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Henry Oliver Lancaster 1913–2001

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1. Some Family History

Henry Oliver Lancaster (HOL) was born in Sydney on 1 February 1913 and died there 2 December 2001. His parents were Edith Hulda ('Edie') Smith (1885–1964) and Llewellyn Bentley Lancaster (1871–1921).

The Macleay River region of New South Wales had been the home of the Lancasters since 1850, beginning with William Henry Lancaster, who from 1850 taught and later served as Postmaster at Kempsey and Frederickton. HOL's grandparents on his father's side were James Henry Lancaster who died in 1879, predeceasing his father William Henry by about a year; and Jane Norton, who was born at Parkmore, Co. Offaly, in Ireland, and came to Australia at age 15 as part of a family contingent. Their children were Clara, William, Albert, Stanley, James, Llewellyn and Eveline.

Llewellyn became a medical practitioner in Kempsey. He had done his training at the University of Sydney. HOL's mother, Edie, was Llewellyn's second wife. She was a nurse who had completed her general nursing training at Kalgoorlie District Hospital, Western Australia.

Edie belonged to a powerful Western Australian family that included the Forrests (one first cousin was a Forrest). Sir John Forrest (1847–1918), the most eminent of the Forrests, is an outstanding figure in Australian history. Western Australia's first premier (1890–1901), he entered the first Federal Parliament in 1901 after successful negotiations on Western Australia's behalf. In 1869 he had led a search expedition for Ludwig Leichhardt; in 1870 an expedition from Perth along the Great Australian Bight to Adelaide; and in 1874 he completed a 4,300 kilometre crossing of the Australian continent. HOL in his retirement often spoke with pleasure of the Forrest family connection.

Edie, HOL's mother, was the second child and second daughter of Matilda Hulda Mattner and Harry Smith, whom she had married 4 August 1883.

Harry Smith's real name was Christian Jorgenson or Jensen. As a youth Harry had shipped out from Moss in Norway as a seaman, and left his ship on arrival in Australia. Matilda was the eldest of twelve children of Pauline Emilie Marks, born in Bomst, East Prussia (now Babimost, in Poland) in 1843, and Carl Wilhelm Mattner, who had reached South Australia in 1857. South Australia's history and society is rich in German names. The Mattners achieved considerable standing; for example, one of South Australia's Senators to Federal Parliament from the 1940s, last elected in 1961, was Edward William Mattner (1893–1977), and there is a privately-printed book, *The Mattners in Australia 1839–1980*. HOL was proud of this heritage, and in his retirement spoke to ES of a large Mattner family reunion which he had attended.

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* Eugene Seneta, FAA, succeeded H.O. Lancaster as Professor and Head of Mathematical Statistics at the University of Sydney in 1979. Denoted by 'ES' in the sequel.
** Geoff. Eagleson was one of Lancaster's first PhD students in Mathematical Statistics (PhD, University of Sydney, 1967).

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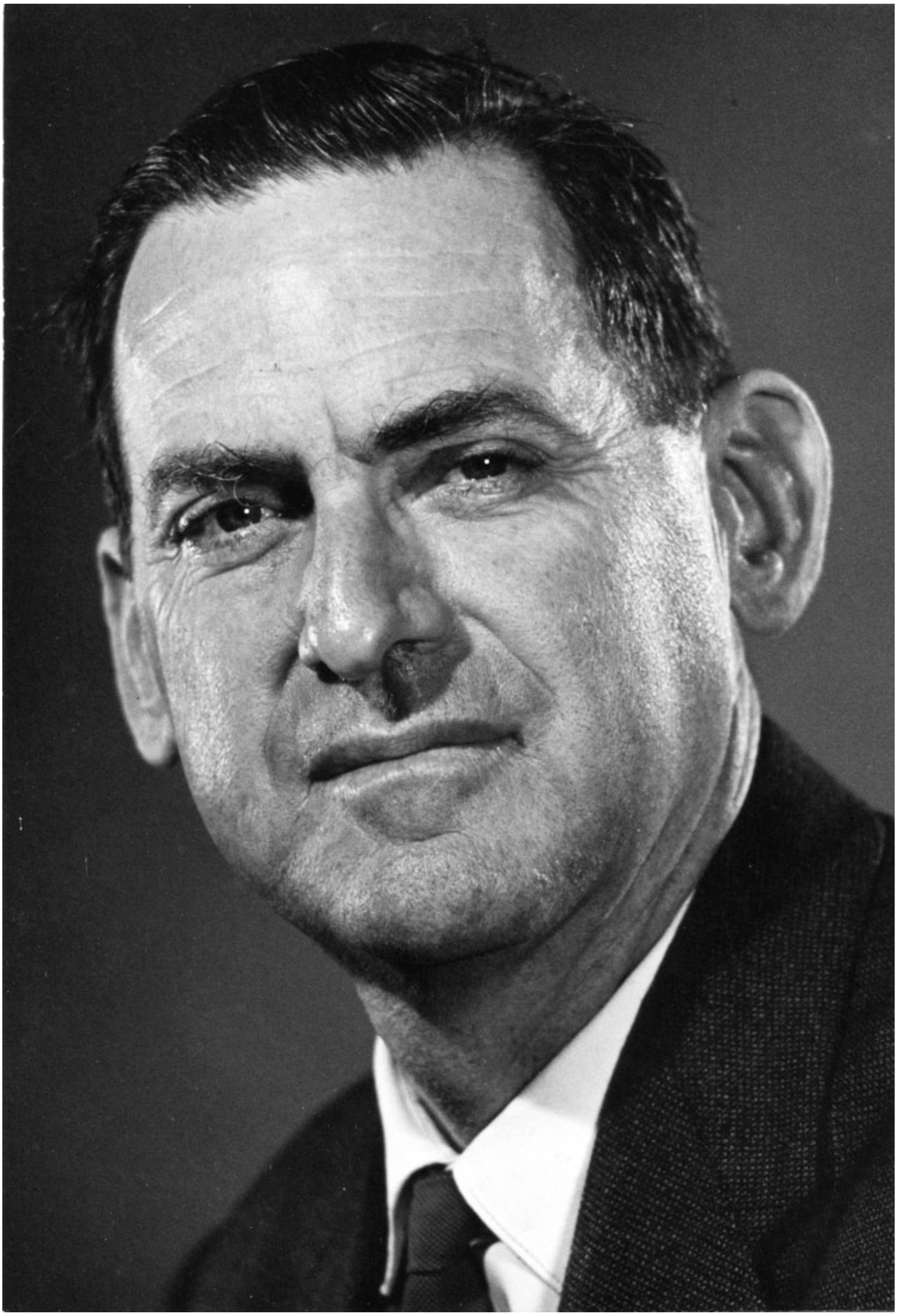

2. Early Years. Formation

HOL's father, Dr Llewellyn Bentley Lancaster, had a wide range of interests that encompassed the entrepreneurial, after his medical and hospital activities. After his decease the local district newspaper spoke of his 'evincing a deep scientific interest in agriculture and stock raising, entering into political arguments, taking prominently active participation in the establishment of butter factories, drainage unions, water supply schemes and other projects for the advancement of the district'.

On the medical side, soon after his return to the Kempsey district, an X-ray unit was installed at the local hospital in 1902, only a few years after the discovery of X-rays, through public subscription and his efforts and that of his senior colleague, Dr Casement. Llewellyn owned one of the first motor cars in the district, the 'noisy approach' of which writes his son Richard, HOL's younger brother and family historian, 'could be heard from a considerable distance to the satisfaction of the patient's family' (R.L. Lancaster, 2002).

Llewellyn had a love of horse racing and, as a member of the Australian Jockey Club (AJC) regularly visited Randwick in Sydney. He was also president of the Macleay River Jockey Club. Another deep interest and recreation was chess, which he took up as a medical student. He played a challenge match against W.S. Viner for the Australian Chess Championship in 1913, losing with honour by seven games to two. His wife Edie had accompanied him to Sydney, and HOL was born there during the time Llewellyn was playing in the championship.

The marriage of Dr Llewellyn Bentley Lancaster and Edith Hulda Smith produced four offspring: Geoffrey Bentley (11/4/1911– ?); Henry Oliver (1/2/1913–2/12/2001); Richard Lonsdale (b. 12/4/1914); and Loaline Lonsdale (14/3/1919–2/2/1977).

HOL showed an exceptional ability with mental arithmetic in his earliest years, entertaining his father's professional guests with his skill. However, a great problem emerged before his fifth birthday. He stuttered. This persisted through his schooling, and caused him great embarrassment. It affected his performance in the *viva voce* examinations in the early years of his medical studies. As to the cause, he wrote in his autobiography (**II**.83)*: 'I have always believed that it was partly due to dominant personalities.'

