4

High Voltage Transient Analysis

4.1 Surges on Transmission Lines

Due to a variety of reasons, such as a direct stroke of lightning on the line, or by indirect strokes, or by switching operations or by faults, high voltage surges are induced on the transmission line. The surge can be shown to travel along the overhead line at approximately the speed of light. These waves, as they reach the end of the line or a junction of transmission lines, are partly reflected and partly transmitted. These can be analysed in the following manner.

Consider a small section of the transmission line, of length **dx**.

Let the voltage variation across this section at any instant of time be e to $e + \frac{\partial e}{\partial x}.dx$, and let the current vary similarly.

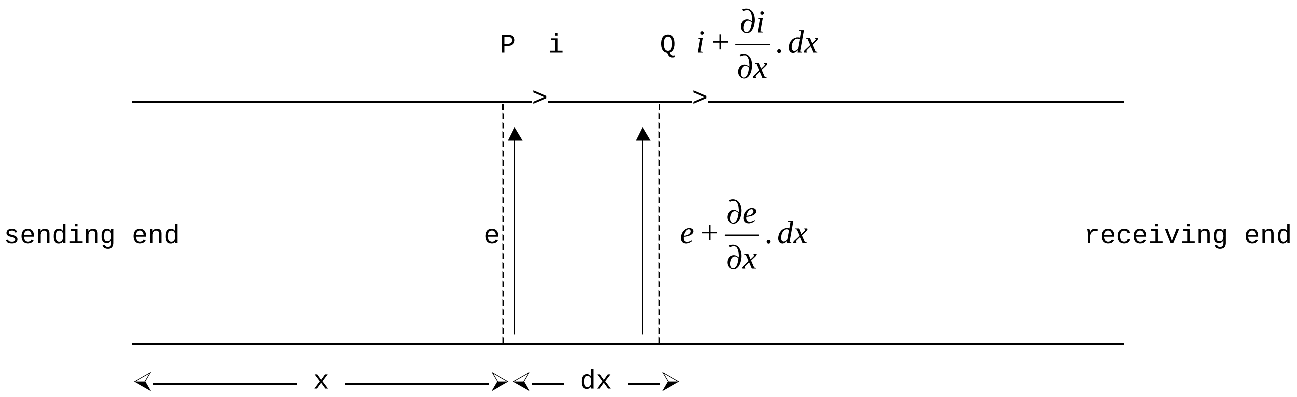

Figure 4.1 - Element of transmission line

Due to the surge, the voltage build-up in the line travels along the line and would cause damage to the transformer and other terminal equipment.

Let e = instantaneous voltage (varies with both distance and time)
i = instantaneous current (varies with both distance and time)
r = resistance of line per unit length
l = inductance of line per unit length
c = capacitance of line per unit length
g = conductance of line per unit length

The voltage drop across PQ and the corresponding current through it are given by the following equations.

$$v = -\frac{\partial e}{\partial x}.dx = r\,dx\,i + l\,dx\frac{\partial i}{\partial t}$$

$$i = -\frac{\partial i}{\partial x}.dx = g\,dx\,e + c\,dx\frac{\partial e}{\partial t}$$

Eliminating **dx** gives us the partial differential equations

$$-\frac{\partial e}{\partial x} = r\,i + l\frac{\partial i}{\partial t} \qquad (1)$$

$$-\frac{\partial i}{\partial x} = g\,e + c\frac{\partial e}{\partial t} \qquad (2)$$

Differentiating equation (1) with respect to x, and equation (2) with respect to t, and eliminating **i**, we have

$$\frac{\partial^2 e}{\partial x^2} = l.c\frac{\partial^2 e}{\partial t^2} + (c.r + l.g)\frac{\partial e}{\partial t} + g.r\,e$$

A very similar partial differential equation can be obtained for **i**.

In practical power lines, the resistance r is much less than the inductance l, and the conductance g is negligible. When these are neglected, the equation reduces to

$$\frac{\partial^2 e}{\partial x^2} = l.c\frac{\partial^2 e}{\partial t^2}$$

It is usual to substitute l.c = 1/a2, where a has the dimension of velocity. In this case the equation becomes

$$a^2\frac{\partial^2 e}{\partial x^2} = \frac{\partial^2 e}{\partial t^2}$$

The solution to this second order partial differential equation can be written in the form of two arbitrary functions.

Consider the function e = f(x - at) . For this

$$a^2\frac{\partial^2 e}{\partial x^2} = a^2\,f''(x - a\,t),\text{also}\,\frac{\partial^2 \mathrm{e}}{\partial \mathrm{t}^2} = \mathrm{f''(x - a\,t).(- a)^2}$$

It is thus seen that this function satisfies the partial differential equation.

Similarly, consider the function e = F(x + at) . For this

$$a^2\frac{\partial^2 e}{\partial x^2} = a^2\,F''(x + a\,t),\text{also}\,\frac{\partial^2 \mathrm{e}}{\partial \mathrm{t}^2} = \mathrm{F''(x + a\,t).(a)^2}$$

This too is seen to satisfy the partial differential equation.

Thus the general solution to the partial differential equation is

e = f(x - at) + F(x + at)

where **f** and **F** are two arbitrary functions of **(x-at)** and **(x+at)**. These two functions can be shown to be **forward** and **reverse** traveling, as follows.

Consider a point $\mathbf{x_1}$ at an instant $\mathbf{t_1}$ on a transmission line.

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                <———————— x₁ ————————> t₁
sending end ——————————————————————————x——————————————————————— receiving end
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Figure 4.2 - Position (x_1, t_1) on transmission line

The value of the function **f(x-at)** at position $\mathbf{x_1}$ and time $\mathbf{t_1}$ would be

$$e_1 = f(x_1 - a\,t_1)$$

At any time **t** afterwards (i.e. at time $\mathbf{t+t_1}$), the value of this same function at the position **x** would be given by

$$e_2 = f[x - a\,(t+t_1)] = f(x-at + a\,t_1)$$

This latter voltage $\mathbf{e_2}$ would be equal to $\mathbf{e_1}$ at the position $\mathbf{x_1 = x - at}$.

Now **a.t** is the distance traveled by a wave traveling with velocity **a** in the forward direction in a time **t**. Thus it is seen that the voltage at a distance **a.t** in the forward direction is always equal to the value at the earlier position at the earlier time for any value of time. Thus the function **f(x-at)** represents a **forward wave**. Similarly, it can be seen that the function **F(x+at)** represents a **reverse wave.**

The effect of resistance and conductance, which have been neglected would be so as to modify the shape of the wave, and also to cause attenuation. These are generally quite small, and the wave travels with little modification. In fact this effect can be separately included in the analysis as will be shown later.

4.1.1 Surge Impedance and Velocity of Propagation

Consider the forward wave **e = f(x-at)**. The corresponding current wave **i** can be determined from equation (1) as follows.

$$l\,\frac{\partial e}{\partial t} = -\,\frac{\partial e}{\partial x} = -\,f\,'(x - a\,t)$$

$$\therefore i = \frac{1}{a\,l}\,f\,(x - at) = \frac{1}{a\,l}\,.\,e = \sqrt{\frac{c}{l}}\,.\,e$$

$$\text{i.e.}\quad e = \sqrt{\frac{l}{c}}\,.\,i = Z_0\,.\,i \quad \text{where} \quad Z_0 = \sqrt{\frac{l}{c}}$$

$\mathbf{Z_o}$ is known as the surge (or characteristic) impedance of the transmission line.

