3D F.E.M. analysis of a Roman arch bridge

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ABSTRACT: The rehabilitation and conservation has shown in recent years the need of reliable methods for assessing masonry arch bridges: it is important not only to maintain ancient structures in good conditions, but also, when necessary, to be able to estimate their safety factor as accurately as possible.

Starting from a real case, this paper presents the results of a 3D FEM analysis of a stone masonry arch bridge, performed involving non-linear material behaviour, in which the structural role of the spandrel walls and filling is involved.

1 INTRODUCTION

Masonry arches were built since the beginning of the earliest civilization, but the greatest examples of their use were the arch bridges built in the Roman age. Anyone approaching the study of masonry arch bridges will be struck by the diversity of structural models and materials employed in the Roman solution of bridging a gap with an arch. Many of them still exist and some remain in service to this day, together with the considerable number of masonry arch bridges built during the centuries until the First World War.

The analysis methods proposed for masonry arches until the second half of last century were essentially based on the techniques of graphic statics and on the principle of structural mechanics developed at that time, and it is likely that many arch bridges were designed by that methods (Hendry, 1995). The advances of structural mechanics, and in particular the development of the elasticity theory has focused the attention of the engineers on the new possible structures, so that iron and reinforced concrete have become the favourite materials, whereas masonry has lost its principal role in the building structures. In fact from the early years of this century, very few masonry arch bridges have been built and the knowledge of the related design methods have ceased to form part of the civil engineers' stock-in-trade.

Together with the lessened understanding of the behaviour of these structures, the need of rehabilitation and conservation (Page, 1993; Melbourne, 1991) has shown in recent years a lack of reliable methods for assessing masonry arch bridges. In fact, it is important not only to maintain ancient structures in good conditions, but also, when necessary, to be able to estimate their safety factor as accurately as possible (Broomhead and Choo, 1992, Das, 1993). The eventual distress of such structures can be linked to different causes, such as an exceptional event or deterioration due to the effects of traffic and weathering. Since, in general, the stone masonry bridges dead loads are dozen times the live ones, the distress map cannot change significantly its shape for the current employment of the structures, even if the traffic they are now required to carry is much heavier than that envisaged by the designer. Thus, in several arch bridges crack patterns can be observed in the areas in which the load capacity of the structure is more involved. If the load history of the bridge excludes exceptional cases, such as the seismic load or foundation movement, in which the shape of the map is strongly different, the "deterioration map" coincides with the high stresses distribution and with the pattern of ideal hinges of rupture mechanism.

The theoretical modelling of arch bridges considers two main different approaches: a 2-dimensional one, based on preelastic theories (Harvey, 1988, Sinopoli et al., 1997), and the 3-dimensional Finite Element Method approach. The former is based on a classical 'limit' analysis, after Heyman, who developed the method at the end of sixties (Heyman, 1966, 1969) starting from the 19th century treatises. As a result of recent studies in structural mechanics, the latter has shown a great flexibility and a wide range of application fields (Zienckiewicz and Taylor, 1991; Choo et al., 1992). The recent development of the method has induced a perhaps excessive trust in the numerical tools of structural analysis, even if the F.E.M. can be usefully employed for the analysis of masonry arches (Roca et al., 1995).

This paper presents the results of a F.E.M. analysis, performed involving non-linear material behaviour, for a stone masonry arch bridge. It is shown that a substantially good evaluation both of the load carrying capacity of the arch and the necessity of rehabilitation can be reached making use of the results of F.E.M.: the numerical analysis can give a 3-dimensional map of the stress and strain distribution, so that a sort of "intervention map" can be designed. The low values of stresses confirm the fact that the material failure can be considered absent, and limited areas of plastic deformations can be recognized.

2 THE STRUCTURAL ANALYSIS

The case study is the Roman arch bridge in Pont Saint Martin, Aosta, Italy, that is a stone bridge in the Lys valley, nowadays in perfect conditions, for which several structural analysis have been proposed in the last decade (Franciosi, 1986). The great blocks by which it is build, arranged in a perfect ordered texture, were obtained from stone locally available, as it was usual in the Roman Age. The incomplete circular arch has a span of 31,4 m and a radius of 16,5 m (Figure 1). The bridge was chosen because the geometry (single span, shape as segment of a circle, range of the span) and the material (cut stones) are among the most diffused (Page et al., 1991).