Through force of will and evolving confidence derived from the strength of his scholarship he eventually cured himself of the stammer, but some after-effects persisted throughout his life. He spoke in a somewhat halting, deliberate manner, sometimes coming out with the wrong word, perhaps because he could not quickly enunciate the right one, and then, after a pause, trying to explain what he meant. This attempt would characteristically begin with: 'Well, at any rate…' —a phrase very frequent in his lectures. Difficulty in communication coupled with an essential shyness sometimes gave the wrong impression, and led to a reclusiveness in later life.

During HOL's early years the family was prosperous and comfortable, but his father, Llewellyn, died a relatively young man on 8 December 1921 from an attack of pleurisy and pneumonia, the result of an influenza epidemic two years earlier. Because of financial difficulties that became apparent after probate, HOL's mother, a strong woman, returned to nursing in Sydney. From November 1923 Oliver and his younger brother Richard ('Rick') became boarders at St George's

* HOL's publications are consecutively numbered within each of two separate lists at the conclusion of this biographical memoir. Thus **II**.83 refers to item 83 in list **II**. References in the text to other works are in the usual name and date format, and are to be found under 'References'.

Hostel, a church institution run by the (Anglican) Diocese of Grafton. Here HOL was to stay for six years. He attended West Kempsey Intermediate High School in which he was fortunate in his teachers.

He achieved considerable distinction in his Leaving Certificate examination in November 1929. Throughout his schooling, in which he excelled, his mathematical skills flowered. One of his high-school classmates, competitor, and life-long friend was Alan Heywood Voisey (1911–1995), later Professor of Geology (1954–1965) at the University of New England and then foundation Professor of Geology and Head of the School of Earth Sciences at Macquarie University, Sydney.

HOL was an enthusiastic cricketer at school and later revelled in Bradman's prodigious feats and averages. Peter Armitage, whom he met on arrival at the London School of Hygiene and Tropical Medicine in 1948, recalls that, having introduced himself on his first day, HOL immediately left to watch the Australian cricket team at Lords. Very much in character, HOL died peacefully in his sleep on a Sunday after watching a game of cricket on television in the company of his youngest son Jon, at Ocean View Nursing Home at Mona Vale.

HOL's recreations as a young man encompassed body surfing, billiards (even playing for money), bridge, and chess; and in later life, bowls. On weekend visits to him at his retirement home ('The Garrison', Mosman), HOL would often be found watching on television a Rugby League game of his team, the Manly Sea Eagles. He had a continuing attachment to the Manly area of Sydney, going back to his student days at the University of Sydney.

His interest in chess was another echo of his father's broad interests, and of the circumstances of his own birth. In retirement in 1996 he showed ES a 1977 photograph entitled 'The Chess Players' that pictures HOL and another Sydney graduate, J.W. (later Sir John) Cornforth, Nobel Laureate in Chemistry in 1975, playing. Watching is the physicist and radio astronomer Bernard Mills. The photograph was taken forty years after a New South Wales Chess Championship in which HOL defeated Cornforth, with Bernie Mills, then a schoolboy, in the audience.

After learning Latin and French at school, HOL taught himself to read German and Russian by reading bilingual Bibles. This enabled him to cover wider literature sources in later life than most of his scientific peers.

HOL and his brother Richard remained close throughout their lives. Richard edited HOL's autobiography (**II**.83), and much of the family history of our previous section comes from his unpublished *The Family of the Lancasters with the Nortons*. His company through regular visits and support during HOL's later retirement years contributed much to HOL's positive view of his past life.

3. 1930–1946. University of Sydney. Hospitals, War Service

On leaving school, HOL wished to study science, in particular mathematics; and there was a sentimental leaning towards medicine. His mother opposed the latter choice, fearing that an associated sentimental attachment to the Macleay River area, which she regarded as the wilds, would bring him there to practise. It was decided that Oliver would become an actuary (possibly because of the potential for income), and he began as a trainee in Sydney with the MLC (Mutual Life and Citizens) insurance company, starting an evening course in Economics at the University of Sydney. After four weeks he was able to transfer to a full-time Arts course, obtaining amongst other good results a High Distinction in Mathematics I, but dismissing on account of bleak career prospects and nervousness about lecturing to large classes his desire to become a professional mathematician. So, in the hope of an honourable professional life and a proper reward, he enrolled in first year Medicine in 1931, with financial support from his mother near whom in Manly he boarded. Clinical studies in Medicine began in 4th year and continued through the 5th and 6th years of the course, and HOL had generally good memories of these, recollecting positively J.C. Windeyer, the Professor of Obstetrics; Harvey Sutton, Professor of Preventative Medicine; Professor C.G. Lambie in Medicine; and H.D. Wright and N.E. Goldsworthy in bacteriology. Professor A.N.St.G. Burkitt was his teacher in Anatomy in 3rd year, a subject HOL did not like although he enjoyed Burkitt's tales of the great days of the Sydney University Anatomy Department in the 1890s.

Harvey Vincent Sutton (1882–1963) played a role in HOL's post-war career, and was one of his heroes.

Having completed the medical degree in 1936, HOL spent 1937 to 1939 at Sydney Hospital, first as Resident Medical Officer and then as Pathologist and Senior Medical Officer. The Kanematsu Memorial Institute of Pathology was located at Sydney Hospital, with J.C. (later Sir John) Eccles (1903–1997), Nobel Laureate in Medicine in 1963, as Director, and had a strong research team that included Bernard Katz (b. 1911) who in 1970 also became a Nobel Laureate, and Stephen W. Kuffler (1913–1980), later a foreign member of the Royal Society of London and a member of the US National Academy of Sciences. The afternoon teas, which Eccles and his staff often attended, were very stimulating to the young graduate. And he clearly made an impression on Eccles, who wrote a reference for him dated 11 December 1939, when HOL's term of three years at Sydney Hospital was about to end. HOL, in personal autobiographical notes in his file at the Australian Academy of Science, says that the two years 1938 and 1939 were perhaps the happiest in his life.

In 1940 HOL obtained a Junior Fellowship in Medicine at Prince Henry Hospital, Sydney. He writes in his autobiography that Dr C.J. Walters, the Superintendent of the hospital, was not well disposed to the Junior Fellowships scheme, and the shortage of medical officers brought about by war pressures also helped prevent its proper development. Consequently HOL left. After a few further months at Royal North Shore Hospital as resident pathologist, on 31 July 1940 he joined the Australian Imperial Forces as Medical Officer, and served until April 1946 with the rank of Captain and later Major.

While at Prince Henry Hospital, he met Joyce Mellon who was training as a nurse. They married on 20 December 1940. His son Paul (the first of 5 children, all sons) was born in September 1941.

HOL served as army pathologist at the 9th Australian General Hospital outside Alexandria in Egypt, then moved with it to Nazareth. In February 1942 he returned to Australia and worked in the 117th Australian General Hospital in Townsville, laying the groundwork for his first published papers (on incidence of eosinophilia, usually caused by hook worm infestation), joint publications with Major T.E. Lowe. These appeared in the *Medical Journal of Australia* in 1944. He also investigated the incidence of intestinal protozoal infections. About some presumably unpublished analysis he later wrote (**II**.83, p.16): 'Here a statistical problem was to compare incidence of infection in the 4 classes of troops, according to whether or not they had been in the Middle East or New Guinea, a problem in $2 \times 2 \times 2$ tables …'.

He had tried to read in 1939 Udny Yule's *Introduction to the Theory of Statistics* but the appearance of chi-squared, closely linked to the analysis of such tables, suggested that he needed to extend his mathematical studies. In the event, chi-square analysis of cross-tabulated categorical data was to be one of the dominant themes of his work in mathematical statistics (see Section 10 below).

In August 1944 HOL was seconded to the Australian New Guinea Administrative Unit (ANGAU) as pathologist, attached to headquarters at hospitals in Port Moresby and Lae. In the Australian War Memorial's collection, Canberra, there is a fine pencil sketch (AWM Art 22670) from this period by the war artist Nora Heysen, entitled 'Pathologist (Major Henry Oliver Lancaster) 1944'. HOL made a survey of more than a thousand native troops and civil workers from different areas, and in 1945 'surprised the army Director of Pathology, Colonel E.V. Keogh, by reporting the results in systematic form with properly drawn graphs and means and standard deviations correctly computed'.

Esmond Venner Keogh (1895–1970) was later to play a major role in HOL's sojourn at the School of Public Health and Tropical Medicine in Sydney.

HOL like many returned servicemen almost never spoke with his family of his experiences in the war, and after the early years he rarely attended Anzac Day services. At the Ocean View Nursing Home where he died, the staff, however, told of his recurring nightmares of firing soldiers running over the desert towards him.

