Locking in a Tournament Win: Build a Better Surrender Trap

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It is the final hand of a blackjack tournament and only you and one other player remain. You have 10000, and your opponent has 8700, but you must bet first. He expects you to bet 1200, but instead you bet 2000. Thinking that he can triumph if you lose the hand, your opponent "takes the low"[1] and bets the minimum 100. The cards are dealt and then you surrender, leaving yourself with 9000. Your surprised opponent can't catch you unless he wins three bets. He fails, you take the trophy, and get the girl.

The above description is what most tournament players think of when they hear "surrender trap." There is a major problem with the story though—your opponent might not bet the minimum to take the low. If he follows the time-honored rule of holding back one chip more than you have held back, he would bet 600 and leave himself with 8100 on the rail, a black chip more than the 8000 you have left on the rail. If you both lose your bets, he wins the tournament by one chip. Unlike the minimum 100 bet, this 600 bet poses a threat: if your opponent wins his hand, his bankroll will increase to 9300, compared to the 9000 you will have if you surrender. Overbetting your lead fooled him, but didn't lock him out.

So what size bet will give you the lock if it fools a player who bets the maximum amount possible to take the low? This question is more cumbersome than it seems, because of several complications:

1. BR2 can double down or get a blackjack, or even win up to eight bets in a DAS RS4 game.

2. If the dealer has a blackjack, BR1 won't have the opportunity to surrender.

3. If the dealer has an Ace up, the players must consider insurance.

4. Players are required to bet in minimum increments.

To deal with this last problem, we will use a categorization methodology that could be useful for other questions involving betting increments. We will start by examining a theoretical category where a player can bet any positive amount he chooses, even the tiniest fraction of a cent that he can imagine. This category is labeled "Infinitesimal" in the chart following. Then we proceed to the real world, where the player's bets must be divisible by some minimum increment. For instance, some of the major tournaments give each player an initial bankroll of 10000 in tournament chips, but require bets to be made in increments of 100. (A 100000/500 scheme is also becoming prevalent.) Because of the characteristics of blackjack, we know that players must have multiples of half increments. For instance, with an increment of 100, a player may wind up with some odd multiple of 50, due to naturals, surrender, and insurance, but further division is impossible (assume no tipping, insurance allowed in half increments, doubling for less still in full increments, and naturals paying either 3:2 or 2:1). This constraint makes the problem tractable. We can break down the situations according to two possibilities—BR1's lead is divisible by the betting increment (for example, a lead of 700), or is divisible by the half increment (for example, a lead of 650). These categories are denoted by "Full" or "Half" in the middle column of the chart below.

The consideration of units and the other complications above creates eighteen different cases. We need a thorough analysis, because the surrender trap has taken on increased relevance with the introduction of tournaments that feature "elimination blackjack,"™ where the lowest-bankrolled

[1] To "take the low" means to make a bet so that you will be the leader if everyone loses the hand, or, more accurately, you will be the leader if none of your opponents win their hands. To "take the high" means to make a bet so that you will be the leader if you win your hand, even if your opponents also win their hands. To "have the swing" means that you will become the leader only if you win your hand while the current leader loses his hand. Because losing a hand is more likely than winning in a typical blackjack game, taking the low against a single opponent is usually a better play than taking the high in that situation.

player is eliminated at the conclusion of, say, round 8, round 15, and round 22 of a 30-round tournament.[2] An elimination round is similar to the final round in that the lowest-bankrolled player meets with Death—survival becomes paramount, and the surrender trap has shown to be powerful, even in fields of supposedly strong competitors.

Analyzing BR1's Surrender Trap: 18 Cases		
How Many Bets Can BR1 Allow BR2 to Win?	Lead is a Multiple of What Increment?	Does BR1 Buy Insurance Against Ace?
1	Infinitesimal	No
1	Infinitesimal	Yes
1	Full	No
1	Full	Yes
1	Half	No
1	Half	Yes
1.5	Infinitesimal	No
1.5	Infinitesimal	Yes
1.5	Full	No
1.5	Full	Yes
1.5	Half	No
1.5	Half	Yes
2	Infinitesimal	No
2	Infinitesimal	Yes
2	Full	No
2	Full	Yes
2	Half	No
2	Half	Yes

We will use the following additional notation in the mathematical derivation of the optimal bets for the first six cases:

BR1	Bankroll 1 (leader)	ε	a very small, positive amount (*really, really* small)
BR2	Bankroll 2 (second place)	δ	a very small, positive amount (you cannot imagine)
B1	Bet by BR1	x	minimum betting increment, 100 in the examples
B2	Bet by BR2	L	lead going into elimination round $\equiv$ BR1 $-$ BR2
I	Insurance bet by BR1		

