Bailouts and Financial Fragility

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Abstract

Should policymakers be prevented from bailing out investors in the event of a crisis?
I study this question in a model of financial intermediation with limited commitment.
When a crisis occurs, the efficient policy response is to use public resources to augment the
private consumption of those investors facing losses. The anticipation of such a "bailout"
distorts ex ante incentives, leading intermediaries to choose arrangements with excessive
illiquidity and thereby increasing financial fragility. Prohibiting bailouts is not necessar-
ily desirable, however: it induces intermediaries to become too *liquid* from a social point
of view and may, in addition, leave the economy *more* susceptible to a crisis. A policy of
taxing short-term liabilities, in contrast, can both improve the allocation of resources and
promote financial stability.

Key words: bank runs, bailouts, moral hazard, financial regulation

1 Introduction

The recent financial crisis has generated a heated debate about the effects of public-sector bailouts of distressed financial institutions. Most observers agree that the anticipation of being bailed out in the event of a crisis distorts the incentives faced by financial institutions and their investors. By insulating these agents from the full consequences of a negative outcome, an anticipated bailout results in a misallocation of resources and encourages risky behavior that may leave the economy more susceptible to a crisis. Opinions differ widely, however, on the best way for policy makers to deal with this problem. Some observers argue that the primary focus should be on credibly committing future policy makers to not engage in bailouts. Such a commitment would encourage investors to provision for bad outcomes and, it is claimed, these actions would collectively make the financial system more stable. Swagel (2010), for example, argues "[a] resolution regime that provides certainty against bailouts will reduce the riskiness of markets and thus help avoid a future crisis." Credibly restricting the actions of future policy makers is difficult, of course, and it is not clear to what extent a strict no-bailouts commitment is feasible. Nevertheless, many current reform efforts have embraced the view that such commitments are desirable and should be pursued where possible. A leading example is given in the preamble of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010, which states that the Act aims "to promote financial stability ... [and] to protect the American taxpayer by ending bailouts."

Would it actually be desirable – if feasible – to commit policy makers to never bail out financial institutions? Would doing so be an effective way to promote financial stability? I address these questions in a model of financial intermediation and fragility based on the classic paper of Diamond and Dybvig (1983). In particular, I study an environment with idiosyncratic liquidity risk and limited commitment, as in Ennis and Keister (2009a). Intermediaries perform maturity transformation, which leaves them illiquid and potentially susceptible to a self-fulfilling run by investors. I introduce fiscal policy into this framework by adding a public good that is financed by taxing households' endowments. In the event of a crisis, some of this tax revenue may be diverted from public production and instead given as private consumption to investors facing losses. These "bailout" payments aim to improve the allocation of the remaining resources in the economy, but have undesirable effects on ex ante incentives.

I begin the analysis by characterizing a benchmark allocation that represents the efficient distribution of resources in this environment conditional on investors running on the financial system in some state of nature. I show that this allocation always involves a transfer of public resources to private investors in that state. In other words, a bailout is part of an *efficient* social insurance arrangement in this setting. The logic behind this result is straightforward and quite general.[1] In normal times, the policy maker chooses the tax rate and the level of public good to equate the marginal social values of public and private consumption. A crisis results in a misallocation of resources, which raises the value of private consumption for some investors. The optimal response must be to decrease public consumption and transfer resources to these investors – a bailout.

In a decentralized setting, the anticipation of this bailout distorts the incentives of investors and their intermediaries. As a result, intermediaries choose to perform more maturity transformation, and hence become more illiquid, than in the benchmark allocation. This excessive illiquidity, in turn, implies that the financial system is more *fragile* in the sense that a self-fulfilling run can occur in equilibrium for a larger range of parameter values. The incentive problem created by the anticipation of a bailout thus has two negative effects: it both distorts the allocation of resources in normal times and increases the financial system's susceptibility to a crisis.

A no-bailouts policy is not necessarily desirable, however. Such a policy does lead intermediaries to become more liquid by performing less maturity transformation. However, it also implies that investors suffer a larger fall in private consumption when a crisis occurs. When the probability of a crisis is sufficiently small, a no-bailouts commitment is strictly inferior to a discretionary policy regime – it lowers equilibrium welfare without improving financial stability. For higher probabilities of a crisis, a no-bailouts policy may or may not be preferable, depending on parameter values, but it will never achieve the benchmark efficient allocation. Interestingly, for some economies that are not fragile in a discretionary regime, a no-bailouts policy would *introduce* the possibility of a self-fulfilling run.