4. 1946–1959. School of Public Health And Tropical Medicine, Sydney. Rockefeller Fellow, School of Hygiene and Tropical Medicine, London

With the support of E.V. Keogh, HOL was given a temporary appointment on 8 April 1946 as Lecturer in Medical Statistics at the School of Public Health and Tropical Medicine (SPHTM) at the University of Sydney by Professor Harvey Sutton, who was the first Director (1930–1947) of the School. He was expected to spend his first year in acquiring expertise in medical statistics. In fact, he spent much of his time completing his undergraduate mathematics education by attending Mathematics III (he was awarded the BA in 1947), as well as reading the English medical statisticians (Farr, Greenwood, Bradford-Hill) and American statisticians/epidemiologists (Dublin, Lotka, Pearl and Frost). In applied mathematics he heard 'very clear lectures' from Professor K.E. Bullen (1906–1976) 'who was a help and encouragement to me then and for many years later'. In pure mathematics his lecturer was Professor T.G. Room (1902–1986) and the topic principally geometry. He later wrote **II**.83, p.18) 'I have always been greatly impressed by Room's lectures and his virtuosity'. HOL found the elementary statistical texts that he was reading at the time to be lacking in precision, but seems to have found the book of Maurice G. (later Sir Maurice) Kendall (1907–1983) *The Advanced Theory of Statistics*, Vol.1, first published by Griffin, London in 1943, the most helpful—as had, at about the same time, his eventual long-time Australian contemporary statistical luminary, P.A.P. Moran (1917–1988) (see Heyde (1992)).

In 1947 HOL did the Pure half of the Mathematics IV course, and obtained a Rockefeller Fellowship in Medicine that was to take him to London for post-graduate training in medical statistics in the latter part of that year. However, his third son Llewellyn was born in 1947, and the trip was postponed for a year.

A. (later Sir Austin) Bradford-Hill (1877–1991) had succeeded Major Greenwood at the London School of Hygiene and Tropical Medicine (LSHTM) and was Professor of Medical Statistics there when HOL, travelling without his family, arrived in August 1948 for a stay of about twelve months. He was assigned to share a room with Peter Armitage, who was to become an outstanding biostatistician, a President of the Royal Statistical

Society, and a friend for life. HOL was living in London House, a hostel for Commonwealth visitors and students in Bloomsbury, near LSHTM. He had arrived with notes on a variety of topics in medical statistics. These included analysis of amoebic surveys (involving over-dispersed binomial distributions), control of routine blood counting, partition of the chisquare-statistic in contingency tables in order to identify particular contrasts (see Section 10), the use of what is now known as Lancaster's mid-P test for discrete test statistics (see Seneta (2003)), and the sex ratios in families.

In regard to the last subject, in London he obtained 1889 data of A. Geissler of Saxony, and produced a paper published in 1950 in *Annals of Eugenics*. Armitage (2002) writes that HOL found no evidence of genetic variability in the probability of a male, although he had four sons and was later to have a fifth! HOL's paper was attacked in a later number of the same journal by the eminent Italian statistician Corrado Gini (1884–1965), who had written a dissertation on the subject and who according to HOL considered it improper that anyone should criticize the work of German statisticians. Gini's cause was taken up by A.W.F. Edwards in the same journal, which caused HOL some undue distress at the time. The work on the over-dispersed binomial, HOL later discovered, also overlapped with earlier work of Gini.

Papers on all the topics HOL had brought with him were published in 1949 and 1950 in prestigious English journals. Of the senior people at LSHTM the major influence on HOL was the mathematical statistician J.O. Irwin (1898–1982) who was particularly interested in HOL's work leading to his first paper in mathematical statistics (**II**.1), on the partition of chi-square, and who wrote a sequel to it (Irwin, 1949). Irwin was something of a recluse. In the early weeks of HOL's stay Irwin's disappointment that it was Bradford-Hill and not himself who had succeeded Major Greenwood was still much in evidence. Bradford-Hill was an excellent choice in many respects, pleasant and helpful, but he lacked a knowledge of the mathematical side of medical statistics. HOL had hoped to gain a great deal from Greenwood, who was then retired but still occupying a room; but Greenwood didn't seem to recognize HOL's talent in epidemiological theory, and HOL's reticent personality did not permit him to force closer acquaintance.

Under the Rockefeller Fellowship scheme there was no need to worry about studying for a degree, and HOL attended as many meetings of the Royal Statistical Society as possible and also attended lectures at the LSHTM and at University College nearby. Outside LSHTM he heard lectures from the great names in statistics and genetics, Hermann Otto Hartley (né Hirschfeld) (1912–1980) on analysis of variance, Egon S. Pearson (1885–1980) on general statistical theory, J.B.S. Haldane (1892–1964) on genetics, and Lionel S. Penrose (1898–1972) on human genetics. Penrose encouraged his work on the sex-ratio. (In his correspondence with Peter Armitage in later years, HOL came to the curious ranking of 'medical' statisticians: 1. R.A. Fisher; 2. L.S. Penrose, with the others as 'also ran'.)

In spite of the friendship of Peter Armitage and his wife Phyllis, who tried to draw him into their circle of friends, HOL in time became increasingly lonely, and dissatisfied with his work environment, the situation being exacerbated by his separation from his family and the fact that his fourth son, Andrew James, was due to be born in Sydney in January 1949. Another reason was that before his departure from Sydney, the new director (1947–1968) of the SPHTM, Edward (later Sir Edward) Ford (1902–1986), had told him that his work would be subsidiary to planned directions of expansion. As a result of HOL's concerns about going home to a position commensurate with his abilities and productivity, someone (possibly Bradford-Hill; or Charles Kellaway (1889–1952), former Director of the Walter and Eliza Hall Institute, Melbourne) wrote to Ford to ask what position HOL might be offered on his return. The response was non-committal. Terribly upset by this, in late spring or early summer in 1949, HOL suddenly went missing from the LSHTM, and Peter Armitage found him in a depressed state in his rooms in London House, determined to go to Cambridge to work with the great mathematical statistician and geneticist R.A. (later Sir Ronald) Fisher (1890–1962). Bradford-Hill arranged the transfer to Cambridge; but HOL did not find in Cambridge or in Fisher what he had hoped, and returned to LSHTM. Bradford-Hill, a kindly man solicitous of HOL's mental welfare, did not take offence.

On his return to Australia in August 1949, HOL embarked on the career of medical statistician with the Commonwealth Health Department, taking up duties again at the SPHTM at the University of Sydney. Salaries at SPHTM were paid by the Health Department but some officers held university titles. The university staff treated HOL as a public servant while the Health Department treated him as a raw recruit. His relations with Ford were strained, and his desire to submit an MD thesis to strengthen his academic standing was repeatedly not supported.

HOL's duties were to lecture to postgraduate students on the subject of medical statistics within the Diploma of Public Health and the Diploma of Tropical Medicine. Not wanting to illustrate applications of statistics with non-Australian data, he considered it his main task to put in order and publish the medical statistics of Australia, since they had never been dealt with by a competent medical man. His focus on the collection and statistical analysis of Australian data resulted in some remarkable successes that will be discussed in our following section.

In April 1951 HOL wrote to M.G. Kendall that he had been advised to consider applying for a DSc. Kendall's cautious reply steered him to the PhD, which was awarded in the Faculty of Science in 1953 for a thesis entitled *The Distribution of X^2 in Discrete Distributions*. Professor Room was appointed as his supervisor for this until the return of Harry Mulhall, who was absent in Cambridge doing his own PhD under J. Wishart. It does not appear that Mulhall eventually took over supervision.

By about 1954, when HOL's position was becoming truly anomalous, Ford had begun speaking about promotion to Reader or Associate Professor, but nothing happened. HOL's promotion to Associate Professor in Medical Statistics finally occurred on 2 March 1959, after legal advice had been sought on both sides. When HOL took up duty as Professor of Mathematical Statistics shortly afterwards on 19 June 1959, the university had no space for him and the SPHTM had no one else to teach statistics to their diploma classes. HOL agreed to take the courses and so kept his room in the depths of the SPHTM. He writes (**II**.83, p. 30): 'As I walked over to the Professorial Board [meeting] on my first occasion, Ford joined me and so we entered together. Probably this was a good thing, as the Board had had enough of the Room–Bullen debates.'

HOL's oldest son, Paul, a prominent medical statistician in his own right who worked at the SPHTM for many years, once said to ES that in spirit his father never left the SPHTM. Indeed, in spite of the difficulties in recognition that he encountered at that School, his ambition, drive, and energy resulted in his best research work being done there in both medical and mathematical statistics. His correspondence file of those times contains letters from Joseph Berkson, A.C. Aitken, M.G. Kendall, A.E. (Alf) Cornish, F.M. Burnet, T.M. Cherry, Hugh Wolfenden, and E.S. Pearson. Cornish's letter (6 March 1957) is concerned with HOL's candidature for membership of the International Statistical Institute with the proposed support of Cornish from Australia, J.O. Irwin and M.G. Kendall from the U.K., W.G. Cochran and S.S. Wilks from the US, and possibly Bradford-Hill. HOL was elected in 1961 and regarded his membership with great pride. It was the first of many international honours. It is clear that two of HOL's great senior supporters in mathematical statistics were J.O. Irwin and M.G. Kendall. Of Kendall he often said, in admiration, that he belonged to no statistical 'school', a flexibility to be admired. HOL described Irwin as a lucid thinker and good mathematician. Indeed the times were an era of great efflorescence for mathematical statistics.