Case 1: BR2 wins 1 bet, Infinitesimal Bets Allowed, No Insurance

First, BR1 must overbet his lead in order to trick BR2 into trying to take the low, so B1 $>$ L, an expression we term the "bait condition." Furthermore, BR1 wants to be in the lead after surrendering, even if BR2 wins his bet, so BR1 $- \frac{1}{2}$B1 $>$ BR2 + B2, an expression called the "lead condition." For BR2's betting decision, he wants to bet as much as possible while maintaining the low, so that even if he and BR1 both lose their bets, he will have the lead: BR2 $-$ B2 $=$ BR1 $-$ B1 $+ \varepsilon \implies$ B2 $=$ B1 $- L - \varepsilon$. By substituting for B2 in BR1's lead condition, we obtain B1 $< (4/3)L + (2/3)\varepsilon \implies$ B1 $\leq (4/3)L$. Combining this final result with the initial bait condition that BR1 must overbet his lead tells us that BR1 should select B1 such that $L <$ B1 $\leq (4/3)L$, in order to lock out BR2 if BR2 is fooled into trying to take the low.

Example of BR1 making appropriate bet. Suppose BR1 has 10000 and BR2 has 9100, and BR1 bets 1200. Then BR2 bets 300 $- \varepsilon$ to take the low. BR1's surrender leaves him with 9400. The most BR2 can have by winning one bet is 9400 $- \varepsilon$. Because BR1 satisfied the condition that $L <$ B1 $\leq (4/3)L$, the surrender trap locked out BR2.

[2] The phrases "elimination blackjack" and "elimination round" are trademarks of the Ultimate Blackjack Tour.

Example of BR1 failing to trap. Now suppose BR1 has 10000 and BR2 has 9100, as before, but BR1 bets 1201 (too much!) and then BR2 follows with a bet of $301 - \varepsilon$. If BR1 surrenders, his bankroll drops to 9399.5. If BR2 wins his bet, he has $9401 - \varepsilon$, which exceeds BR1's total (since ε is a very tiny number). Even though BR2 was tricked into taking the low, he wasn't locked out.

Case 2: BR2 wins 1 bet, Infinitesimal Bets Allowed, Insurance

If the dealer gets a Ten-up blackjack, there is nothing BR1 can do—he is caught by his own trap and loses, because he has overbet his lead. If the dealer has an Ace up, BR1 can buy insurance. BR1 wants to buy enough insurance so that if he wins the insurance bet, he will have more money than BR2 had going into the round. BR1 must stay above BR2's bankroll, because BR2 might not lose any money on the hand (BR2 might fully insure). Furthermore, BR1 wants to be ahead even if he loses the insurance bet, surrenders the initial bet, and BR2 wins his bet. Let's go through the equations one at a time.

BR1 must buy enough insurance so that he will have a lead if the dealer has blackjack: $\mathrm{BR1} - \mathrm{B1} + 2I > \mathrm{BR2}$, or $\mathrm{BR1} - \mathrm{B1} + 2I = \mathrm{BR2} + 2\delta$. Solving for the insurance bet gives $I = \frac{1}{2}(\mathrm{BR2} - \mathrm{BR1} + \mathrm{B1} + 2\delta) = \frac{1}{2}(\mathrm{B1} - L) + \delta$. Winning this insurance bet will leave BR1 with more than BR2, even if BR2 fully insures. If BR1 loses his insurance bet and then surrenders, he drops to $\mathrm{BR1} - I - \mathrm{B1}/2$, but wants this quantity to exceed the amount BR2 will have if he wins his bet, so our lead condition is $\mathrm{BR1} - [\frac{1}{2}(\mathrm{B1} - L) + \delta] - \mathrm{B1}/2 > \mathrm{BR2} + \mathrm{B2}$. BR2 bets as much as possible while still maintaining the low, so $\mathrm{B2} = \mathrm{B1} - L - \varepsilon$, as always. Substituting this value of B2 into the lead condition tells us that $\mathrm{B1} < (5/4)L + \varepsilon/2 - \delta/2$. Choose $\delta < \varepsilon$ (this choice is possible because δ is part of BR1's insurance wager which is decided *after* BR2 has chosen ε) and we have $\mathrm{B1} \leq (5/4)L$.

So, BR1's bet must be in the range $L < \mathrm{B1} \leq (5/4)L$. Recall that without insurance in Case 1, the range was $L < \mathrm{B1} \leq (4/3)L$, allowing BR1 to bet one third more than his lead. Now, to have extra chips for insurance, BR1 can bet only one fourth more than his lead.