Figure 1: The Roman Arch Bridge of Pont St Martin

Together with the geometry, an accurate knowledge of the way in which it has been built is the first step towards the determination of the stress and deformation states of the bridge. An accurate analysis of geometry and constituents of the bridge has been performed: the different materials of both stone spandrels and parapets on the edges and backfill (concrete with large size aggregates) with pavimentum on the centre of the vault have been considered (Frunzio and Monaco, 1998.a).

A scheme of the bridge cross section is shown in figure 2.

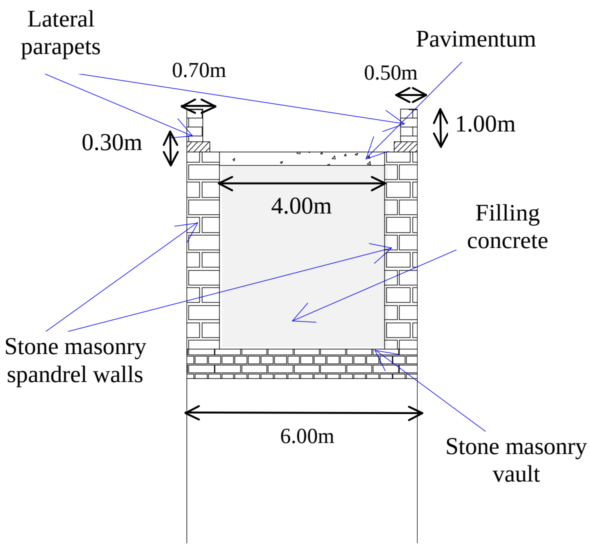

Figure 2: Bridge cross section

The bridge has been considered composed by four different materials for the different structural elements: the arch, the spandrel walls, the fill and the foundations, with the characteristics reported in the following table:

Table 1: Mechanical Characteristics

	Young Modulus (N/mm^2)	Poisson ratio	Cohesion (N/mm^2)	Friction Angle (°)
Arch	3,000	0.2	1.2	50
Spandrel walls	2,500	0.2	1.0	48
Fill	1,500	0.05	0.5	32
Foundation	7,000	0.25	1.8	58

The Drucker-Prager criterion was assumed as failure criterion for all the materials. To evaluate the elastic parameters, the stone masonry has been considered as a material obtained after a homogenization procedure, regarding the assemblage of stone blocks and mortar as a composite medium. The homogenized characteristics have been obtained by means of the classical differential scheme (Aboudi, 1991). The method is based on the idea that the composite is constructed explicitly from an initial material (stone) through a series of incremental additions (mortar). Due to the lack of experimental data, the Poisson's ratio was assumed equal to 0.2, although it has been shown that a variation in the Poisson's ratio provides sensible variation in the evaluation of the safety degree (Frunzio et al., 1998.b).

The analysis has been performed for the dead load only and the FEM mesh, involving both solid and tetrahedra elements, has been designed according the scheme picted in figure 3, where the four constituents materials are shown too. A total number of 28,388 elements and 11,068 nodes have been considered.