5. Medical Statistics

Initially at SPHTM HOL continued his interest in blood counting, analysing some blood counts that he had taken in 1946 in Townsville. He writes (**II**.83, p. 23): 'It was the blood counting statistical tests that led me to the theory of chi-squared and has really introduced me to a life-long interest in a particular branch of mathematical statistics'. We return to the chi-squared (χ^2) topic in Section 10. A manifestation of this medical-statistics theme is at least one of the papers published in 1950 in *J. Hyg. Camb.* (**II**.3,4); these two papers HOL himself listed under his publications in Mathematical rather than Medical Statistics.

At about the same time Dr (later Sir) Kempson Maddox (1901–1990) asked him to prepare a statistical survey of diabetes in the Australian population and in his diabetic clinic. Four papers resulted. The first of these (**I**.3), jointly with Maddox, published in the *Medical Journal of Australia* (MJA in the sequel), 'reassured the then editor of this journal, Dr M. Archdall, that medical statistics had a place in medicine and thereafter he allowed me practically a free hand in submitting papers, indeed about 50 of them, to his journal'. Among the findings of these articles, which initiated what HOL described as his *Mortality Series*, were that recorded mortality rates from diabetes were high in Australia, and that under a commonly held genetic hypothesis the rates implied that 30 per cent of genes were recessive and that cancer of the pancreas and diabetes were positively associated.

The *Mortality Series* was the manifestation of what HOL considered as his main task at SPHTM, namely to put in order, teach and publish the medical statistics in Australia. Most of the work in the *Series* subsequent to the diabetes papers was done without collaborators.

The most striking finding in his series of investigations on the prevalence of cancer was that melanoma (black mole cancer) was associated with latitude in Australia: that is, malignant melanoma of the skin caused higher death rates for people living nearer the equator, in Queensland, than in the more southern states. This appeared in the *MJA* in 1956 under the title 'Some geographical aspects of the mortality from melanoma in Europeans' (**I**.42). HOL was the first to demonstrate this association between latitude and prevalence quantitatively, although there were qualitative views (for example by V.J. McGovern in *MJA* in 1952, and Dr A.G.S. Cooper of the Queensland Radium Institute) that served as motivation. HOL's study was not confined to Australia and there is analysis by latitude of data from New Zealand, the British Isles, various European countries, the US, Canada and South Africa. The list of Acknowledgments for the data is extensive, and includes one of the important names of statistics, R.C. Geary (1896–1983), then

Registrar-General of Eire. The danger of intense ultra-violet radiation has now passed into standard knowledge in Australia, with its discoverer forgotten. The association between ultra-violet radiation and skin cancer is standard doctrine in dermatology.

Another paper (**I**.49) published in 1957 in *Lancet* within the mortality series was on tuberculosis, giving special emphasis to generation (cohort) analysis, a demographic technique new at the time (**I**.89).

The most striking of HOL's discoveries of this period, however, was outside the mortality series, and is increasingly the only one for which he is medically remembered. This was his landmark paper on rubella deafness (**I**.13). In 1941, the Sydney ophthalmologist N.M. (later Sir Norman) Gregg (1892–1966) had observed that many cases of cataract were the results of maternal rubella in the first month of pregnancy, due, Gregg and others thought, to a new, highly virulent form of rubella in pregnancy in Australia in the years 1938–1941. Another congenital defect, deafness, was described in detail by workers from South Australia in 1943. HOL's follow-up was the result of the idea he had, on passing the old New South Wales Institution for the Deaf and Dumb and the Blind, of examining its well-kept admission records since its opening in 1861. He followed this by a careful examination of similar institutions in other states, and of Australian census records for 1911, 1921 and 1933, where he found (corresponding) peaks in the age distribution of deaf people that he connected with births in 1898 and 1899, a time of known rubella epidemics in Australia. These observations were aided by knowledge of the fact that in small, relatively isolated populations like Australia's, epidemics tend to die out, so can be observed to come and go. A passage from his autobiography (**II**.83, p. 34) is illuminating here:

> In England and the large continental masses, the females all contracted rubella in the first few years of life. In Australia, however, the population was not large enough to maintain the rubella epidemic and so it died out. It followed that if the females survived up to adult life without having had rubella, [and] the rubella was introduced into Australia from outside, … there were epidemics in which women of childbearing age were attacked. Children subsequently born had congenital defects such as congenital cataract.

Thus HOL dispelled the illusion that what had happened in 1940 was a new phenomenon and in effect established a causal connection between 'ordinary' rubella and congenital deafness. When recollecting his rubella conclusions in contrast to what had been thought earlier, he quoted, with some satisfaction, Occam's Razor: *Entitia non sunt multiplicanda*.

The rubella-deafness study and the melanoma study are both careful quantitative and highly original follow-up studies to clinical observation, and are great success stories for statistical analysis, in being definitive and totally convincing in their conclusions, and impinging directly on public health.

HOL gave due credit to the commentary in the census reports for the censuses of 1911, 1921 and 1933 by the Commonwealth Statisticians George Handly Knibbs (1858–1929), Charles Henry Wickens (1872–1939), and Roland Wilson (1904–1996) who reported in 1942. HOL had followed their lead. He expressed continuing gratitude and admiration for Wickens, reading an oration on the centenary of his birth, published as **II**.49.

Knibbs had been first Commonwealth Statistician (from 1906); Wickens had been 'Compiler' in the Commonwealth Bureau of Census and Statistics under Knibbs and succeeded him in 1922. Some of Wickens' compilations, especially in respect of total mortality, saved HOL much labour.

6. 1959–1978. Professor of Mathematical Statistics, University of Sydney

The Department of Mathematical Statistics at the University of Sydney was founded in 1959 on the recommendation of the Murray Committee inquiry into tertiary education in Australia. Applications for a (Foundation) Chair of Mathematical Statistics were to close on 9 March 1959. HOL, now Associate Professor of Medical Statistics, was reluctant to apply, believing he had left it too late to become a professional mathematician:

> I did not wish to remain primarily a medical statistician presiding over a group of mathematical statisticians. Nevertheless if successful I would be free of the SPHTM. My brother Richard wanted me to apply and so did Keith Bullen … I was not all that confident about getting the Chair because of strong competition but I was not depressed about the possible outcomes.

In the event, he did apply. In his application under the heading 'Research', he said that he was writing a monograph: on the *Distribution of* X^2 *in Discrete Distributions*:

> As a research program, I propose to continue these investigations into the tests of significance and the mathematical form of statistical distributions. Several of the methods introduced in my PhD thesis have now become standard practice: e.g. the partition of X^2 by orthogonal matrices, the tests in complex contingency tables and the combination of the probabilities from different experiments.

His referees were J.O. Irwin, J. Berkson (an American biometrician/medical statistician of very considerable eminence in statistics) and M.G. Kendall.

Kendall was to cite seven of HOL's papers (from 1949 to 1960) in Volume 2: *Inference and Relationship* (1961), of his three-volume edition (with A. Stuart) of *The Advanced Theory of Statistics*, and the contents contain substantial exposition of Lancaster's work. The section *Partitions of* X^2: *canonical components*, on pp. 574–8, is a particular case in point. At the time of HOL's application for the Sydney Chair in early 1959, this book would have been well into preparation and HOL mentioned, in support and anticipation, his 'satisfaction of seeing much of this material incorporated…in…Kendall and Stuart'.

R.A. Fisher's advice was also sought, and this may have been what tipped the scales. HOL later wrote: 'Some people believed that writing on the distribution of the sphere, the subject of an important Fisher paper, had cost a rival three Chairs'. That Australian rival, Geoffrey S. Watson, now deceased, went on to important Chairs in Statistics in US universities.

On his return from sabbatical leave, having completed a Cambridge PhD, Harry Mulhall (1915–1995) joined the new Department of Mathematical Statistics, transferring from Applied Mathematics where he had kept mathematical statistics alive in the old Department of Mathematics (see Phipps and Seneta (1996)). (An account of the teaching of statistics at the University of Sydney prior to 1959 is in Seneta (2002*c*)). HOL (**II**.83, p. 30) describes Mulhall as 'a fine mathematical scholar and possibly Australia's most experienced lecturer in statistics…; he continued to play an important role in the teaching and organisation of the department'.

Mulhall's teaching helped to attract a galaxy of students who went on to achieve eminence as statisticians in Australian and overseas universities, the CSIRO, the Commonwealth Bureau of Census and Statistics, and the Australian Department of Health and Community Services. More detail, with names and photographs may be found in Seneta (2002*c*). Graduates of HOL's Department still form a portion of the mathematical statistics component-group of the University's present School of Mathematics and Statistics, formed in 1991.