The surge current **i** traveling along the line is always accompanied by a surge voltage $\mathbf{e = Z_o\,i}$ traveling in the same direction. For a reverse wave, it can be similarly shown that the surge current **i** is associated with a surge voltage $\mathbf{e = -\,Z_o\,i}$.

For a transmission line, with conductors of radius **r** and conductor spacing **d**, it can be shown that the inductance per unit length of the line is given by

$$l = \frac{\mu_0}{2\,\pi}\left[\frac{\mu_r}{4} + \log_e\frac{d}{r}\right] \quad \text{H/m}$$

Since the internal flux linkage is small, if this is neglected

$$l = \frac{\mu_0}{2\pi} \log_e \frac{d}{r} \quad \text{H/m}$$

The capacitance **c** per unit length is given by

$$c = \frac{2\pi\,\varepsilon_0\,\varepsilon_r}{\log_e \dfrac{d}{r}} \quad \text{F/m}$$

$$\text{for air} \quad \varepsilon_r = 1$$

$$\therefore \; l\,.\,c = \mu_0\,\varepsilon_0 = \frac{1}{a^2}$$

$$\text{but} \; \frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \text{velocity of light}$$

Therefore the velocity of propagation of the wave **a** is equal to the velocity of light. [Note: If the resistance of the line was not neglected, the velocity of propagation of the wave would be found to be slightly less than that of light (about 5 to 10%)].

For a cable, the dielectric material has a relative permittivity $\boldsymbol{\varepsilon_r}$ different from unity. In this case, the above derivation would give the velocity of propagation in a cable as

velocity of propagation = velocity of light/$\sqrt{\varepsilon_r}$

For commercial cables, ε_r lies between about 2.5 and 4.0, so that the velocity of propagation in a cable is about **half** to **two-third** that of light.

The surge impedance of a line can be calculated as follows.

$$Z_0 = \sqrt{\frac{l}{c}} = \sqrt{\frac{\mu_0}{\varepsilon_0}\left(\frac{\log_e d/r}{2}\right)^2}$$

Substituting the velocity of light as $\mathbf{3 \times 10^8}$ **m/s** and simplifying gives

$$Z_o = 60 \log_e (d/r) \qquad \Omega$$

For an overhead line, for practical values of conductor radius **r** and spacing **d**, the surge impedance $\mathbf{Z_o}$ is of the order of 300 to 600 Ω.

For a cable, the corresponding surge impedance would be given by the expression

$$Z_o = 60/\sqrt{\varepsilon_r} \,.\, \log_e (d/r) \qquad \Omega$$

which has values in the region of 50 to 60 Ω.

4.1.2 Energy stored in surge

The energy stored in a traveling wave is the sum of the energies stored in the voltage wave and in the current wave.

$$\text{Energy} = \tfrac{1}{2}\, c\, e^2 + \tfrac{1}{2}\, l\, i^2$$

$$e = \sqrt{\frac{l}{c}}\, i, \quad \text{i.e. } c\, e^2 = l\, i^2, \quad \text{i.e. } \tfrac{1}{2} c\, e^2 = \tfrac{1}{2} l\, i^2$$

$$\therefore \text{total energy} = c\, e^2$$

But for a surge, $\mathbf{e = Z_o\, i}$, so that we have

It is seen that half the energy of the surge is stored in the electrostatic field and half in the electromagnetic field.

4.2 Reflection of Traveling waves at a Junction

When a traveling wave on a transmission line reaches a junction with another line, or a termination, then part of the incident wave is reflected back, and a part of it is transmitted beyond the junction or termination.

The **incident wave**, the **reflected wave** and the **transmitted wave** are formed in accordance with Kirchhoff's laws. They must also satisfy the differential equation of the line.

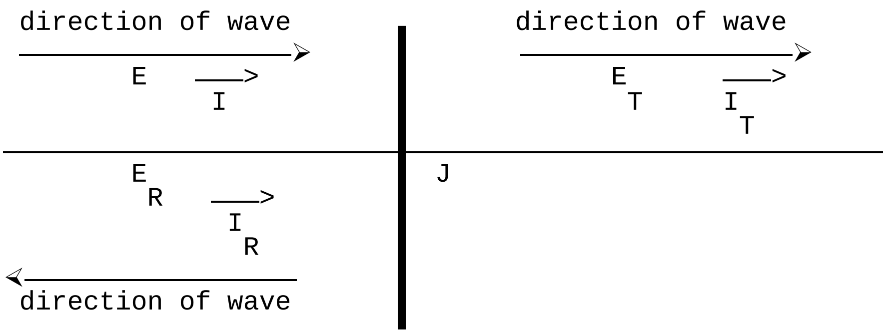


Figure 4.3 - Reflection at a junction

Consider a step-voltage wave of magnitude **E** incident at junction **J** between two lines of surge impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$. A portion $\mathbf{E_T}$ of this surge would be transmitted and a portion $\mathbf{E_R}$ would be reflected as shown in figure 4.3.

There is no discontinuity of potential at the junction J. Therefore

$$E + E_R = E_T$$

There is also no discontinuity of current at the junction. Therefore

$$I + I_R = I_T$$

Also, the incident surge voltage **E** is related to the incident surge current **I** by the surge impedance of the line $\mathbf{Z_1}$. Similarly the transmitted surge voltage $\mathbf{E_T}$ is related to the transmitted surge current $\mathbf{I_T}$ by the surge impedance of the line $\mathbf{Z_2}$ and the reflected surge voltage $\mathbf{E_R}$ is related to the reflected surge current $\mathbf{I_R}$ by the surge impedance of the line $\mathbf{Z_1}$.

However it is to be noted that the reflected wave is a reverse wave. Thus we can write

$$E = Z_1 I, \quad E_T = Z_2 I_T, \text{ and } E_R = -Z_1 I_R$$

Substituting these values gives

$$E/Z_1 - E_R/Z_1 = E_T/Z_2 = (E + E_R)/Z_2$$

This gives on simplification

$$E_R = \frac{Z_2 - Z_1}{Z_2 + Z_1} . E$$

Similarly, the transmitted surge may be written as

$$E_T = \frac{2\,Z_2}{Z_2 + Z_1} . E$$

Thus we have obtained the transmitted wave E_T and the reflected wave E_R in terms of the incident surge E. Since both these surges are a definite fraction of the incident surge, a transmission factor **α** and a reflection factor **β** are defined as follows.

$$\alpha = \frac{2\,Z_2}{Z_2 + Z_1}, \quad \beta = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

When the junction is matched (i.e. $Z_1 = Z_2$), then there is no reflection and the reflection factor can be seen to be zero.

When the line Z_1 is open circuited at the far end (i.e. $Z_2 = \infty$), then the full wave is reflected back and the reflection factor becomes 1.

When the line Z_1 is short circuited at the far end (i.e. $Z_2 = 0$), then no voltage can appear and the full wave is reflected back negated so as to cancel the incident wave and the reflection factor becomes - 1.

4.2.1 Open circuited line fed from a infinite source

For this case $Z_2 = \infty$ and $\beta = 1$.

When a voltage surge **E** arrives at the junction J, which is on open circuit, it is reflected without a change in sign (i.e. **E**).

Also, a current surge (**- I**) of opposite sign to the incident (**I**) is reflected so that the transmitted current is zero.

If the line is fed from a constant voltage source **E**, then as the reflected voltage surge (**E**) arrives at the generator end, since the generator maintains the voltage at its end at voltage **E**, it send a voltage surge of **-E** back to the line so as to keep its voltage at **E**.