Example of BR1 making appropriate bet. Suppose BR1 has 10000 and BR2 has 8000. A bet of $(5/4)L$ would be 2500, inducing BR2 to bet $500 - \varepsilon$ to take the low. If the dealer shows an Ace, BR1 then buys $250 + \delta$, where $\delta < \epsilon$, so $250 + \frac{1}{2}\varepsilon$ is fine. If the dealer has blackjack, BR1 would have $8000 + \varepsilon$, which exceeds BR2's best possible chip stack of 8000. If the dealer does not have blackjack, BR1 surrenders to have $8500 - \frac{1}{2}\varepsilon$, while BR2 will have only $8500 - \varepsilon$, even if he wins his bet.

Example of BR1 failing to trap. Now suppose BR1 has 10000 and BR2 has 8000, but BR1 overbets by betting 2501. BR2 follows with $501 - \varepsilon$. Make sure, for this example, that BR2 chooses $\varepsilon \leq 2$, so $\mathrm{B2} = 500.5$ would suffice. Against an Ace, BR1 needs to buy at least $250.5 + \delta$ in insurance. If the dealer has blackjack, BR1 will still win, because he would have more than 8000, which is what BR2 started with. The problem is that if there is no blackjack, BR1's subsequent surrender leaves him with only $8499 - \delta$, while BR2 can reach a higher total of $8501 - \varepsilon$ (remember that BR2 chose $\varepsilon \leq 2$).

Case 3: BR2 wins 1 bet, Lead is a Full Increment, No Insurance

In the next several examples, bets must be in multiples of the minimum betting increment x. If the lead is a multiple of this increment, then $L/x = w$, where w is a whole number. For the trap to work, BR1 must bet enough so that BR2 thinks that the low is available, so BR1 must bet at least his lead plus two minimum increments, creating a minor modification to the bait condition: $\mathrm{B1} \geq L + 2x$. The lead condition still requires BR1 to hold the lead after he surrenders, even if BR2 wins his bet: $\mathrm{BR1} - \frac{1}{2}\mathrm{B1} > \mathrm{BR2} + \mathrm{B2}$. As usual, BR2 bets as much as possible while still covering the low: $\mathrm{B2} = \mathrm{B1} - L - x$. Substituting this expression for B2 into the lead condition produces $\mathrm{B1} < (4/3)L + (2/3)x$, for a final result that BR1's bet must satisfy $L + 2x \leq B1 < (4/3)L + (2/3)x$.

This is possible only if $L > 4x$, and under our assumption that the lead is an exact multiple of the betting increment, $L \geq 5x$.

Example of large enough lead. Suppose BR1 has 10000 and BR2 has 9500, and the betting increment is $x = 100$. The rule for BR1's bet says $700 \leq \mathrm{B1} < (4/3) \cdot 500 + (2/3) \cdot 100 \approx 733$. With increments of 100, BR1's only choice is to bet B1 = 700. BR2 bets 100, allowing him to reach 9600, which will not be enough to beat the 9650 that BR1 will have after surrendering. The trap worked because $L \geq 5x$.

Example of insufficient lead. Suppose BR1 has 10000 and BR2 has 9600. The rule requires $600 \leq \mathrm{B1} < 600$, an impossible condition, but let's see how it happens. BR1 knows that he must bet at least 600 to bait BR2 into betting 100. After surrendering his 600 bet, BR1 has 9700, allowing BR2 to catch him (a tie at 9700) by winning the hand. The trap did not succeed, because a lead of $L = 4x$ is not enough.

Case 4: BR2 wins 1 bet, Lead is a Full Increment, Insurance

As before, we have to lay the bait, so $\mathrm{B1} \geq L + 2x$. The amount overbet, $\mathrm{B1} - L$, must be recovered via insurance. Since insurance pays 2:1, we must bet half of the amount overbet, and then we want to actually lead BR2 by a chip, so we add an additional $x/2$ of insurance (insurance bets can be in half increments): $I = (\mathrm{B1} - L)/2 + (1/2)x$. BR1 wants to have the lead even if he loses the insurance bet, surrenders his main bet, and BR2 wins a bet: $\mathrm{BR1} - [(\mathrm{B1} - L)/2 + (1/2)x] - \mathrm{B1}/2 > \mathrm{BR2} + \mathrm{B2}$. As always, we substitute into this expression the assumption that BR2 takes the low by betting $\mathrm{B2} = \mathrm{B1} - L - x$. Solving for B1 and combining with the bait condition produces the rule that $L + 2x \leq \mathrm{B1} < (5/4)L + (1/4)x$, which has a solution as long as $L \geq 8x$.