The idea that a no-bailouts policy can make the financial system more fragile runs counter to conventional wisdom, but the mechanism behind this result is easy to understand. Bailouts provide insurance – they lessen the potential loss an investor faces if she does not withdraw her funds and a crisis occurs. Removing this insurance increases each individual's incentive to withdraw early if she expects others to do so, which makes the financial system more susceptible to a run.

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[1] The idea that bailouts can be part of a desirable social insurance arrangement also appears, in different forms, in Green (2010) and Bianchi (2012).

This argument is familiar in the context of retail banking, where government-sponsored deposit insurance programs are explicitly designed to promote stability by limiting depositors' incentive to withdraw. Despite this similarity, discussion of this insurance role of bailouts and its effect on investor withdrawal behavior has been largely absent in both the existing literature and the current policy debate.

An optimal policy arrangement in the environment studied here requires permitting bailouts to occur, so that investors benefit from the efficient level of insurance, while offsetting the negative effects on ex ante incentives. If a Pigouvian tax can be levied on intermediaries' short-term liabilities, an appropriately-chosen tax rate will implement the benchmark efficient allocation. Note that, in addition to improving the allocation of resources, this policy has a *macroprudential* component: it decreases financial fragility relative to either the discretionary or the no-bailouts regime. The results here thus argue for a shift in the focus of regulatory reform away from attempts to commit future policy makers to be "tough" in times of crisis and toward developing more effective policy tools for correcting distorted incentives.

There is a growing literature on the incentive effects of financial-sector bailouts and optimal regulatory policy in the presence of limited commitment. In many of the settings that have been studied, bailouts serve no useful purpose from an ex ante point of view. Chari and Kehoe (2010), for example, study an environment in which committing to a no-bailout policy would generate the constrained-efficient allocation of resources. When such commitment is infeasible, they show how renegotiation of contracts (i.e., "a bailout") tends to undermine ex ante incentives and how regulation of private contracts can be welfare improving. In a similar vein, Farhi and Tirole (2012) study a setting where the policy maker would like to commit to not lower interest rates in the event of a crisis. In the absence of commitment, the anticipation of this type of bailout distorts banks' incentives and introduces a role for regulation.[2]

In the environment studied here, in contrast, committing to a no-bailout policy is never fully optimal because bailout payments provide socially-valuable insurance. As a result, the paper presents a richer view of the issue in which the ex ante incentives generated by bailouts are not entirely negative. In particular, the insurance provided by a bailout encourages intermediaries to undertake

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[2]   Other related work includes Gale and Vives (2002), who study dollarization as a device for limiting a central bank's ability to engage in bailouts, Niepmann and Schmidt-Eisenlohr (2010), who examine the strategic interaction between governments when bailouts have international spillover effects, Ranciere and Tornell (2011), who show how the anticipation of a bailout can lead to welfare-reducing financial innovation, and Nosal and Ordoñez (2012), who study how uncertainty about the government's information set can mitigate the moral hazard associated with bailouts.

socially-valuable maturity transformation *and* makes investors more willing to stay invested. The problem, of course, is that intermediaries have an incentive to go too far and become too illiquid, which in turn makes investors more anxious to withdraw in a crisis. The analysis shows how policy makers must seek to balance these concerns, reigning in the incentive for excessive illiquidity without discouraging desirable activity or increasing investors' incentive to withdraw.

The next section presents the model. Section 3 studies equilibrium outcomes in a benchmark case where the financial system is operated by a benevolent planner. Sections 4 and 5 study outcomes under discretion and under a no-bailouts policy, respectively, and Section 6 presents an example to illustrate the results. Section 7 shows how a tax on short-term liabilities can implement the benchmark efficient allocation. Finally, Section 8 offers some concluding remarks.

2   The Model

The analysis is based on a version of the Diamond and Dybvig (1983) model augmented to include limited commitment and a public good. This section describes the basic elements of the model and defines financial fragility in this setting.