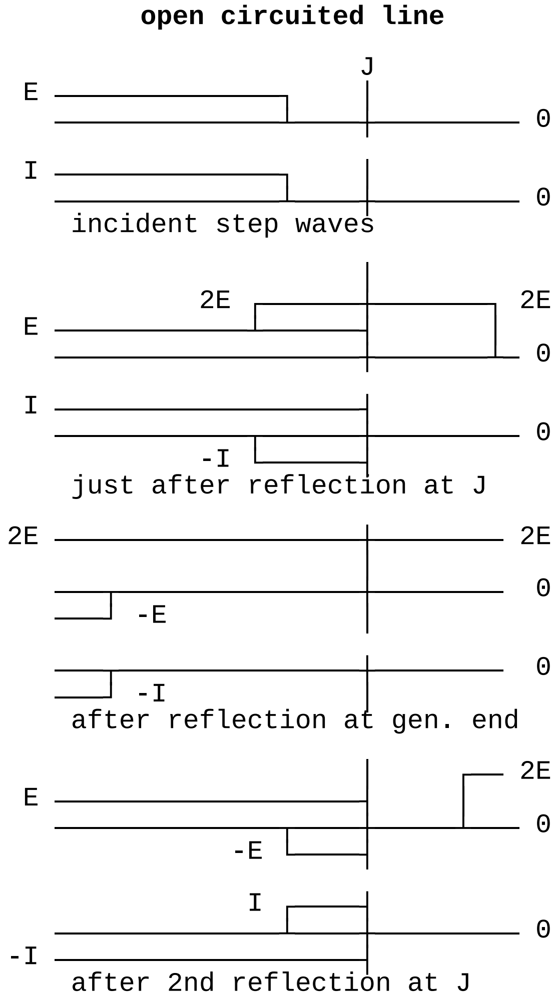


Figure 4.4 - Reflections under open circuit conditions

[This could also have been deduced by considering that a constant voltage source has a zero internal impedance, and the reflection coefficient can be calculated based on $Z_2 = 0$].

The voltage surge **- E** is accompanied by a current surge **- I**. The surge voltage **- E** as it reaches the open junction J, is reflected again without a change in sign, and accompanied by a current **+ I** so as to make the transmitted current again zero. Once these voltage and current waves reach the generator, the instantaneous voltage and current will be zero, and the line would once again be uncharged. The generator now sends a voltage surge **E** accompanied by a current surge **I**, and the who process described repeats again.

4.2.2 Short Circuit Line fed from an infinite source

For this case $Z_2 = 0$ and $\beta = -1$.

When a voltage surge **E** arrives at the junction J, which is on short circuit, it is reflected with a change in sign (**- E**), so as to cancel the incoming surge. Also, a current surge **I** of the same sign as the incident (**I)** is reflected so that the transmitted current is doubled (**2I**).

If the line is fed from a constant voltage source **E**, then as the reflected voltage surge (- **E**) arrives at the generator end, it send a voltage surge of **E** back to the line so as to keep its voltage at **E**.

The voltage surge **E** is again accompanied by a current surge **I** so that the transmitted current becomes **3I**. The surge voltage **E** as it reaches the junction J, is reflected again with a change in sign, and accompanied by a current **I** so as to make the transmitted current again increase by **I** to **4I**. At successive reflections, the current keeps on building. [This is to be expected as a short circuited line with zero line resistance and zero source resistance, fed from a constant voltage source will finally tend to zero.

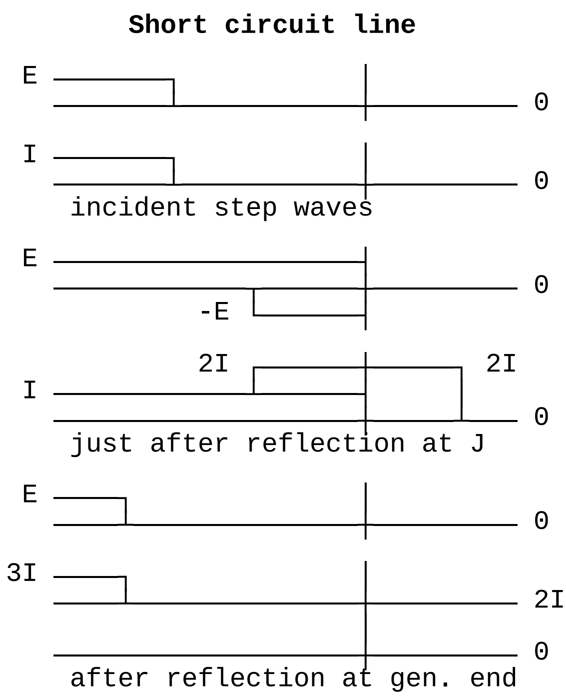


Figure 4.5
- Reflections under short circuit conditions

However the increase is in a step like manner rather than in a linear manner]. In practice, due to the resistance of the line, the current does not keep on building, but each successive current surge is lower than the earlier one due to attenuation. Thus the final current tends to a limiting value determined by the line resistance.

In the above transient, the voltage **E** has been assumed constant at the generator end. In practice, such an assumption is generally valid, owing to the fact that the very high velocity of propagation does not normally cause the system voltage to vary significantly during the period of interest for reflections.

Example: Consider a line 30 km long, operating at a frequency of 50 Hz.

Assuming the velocity of propagation to be $\mathbf{3 \times 10^8 m/s}$, the travel time for single transit of the line would be $\mathbf{30 \times 10^3 / 3 \times 10^8\ s}$ = 100 μs.

During this time, the change in the phase angle of the 50 Hz voltage would be $\mathbf{\omega t = 2\ \pi\ .\ 50\ .\ 10^{-4}\ rad\ = 10^{-2}\ .\ 180^o = 1.8^o}$.

If we consider the peak value of the sinusoidal voltage as **1 pu**, then if we deviate from this position by **1.8°**, then the corresponding voltage would be **cos 91.8° = 0.9995 pu**. During this interval, it can be seen that the variation of the voltage is negligibly small and the step approximation can be considered valid. Even in other instances, the step analysis is useful because other waveforms can be considered as made up of step surges.

4.3 Bewley Lattice Diagram

This is a convenient diagram devised by Bewley, which shows at a glance the position and direction of motion of every incident, reflected, and transmitted wave on the system at every instant of time. The diagram overcomes the difficulty of otherwise keeping track of the multiplicity of successive reflections at the various junctions.

Consider a transmission line having a resistance **r**, an inductance **l**, a conductance **g** and a capacitance **c**, all per unit length.

If γ is the propagation constant of the transmission line, and
E is the magnitude of the voltage surge at the sending end,

then the magnitude and phase of the wave as it reaches any section distance **x** from the sending end is $\mathbf{E_x}$ given by.

$$E_x = E\,.\,e^{-\gamma x} = E\,.\,e^{-(\alpha + j\beta)x} = E\, e^{-\alpha x}\, e^{-j\beta x}$$

where

$\mathbf{e^{-\alpha x}}$ represents the attenuation in the length of line x
$\mathbf{e^{-j\beta x}}$ represents the phase angle change in the length of line x

Therefore,

α = attenuation constant of the line in neper/km
β = phase angle constant of the line in rad/km.

It is also common for an attenuation factor **k** to be defined corresponding to the length of a particular line. i.e. $\mathbf{k = e^{-\alpha l}}$ for a line of length **l**.