Example of large enough lead. With a 100 betting increment, the last condition suggests that the lead must be 800 or more in order for BR1 to lock out an unsuspecting BR2, while still protecting against the dealer's Ace-up blackjack. Suppose BR1 has 10000 and BR2 has 9200, producing a recommendation that $1000 \leq \mathrm{B1} < 1025$. After BR1 bets 1000 and BR2 bets 100, BR1 can still afford to bet the 150 in insurance, $I = (1000 - 800)/2 + (1/2) \cdot 100$. Loss of the insurance and subsequent surrender leaves BR1 with 9350, more than BR2 will have after winning a single bet.

Example of insufficient lead. Suppose BR1 has 10000 and BR2 has 9300. BR1 tries to lay the trap by betting 900, inducing the reactive bet of 100. If BR1 loses his 150 in insurance (which would give him 9400 if the dealer has blackjack), he will have 9400 after surrendering. BR2 will catch him by winning the hand, resulting in a tie at 9400. With a lead of only $L = 7x$, BR1 cannot lock out BR2.

Case 5: BR2 wins 1 bet, Lead is a Half Increment, No Insurance

Here we assume that the lead is not divisible by a full betting increment, but is divisible by a half increment. It must be divisible by half increments, because only half increments and full increments can be won or lost in a blackjack game.[3] Mathematically these assumptions imply $L/(x/2) = z$, where z is an odd number.

To lay the bait, subject to the restriction that his bet must be in full increments, BR1 bets at least $\mathrm{B1} \geq L + x + (1/2)x$, and wants to be ahead after surrendering: $\mathrm{BR1} - \mathrm{B1}/2 > \mathrm{BR2} + \mathrm{B2}$. BR2 will bet the maximum amount possible while maintaining the low: $\mathrm{B2} = \mathrm{B1} - L - (1/2)x$. Substituting this bet into the previous condition, simplifying, and combining with the bait condition tells us that BR1's bet must satisfy: $L + x + (1/2)x \leq \mathrm{B1} < (4/3)L + (1/3)x$, which has a solution if $L > (7/2)x$. Since this case assumes that the lead is a half-increment multiple, we can rewrite this condition as $L \geq (9/2)x$.

[3] "Insurance for less" is usually allowed, but still required to be made in half increments. Similarly, doubling for less is possible, but usually restricted to full increments.

Example of large enough lead. If BR1 has 10000 and BR2 has 9550, we can apply this case. BR1 must choose B1 in the range $600 \leq \mathrm{B1} < 633$, so 600 is his choice. BR2 bets the minimum, 100. After surrendering, BR1 has 9700, which beats the 9650 that BR2 could reach by winning the hand. The trap works because the lead is 450 or more (9/2 of 100).

Example of insufficient lead. Suppose BR1 has 10000 and BR2 has 9650. BR1's bait bet is 500, leaving him with 9750 upon surrendering. BR2's 100 bet allows him to tie at 9750 by winning the hand. Because $L = (7/2)x$, BR1 cannot lock out BR2.

Case 6: BR2 wins 1 bet, Lead is a Half Increment, Insurance

We complicate the previous case by adding an insurance decision. The insurance must be enough to protect BR1, who overbet his lead, so $I = (\mathrm{B1} - L)/2 + (1/4)x$. The bait condition is the same as the previous case, $\mathrm{B1} \geq L + x + (1/2)x$, and the lead condition adds in the loss of the insurance wager: $\mathrm{BR1} - [(\mathrm{B1} - L)/2 + (1/4)x] - \mathrm{B1}/2 > \mathrm{BR2} + \mathrm{B2}$. BR2 takes the low: $\mathrm{B2} = \mathrm{B1} - L - (1/2)x$. Substituting, simplifying, slicing, dicing, wheeling, dealing, and combining gives us: $L + (3/2)x \leq \mathrm{B1} < (5/4)L + (1/8)x$, which can be satisfied if the lead is large enough, $L \geq (13/2)x$.

Example of large enough lead. If BR1 has 10000 and BR2 has 9350, BR1 bets 800, satisfying $800 \leq \mathrm{B1} < 825$. If BR1 buys 100 of insurance and surrenders, he is left with 9500, more than BR2's possible 9450. The lead of 650 satisfies the requirement that $L \geq (13/2)x$.