2.1   The environment

There are three time periods, $t = 0, 1, 2$, and a continuum of investors, indexed by $i \in [0, 1]$. Each investor has preferences given by

$$U\left(c_1, c_2, g; \omega_i\right) = u\left(c_1 + \omega_i c_2\right) + v\left(g\right),$$

where c_t is consumption of the private good in period t and g is the level of public good, which is provided in period 1. The functions u and v are assumed to be strictly increasing, strictly concave, and to satisfy the usual Inada conditions. In addition, the coefficient of relative risk aversion for the function u is assumed to be constant and greater than one. The parameter ω_i is a binomial random variable with support $\Omega_i \equiv \{0, 1\}$ whose value is realized in period 1 and is private information. If ω_i is zero, investor i is said to be *impatient*. Let π denote the probability with which each individual investor will be impatient. By a law of large numbers, π is also the fraction of investors in the population who will be impatient.

Each investor is endowed with one unit of the private good in period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into private consumption in

the later periods. A unit of the good invested in period 0 yields $R > 1$ units in period 2, but only one unit in period 1. This investment technology is operated in a central location, where investors can pool resources in an intermediation technology to insure individual liquidity risk. Investors are isolated from each other in periods 1 and 2 and no trade can occur among them. Upon learning her preference type, each investor chooses either to contact the intermediation technology in period 1 and withdraw funds or to wait and withdraw in period 2. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the investment technology in period 0.

Investors who choose to withdraw in period 1 arrive one at a time to the central location in the order given by their index i. In other words, investor $i = 0$ knows that she has the opportunity to be the first investor to withdraw in period 1, and investor $i = 1$ knows his withdrawal opportunity in period 1 will be the last. An investor's position in the order is private information and her action is only observable when she chooses to withdraw.[3] As in Wallace (1988, 1990), an investor must consume immediately upon arrival. This sequential-service constraint implies that the payment made to the investor can only depend on the information received by the intermediation technology up to that point. In particular, this payment can be contingent on the number of withdrawals that have taken place *so far*, but not on the *total* number of early withdrawals that will occur because this latter number will not be known until the end of the period.

Welfare is measured by the equal-weighted average of investors' expected utilities,

$$W = \int_0^1 E\left[U\left(c_1\left(i \right), c_2\left(i \right), g; \omega_i \right) \right] di.$$

We can think of investors as being assigned their index i randomly at the beginning of period 0, in which case this expression measures the expected utility of each investor *ex ante*, before any individual-specific characteristics are revealed.

2.2 The decentralized economy

In the decentralized economy, the intermediation technology is operated by a large number of identical intermediaries. Each intermediary correctly anticipates that a fraction π of its investors will be impatient and behaves competitively in the sense that it considers its own effect on economy-wide

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[3] In Diamond and Dybvig (1983), investors first choose when to withdraw and then are randomly assigned positions in the withdrawal order. Green and Lin (2003) introduced the approach of allowing investors to be informed about their position in the withdrawal order prior to deciding, which is useful for studying the dynamics of a withdrawals (see also Ennis and Keister 2009b). The approach here follows that in Ennis and Keister (2010).

resource constraints to be negligible. Intermediaries act to maximize the expected utility of their investors at all times.[4] However, as in Ennis and Keister (2009a, 2010), they cannot commit to future actions. This inability to commit implies that they are unable to use the type of suspension-of-convertibility plans discussed in Diamond and Dybvig (1983) or the type of run-proof contracts studied in Cooper and Ross (1998). Instead, the payment given to each investor will be a best response to the current situation when she withdraws.

The public good is provided by a benevolent policy maker who taxes endowments in period 0 at rate τ and places this revenue into the investment technology. In period 1, the policy maker can use these resources to produce the public good and, if a crisis is underway, to make transfer ("bailout") payments to financial intermediaries.[5] The policy maker is also unable to commit to future actions and will choose these bailout payments as a best response to the situation at hand.

2.3 Financial crises

In order to allow a crisis to occur with nontrivial probability, I introduce an extrinsic signal on which investors can potentially condition their actions. Let $S = \{\alpha, \beta\}$ be the set of possible states and $(1 - q, q)$ the probabilities of these states, respectively. Investor i chooses a strategy that assigns an action to each possible realization of her preference type ω_i and of the state

$$y_i : \Omega_i \times S \rightarrow \{0, 1\},$$

where $y_i = 0$ corresponds to withdrawing early and $y_i = 1$ corresponds to waiting until period 2. Let y denote a profile of strategies for all investors. The state s can be thought of as representing investor sentiment; it has no fundamental impact on the economy, but in equilibrium it may be informative about the withdrawal plans of other investors.