The propagation constant of a line γ of a line of series impedance **z** and shunt admittance **y** per unit length is given by

$$\gamma = \sqrt{z.y} = \sqrt{(r + j\,\omega\, l)(g + j\,\omega\, c)}$$

Similarly the surge impedance of the line (or characteristic impedance) $\mathbf{Z_o}$

$$Z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{(r + j\,\omega\, l)}{(g + j\,\omega\, c)}}$$

When a voltage surge of magnitude **unity** reaches a junction between two sections with surge impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$, then a part $\boldsymbol{\alpha}$ is transmitted and a part $\boldsymbol{\beta}$ is reflected back. In traversing the second line, if the attenuation factor is **k**, then on reaching the termination at the end of the second line its amplitude would be reduced to $\mathbf{k.\alpha}$. The lattice diagram may now be constructed as follows. Set the ends of the lines at intervals equal to the time of transit of each line. If a suitable time scale is chosen, then the diagonals on the diagram show the passage of the waves.

In the Bewley lattice diagram, the following properties exist.

(1) All waves travel downhill, because time always increases.

(2) The position of any wave at any time can be deduced directly from the diagram.

(3) The total potential at any point, at any instant of time is the superposition of all the waves which have arrived at that point up until that instant of time, displaced in position from each other by intervals equal to the difference in their time of arrival.

(4) The history of the wave is easily traced. It is possible to find where it came from and just what other waves went into its composition.

(5) Attenuation is included, so that the wave arriving at the far end of a line corresponds to the value entering multiplied by the attenuation factor of the line.

4.3.1 Analysis of an open-circuit line fed from ideal source

Let τ is the time taken for a wave to travel from one end of the line to the other end of the line (i.e. single transit time) and **k** the corresponding attenuation factor.

Consider a step voltage wave of amplitude unity starting from the generator end at time **t = 0.** Along the line the wave is attenuated and a wave of amplitude **k** reaches the open end at time τ. At the open end, this wave is reflected without a loss of magnitude or a change of sign. The wave is again attenuated and at time 2τ reaches the generator end with amplitude k^2. In order to keep the generator voltage unchanged, the surge is reflected with a change of sign ($-k^2$), and after a time 3τ reaches the open end being attenuated to $-k^3$. It is then reflected without a change of sign and reaches the generator end with amplitude $-k^4$ and reflected with amplitude $+k^4$. The whole process is now repeated for the wave of amplitude k^4.

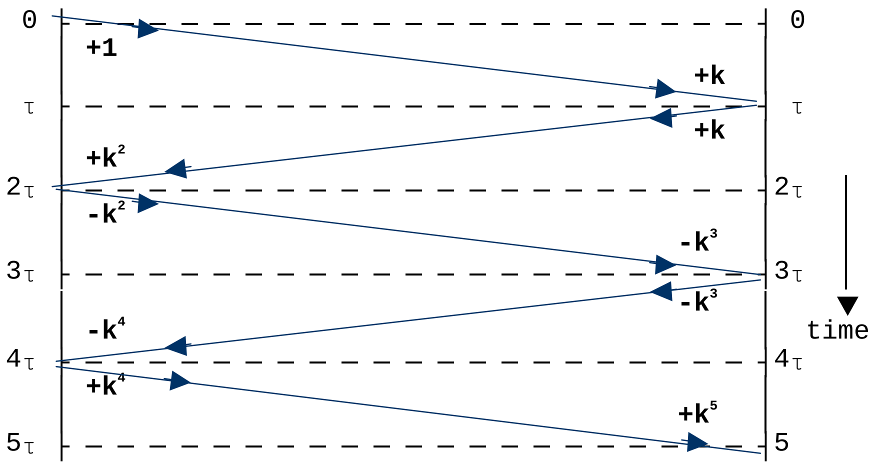


Figure 4.6 - Lattice diagram for an open-circuited line

The corresponding lattice diagram is shown in figure 4.6. At the receiving end of the line, the transmitted surge is twice the incident surge. [This can be obtained from either the transmission coefficient, or by adding the incident and reflected surges which make up the transmitted surge]. At any given instant, the voltage at this end is the summation of the surges arriving until that instant of time.

Thus the voltage at the open end after the n^{th} reflection is given by

$$V_r = 2\,(k - k^3 + k^5 - k^4 + \ldots.\ k^{2n-1})$$

This is a geometric series which has the summation given by

$$V_r = 2\,k\,\frac{1-(-k^2)^n}{1-(-k^2)}$$

for the final value $t \rightarrow \infty$, $n \rightarrow \infty$

$$\therefore V_r = \frac{2\,k}{1+k^2}$$

$$\because (1-k)^2 > 0,\ 1+k^2 > 2\,k$$

$$\text{i.e.} \frac{2\,k}{1+k^2} < 1$$

It is thus seen that when attenuation is present, the receiving end voltage is less than the sending end voltage. The reason for this is that there is a voltage drop in the line due to the shunt capacitive currents flowing in the line even on open circuit. However, since **k is very close to** 1, the reduction is very very small. [For example, even for k = 0.90, the corresponding reduced value is 2x0.90/(1+0.90^2) = 0.9944].

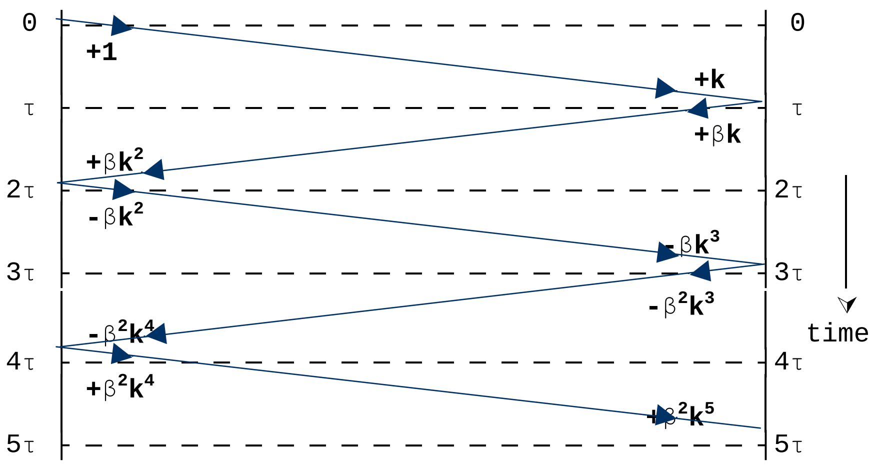


Figure 4.7 - Lattice diagram for a resistive termination

Let us now consider a line terminated through a resistance **R.** The corresponding reflection coefficient at the receiving end would be **β = (R-Z$_1$)/(R+Z$_1$)** and the reflection factor at the sending end would still be **-1.** Thus in addition to the attenuation occurring on the line, there is a non complete reflection occurring at the far end of the line. This would have a lattice diagram as shown in figure 4.7.

The final voltage attained at the resistive termination will now depend on both the attenuation and the reflection coefficient. From the lattice diagram it can be seen that this value can be calculated as follows.

$$V_r = k + \beta\, k - \beta\, k^3 - \beta^2\, k^3 + \beta^2\, k^5 + \beta^3\, k^5 - ......$$

$$V_r = \frac{(1+\beta\,)\,k}{1+\beta\,k^2}$$

$$\because k<1,\ \beta\,k<1,\ (\,1-\beta\,k\,)(\,1-k\,)>0\,,$$

$$1+\beta\,k^2 - k - \beta\,k\ >0$$

$$\text{i.e.}\ \ \frac{(1+\beta)\,\text{k}}{1+\beta\,\text{k}^2}<1$$

Therefore, again, the receiving end voltage is less than the sending end voltage. However if there is no attenuation (k = 1), then the receiving end voltage tends to unity as shown in figure 4.8.