As regards research, HOL's activity was true to his word as set out in his application. He later summarized it as focussing on:

> the application of orthogonal functions to statistical problems and included two joint papers, with M.A. Hamdan and with G. Eagleson [his doctoral students]. For a short time it could be said that, with these two authors and R.C. Griffiths, a 'school' was in existence. The orthogonal theory has been summed up in my book *The Chi-Squared Distribution* [1969] and in [the survey paper] 'Orthogonal models for contingency tables [1980]'. In this second monograph, full use is made of the correlation generating function, which has very nice properties in the Meixner classes, namely the binomial, negative-binomial, Poisson, normal, gamma and inverse hyperbolic distributions. This monograph and the note on the 'Development of the notion of statistical dependence', reprinted [1977] from the *Mathematical Chronicle* (New Zealand) [1972] were with others part of a general survey which I had hoped to publish. However, statistical dependence proved too large a topic to cover in one book.

Lancaster achieved scholarly distinction in at least four fields: medical and public

HOL in 1978 (year of retirement from the University of Sydney).

health statistics, mathematical statistics, history and biography of medicine and of statistics, and statistical bibliography. The bulk of his creativity during his period as Professor of Mathematical Statistics was devoted to the latter three areas.

Of the topics covered in his seven 'favourite' papers in mathematical statistics, two, 'Lancaster's mid-P' and 'Normality and independence', are discussed extensively in Seneta (2002*c*). In the present memoir we have thought it appropriate to include an expository technical section, Section 10, which does not substantially overlap with that account and is related to HOL's beloved topic of the chi-square distribution, his interest in which had its origins, as did almost all his work, in his earliest papers of 1949 in mathematical statistics.

In the history and biography of statistics, we have already mentioned Lancaster's paper (**II**.49) devoted to Wickens. An earlier biographical study, of 1962, entitled 'An early statistician–John Graunt (1620–1674)' (**I**.65) is meticulously researched, clearly written, and well-documented; sadly, it is so little known that it was missed as a source and citation for the Graunt entry in the collection of sketches edited by Heyde and Seneta (2001).

An article by HOL in 1966, characteristically entitled 'Forerunners of the Pearson χ^2'(**II**.31) and personal contacts with HOL relating to one of its protagonists, the French statistician I.-J. Bienaymé (1796–1878) led Heyde and Seneta (1972, 1977) to a striking historical revelation about the origin of certain fundamental results such as the Criticality Theorem of branching processes. (The Bienaymé discovery story is told in Seneta (1979).)

HOL's 1996 autobiography has an interesting concluding section (pp. 37–42) entitled 'Further Thoughts' in which he specifically summarizes, from a late perspective, some memories of his work on bibliography, biography and history. He writes (p. 38):

> Before I was elected to the Chair I used to take afternoon tea about once a week with colleagues in the mathematics department in the Physics Building where I used to hear tales of Cambridge from some of the graduates, usually about G.H. Hardy. So I resolved to learn about the mathematicians of other places and I thought a bibliography would be suitable.

Steps in this direction were already evident in his famous paper on characterization of normality (**II**.18, p. 375), where he says: 'The history of this theorem began with an anonymous review by Herschel in the *Edinburgh Review* for 1850'. It may well be that the preparation of the three volumes of *Bibliography of Statistical Literature* published in 1962, 1965 and 1968, compiled by his mentor M.G. Kendall and Alison Doig (especially Kendall and Doig (1968)) played a key role in motivating HOL's work in history and bibliography.

The compilation of his massive and detailed card index (in which Statistics students were made to play a part), working from his office in the Carslaw Building, was a long-term statistical labour of love that resulted in his *Bibliography of Statistical Bibliographies* (1968), followed by 21 addenda over the succeeding years. At the same time, apart from professorial responsibilities, he was running from his office the *Australian Journal of Statistics* with the help of G.K. Eagleson and his dedicated secretaries, Janet Fish and later Elsie Adler. More details on his bibliographical activity may be found in Seneta (2002*a*). Out of this activity came his celebrated maxim: 'Every statistician should write his own obituary'.

HOL had an understandable continuing interest in his predecessor, the first professor of mathematics at the University of Sydney, Morris Birkbeck Pell (1827–1879). HOL published a study of Pell (**II**.57) in 1977, and this extended to an extensive study published in 1986 entitled ‘The Departments of Mathematics in the University of Sydney’ (**II**.74).

There were a number of articles dealing with the history of the New South Wales Branch of the Statistical Society of Australia; these are discussed in Seneta (2002*c*). He wrote several articles and encyclopaedia entries on the history of mortality surveys in Australia before his retirement in 1978; but would have had less time to devote to history and biography during the tenure of his Chair due to his other commitments.

Much work of this kind on history and biography, however, was done after his retirement, especially in regard to statistics in medicine, and we continue the story there (Section 8). However, we should mention here his standing in the history field, in that he was asked to write the entry entitled ‘History of Statistics’ for the *Encyclopaedia of Statistical Sciences*; Volume 8 containing this item (**II**.78) was published in 1988.

HOL was a true believer, in his quiet way, in the importance of statistics in science and in the world and was ever its defender. At a reception at Old Parliament House in Canberra, he was once introduced to HRH Prince Philip as Professor of Mathematical Statistics at the University of Sydney. ‘Ah’, said HRH, ‘There are “Lies, damned lies, and statistics” ’, quoting the well-known saying of Disraeli and Mark Twain. Quick as a flash HOL replied: ‘Figures fool when fools figure’. This must have been a supreme moment in HOL’s eventual mastery of his childhood stammer.

7. HOL and the Australian Academy of Science

HOL was elected a Fellow of the Australian Academy of Science (FAA) and awarded the Academy’s Thomas Ranken Lyle Medal for Mathematics and Physics, both in 1961.

His friend and colleague P.A.P. Moran (1917–1988), was Professor of Statistics at the Institute of Advanced Studies (IAS), Australian National University (ANU), having returned from England in 1952. The ANU had been founded in 1949 as a purely research institution, ‘to secure Australia a place in international research, and to attract back to Australia some of the many expatriates who had made names for themselves abroad’ (Heyde (1992), p. 19). One of the functions of the IAS was to provide supervision of PhD candidates within Australia, it having been traditional for such candidates to go overseas. Moran’s Department of Statistics become a focus for students who had completed a Master’s degree in Australia or New Zealand, and many of them came from Sydney, presumably with HOL’s encouragement.

HOL had known of the ‘English’ statistician Moran at Oxford during his sojourn in London, possibly through M.G. Kendall. It was inevitable that close academic contact should develop. The first volume of the *Journal of the Australian Mathematical Society*, 1959–1960, carried three articles by HOL including the highly important probabilistic treatment (**II**.18) of the interaction between independence and normality of distribution in linear and quadratic forms in independent random variables, leading to characterization of the normal. Moran was thus very likely the referee, being a natural choice since there were few senior probabilists in Australia at the time, although E.J.G. Pitman (1897–1993) is also a definite possibility.

The Australian Academy of Science’s first set of Fellows, elected in 1954, included the pure mathematician Eric Stephen Barnes (1924–2000), and E.J.G. Pitman as the only mathematical statistician. Barnes knew HOL at the University of Sydney. HOL’s nomination for Fellowship was signed by E.J.G. Pitman, T.G. Room, and E.S. Barnes. The nomination was considered in 1959 and in 1960, with election in 1961.

HOL's citation for election reads:

> Senior Lecturer in Medical Statistics in the University of Sydney, distinguished for his studies of Australian mortality, and for his related statistical studies. The current views on the epidemiology of deaf-mutism are largely based on his enquiries in Australia and elsewhere. He has produced important evidence that melanoma is caused by sunlight. He has studied the statistics of laboratory methods, haematological counting, and amoebic surveys, in particular. This work has led him to an extensive enquiry into the theory and application of Pearson's X^2 test, obtaining new tests of goodness of fit and revealing many interrelations between other tests. He started his career as a pathologist and has subsequently applied mathematical methods in his work to increasing extent.

The citation was written prior to October 1958, but no doubt later papers such as **II**.18, strikingly mathematical and important, helped ensure election to Section 1 (Mathematics). HOL in private conversation with ES did not believe that much attention was paid to his more medical work in his election, or in his Lyle Medal award in 1961, in which Barnes, who had been awarded the medal in 1959, was instrumental as member of the selection committee.

P.A.P. Moran was elected to the Fellowship in 1962, one year after HOL, and awarded the Lyle Medal in 1963, jointly with G.R.A. Ellis. The Convenor of the selection committee for the medal on this occasion was HOL.

HOL had a deep pride in his membership of the Academy and for many years participated actively in its affairs. His file at the Academy contains extensive autobiographical and personal notes. Only some of the personal biographical material is encompassed by (**II**.83) and by the present memoir, by his implicit wish. There are in his file letters to the Academy on various matters. Two are discussed in our Section 9. In the last decade of HOL's life he expressed some dissatisfaction with the changes being implemented in the Academy's procedures, which he thought detracted from its elite nature.