Example of insufficient lead. Suppose BR1 has 10000 and BR2 has 9450. BR1's bait bet is 700. Against a dealer's Ace, BR1 is afraid of losing to a blackjack and dropping to 9300, lower than BR2's 9350. Since BR2 could buy full insurance to remain at 9450, BR1 needs to buy 100 of insurance to get back up to 9500. Losing the 100 of insurance and half his original bet through surrender leaves BR1 with 9550, the same amount that BR2 could reach by winning the hand. Because the lead is only $(11/2)x$, BR1 cannot lock out BR2.

Manufacturing the Trap: BR1's Betting Range for all 18 Cases

The following summary chart shows: (1) BR2's bet if he bets the maximum amount possible while taking the low; (2) BR1's range of possible bets that could fool BR2 into taking the low, cover the specified number of bets won by BR2 (1, 1.5, or 2), and still allow funds for insurance; (3) The size of the lead sufficient to allow BR1 to make the appropriate bet; and (4) BR1's insurance bet that guarantees that he will not lose more than his lead if the dealer has an Ace-up blackjack.

How BR1 Can Lay the Surrender Trap				
		How BR1 Should Choose his Bet, B1		
How Many Bets Can BR1 Allow BR2 to Win?	Lead is Multiple of What Inc.?	Range of Possible Bets	How Big Does the Lead Have to Be?	BR1's Insurance Bet I
1	Inf.	$L < \text{B1} \leq (4/3)L$	Any lead OK	—
1	Inf.	$L < \text{B1} \leq (5/4)L$	Choose $\delta < \varepsilon$	$(\text{B1} - L)/2 + \delta$
1	Full	$L + 2x \leq \text{B1} < (4/3)L + (2/3)x$	$L \geq 5x$	—
1	Full	$L + 2x \leq \text{B1} < (5/4)L + (1/4)x$	$L \geq 8x$	$(\text{B1} - L)/2 + (x/2)$
1	Half	$L + (3/2)x \leq \text{B1} < (4/3)L + (2/3)(x/2)$	$L \geq (9/2)x$	—
1	Half	$L + (3/2)x \leq \text{B1} < (5/4)L + (1/4)(x/2)$	$L \geq (13/2)x$	$(\text{B1} - L)/2 + (x/4)$
1.5	Inf.	$L < \text{B1} \leq (5/4)L$	Any lead OK	—
1.5	Inf.	$L < \text{B1} \leq (6/5)L$	Choose $\delta < (3/2)\varepsilon$	$(\text{B1} - L)/2 + \delta$
1.5	Full	$L + 2x \leq \text{B1} < (5/4)L + (3/4)x$	$L \geq 6x$	—
1.5	Full	$L + 2x \leq \text{B1} < (6/5)L + (2/5)x$	$L \geq 9x$	$(\text{B1} - L)/2 + (x/2)$
1.5	Half	$L + (3/2)x \leq \text{B1} < (5/4)L + (3/4)(x/2)$	$L \geq (11/2)x$	—
1.5	Half	$L + (3/2)x \leq \text{B1} < (6/5)L + (2/5)(x/2)$	$L \geq (15/2)x$	$(\text{B1} - L)/2 + (x/4)$
2	Inf.	$L < \text{B1} \leq (6/5)L$	Any lead OK	—
2	Inf.	$L < \text{B1} \leq (7/6)L$	Choose $\delta < 2\varepsilon$	$(\text{B1} - L)/2 + \delta$
2	Full	$L + 2x \leq \text{B1} < (6/5)L + (4/5)x$	$L \geq 7x$	—
2	Full	$L + 2x \leq \text{B1} < (7/6)L + (3/6)x$	$L \geq 10x$	$(\text{B1} - L)/2 + (x/2)$
2	Half	$L + (3/2)x \leq \text{B1} < (6/5)L + (4/5)(x/2)$	$L \geq (13/2)x$	—
2	Half	$L + (3/2)x \leq \text{B1} < (7/6)L + (3/6)(x/2)$	$L \geq (17/2)x$	$(\text{B1} - L)/2 + (x/4)$
BR2's betting decision is straightforward. He holds back a tiny amount more than BR1 has held back, implying that his bet is $\text{B2} = \text{B1} - L - \varepsilon$ (infinitesimal case), or $\text{B2} = \text{B1} - L - x$ (full-unit), or $\text{B2} = \text{B1} - L - (x/2)$ (half-unit).				
BR1's insurance bet is always just enough to recapture the amount he overbet his lead, plus a little extra to strictly surpass BR2's current chip total.				