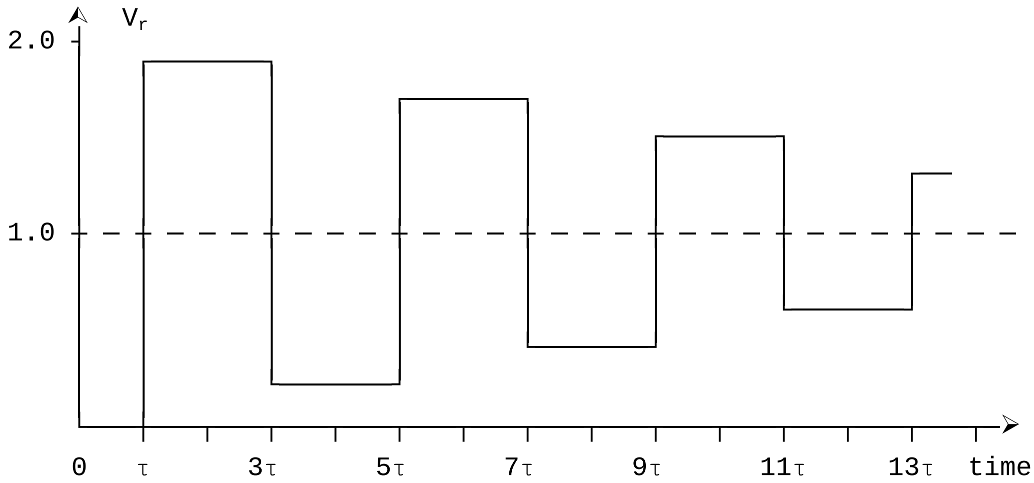


Figure 4.8 - Receiving end voltage with resistive termination

4.2.3 Reflections at 3 substation system

Consider 3 substations (1), (2) and (3) connected by lines Z_1, Z_2, Z_3 and Z_4, as shown in figure 4.9. Let α and α' be the transmission coefficients for a wave incident at the substation from the left hand side and the right hand side respectively, and let β and β' be the corresponding reflection factors.

In the Bewley lattice diagram, the junctions must be laid off at intervals equal to the time of transit of each section between junctions. (If all lines are overhead lines, then the velocity of propagation may be assumed to be the same and the junctions can be laid off proportional to the distance between them. Otherwise this is not possible).

The lattice diagram is shown together with the system diagram, and a unity magnitude surge is assumed to arrive from outside the on line 1.

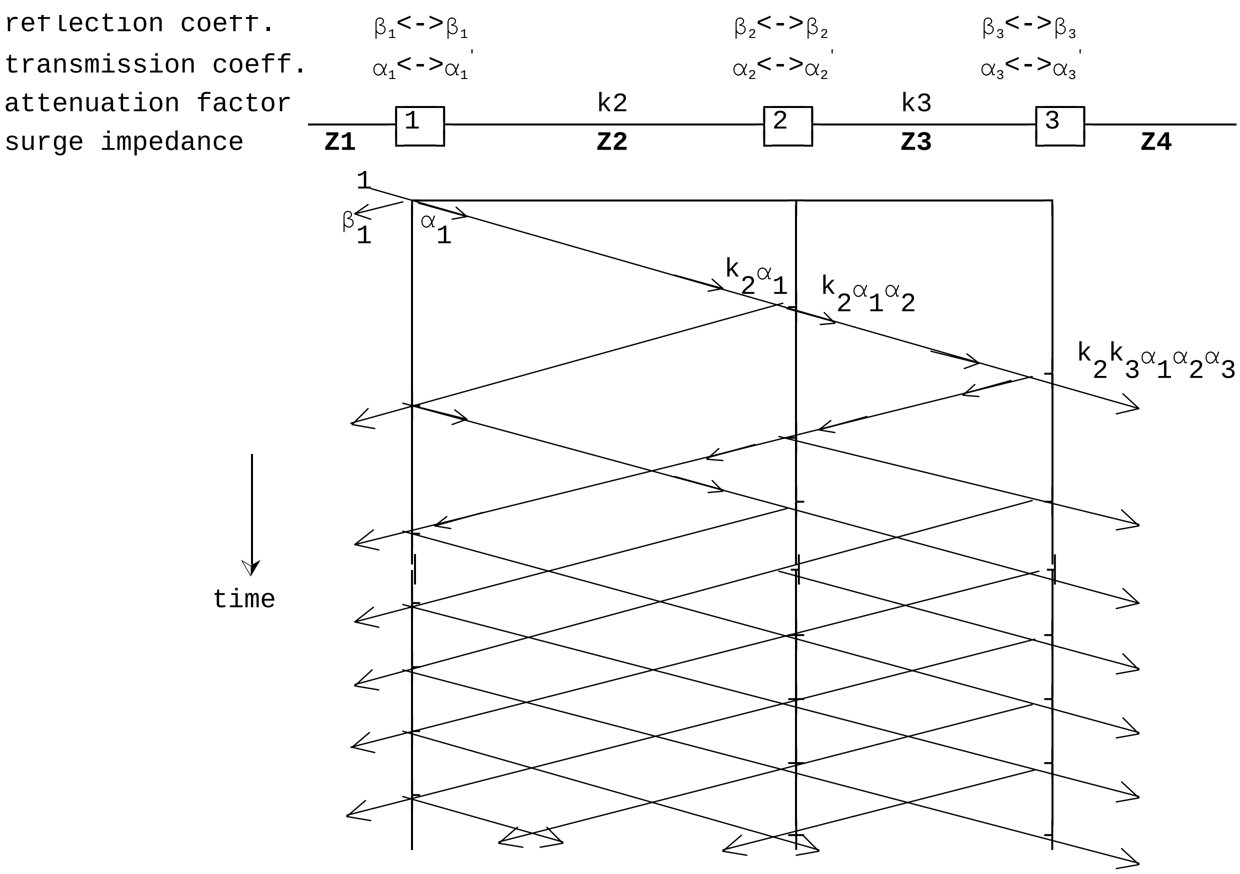


Figure 4.9 - Bewley Lattice Diagram

Example:

3 substations **A, B** and **C** are spaced 75 km apart as shown in figure 4.10. **B** and **C** are connected together by a cable (velocity of propagation 2 x 10^8 m/s), and the remaining connections are all overhead lines (velocity of propagation 3 x 10^8 m/s). The attenuation factors and the surge impedances of the lines are shown alongside the lines. The overhead lines beyond **A** and **C** on either side are extremely long and reflections need not be considered from their far ends. Determine using the Bewley lattice diagram the overvoltages at the 3 substations, at an instant **1_** ms after a voltage surge of magnitude unity and duration ¾ reaches the substation **A** from the outside.

The transmission and reflection coefficients can be calculated as follows.

At **A**,

$$\beta_1 = \frac{600 - 400}{600 + 400} = 0.2 \qquad \beta_{1'} = -0.2$$

$$\alpha_1 = \frac{2 \text{ x } 600}{1000} = 1.2 \qquad \alpha_{1'} = \frac{2\text{x}400}{1000} = 0.8$$

similarly, at **B**

$$\beta_2 = \frac{66\frac{2}{3} - 600}{66\frac{2}{3} + 600} = -0.8 \qquad \beta_{2'} = 0.8$$

$$\alpha_2 = \frac{2 \text{ x } 66\frac{2}{3}}{666\frac{2}{3}} = 0.2 \qquad \alpha_{2'} = \frac{2\text{x}600}{666\frac{2}{3}} = 1.8$$

similarly, at **C** it can be shown that

$\beta_3 = 0.8$, $\beta_3' = -0.8$, $\alpha_3 = 1.8$, and $\alpha_3' = 0.2$.