Under all of the policy regimes considered here, the model has an equilibrium with $y_i(\omega_i, s) = \omega_i$ for all i in both states. In other words, there is always a "good" equilibrium in which investors withdraw in period 1 only if they are impatient. Since no crisis occurs in this equilibrium, no bailout

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[4] In reality, there are important agency problems that cause the incentives of financial intermediaries to differ from those of their investors and creditors. I abstract from these agency problems here in order to focus more directly on the distortions in investors' incentives that are created by the anticipation of a bailout.

[5] Notice that this type of bailout policy is entirely consistent with the sequential service constraint, since all taxes are collected before any consumption takes place. I assume the sequential service constraint applies to the policymaker as well as to the intermediaries and, hence, the approach here is not subject to the Wallace (1988) critique of Diamond and Dybvig (1983). Other papers have introduced taxation into the Diamond-Dybvig framework in a similar way; see, for example, Freeman (1988), Boyd et al. (2002), and Martin (2006). The goal of fiscal policy in those papers, however, is to fund a deposit insurance system rather than to pursue an independent objective like the provision of a public good.

payments are made and no incentive distortions arise. As a result, this equilibrium implements the first-best allocation of resources. The question of interest is whether there exist other, "bad" equilibria in which some patient investors *run* by withdrawing early in some state. Without loss of generality, I focus on strategy profiles in which a run occurs in state β.

Definition 1: *The financial system is* fragile *if there exists an equilibrium strategy profile with* $y_i(1, \beta) = 0$ *for a positive measure of investors.*

Fragility thus captures the idea that the financial system is potentially susceptible to a run based on shifting investor sentiment.

2.4 Timing of decisions

The timing of decisions reflects both the information structure of the environment and the lack of commitment. In period $0,$ the policy maker chooses how much tax revenue to collect and investors deposit their after-tax endowments with intermediaries. Intermediaries make no decisions in this stage, since there is a single asset (and thus no portfolio choice) and they cannot commit to any future payment scheme.

At the beginning of period $1,$ investors are isolated from each other and from the intermediaries. After observing her own preference type ω_i and the state s, each investor chooses whether to withdraw in period 1 or to wait. Those investors who chose to withdraw then begin to arrive at their intermediaries one at a time, in the order determined by the index i. The amount of consumption a withdrawing investor receives is determined by her intermediary as a best response to the current situation when she arrives. In particular, note that this payment is determined *after* investors have made their withdrawal decisions and thus cannot be used as a tool to influence withdrawal behavior. A withdrawing investor consumes as soon as she receives the payment from her intermediary and returns to isolation.

While investors observe the realization of s at the beginning of period $0,$ intermediaries and the policy maker observe the state with a lag, after a fraction $\theta \in (0, \pi]$ of investors have withdrawn. If the state is β and a crisis is underway, the policy maker can choose to bail out intermediaries by transferring some tax revenue to each of them. Intermediaries combine this transfer with their own remaining funds and continue to serve withdrawing investors, making payments that are based on their updated information. The parameter θ thus measures the speed with which both intermediaries and the policy maker are able to react to an incipient crisis.

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By the end of period 1, all investors who chose to withdraw early have been served and the policy maker uses its remaining funds to provide the public good. In period 2, those investors who have not yet withdrawn will contact their intermediary. Since all uncertainty has been resolved at this point, an intermediary will choose to divide its remaining resources evenly among these investors.

2.5   Discussion

The model follows Green and Lin (2003), Peck and Shell (2003) and other recent work in allowing intermediaries to offer any payment schedule that is consistent with the information structure of the environment. Under this approach, a fully-anticipated financial crisis cannot occur. If intermediaries expected all investors to withdraw early in both states, for example, their best response would be to simply give each investor her initial deposit back when she withdraws. When an intermediary follows this policy, however, an individual patient investor has no incentive to join the run; she would prefer that the intermediary keep her funds until period 2 and earn the return $R > 1$. In order for a crisis to arise in this setting, therefore, it must be the case that intermediaries and the policy maker are initially uncertain about investors' actions.

Moreover, they must *remain uncertain* about these actions while some withdrawals are being made. If intermediaries were able to observe the state before any withdrawals take place – that is, if θ were zero – the above logic would apply once the state is revealed. When θ is positive, however, intermediaries must make payments to some investors before observing the state. In this case, once an intermediary discovers a run is underway it may not longer be feasible to give each remaining investor her initial deposit back in period 1. A patient depositor may then prefer to join the run, depending on her position in the withdrawal order.