Note that some bets will satisfy more than one of the 18 cases. If BR1 can cover a 2-bet win by BR2, then he can cover a blackjack or a single-bet win by BR2 as well. Furthermore, if BR1 ends up not buying any insurance after making a bet so that he could buy insurance, sometimes he will be protected against larger wins by BR2. Suppose BR1 initially intends to cover a 1-bet win by BR2 while saving enough chips for insurance. BR1 now has the extra chips to insure, but perhaps he should worry instead about BR2's double down (for example, upon seeing that BR2 has a hard 11), or perhaps the dealer does not have an Ace up, and BR1's chips allocated for insurance are now extra. The following chart shows the coverage provided by the recommended bet at each size lead:

Recommended Trapping Bet for BR1 Betting Increment is 100								
Lead	Bet	Coverage	Lead	Bet	Coverage	Lead	Bet	Coverage
0-400	—	—	1700	*2000	2+I	3000	3500	2+I
450	600	1	1750	*2000	2+I	3050	3500	2+I
500	700	1	1800	2000	2+I	3100	3500	2+I
550	700	1.5	1850	2000	2+I	3150	3500	2+I
600	800	1.5	1900	2100	2+I	3200	3500	2+I
650	800	2,1+I	1950	2100	2+I	3250	3500	2+I
700	900	2	2000	2200	2+I	3300	*4000	2!
750	900	2,1.5+I	2050	2200	2+I	3350	*4000	2!
800	1000	2,1+I	2100	*2500	2!	3400- 3850	4000	2+I
850	1000	2+I	2150	*2500	2+I	3900- 4250	4500	2+I
900	1100	2,1.5+I	2200	*2500	2+I	4300- 4850	*5000	2+I
950	1100	2+I	2250	*2500	2+I	4900- 5100	5500	2+I
1000	1200	2+I	2300	2500	2+I	5150- 5850	*6000	2+I
1050	1200	2+I	2350	2500	2+I	5900- 5950	6500	2+I
1100	1300	2+I	2400	2600	2+I	6000- 6350	7000	2+I
1150	1300	2+I	2450	2600	2+I	6400- 6800	7500	2+I
1200	*1500	2!	2500	3000	2!	6850- 7650	8000	2+I
1250	*1500	2!	2550	3000	2!	7700- 8550	9000	2+I
1300	1500	2+I	2600	3000	2+I	8600- 9850	10000	2+I
1350	1500	2+I	2650	3000	2+I	9900-10250	11000	2+I
1400	1600	2+I	2700	3000	2+I	10300-11100	12000	2+I
1450	1600	2+I	2750	3000	2+I	11150-12800	13000	2+I
1500	1700	2+I	2800	3000	2+I	12850-14850	15000	2+I
1550	1700	2+I	2850	3000	2+I	14900-16850	17000	2+I
1600	1800	2+I	2900	3500	2!			
1650	*2000	2!	2950	3500	2!			
An asterisk in the bet column indicates that the rules of thumb below are not quite exact.								
Coverage refers to the number of bets BR1 can allow BR2 to win. +I indicates that BR1 can also buy insurance. In a few cases, BR1 can elect not to cover a 2-bet win and instead buy insurance. An exclamation point means that BR1 forgoes insurance coverage in order to make a more deceptive bet.								

In showing the coverage provided, the chart tops out at "2+I," even though some of those bets in fact allow coverage of three bets or more. There are very few cases where we sacrifice coverage in order to add deception. In our opinion (subject to revision as we collect more real-world results), if BR2 knows the move, he knows the move. That is, if BR2 is already aware of the surrender trap, he is unlikely to fall for it, regardless of your bet, and if he is unaware of it, he will fall for it the moment he sees that you have overbet your lead. We lean towards making the bet that gives the maximum coverage and worrying less about selling the trap. This trap is so good it sells itself; BR2 will be beating a path to our door to try to take that low. When he does, we want maximum coverage.

Those looking for rules of thumb, instead of charts to memorize, are in luck:

1. For leads less than 2500, bet the lead plus two increments ($L+2x$), and round down if necessary. For instance, with a 1050 lead, bet $1050 + 2(100) = 1250$, which rounds down to 1200, since bets must be in multiples of the increment $x = 100$.

2. For leads less than 6000, use the previous rule of thumb, but then round the bet *up* to the

nearest multiple of 500. For instance, with a lead of 2950, the first rule of thumb gives us a preliminary bet of $2950 + 200 = 3150$, which is rounded down to 3100, but then we round that number *up* to 3500.