8. 1979–2001. Emeritus Professor

On his formal retirement from the University of Sydney in 1978, HOL was made Emeritus Professor and given an office in the basement of the University's Fisher Library, a short walk from his old office in the Carslaw Building. A grant from the Australian Research Grants Committee (ARGC) assisted him to complete a project that had been in his mind for many years, a survey of world mortality (**II.**63). He worked on this alone, aided only by a secretary, Mrs Philippa Holy. The first fruits appeared in 1990 as a large-format book of 605 pages entitled *Expectations of Life*: *A Study in the Demography, Statistics and History of World Mortality* (**I**.92). It is dedicated to Frank J. Fenner and to the memory of Thomas Carlyle Parkinson and Dora Lush, bacteriologists. A book of amazing scholarship and commitment, its bibliography alone takes up pages 505–592, at two columns per page, and contains an estimated 2,800 items. The compilation was done without the aid of electronic data bases. The work as a whole, including its meticulously detailed references, has strong historical colouration, as its subtitle testifies.

HOL was particularly pleased by a review of *Expectations* in *Nature* (Vol. 346, 19 July 1990, p. 352) by Professor Roy Porter (1946–2002) who described the work as a 'magisterial survey'. Later HOL wrote to ES (private communication, 1 July 1994), via Philippa Holy: 'concerned that you may not give sufficient weight to his [HOL's] medical work', in the anticipation, presumably, that ES would be writing his obituary and/or biographical memoir. He recommended that ES consult

Dr Roy Porter and Professor Peter Armitage for an opinion.

HOL had an enduring regard and indeed filial affection for the University of Sydney. His autobiography, with memorabilia and archival material, was deposited in its Archives. He contributed some $27,500 to the University of Sydney, largely to support or supplement the ARGC grants for the writing of *Expectations*. He held a strong affection and loyalty also to the Australian Academy of Science, whose Basser Library also holds his autobiography in its archives, along with his huge collection of bibliographical cards. His miscellaneous gifts to the Academy amounted to about $5000.

HOL's last book, published in 1994, was entitled *Quantitative Methods in Biological and Medical Sciences*: *A Historical Essay* (**I**.94). Its title encapsulates neatly the various themes of his life-long work, and again it is rich in the breadth of its coverage. It is dedicated to the biologist Ernst Mayr (b. 1904), whose book of 1982, *The Growth of Biological Thought*: *Diversity*, *Evolution and Inheritance*, HOL was often found to be reading in his later retirement at 'The Garrison'. HOL described Mayr as Grandmaster of the history of biological science, and claimed to have found what might be called the chi-squared distribution in Mayr's text. In HOL's last published papers in medical statistics, 'Semmelweis: a Re-reading of Die Aetiologie' (**I**.95), and 'Mathematics in medicine and biology. Genetics before Mendel, Maupertuis and Réaumur' (**I**.96), he was, as ever, concerned with the proper attribution of credit and priority and the dismantling of scientific myth. HOL's own view on historiography is encapsulated by a comment to ES (22 August 1999): 'Historians should go to primary sources. Reading secondary sources gives the wrong impression of people like Semmelweis.'

Apart from the two books just mentioned, HOL's very productive retirement years produced some 36 items. On the whole these were of a general, historical, survey and bibliographical nature, commensurate with his senior standing in the medical and statistical areas, but they also included some technical articles in the *Australian Journal of Statistics*. His last technical paper (**II**.77) in 1987 concluded his preoccupation over four decades with the interaction of normality of distribution and independence.

He concluded his own story (**II**.83, p. 42) thus:

> I think I have lived in quite a happy time. Life in an Australian country town when I was a boy was very pleasant, … I had opportunities to do the type of work I wished to do, with ample opportunity for sport and in many ways I would not have liked things to have been changed very much. It had been a great satisfaction that interest in the histories of science and of mortality have led me to take a heightened interest in the general history of mankind. A continued interest in some general problem of the world at large, related to an individual's technical expertise, can be its own reward.

A favourite memory of ES, so characteristic of HOL's later years and of his deeply Australian persona, was a comment that HOL made in 1996 at 'The Garrison', that he liked reading *The Australian Encyclopedia* because it 'keeps me in touch with what used to be'.

9. Epilogue

HOL was much honoured in his lifetime. His degrees, all from the University of Sydney, were MB,BS (1937); BA (1947); PhD (1953); MD (1967); DSc (1971).

A full account of his honours from national and international statistical societies is in Seneta (2002*c*). Those by the Statistical Society of Australia included the Pitman Medal for 1980, given every two years for distinction in research in statistical science, and named in honour of

Professor Edwin J.G. Pitman who was its first recipient in 1978. Since 1979 there has been an annual *H.O. Lancaster Lecture* at meetings of the New South Wales Branch of the Statistical Society of Australia.

As already mentioned, HOL was elected a Fellow of the Australian Academy of Science in 1961, and awarded its Lyle Medal in the same year. He was made an Officer of the Order of Australia (AO) in 1992 for services to science and education. On being congratulated by the Academy's then President, Professor D.P. Craig, he replied (3 February 1992) still in a firm writing hand:

> Dear David,
>
> Please accept my thanks to the Council and Fellows of the Academy for their kind congratulation on my attaining the rank of Officer in the General Division of the Order of Australia, of which I am very proud. I thank you also for your kind congratulations.
>
> One of my friends called me a 'quiet achiever' and I am happy to accept his words. There is a good deal to be done that is not spectacular, even turning up regularly at meetings of my societies as well as the chores of editing and refereeing, but with this award I feel very glad to have done these tasks.

HOL's children from his marriage to Joyce Mellon were Paul, Peter, Llewellyn, Andrew and Jon. The marriage ended late in HOL's career, and at age 66 he married Nancy Gee, but they divorced shortly afterwards. In his eulogy at his father's funeral, Paul said: 'While Dad rarely expressed openly to any of us what he thought about his family's personal and professional achievements, or, indeed, their shortcomings, he was quietly proud of how they etched out their lives.'

HOL was one of the small band of Australian statisticians who made Australia one of the powerhouses of the statistical world in the 1950s, 1960s and 1970s. They left a strong legacy. HOL lived for science, and specifically Australian science. Of him it can truly be said, as of the others: 'There were giants upon the earth in those days.'

10. Chisquare

Among Lancaster's many contributions to mathematical statistics there is a major area that stands out due to the depth of his analysis and the impact his work has had on the development of further research. The first strand of this area is the decomposition of the X^2 goodness-of-fit statistic to which Lancaster was drawn from his work in pathology and epidemiology. His research in the decomposition of X^2 led naturally into a series of more theoretical investigations that constitute the second strand of his major area of interest: the structure of bivariate distributions.

When Lancaster first started to work in mathematical statistics, the standard way to assess independence in a contingency table was to use the X^2 goodness-of-fit statistic which is asymptotically distributed, under independence, as a χ^2 variable. Contingency tables arise when two or more categorical variables are measured on the same object and the joint frequencies of the different possible combinations are displayed in tabular form. The simplest case, a two-way contingency table, is when there are only two variables being considered.

Thus, for example, in order to trial a new vaccine against parasitic infection, a group of animals can be randomly separated into a control and a treated group. For each animal some outcome variable, such as whether the animal became infected or not when exposed to the parasite, can be recorded. The results of this experiment can be displayed in a 2×2 table of which an example is given in Table 1.

If the vaccine has no protective power, the observed frequencies in the contingency table will reflect the hypothesis of independence between the administration of the vaccine and the outcome of the trial.

Table 1. Example of a two-way table

	Infected	Not infected	Total
Treated	3	32	35
Control	21	10	31
Total	24	42	66

Lancaster's interest in the analysis of contingency tables is not surprising; in the early part of his career as a pathologist he was generating such tables from epidemiological data.

The X^2 goodness-of-fit statistic for an $r \times s$ contingency table is the sum of the squared standardized deviations of cell frequencies from their expected values. Under the hypothesis of independence, it is asymptotically distributed as a χ^2 variable with $(r–1)(s–1)$ degrees of freedom. If we denote the cell frequencies by $\{f_{ij}, i = 1, \ldots, r; j = 1, \ldots, s\}$ and their expected values by $\{e_{ij}, i = 1, \ldots, r; j = 1, \ldots, s\}$, then the X^2 statistic is calculated as:

$$X^2 = \sum_{i=1}^{r} \sum_{j=1}^{s} (f_{ij} - e_{ij})^2 / \mathrm{e}_{ij}. \qquad (1)$$

One drawback of using this test on contingency tables that are more complex than the simple 2×2 one is its global nature. A significantly large value of the X^2 statistic calculated under the assumption of independence provides evidence of a lack of independence but no detailed information as to precisely where the independence hypothesis is likely to have been violated.

In his first mathematical paper (**II**.1) he showed how a single, global X^2 with multiple degrees of freedom can be decomposed into mutually independent components, each one distributed asymptotically as a χ^2 with one degree of freedom and each one appropriate for testing a particular contrast.