Previous work has assumed that intermediaries and policy makers never observe the state, but instead must infer the state from the flow of withdrawals. In the equilibria I study below, they are initially unable to make any inference about the state as investors begin to withdraw, but would be able to infer that a run is underway if the fraction of investors withdrawing goes above π, the fraction of impatient investors. In the special case of $\theta = \pi$, therefore, the approach I introduce here is equivalent to that taken in the existing literature. For $\theta < \pi$, this more general approach allows one to study how equilibrium outcomes depend on the speed of the policy reaction to an incipient crisis. In Section 6, for example, I show how the minimal value of θ necessary to generate financial fragility varies across policy regimes.

Some of the existing literature avoids the issue of how quickly intermediaries and policy makers can identify a run by assuming that investors must be given a pre-specified payout until an intermediary is completely out of funds. Under this "simple contracts" approach, intermediaries and policy makers are assumed to be unable to react to any information they receive and, hence, the issue of when they can observe or infer the state is irrelevant.[6] While this approach is often analytically convenient, it is at odds with the fact that the liabilities of financial intermediaries are routinely altered during periods of crisis. In the Argentinean crisis of 2001-2, for example, bank deposits were partially frozen, dollar-denominated deposits were forcibly converted to pesos at an unfavorable exchange rate, and some deposits were replaced with long-term bonds.[7] The approach here of allowing intermediaries and the policy maker to react efficiently to the information they receive shows how both the nature and the timing of this reaction are important determinants of the financial system's susceptibility to a crisis.

3 Efficient Allocations and Bailouts

In this section, I study equilibrium in a version of the model where the intermediaries and policy maker are replaced by a single, benevolent planner. This planner operates both the intermediation technology and the public sector, but cannot control investors' actions and faces all of the informational constraints described above. The planner aims to maximize welfare, taking the profile of withdrawal strategies y as given. It will allocate resources efficiently conditional on investors' behavior and, hence, this case forms a useful benchmark for the analysis that follows.

I begin by proposing a particular strategy profile in which some patient investors withdraw early in state β. After deriving the planner's best response to this strategy profile, I show that the financial system is fragile if and only if there is an equilibrium in which investors follow this particular profile. Finally, I highlight a key property of the resulting equilibrium allocation: the planner chooses to (partially) bail out investors in the event of a crisis.

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[6]   See, for example, Postlewaite and Vives (1987), Cooper and Ross (1998), and Allen and Gale (2004), among many others. The approach I introduce here could be viewed as a generalization of the simple-contracts approach in which the face value of investors' claims can be reset as a function of the state after θ withdrawals. However, it bears emphasizing that the form of the payment schedule here represents a best response by intermediaries to the information environment and not an assumption. In additon, the lag θ is a restriction on the flow of information that is present under all policy regimes, including the benchmark case where the financial system is run by a benevolent planner.

[7]   See Ennis and Keister (2009a) for a brief discussion and Dominguez and Tesar (2007) for a more detailed description of these events.

3.1 A partial-run strategy profile

Consider the following strategy profile

$$y_i (\omega_i, \alpha) = \omega_i \qquad \text{for all } i$$

$$y_i (\omega_i, \beta) = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \quad \text{for } \left\{ \begin{array}{c} i \leq \theta \\ i > \theta \end{array} \right\}. \qquad (1)$$

In other words, suppose that in state α all patient investors wait to withdraw, but in state β those investors whose opportunity to withdraw arrives before the planner observes the state choose to withdraw early. How would the planner allocate resources if it anticipated this behavior?

3.2 The q-efficient allocation

As the first θ withdrawals are taking place, no information about the state is revealed and, hence, the planner will choose to give the same level of consumption c_1 to each of these investors. Once the state is revealed, the planner is able to anticipate how many additional investors will withdraw in period 1 and chooses to give a common amount c_{1s} to each of them. The remaining investors will withdraw in period 2 and each receive c_{2s}. Finally, the planner will provide a level g_s of the public good. The planner's best response to the profile (1) is thus summarized by a vector

$$\mathbf{c} \equiv \left(c_1, \{c_{1s}, c_{2s}, g_s\}_{s=\alpha,\beta} \right).$$