The single transit times of the two lines connecting the substations are

$\tau_{AB} = 75 \times 10^3/3 \times 10^8 = ¼$ ms, $\tau_{BC} = 75 \times 10^3/2 \times 10^8 = \_$ ms .

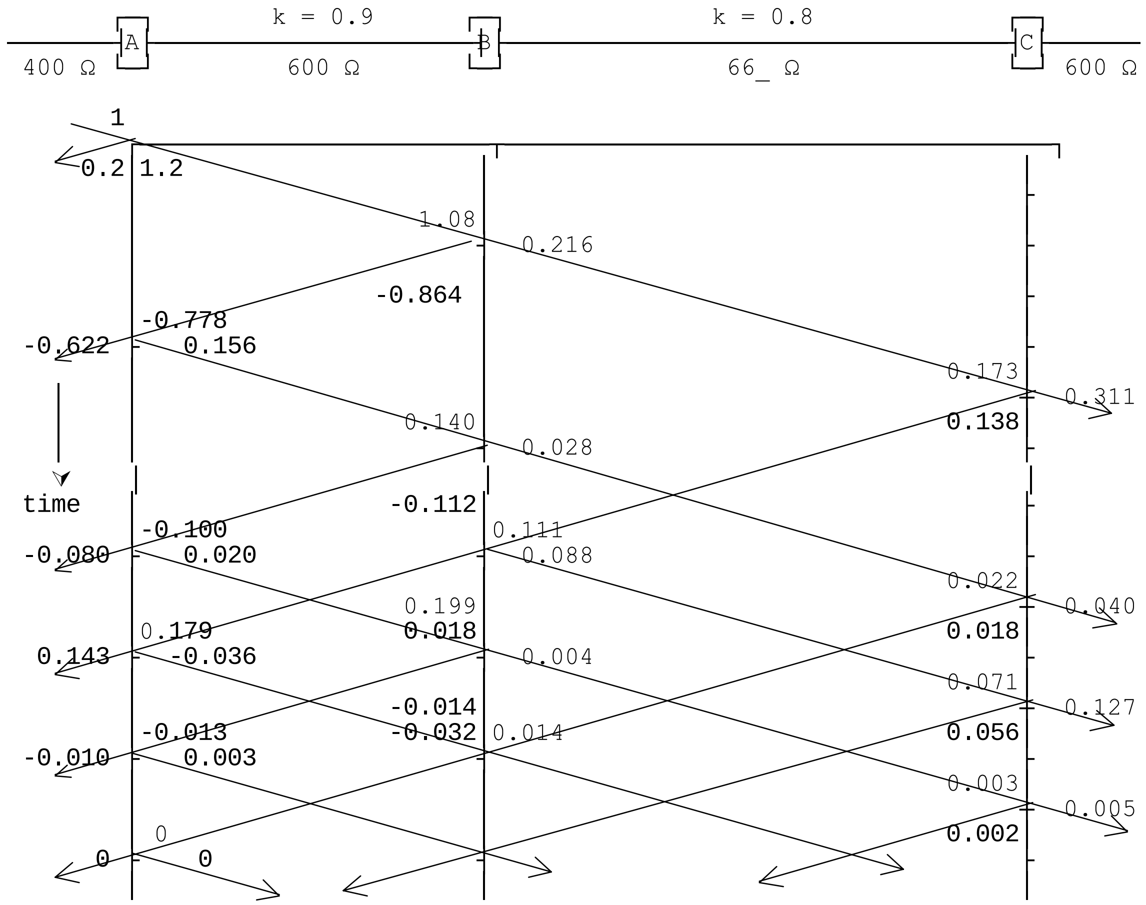


Figure 4.10 - Example of application of Bewley Lattice diagram

The lattice diagram must be set out, such that the intervals **AB** and **BC** are in the proportions to the times 1/4 ms and 3/8 ms respectively. These are shown in figure 4.10 .

Since the step voltage incident at substation **A** is of duration 3/4 ms, only reflections that have occurred after 3/4 ms prior to the present will be in existence.

at time **t = 1_** ms,

voltage at junction **A** = - 0.080 + 0.143 -0.010 = 0.053 pu
voltage at junction **B** = 0.199 + 0.004 + 0.032 + 0 = 0.235 pu

Since a surge arrives at junction **C** at the instant of interest, we can define values either just before or just after the time.

voltage at junction **C** at t^- = 0.040 + 0.127 = 0.167 pu
voltage at junction **C** at t^+ = 0.040 + 0.127 + 0.005 = 0.172 pu

4.4 Reflection and Transmission at a T-junction

When the intersection occurs between more than two lines, the analysis can be done as follows. Consider the connection shown in figure 4.11.

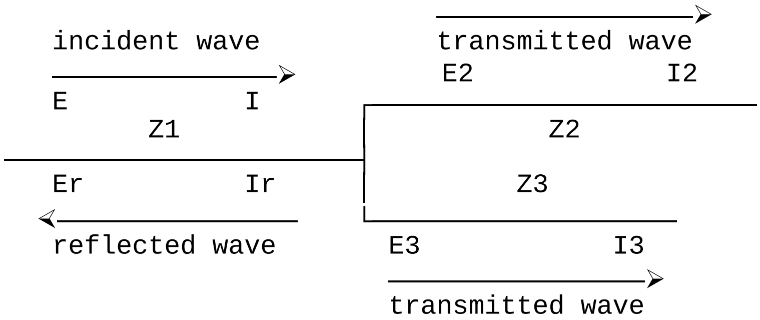


Figure 4.11 - Reflections and transmissions at T-junction

If a surge voltage of magnitude **E** is incident on the junction with two other lines (Z_2 and Z_3) from a line (Z_1), then the transmitted and reflected surges would be as shown. For these

$$E = Z_1\, I,\quad E_r = -\, Z_1\, I_r,\quad E_2 = Z_2\, I_2\ ,\ \text{and}\quad E_3 = Z_3\, I_3$$

Also, considering the fact that the total voltage and the current on either side of the junction must be the same,

$$E_2 = E_3 = E_T = E_r + E,\quad \text{and}\quad I_r + I = I_2 + I_3$$

These may be solved to give the following expressions for the transmitted and reflected surges.

$$\frac{2\,E}{Z_1} = E_T\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$\text{i.e. } E_T = \frac{\dfrac{2}{Z_1}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}}\,E\ ,\quad \text{similarly}\quad E_R = \frac{\dfrac{1}{Z_1} - \dfrac{1}{Z_2} - \dfrac{1}{Z_3}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}}\,E$$

The method can be extended to junctions with more than 3 lines. However, there is an easier method of analysis to obtain the same result.

For a surge, the voltage and the current are always related by the surge impedance, independent of the termination of the line at the far end. Thus for analysis purposes, the line behaves similar to a load of impedance $\mathbf{Z_0}$ connected between the start of the line and the earth. Thus when a single line ($\mathbf{Z_1}$) feeds two other lines (Z_2 and Z_3), the resultant reflections and transmissions could be obtained by considering both these lines as impedances connected from the junction to earth. That is, these two lines behave for surge purposes as if their surges impedances were connected in parallel.