He proved this by first showing that the joint probability function for an $r \times s$ table, assuming independence of the frequencies $\{f_{ij}\}$, given fixed row and column sums $\{f_{i\cdot}\}$ and $\{f_{\cdot j}\}$ (f being the total frequency):

$$\frac{\prod_{i=1}^{r} f_{i\cdot}! \prod_{j=1}^{s} f_{\cdot j}!}{f! \prod_{i,j} f_{ij}!}$$

can be written as a product of such probability functions for (2×2) tables. This provides asymptotically a decomposition of the associated X^2 goodness-of-fit (1) statistic into $(r–1)(s–1)$ asymptotically independent X^2 statistics, each one asymptotically distributed as a χ^2 variable with one degree of freedom.

In the same paper, under the influence of J.O. Irwin (see our Section 4), he gave a method for partitioning X^2 *exactly* into $(r–1)(s–1)$ components each corresponding to a (2×2) table.

This was done with the help of a family of orthogonal matrices, the Helmert matrices, which were to play a large role in his future work. Irwin's (1949) commentary on the exact decomposition immediately followed Lancaster's. Later Lancaster (II.26) reprised this work. The exact decomposition result is best understood in the context of finite discrete bivariate probability tables, which can be derived from an $(r \times s)$ frequency table by dividing all entries by the total frequency f. We shall return to the decomposition result after discussing bivariate probability tables.

The analysis is more complex with contingency tables in three or more dimensions. However, by using the appropriate Helmert matrices to transform the original data, Lancaster (**II**.7) was able again to decompose the global X^2 goodness-of-fit statistic into components, analogous to those in an analysis of variance. Thus, in a three-dimensional table he was able to create independent contrasts for the three main effects, the three first-order interactions and the second-order interaction. In doing so he showed how, in an analysis of goodness of fit, it is possible to extract detailed information that is both interpretable and analogous to the decompositions of the sums of squares in the analysis of variance.

In 1953 Lancaster published a paper (**II**.11) explaining how the classical χ^2 goodness-of-fit statistic is related to tests based on the deviations of sample moments from their expected values. He achieved this through a representation of X^2 based this time not on a Helmert matrix, but on an orthogonal matrix generated from the orthogonal polynomials associated with the marginal distributions.

If $p_{i}\cdot$, $i = 1, \ldots, r$ is a discrete distribution on r real numbers x_i, $i = 1, \ldots, r$, then the set of orthogonal polynomials associated with it, $\{g_r(x), r = 0, \ldots (r–1)\}$, are defined by the conditions:

- $g_0(x) = 1$,
- $g_r(x)$ is a polynomial in x of degree r, $r > 0$, and
- $\sum_{i=1}^{r} g_m(x_i) g_n(x_i) p_{i}\cdot = \delta_{mn}$.

An orthogonal matrix, $\{G = g_{ij}\}$, can be formed from these orthogonal polynomials by taking:

$$g_{ij} = g_{i-1}(x_j)\sqrt{p_j\cdot}, \text{ for } i, j = 1, \ldots, r.$$

Given an $r \times s$ bivariate probability table:

$$P = \{p_{ij}, i = 1, \ldots, r, j = 1, \ldots, s\},$$

it will have marginal distributions defined by:

$$\left\{p_{i}\cdot = \sum_{j=1}^{s} p_{ij}, i = 1, \ldots, r\right\} \text{ and}$$

$$\left\{p_{\cdot j} = \sum_{i=1}^{r} p_{ij}, j = 1, \ldots, .s\right\}.$$

If the marginal distributions are associated with the values:

$$\{x_i, i = 1, \ldots, r\} \text{ and } \{y_j, j = 1, \ldots s\},$$

then we can define two orthogonal matrices G and H, by using the values of the orthogonal polynomials on the marginal distributions as indicated above.

Now consider the matrix $Q = \{q_{tk}\}$, $t = 1, \ldots, r$, $k = 1, \ldots, s$ defined by

$$q_{tk} = \frac{p_{tk} - p_{t}\cdot p_{\cdot k}}{\sqrt{p_{t}\cdot p_{\cdot k}}}$$

Then GQH^T has every entry in its first row and column zero, since polynomials of degree greater than 0 are orthogonal to the constant vector. In general, for fixed (i,j), $i = 2, \ldots, r$; $j = 2, \ldots, s$ $(g_{i-1}(x_m), h_{j-1}(y_n))$, may be regarded as the values taken by a pair of random variables (U_i, V_j) with probability p_{mn}. Since, on account of the orthonormality conditions the random variables are already individually standardized to mean zero and variance 1, the elements of GQH^T are correlations.

Next, when a matrix A is multiplied by an orthogonal matrix O, the sum of squares of its entries is also preserved:

$$\begin{aligned} tr(A^T A) = tr(O^T O A^T A) &= tr(O A A^T O^T) \\ &= tr(OA(OA)^T) \\ &= tr((OA)^T OA). \end{aligned}$$

Thus

$$\begin{aligned} \varphi^2 &= \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{(p_{ij} - p_{i}\cdot p_{\cdot j})^2}{p_{i}\cdot p_{\cdot j}} \\ &= \sum_{i=1}^{r} \sum_{j=2}^{s} (\{GQH^T\}_{ij})^2 \end{aligned}$$

is the sum of the squares of these $(r–1)(s–1)$ correlations. The use of the left-hand side, φ^2, as a measure of dependence in a bivariate distribution $\{p_{ij}\}$ dates back to the beginnings of Karl Pearson's work in mathematical statistics and is called the *contingency.*

Now let us return to the $(r \times s)$ contingency table, with underlying probability distribution $\{p_{ij}\}$, and put $p_{ij} = f_{ij} / f$, and

$$\hat{q}_{tk} = \frac{\hat{p}_{tk} - \hat{p}_{t}\cdot\hat{p}_{\cdot k}}{\sqrt{\hat{p}_{t}\cdot\hat{p}_{\cdot k}}}$$

where

$$\hat{p}_{t}\cdot = \sum_{k=1}^{s} \hat{p}_{tk} = f_{t}\cdot / f$$

and similarly for $\hat{p}_{\cdot k}$. We know that under the assumption of independence ($p_{ij} = p_{i\cdot}\, p_{\cdot j}$) that

$$f \sum_{t=1}^{r} \sum_{k=1}^{s} \hat{q}_{tk}^2$$

has asymptotically (as $f \to \infty$) a χ^2 distribution with $(r-1)(s-1)$ degrees of freedom. On the other hand with the use of suitable (random) orthogonal matrices $\hat{G}, \hat{H}$ the $(r-1)(s-1)$ degrees of freedom can be allocated to squares, each multiplied by f, of correlations between orthogonal polynomials.

Each of these summands in an 'exact' decomposition will have asymptotically the distribution of the square of a standard normal random variable, i.e. a χ^2 distribution with one degree of freedom.

This decomposition depends on the orthogonality of G and H. The particular form of G and H allows the individual components to be identified as correlations between orthogonal polynomials. Other orthogonal matrices can be used to transform $\hat{Q}$, each one providing another representation of the X^2 statistic. Choosing an orthogonal matrix based on some alternative model will simplify the search for how the independence hypothesis breaks down.

The second strand of Lancaster's major area of interest may be seen to derive from the preceding by refocussing on a bivariate probability distribution table $P = \{p_{ij}\}$, $i = 1, \ldots, r$, $j = 1, \ldots, s$, and on the contingency, φ^2, which may be written as

$$\varphi^2 = \sum_{i,j} \frac{p_{ij}^2}{p_{i\cdot} \cdot p_{\cdot j}} - 1. \qquad (2)$$

Write in vector notation $\mathbf{x} = \{p_{i\cdot}\}$, $\mathbf{y} = \{p_{\cdot j}\}$, and put $B = \{b_{ij}\}$ where

$$b_{ij} = p_{ij} / \sqrt{p_{i\cdot} . p_{\cdot j}}$$

so

$$B = (\operatorname{diag} \mathbf{x}^{-\frac{1}{2}}) P (\operatorname{diag} \mathbf{y}^{-\frac{1}{2}}).$$

Note that the symbols $\mathbf{x}$ and $\mathbf{y}$ just introduced have no relation in meaning to the x_i's and y_j's occurring earlier in this account. Since

$$Z = \{z_{ij}\} = B^T B$$

is $(s \times s)$, symmetric and non-negative definite, it has real non-negative eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_s \geq 0$. It turns out that $\lambda_1 = 1$. Let $\mathbf{k}_i$ be the (normed) right eigenvector of $Z = B^T B$ corresponding to the eigenvalue λ_i, $i = 1, 2, \ldots s$. We may take $\mathbf{k}_1 = (\operatorname{diag} \mathbf{y}^{1/2})\, \mathbf{1}$. Now $Z_1 = B\, B^T$ has the same non-zero eigenvalues as Z, and corresponding to the eigenvalue $\lambda_i > 0$ the corresponding eigenvector of Z_1 is $B\mathbf{k}_i$, with $B\mathbf{k}_1 = (\operatorname{diag} \mathbf{x}^{1/2})\mathbf{1}$. The number of strictly positive eigenvalues is in general $m \leq \min(r,s)$. Some straightforward manipulations give the spectral expansion

$$P = \mathbf{x}\mathbf{y}^T \left\{ 1 + \sum_{i=2}^{m} \sqrt{\lambda_i} \left((\operatorname{diag} \mathbf{x}^{-\frac{1}{2}})\ \frac{B\mathbf{k}_i}{\sqrt{\lambda_i}} \right) (\mathbf{k}_i^T (\operatorname{diag} \mathbf{y}^{-\frac{1}{2}})) \right\}. \qquad (3)$$