3. For leads greater than or equal to 6000, use the chart.

If that's still not simple enough, keep in mind that the single rule of betting the lead plus two increments ($L + 2x$), and rounding down when necessary, will always produce a viable trapping bet. Equivalently, we can try to bet $L + (3/2)x$, and round up to the nearest allowed bet. There are cases, though, when a larger bet is more marketable. To find the upper end of the range of viable bets, notice the case where we allow infinitesimal bets, and where we can buy insurance and still cover a 2-bet win from BR2. A bet $\text{B1} = (7/6)L$ works.

For the real-world complication where bets must be made in minimum increments, we retain this $(7/6)L$ term, and add on a term involving the increment x. But for large leads, the term involving x is trivial, and is used to form an inequality condition anyway, so if we treat it like our little friend ε, we can rewrite the condition $\text{B1} < (7/6)L + (\bullet)x$ as $\text{B1} \leq (7/6)L$, or $\text{B1} \leq L + (1/6)L$. Combining these simplications gives us a slightly less accurate, but perhaps easier to remember, single rule:

$$L + (3/2)x \leq \text{B1} \leq (7/6)L, \text{ for } L \geq 10x$$

The right-hand side instructs us to overbet our lead by 1/6, and then round B1 down to the nearest increment. (Some players overbet by 1/5, but then they do not have a free insurance option.)

For example, suppose the lead is 1200 (how convenient!), with a minimum allowed betting increment of 100. The rule of thumb gives us the range $1350 \leq \text{B1} \leq 1400$. Since 1350 is not a multiple of the increment, we can rewrite the range as $1400 \leq \text{B1} \leq 1400$. It looks like we'll be betting 1400.

A Can of Worms—Yummy yummy!

Notice that the chart of recommended bets includes small leads, but ends at bets of 17000. Let's say that we have a small lead of 7000 (with an initial bankroll of 100000 and a max bet of 50000). On the last hand, we have three candidates for our bet:

1. $\text{B1} = 6700$ to take the *straight low* (some players would bet 6900 to accomplish this goal)

2. $\text{B1} = 8000$ to lay the *surrender trap*

3. $\text{B1} = 13800$ to take the *surrender low*

With the third option, we have bet just under twice our lead, so upon surrendering, we will have the low, but if we do not surrender, we have as much upside coverage as possible. Upside coverage sounds good, but is it worth it? If BR2 naïvely tries to take the low by betting $\text{B2} = 6700$ in response to our 13800 bet, he has fallen for the trap, but his bet is large enough to escape. This is exactly the problem identified in the first paragraph of this chapter, which led us to the result that we need to only slightly overbet our lead. What about that upside coverage? If overbetting our lead tricks BR2 into trying to take the low, then we have the high anyway, and the surrender trapping 8000 bet would have been better. If BR2 goes high, he'll bet the maximum 50000, which cannot be covered by our 13800 bet, even if we double. Betting 13800 instead of 8000 did not give us any useful upside coverage, and it introduced a leak into our surrender trap. The problem here is that our lead is small, so even when we bet nearly double this lead, we do not have any significant upside coverage.

The situation changes when our lead exceeds 16700, a third of the maximum bet of 50000. At this point, that third candidate bet becomes very attractive. A bet of 33400 gives us the surrender low and the high. The surrender low actually gives us a *choice*, after seeing our cards, of surrendering to take the low, or playing out the hand to go for the high. The problem is that our lead is not large enough to allow us to cover both. The moment we surrender, we are locked into the low, and forgo the upside coverage that we had. With our trapping bet at small leads, we had no upside coverage anyway, and we didn't need any if BR2 fell for the trap. With a lead exceeding 16700, brute force outperforms deception, and we are probably better off betting the 33400. Forget about trapping the mouse; just call in the exterminators. There is an added benefit that BR2 might still trap himself with a foolish, minimum-bet attempt at taking low if he sees that you have overbet your lead.

For leads exceeding 25000, a half of the maximum bet, the straight low becomes attractive, because a bet of 25000 gives BR1 the high and the low simultaneously. At this point, the lead is large enough that we do not need to overbet the lead in order to get good upside coverage. There are other thresholds where the lead becomes large enough that other betting options become tempting:

Lead Thresholds for BR1							
BR1's Lead Exceeds What Fraction of Max Bet	Consequence	Attractive Bet, as Fraction of Max Bet	Coverage Provided (How?)				
			Surrender Low	Straight Low	High	Blackjack	Double
1/5	Minor	2/5	x		x (D)		
1/4	Minor	1/2	x		x (BJ*)		
3/10	Minor	6/10	x		x (BJ)	x (D)	
1/3	Major	2/3	x		x	x (D)	
		1/3		x	x (D)		
3/8	Minor	6/8	x		x	x (BJ)	
2/5	Minor	4/5	x		x	x (BJ)	x (D)
		2/5		x	x (BJ)		
1/2	Major	1/2		x	x	x (D)	
		1	x		x	x	x (BJ)
3/5	Minor	3/5		x	x	x (BJ)	
2/3	Major	4/3	x		x	x	x
		2/3		x	x	x (BJ)	x (D)
3/4	Minor	3/4		x	x	x	x (D)
4/5	Minor	4/5		x	x	x	x (BJ)
1	Major	1		x	x	x	x

BJ in parenthesis means that BR1 must get a blackjack or double down to obtain the coverage desired. D in parenthesis means that BR1 must double to obtain the coverage.

