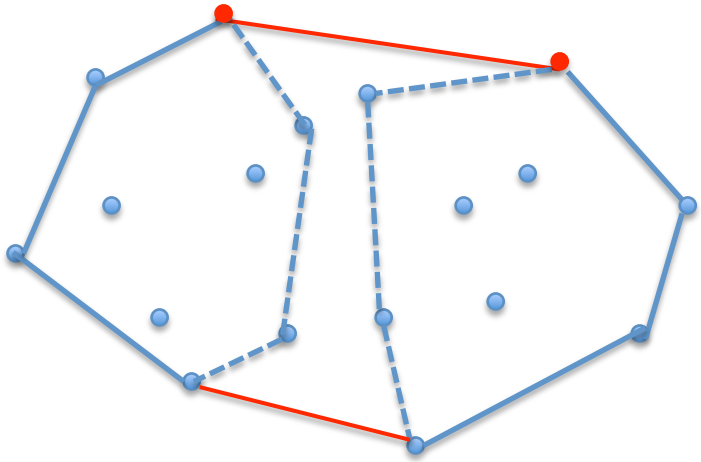
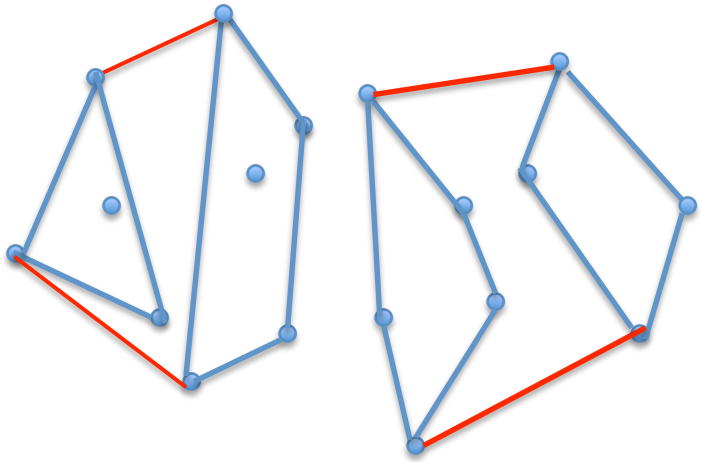


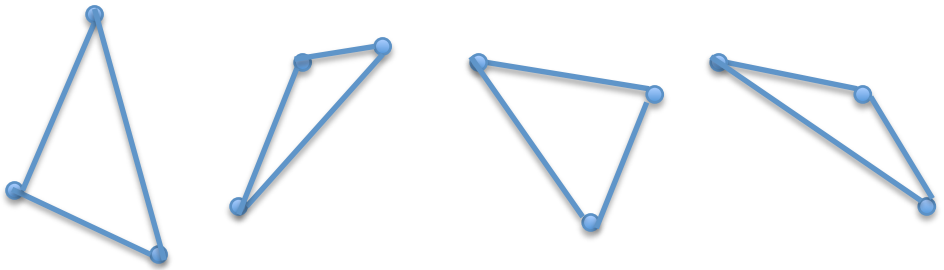
Determine the lower tangente

Recursive decomposition



Basis of the induction

by « brute force » case analysis



4 types of triangles

Cost analysis: $T(n) = 2\,T(n/2) + O(n) = O(n.\log_2(n))$