In deriving this best response, it is useful to divide the problem into two steps. First, let ψ_s denote the quantity of resources the planner uses for the private consumption of the $1 - \theta$ investors who withdraw after the state is revealed, that is

$$\psi_s \equiv 1 - g_s - \theta c_1. \qquad (2)$$

Let $\widehat{\pi}_s$ denote the fraction of these investors who withdraw in period 1. The resources will be distributed to solve

$$V (\psi_s; \widehat{\pi}_s) \equiv \max_{\{c_{1s}, c_{2s}\}} (1 - \theta) (\widehat{\pi}_s u (c_{1s}) + (1 - \widehat{\pi}_s) u (c_{2s})) \qquad (3)$$

subject to the feasibility constraint

$$(1 - \theta) \left(\widehat{\pi}_s c_{1s} + (1 - \widehat{\pi}_s) \frac{c_{2s}}{R} \right) = \psi_s. \qquad (4)$$

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Letting μ_s denote the multiplier on this constraint, the solution to the problem is characterized by the first-order condition

$$u'(c_{1s}) = Ru'(c_{2s}) = \mu_s. \tag{5}$$

Next, the planner will choose the initial payment c_1 and the division of resources between public and private consumption to solve

$$\max_{\{c_1, g_\alpha, g_\beta\}} \theta u(c_1) + (1-q)\left[V(1 - g_\alpha - \theta c_1; \widehat{\pi}_\alpha) + v(g_\alpha)\right] + q\left[V(1 - g_\beta - \theta c_1; \widehat{\pi}_\beta) + v(g_\beta)\right],$$

where the values of $\widehat{\pi}_s$ generated by the strategy profile (1) are given by

$$\widehat{\pi}_\alpha = \frac{\pi - \theta}{1 - \theta} \quad \text{and} \quad \widehat{\pi}_\beta = \pi. \tag{6}$$

The first-order conditions for this problem can be written as

$$u'(c_1) = (1-q)\mu_\alpha + q\mu_\beta, \quad \text{and} \tag{7}$$

$$v'(g_s) = \mu_s, \quad \text{for } s = \alpha, \beta. \tag{8}$$

The first condition says that the marginal value of resources paid out before the planner observes the state should equal the expected future shadow value of resources. The second can be interpreted as the standard Samuelson condition for the efficient provision of a public good in each state.

Let $\mathbf{c}^*$ denote the solution to this problem and let $\left(\mu_\alpha^*, \mu_\beta^*\right)$ denote the corresponding values of the multipliers. I call $\mathbf{c}^*$ the *q-efficient allocation*, since it represents the best possible allocation of resources conditional on the behavior specified in (1), where a run occurs with probability q.

3.3 Fragility

When is the strategy profile (1) part of an equilibrium of the model with a planner-run financial system? An impatient investor will always strictly prefer to withdraw early, as specified in the profile, so we only need to consider the actions of patient investors. Condition (5) implies that $c_{1s}^* < c_{2s}^*$ holds for any value of $\widehat{\pi}_s$ and, hence, a patient investor with $i > \theta$ will prefer to wait until period 2 to withdraw in both states.[8] It is straightforward to show that $c_1^* < c_{2\alpha}^*$ always holds, so

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[8]   This result implies that a run in this model is *necessarily* partial. Once an intermediary observes the state, its reaction will be such that the remaining patient investors have no incentive to withdraw early. See Ennis and Keister (2010) for a related model in which an intermediary never observes the state, but is able to gradually make inferences about it from the flow of withdrawals. In that setting, a run must occur in waves, with only some investors withdrawing in each wave.

that a patient investor with $i \leq \theta$ will strictly prefer to wait in state α. The only question, therefore, is whether a patient investor with $i \leq \theta$ has an incentive to join the run in state β. If she does, she will arrive before the planner observes the state and will receive c_1^*. If she deviates by waiting until period 2, she will receive $c_{2\beta}^*$. We can, therefore, construct an equilibrium in which investors follow (1) and the planner chooses $\mathbf{c}^*$ if and only if the q-efficient allocation satisfies

$$c_1^* \geq c_{2\beta}^*. \tag{9}$$

It is straightforward to show that there exist parameter values such that this condition is satisfied. In other words, financial fragility can arise in this model even when the financial system is operated by a benevolent planner and there are no distortions from the bailout policy. The following proposition shows that this condition is also necessary for fragility to arise; if the inequality is reversed, there is no equilibrium in which patient investors withdraw early in state β under a planner-run financial system. Proofs of results are given in the appendix unless stated otherwise.