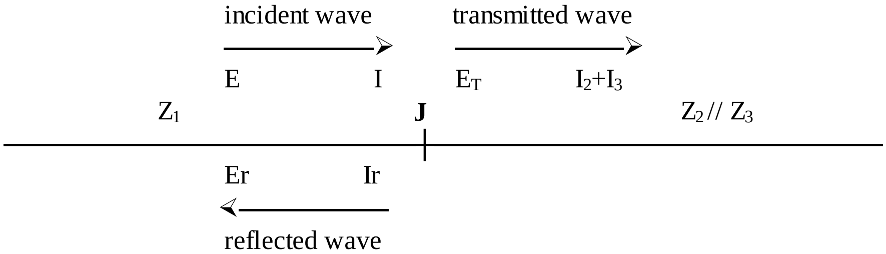


Figure 4.12 - Equivalent 2-branch connection

This gives the transmitted surge as

$$E_T = \frac{2(Z_2 // Z_3)}{Z_1 + (Z_2 // Z_3)} E$$

$$E_T = \frac{\frac{2 Z_2 Z_3}{Z_2 + Z_3}}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} E \quad = \frac{2 Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} E$$

which is basically the same expression as was obtained earlier. Extension of the latter method to a multiline junction is very much easier, as in this case only the parallel equivalent of a larger number of lines needs to be obtained. No new derivations are required.

Example:

An overhead line **A** with a surge impedance 450 Ω is connected to three other lines [overhead lines **B** and **C** with surge impedances of 600 Ω each, and a cable **D** with a surge impedance 60 Ω] at the junction **J**. A traveling wave of vertical front of magnitude 25 kV and very long tail travels on **A** towards the junction **J**. Calculate the magnitude of the voltage and current waves which are transmitted and reflected when the surge reaches the junction **J.** Attenuation in the lines can be neglected.

For an incident surge from **A**, lines **B, C** and **D** are effectively in parallel.

The parallel equivalent impedance is Z_T = 600 Ω//600 Ω//60 Ω = 50 Ω

$$E_T = 2 \times 50 \times 25 \ / \ (450 + 50) = 5 \text{ kV}$$

$$E_r = (50 - 450) \times 25 \ / \ (450 + 50) = - 20 \text{ kV}$$

$$I = 25\text{x } 10^3/450 = 55.56 \text{ A}, \ \ I_r = -(- 20 \times 10^3)/450 = 44.44 \text{ A},$$

$$I_B = 5 \times 10^3/600 = 8.33 \text{ A}, I_C = 5 \times 10^3/600 = 8.33 \text{ A}, \ \ I_D = 5 \times 10^3/60 = 83.33 \text{ A}$$

4.5 Bergeron's Method of Graphical Solution

The lattice diagram method of solution is not easily applied when the load impedance is a non-linear device. In such cases, the graphical method of Bergeron is suitable. This is also based on the partial differential equations

$$-\frac{\partial v}{\partial x} = l \frac{\partial i}{\partial t}, \quad -\frac{\partial i}{\partial x} = c \frac{\partial v}{\partial t}$$

which has the traveling wave solution

v = f(x-at) + F(x+at) where **a** is the wave velocity

The corresponding current can be obtained as follows.

$$-\frac{\partial i}{\partial x} = c\,\frac{\partial v}{\partial t} = c\,[\,-a\;f'(x-a\,t) + a\,F'(x+a\,t)]$$

$$\therefore\; i = a\,c\,[f\,(x-a\,t) - F\,(x+a\,t)], \quad \text{also } Z_0 = \sqrt{\frac{l}{c}} = \frac{1}{a\,c}$$

$$\text{i.e. } i\,Z_0 = f\,(x-a\,t) - F\,(x+a\,t)$$

$$\therefore\; v - i\,Z_0 = 2\,F\,(x+a\,t), \qquad v + i\,Z_0 = 2\,f\,(x-a\,t)$$

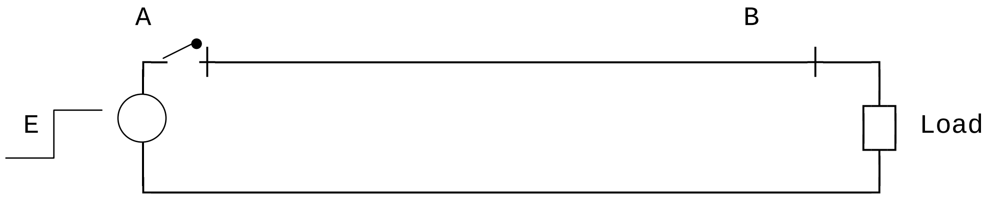
As was learnt earlier, **f(x-at) = constant** represents a **forward traveling wave**, and **F(x+at) = constant** represents a **backward traveling wave**.

From the expressions derived above, it can be seen that **v + Z_0 i = constant** represents a **forward wave** and **v - Z_0 i = constant** represents a **backward wave**. In either case the value of the constant is determined from the history of the wave up to that time.

The Bergeron's method is applied on a **voltage-current** diagram, and is illustrated by means of an example.

Example

A transmission line, surge impedance Z_0 is fed from a constant voltage supply **E** at one end and by a non-linear resistor whose V-I characteristic is known at the other end. Determine the waveform of the voltage at the load end when the initially open line is closed at end **A** at time zero.



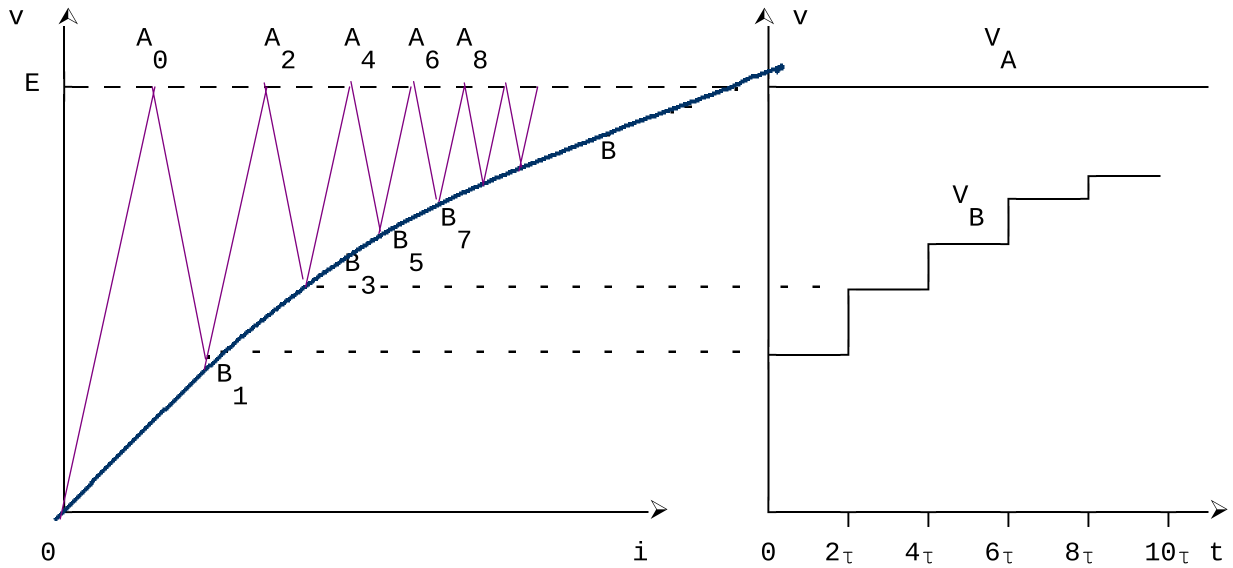


Figure 5.13 - Bergeron's method of solution

The method of constructing the diagram is as follows. The VI characteristic of the source (constant at E), and that of the load (non-linear) are drawn on the V-I diagram. If t = 0 is defined as the instant at which the surge initially reaches the load, then this surge leaves **A** at time $t = -\tau$. This original surge must satisfy two conditions. Firstly, since it is leaving **A**, it must be a point on the source characteristic $\mathbf{A_{-1}}$. Secondly, this is a forward surge. Thus it must be of the form $\mathbf{v + Z_0\, i = constant}$, or a line with slope $\mathbf{-Z_0}$.