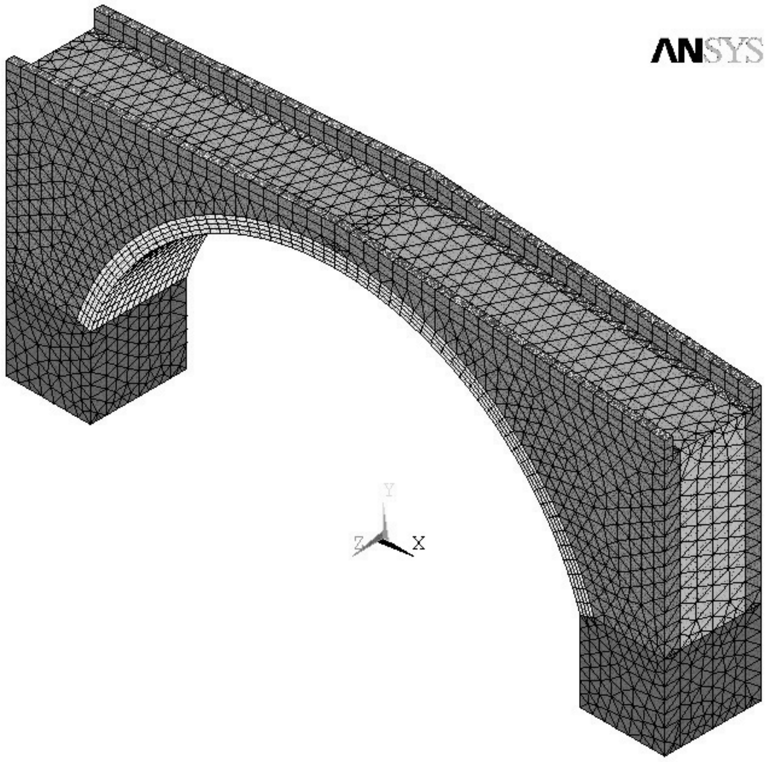

Figure 3: The FEM mesh

In figure 4 a map of the maximum principal stress (more significant in this case for the barrel vault) is represented. As it can be seen, the maximum value in the arch is 14 N/mm^2, lower than both the stone and mortar strength. This confirms that loss of equilibrium is the major cause of global failure: the material failure is absent, as it has been observed in several collapsed stone block masonry structures, such as Selinunte and Agrigento temples. In these cases the collapsed blocks are in perfect conditions, so that the restoration can be done by means of a simple rebuilding. Moreover, the distribution and intensity of stresses is similar to that obtained considering the spandrel and fill as dead load only, and considering the arch supported at the springing.

Since the value of the safety degree cannot be based on the comparison between the masonry strength and the stress evaluated by means of the F.E.M. analysis, the minimum load multiplier for which the displacements make sense is assumed as the safety degree of the bridge. In the present analysis the safety degree is 9,4. Although the mathematical solution of the problem is possible for higher load factors, the present analysis has been carried on until limited increases of the load multiplier give as a result great increments of the maximum displacement.

The safety degree evaluated considering the spandrel and fill as dead load is about one half of that evaluated in the present analysis and, as it has been shown, a little higher than that one evaluated by means of a limit analysis, after Heyman. The concentration of stresses coincides with the hypothesis of six hinges in the final mechanism of the arch elsewhere presented (Frunzio et al., 1998 ad 2001).

The strain distribution is presented in figure 5: localizations of the plastic strain are noticeable in limited areas of the barrel vault, while the spandrels and the foundations are completely free from plastic deformations.

To allow more clarity only the characteristics relative to a quarter of the bridge have been represented in the figures.

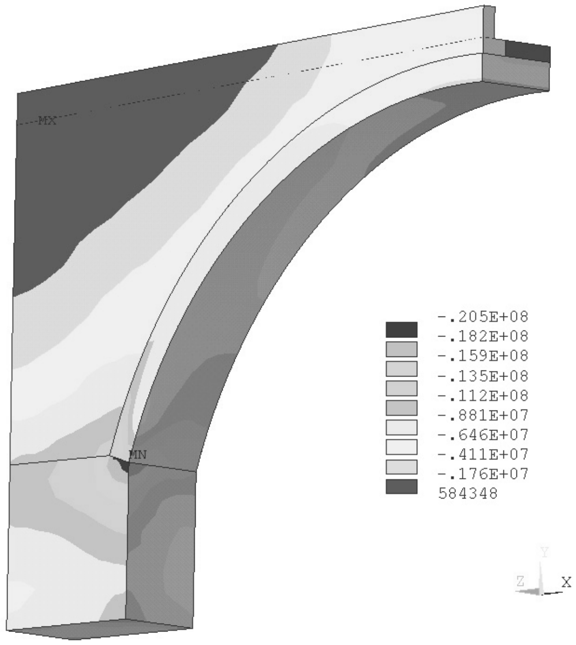

Figure 4: Map of maximum principal stresses [N/m^2]