Proposition 1 *A planner-run financial system is fragile if and only if* (9) *holds.*

In comparing outcomes across different policy regimes, it will be instructive to look at the set of all economies that are fragile under a given regime. An economy is characterized by the parameters $e \equiv (R, \pi, u, v, \theta, q)$. Let Φ^* denote the set of economies that are fragile under a planner-run financial system. Then Proposition 1 establishes that $e \in \Phi^*$ if and only if (9) holds, in which case $\mathbf{c}^*$ represents the allocation of resources in the equilibrium where investors follow (1).

3.4 Bailouts

The next proposition establishes a key feature of the q-efficient allocation $\mathbf{c}^*$: less public good is provided in a crisis than in normal times.

Proposition 2 $g_\beta^* < g_\alpha^*$ *holds for all* $q \geq 0$.

Recall that g_α^* is the quantity of resources initially set aside to provide the public good. If a crisis occurs, some of these resources are instead used to provide private consumption to those investors who have not yet withdrawn. The property $g_\beta^* < g_\alpha^*$ can, therefore, be interpreted as a bailout of the financial system: all investors pay a cost in terms of a lower level of the public good (an "austerity program") in order to augment the private consumption of those agents facing losses

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on their financial investments.[9]

Note that there is no role for liquidity facilities or other forms of lending to the financial sector in this model, even though a crisis is driven by self-fulfilling beliefs. Once the planner realizes a run is underway and some patient investors have already withdrawn, the problem is no longer one of illiquidity in the financial system because all future payments will be adjusted based on this information. Instead, the problem is that the withdrawals by patient investors have created a misallocation of resources that shrinks the set of feasible consumption levels (c_{1s}, c_{2s}) for the remaining investors. The only way to mitigate the losses suffered by these investors is with real transfers from the public sector. Proposition 2 shows that this bailout is part of the *efficient* allocation of resources when a crisis is possible.

The logic behind Proposition 2 also shows that the planner will never completely insure investors against a crisis, in the sense of equating private consumption levels across states, since doing so would leave public consumption inefficiently low in state β.

Corollary 1 $\left(c_{1\beta}^{*}, c_{2\beta}^{*} \right) \ll (c_{1\alpha}^{*}, c_{2\alpha}^{*})$ *holds for all* $q \geq 0$.

This result highlights a key conceptual difference between bailouts and deposit insurance programs. Deposit insurance is an *ex ante* commitment that aims to influence withdrawal behavior by assuring investors that they will not suffer losses. A bailout, in contrast, is an *ex post* response that aims to mitigate the effects of a crisis. Corollary 1 shows that the best response to a crisis is to only provide partial insurance; full deposit insurance is never time consistent.[10]

4 Equilibrium under Discretion

I now turn to the analysis of equilibrium in the decentralized economy, where the financial system is operated by private intermediaries. I begin with the case where policy is discretionary in the sense that bailout decisions are made by the policy maker in period 1 as a best response to the current situation. The resulting bailout policy distorts the incentives of intermediaries, leading

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[9] Note that total government spending is unaffected by a financial crisis in this model, since all tax revenue is collected in the initial period and the government budget is always balanced. What changes during a crisis is the composition of government spending between public services and transfer payments.

[10] In addition, fully insuring deposits may not be feasible ex post here, depending on the size of g_{α}^{*}. Cooper and Kempf (2011) study a model with heterogeneous agents in which a policy maker makes a binary choice between providing full deposit insurance and taking no action. They show that whether deposit insurance will be provided ex post depends on how the policy redistributes wealth across agents.

them to become more illiquid than under the q-efficient allocation derived above. These actions make the financial system more fragile: the set of economies for which a crisis can occur in equilibrium is strictly larger under this policy regime than under a planner-run financial system.

These results are established by first determining the best responses of intermediaries and the policy maker to the strategy profile (1) and to each other's actions. I derive these responses in three steps, establishing (i) what bailout payments the policy maker would choose to make in the event of a crisis, (ii) how the anticipation of these payments distorts intermediaries' choice of c_1, and (iii) what tax rate the policy maker will choose in period 0. These best responses generate an allocation of resources **c**, which then determines investors' withdrawal incentives and the fragility of the financial system.