In fact all of the $\sqrt{\lambda_i}$, $i = 2, \ldots, m$ are correlations, called *canonical correlations*, as can be seen from (3) by taking

$$\mathbf{v} = (\operatorname{diag} \mathbf{y}^{-\frac{1}{2}})\mathbf{k}_i\,, \quad \mathbf{u} = (\operatorname{diag} \mathbf{x}^{-\frac{1}{2}}) \frac{B\mathbf{k}_i}{\sqrt{\lambda_i}}\,,$$

since $\mathbf{u}^T P\mathbf{v} = \sqrt{\lambda_i}$. It can be shown for φ^2 defined by (2):

$$\varphi^2 = \sum_{i=2}^{m} \lambda_i \qquad (4)$$

so that $\lambda_2 = 0$ is equivalent to independence: ($\lambda_2 = 0$ implies that $\lambda_i = 0$, $i \geq 2$.)

Notice that if K is the (orthogonal) matrix whose ith column is $\mathbf{k}_i$ and we write $R = K^T B^T$, $C = K^T$, then

$$RBC^T = K^T B^T BK = \operatorname{diag} \lambda_i$$

so the matrices R and C diagonalize B. Returning to the matrix Q,

$$Q = B - \mathbf{x}^{\frac{1}{2}} (\mathbf{y}^{\frac{1}{2}})^T,$$

we find that the (1, 1) entry of RQC^T is 0 since $\lambda_1 = 1$, as are all the other entries of the 1st row and 1st column, because of the orthogonality conditions.

If we now refer to the contingency table corresponding to the probability distribution $\{p_{ij}\}$ and replace p_{ij} everywhere with $\hat{p}_{ij}$, we find from (4) that the exact decomposition of the chi-square statistic is:

$$X^2 = f\sum_{i=2}^{m} \hat{\lambda}_i ,$$

where $\sqrt{\hat{\lambda}_i}$ is the sample analogue of the canonical correlation.

To obtain an exact decomposition of X^2 paralleling that obtained from $\hat{G}\hat{Q}\hat{H}^T$ above, we may assume without loss of generality that $r \geq s$, and plausibly (since $\hat{P}$ is a data matrix), that $m = s$. Let $\hat{R}^T$ be an $(r \times r)$ orthogonal matrix whose first s columns are $\hat{B}\hat{K}$ and let $\hat{C} = \hat{K}$. Then an exact decomposition of X^2 is obtained from the squares of the $(r -1)(s -1)$ non-zero clements of $\hat{R}\hat{Q}\hat{C}^T$.

Eigenfunction expansions such as (3) of positive matrices can be used to assess the dependence structure of a contingency table for departure from independence and, when there is evidence of departure, to estimate the dependence structure, provided that a specific alternative has been pre-specified. The role of eigenfunction expansions of probability matrices in the derivation of an interpretable decomposition of the X^2 statistic led Lancaster to his extensive research on the structure of bivariate distributions in general.

Eigenfunction expansions can be derived for bivariate continuous distributions (or more precisely for a natural operator defined by them), though in 1953 the only example known was the expansion of the bivariate normal, derived by Mehler in 1866. In this case the canonical variables are the orthogonal polynomials on the normal distribution, the Hermite-Chebyshev polynomials. That is, when the bivariate normal density function is expressed as a weighted sum of products of these polynomials, the expansion is diagonal; when i is not equal to j, the orthogonal polynomial in x of ith degree is uncorrelated with the orthogonal polynomial in y of jth degree.

The Mehler identity provides an elegant way to understand the structure of the dependence between two normal variables in the case when their joint distribution is bivariate normal. The advantage of writing the bivariate normal as a sum of cross-product terms is that it reduces a complex dependence relationship into a sum of simple constituent parts as well as allowing a goodness-of-fit test for the bivariate normal to be constructed (**II**.15).

In this seminal 1958 paper Lancaster began his study of the structure of bivariate distributions. In it he showed how the Mehler expansion was a special case of a more general result. Any bivariate frequency function that satisfies a simple regularity constraint can be expressed as the product of its marginal frequency functions and a sum of cross-products of orthogonal functions that are complete with respect to the marginal distributions. What Lancaster showed was that the orthogonal functions can be chosen so that they are 'canonical': the resulting matrix of coefficients has non-zero terms only along the diagonal. As Lancaster himself noted, the canonical expansion he derived is nothing more nor less than the eigenfunction expansion of an operator that can be generated from the bivariate distribution.

Let $f(x,y)$ denote a bivariate density function with marginal frequency functions, $g(x)$ and $h(y)$. Define a function

$$K(x,y) = \frac{f(x,y)}{\sqrt{g(x)h(y)}} .$$

The regularity constraint then becomes that

$$\varphi^2 + 1 = \iint K^2(x,y)\ dxdy < \infty .$$

$K(x,y)$ can be used to define a conditional expectation operator from $L^2(g)$ to $L^2(h)$ in the following way:

For $\int \alpha^2(x)g(x)dx < \infty$, define

$$\beta(y) = \int K(x,y)\alpha(x)\sqrt{g(x)}dx$$
$$= \int \alpha(x)f(x,y)/\sqrt{h(y)}dx$$
$$= E(\alpha(x)/y)\sqrt{h(y)}.$$

This operator has an eigenfunction expansion that coincides with Lancaster's canonical expansion.

Lancaster used these canonical expansions for two purposes. First he showed how they could be used to construct new bivariate distributions with given marginal distributions. He also used them to derive a goodness-of-fit test for the bivariate normal distribution, associating individual degrees of freedom with the correlations between the canonical variables.

Knowing that most bivariate density functions can be written in a canonical expansion helps understand the structure of probabilistic dependence in general but is not of great value in a particular case. However, when the canonical variables are the associated orthogonal polynomials the expansion is particularly useful. The orthogonal polynomials for a particular distribution satisfy a recurrence relation and are easily computed. Furthermore, the expectations of the orthogonal polynomials can be determined from and determine the moments of their associated distribution. As a consequence the conditional moments are easy to calculate and the regression structure of the bivariate distribution is easy to determine. One question that had concerned many authors was whether canonical expansions, similar to the Mehler identity, could be found for other bivariate distributions.

Lancaster discovered a paper of a German mathematician, Meixner (1934), that turned out to contain the key to answering the above question, at least in the case when the bivariate dependence structure arises from 'random elements in common'. Lancaster's final work on this theme provided a deep and accessible understanding of the structure of those bivariate distributions whose marginals come from the set of distributions studied by Meixner.

A random variable is said to have a distribution belonging to a Meixner class if its associated orthogonal polynomials have a particularly simple generating function $k(x;t)$. More specifically, let X be a centred random variable with moment-generating function φ and a set of orthogonal polynomials, $\{P_m\}$, where $P_m = x^m$ plus terms of lower degree, $m = 1, 2, \ldots$.

Then X is said to belong to a Meixner class defined by $u(t)$ if the generating function of the orthogonal polynomials has a specific form, more precisely if:

$$k(x;t) \equiv 1 + \sum_m t^m P_m(x)/m!$$
$$= \exp[xu(t)]/\psi(u(t)),$$

where $\psi(u(t))$, is the moment-generating function of the distribution.

There are only six distributions that form a Meixner class, depending on the form of $u(t)$. They are the normal, the Poisson, the gamma, the positive and negative binomial, and a hypergeometric. One advantage of looking at the Meixner distributions is that some properties of the associated orthogonal polynomials can be proved for all members of the class together rather than looking separately at the individual cases. Thus, for example, the sets of orthogonal polynomials on a Meixner distribution are complete in the associated L^2 space (Eagleson 1964).

The relationship of the Meixner class to polynomial expansions can be derived for those bivariate distributions that are generated by the random elements-in-common model. If U, V, and W are independent random variables, the two variables defined by:

$$X = U + V$$
$$Y = V + W$$

will be correlated, with correlation coefficient:

$$\text{var}\,(V)\,/\,\sqrt{\text{var}\,(X)\,\text{var}\,(Y)}\,.$$

The canonical expansion of the bivariate density function of (X,Y) will have polynomial eigenfunctions if and only if the U, V and W all belong to the same Meixner class. This result, together with a simple proof of Meixner's original characterization of the members of the Meixner class, is given in **II**.55.

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