Thus this surge is a line with slope $\mathbf{-Z_0}$ leaving $\mathbf{A_{-1}}$. This surge arrives at **B** at time **0**.
At this instant, it must also be a point on the load characteristic as well as on the surge line. Thus it must be the point $\mathbf{B_0}$. At **B** the surge is reflected back. This reflected surge must start from $\mathbf{B_0}$, and also have a slope $\mathbf{+Z_0}$ corresponding to $\mathbf{v - Z_0\, i = constant}$. The surge reaches $\mathbf{A_1}$ at time $\boldsymbol{\tau}$. The process continues. From the diagram, we can determine the voltage at **A** at time $\boldsymbol{-\tau, \tau, 3\tau, 5\tau}$ etc, and the voltage at **B** at time $\mathbf{0}, \boldsymbol{2\tau, 4\tau, 6\tau}$ etc. The voltage waveforms at both **A** and **B** are easily obtained by projecting the values as shown on the diagram on to the right hand side. Similarly, the current waveforms can be obtained by projecting the values below.

4.6 Representation of Lumped Elements in travelling wave techniques

In the Lattice diagram technique, basically only transmission lines can be represented. Since the surge impedance of transmission lines are purely resistive, resistances can also be represented with surge impedance equal to the resistance value, and no travel time.

Inductances and capacitances could be represented, by considering them as very short lines or stub lines. This is done by assuming that an inductance has a distributed capacitance of negligible value to earth, and that shunt capacitances have a negligible series inductance. These assumptions will make the lumped elements stub lines with negligible transmission times. It is usual to select the transmission times corresponding to the minimum time increment Δt.

For the lumped inductance connected in series

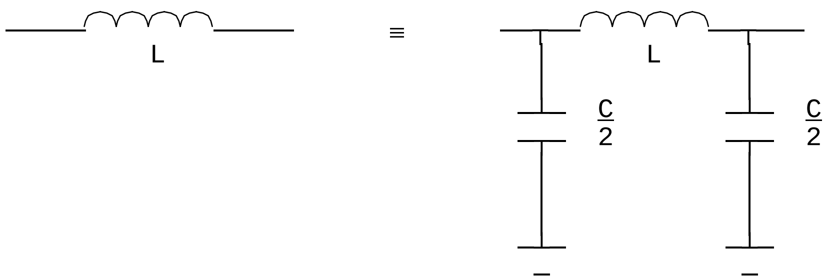


Figure 4.14 - Representation of Inductance

If the travel time of the line is selected corresponding to $\tau = \Delta t$

$$\tau = \sqrt{L\,C} \quad \text{so that} \quad C = \frac{\tau^2}{L} = \frac{(\Delta t)^2}{L}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{L}{\tau} = \frac{L}{\Delta t}$$

Thus a lumped inductance may be represented by a stub line of transit time $\mathbf{\Delta t}$ and surge impedance $\mathbf{L/\Delta t}$.

For the lumped capacitance connected in shunt

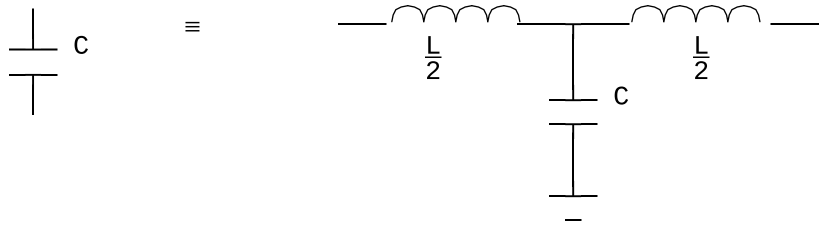


Figure 4.15 - Representation of Capacitance

$$\tau = \sqrt{L\,C} \quad \text{so that} \quad L = \frac{\tau^2}{C} = \frac{(\Delta t)^2}{C}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\tau}{C} = \frac{\Delta t}{C}$$

Thus a lumped capacitance may be represented by a stub line of transit time **Δt** and surge impedance **Δt/C**.

Another method of analyzing in the presence of inductances and capacitances is to use the numerical form as indicated below.

$$v = L\frac{d\,i}{d\,t} \quad \text{for an inductance}$$

$$\frac{v_n + v_{n-1}}{2} = L\,\frac{i_n - i_{n-1}}{\Delta t}$$

$$\text{i.e.} \quad v_n = \frac{2L}{\Delta t}\,i_n - \left(v_{n-1} + \frac{2L}{\Delta t}\,i_{n-1}\right)$$

$$\text{also} \quad i_n = \frac{\Delta t}{2L}\,v_n + I_{eq,n-1}$$

$$\text{where} \quad I_{eq} = i + \frac{\Delta t}{2L}\,v$$

$$i = C\frac{d\,v}{d\,t} \quad \text{for an capacitance}$$

$$\frac{i_n + i_{n-1}}{2} = C\,\frac{v_n - v_{n-1}}{\Delta t}$$

$$\text{i.e.} \quad i_n = \frac{2C}{\Delta t}\,v_n - \left(i_{n-1} + \frac{2C}{\Delta t}\,v_{n-1}\right)$$

$$\text{i.e.} \quad i_n = \frac{2C}{\Delta t}\,v_n + I_{eq,n-1}$$

$$\text{where} \quad I_{eq} = -i - \frac{2C}{\Delta t}$$

4.7 Branch Time Table for digital computer implementation

The Bewley Lattice diagram cannot be implemented directly on the digital computer. When implemented on the computer, a physical diagram is not required to keep track of the travelling waves. The branch time table serves the purpose of the diagram, and keeps track of the voltage of each node, and the reflected and transmitted waves.

4.8 Transform Methods of solving Transients

Simple transient problems may be solved using the Laplace Transform. However, since its inverse transform is not evaluated for complicated transforms, the Fourier transform is preferred. Further, for digital computer application, what is used is the numerical form of the transform, which is obtained by approximating the integrals to summations.

$$F(\omega)=\int_0^{\infty} f(t)\,e^{-j\omega t}\,dt = \sum_0^{T} f(t)\,e^{-j\omega t}\,\Delta t$$

This is used with the transfer function **H(ω)** of a network to give the transform of the response.

$$\xrightarrow[F(\omega)]{} \boxed{H(\omega)} \xrightarrow[R(\omega)]{}$$

Figure 4.16 - Block diagram of system

$$R(\omega) = H(\omega)\,.\,F(\omega)$$

The response in the time domain is then obtained by inverse transformation.

$$r(t)=\frac{2}{\pi}\int_0^{\infty} R(\omega)\,.\,e^{j\omega t}\,.\,d\omega$$

$$r(t)=\frac{2}{\pi}\sum_0^{\Omega} R(\omega)\,.\,e^{j\omega t}\,.\Delta\omega$$

With the approximations introduced, the period of observation T and the maximum frequency Ω are limited to finite values.

The transmission line is usually represented by the frequency dependant two-port admittance parameters, for the determination of the transfer function.

$$\begin{bmatrix} I_S \\ I_R \end{bmatrix} = \begin{bmatrix} A & -B \\ -B & A \end{bmatrix} \begin{bmatrix} V_S \\ V_S \end{bmatrix}$$

where

A = Y_0 coth γl
B = Y_0 cosech γl
Y_0 = surge impedance matrix
γ = propagation matrix