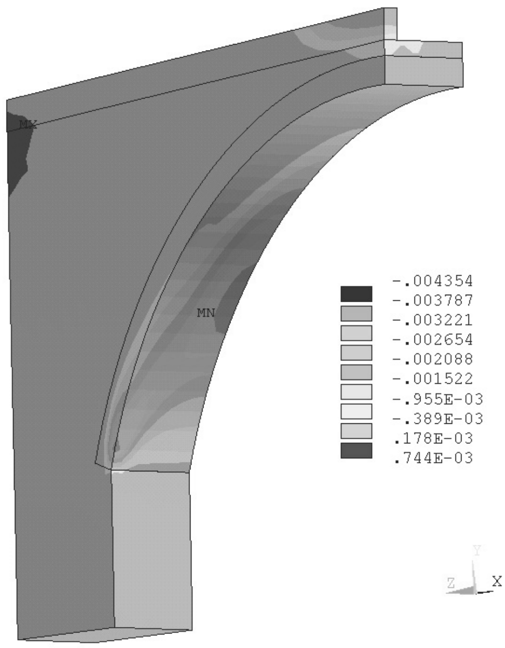

Figure 5: Strain distribution [m/m]

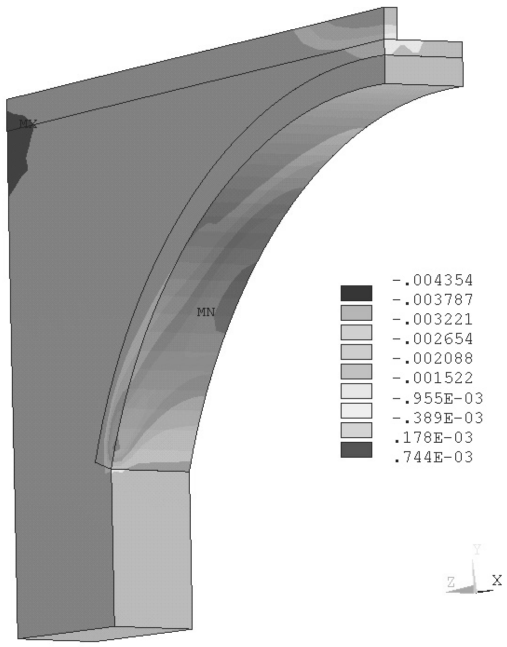

Figure 6: Maximum plastic principal strain [m/m]

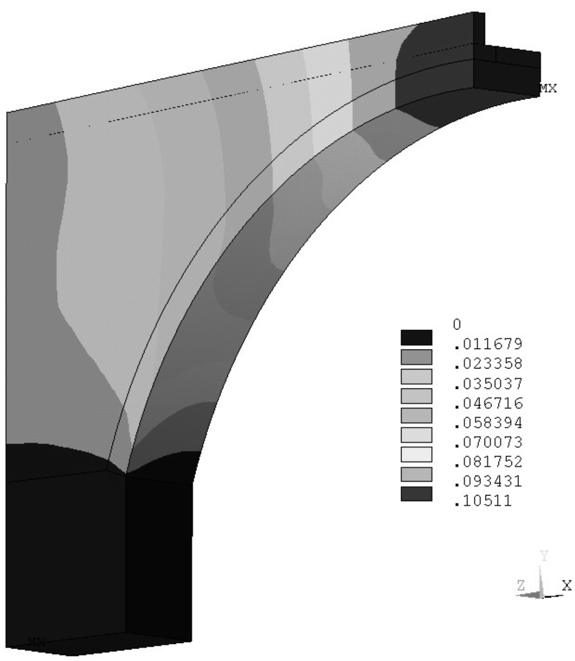

Figure 7: Displacement field [m]

3 CONCLUSIONS

With reference to a Roman stone arch bridge, in this paper is presented a numerical analysis performed by means of a nonlinear F.E.M. algorithm. The results of a F.E.M. analysis can be useful, in case of restoration of a masonry arch, by giving a qualitative map of the "intervention areas". It must be noted that they are strongly dependent on the exactness of mechanical parameters, which often are difficult to evaluate by experimental analyses, especially in the cases of monuments and historical buildings. The low stress levels evaluated assures that the material failure can be considered absent, while their distribution, together with the deformation field, can give information about the failure mechanism: a six-hinges failure mechanism can be recognized.

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