* Doubling for 1/4 of a max bet protects BR1 from BR2's surrender.

BR1's betting decision depends not only on the size of the lead, but on the anticipated betting and playing response from BR2. BR1's playing decision may be complicated if he has overbet his lead to take the surrender low. For now, we are interested in leads less than 1/3 of a max bet, where the surrender trap is such a powerful move.

Cost-Benefit Analysis

Let's imagine that you lay the surrender trap, but BR2 is not fooled. He bets the max. How much did you just cost yourself by laying the trap instead of betting less than your lead to take the straight low? Had you taken the straight low, you would triumph whenever BR2 loses or pushes the hand. By avoiding the trap and betting the max, BR2 is still giving you the low, so that you triumph if he loses his hand, but now, because you have overbet your lead, you are dead if he pushes and you lose a full bet without surrendering. How can this occur? If the dealer has a blackjack, we will not have the option to surrender. Because insurance protects us against Ace-up blackjacks, we need worry only about Ten-up blackjacks, and for BR2 to push in that situation, he must also have a blackjack. The chance of this combination of events is about 1 in 1000, so there is virtually no cost to laying the surrender trap, compared to taking the straight low.

The benefit of laying the surrender trap is significant. If you fool BR2 and cover two bets, your trophy probability is over 95%. If you can cover only one bet, this probability drops significantly, but is still much greater than taking the straight low.

In general, the cost of trapping is so small, compared to the alternative of taking the straight low, and the benefit is a possible lock victory, so you should try it even if there is only a small chance that BR2 will be fooled. Even in high-level, televised tournaments, the surrender trap has been strikingly successful, even on elimination rounds when competitors have ample time to choose their bets. Since covering two bets dramatically increases the probability of success, we recommend that you pay special attention to the critical leads required to cover two bets.

Selling the Trap

Some players will advise you to bet almost twice your lead to sell the surrender trap. While this may sell the availability of the low, it places you in a precarious position. If your bet is large, BR2 can make a fairly aggressive bet while still taking the low, meaning that you will not be able to cover his win after you surrender; BR2 falls into the trap but escapes! If, on the other hand, you bet only slightly more than your lead and protect against his 2-bet win, you will survive around 95% of the time, far greater than the 56% chance afforded by the straight low. You must weigh the probability of BR2 falling into the trap against his chance of surviving that trap.

We recommend that you bet a round number that satisfies the prescribed betting range. For instance, with a lead of 2200, we can cover a 2-bet win and have chips left for insurance if we select any bet B1 satisfying $2400 \leq B1 < 2617$, but the bet of 2500 shown in the above chart is much better than 2400. Because 2500 is a round number, your opponent may think that you made a hasty, unconsidered bet, whereas 2400 seems calculated. When making such a bet, it is best to "color up" the bet and use the highest-denomination chips possible, so that the few chips in the betting circle will appear to represent this round, unconsidered bet. Simultaneously, the chips held back on the rail should be stacked neatly, so that BR2 can count them conveniently! We want BR2 to know exactly what has been held back, so that he will be tricked into thinking that the low is available. Sometimes BR1 will actually make a show of counting out his chips on the rail so that BR2 can easily know the amount. BR2 often knows how much is held back, but forgets that half of the chips in the betting circle can be recovered via surrender; this is the basis for the surrender trap.

Also, remember this: your opponent, though he may see the trap a mile away when sitting on his couch watching the tournament on TV from his living room (yes, that's a *large* living room), is now under extreme pressure. He is trailing in chips and facing elimination, possibly in a big tournament. There may be television cameras and a studio audience, all watching his every bet. He may have had no sleep the night before, and may be playing his second or third match for the day, without even sufficient food. Then add the ultimate pressure: The Clock. Under these conditions, even a skilled opponent can *easily* fall for the trap, or just make a mistake (for instance, if time runs out, many tournaments impose a minimum bet). Under time pressure, many players will make either the minimum bet allowed, or the maximum bet allowed. Pray for the former!