4.1   Bailout policy

Suppose that, after θ withdrawals have been made, the policy maker observes that the state is β and thus knows that a run has occurred as specified in (1). Let $b(j) \geq 0$ denote the bailout payment per investor the policy maker gives to intermediary j. Let $\sigma(j)$ denote the distribution of investors across intermediaries, so that the total size of the bailout package is

$$b \equiv \int b(j)\, d\sigma(j).$$

The policy maker anticipates that each intermediary will allocate whatever resources it has available according to (3) and, therefore, chooses the bailout payments to solve

$$\max_{\{b(j)\}} \int V\left(\psi_\beta(j); \widehat{\pi}_\beta\right) d\sigma(j) + v(\tau - b)$$

subject the resource constraints

$$\psi_\beta(j) = 1 - \tau - \theta c_1(j) + b(j) \quad \text{for all } j.$$

These constraints state that the resources available for the private consumption of intermediary j's remaining investors, measured in per-investor terms, equal the initial deposit $1 - \tau$ minus the payment $c_1(j)$ made to a fraction θ of investors, plus the bailout payment $b(j)$.

Because the function V is strictly concave, the solution to this problem must equalize the value of $\psi_\beta(j)$ across all intermediaries. For a given size of the total bailout package b per investor, this

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entails setting

$$b(j) = b + \theta(c_1(j) - \overline{c}_1) \qquad \text{for all } j,$$

where $\overline{c}_1 \equiv \int c_1(j) \, d\sigma(j)$ is the average level of $c_1(j)$ in the economy. The value of ψ_β for intermediary j will then depend only on aggregate conditions,

$$\psi_\beta(j) = 1 - \tau - \theta\overline{c}_1 + b,$$

which the intermediary takes as given, and not on its own choice $c_1(j)$.

The incentive problems caused by this policy are clear: an intermediary with fewer remaining resources (because it set $c_1(j)$ higher) will receive a larger bailout payment. This larger payment implies that fewer funds are available for making transfers to other intermediaries and for public consumption. In equilibrium, of course, all intermediaries will choose the same value of $c_1(j)$ and receive the same bailout payment b per investor, but this value will be higher than is socially desirable because of the external effect each intermediary's choice has in state β.[11]

The first-order condition for choosing the total size of the bailout package is

$$v'(\tau - b) = \mu_\beta, \tag{10}$$

which says that the policy maker will equate the marginal value of public consumption in state β to the marginal value of private consumption for the remaining investors in each intermediary.

4.2 Distorted incentives

Since all intermediaries face the same decision problem, I omit the j index from here onward. When the first investor arrives to withdraw in period 1, each intermediary will choose c_1 to maximize

$$\theta u(c_1) + (1-q)V(1 - \tau - \theta c_1; \widehat{\pi}_\alpha) + qV(1 - \tau - \theta\overline{c}_1 + b; \widehat{\pi}_\beta).$$

The first-order condition for this problem is

$$u'(c_1) = (1-q)\mu_\alpha. \tag{11}$$

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[11] Note that, in principle, a similar incentive problem could arise in state α if the policymaker made bailout payments to intermediaries that chose an unusually high level of $c_1(j)$ in that state as well. I assume that bailout payments are only made in the event of a financial crisis. This assumption could be justified by reputation concerns, which will be significant for decisions made in normal times but much less important for a policymaker facing a rare event like a financial crisis.

Comparing this equation with (7) illustrates the distortion: here, c_1 is being chosen to equate the marginal value of resources before the state is known to the shadow value of resources *in the no-run state*, ignoring the value of resources in the event of a run. The larger the probability of a run q is, the more distorted the resulting allocation of resources becomes.

Using (5) together with (11), we see that the solution to this problem will satisfy $c_1 \leq c_{2\alpha}$ as long as

$$q \leq \frac{R-1}{R}. \tag{12}$$

If this inequality were reversed, the incentive distortion would be so large that a patient investor would prefer to withdraw early even if all other patient investors choose to wait. In such cases, the profile in (1) is not consistent with equilibrium, but other, more complex partial-run strategy profiles may be. To avoid these complications, I restrict attention to the case (12), where the probability of a crisis is sufficiently small, throughout the analysis.

4.3   The tax rate

When setting the tax rate in period 0, the policy maker anticipates that the level of c_1 chosen by intermediaries and the size of the bailout package b will be functions of τ as derived above. The policy maker will choose τ to maximize

$$\theta u\left(c_1\right) + \left(1-q\right)\left[V\left(1-\tau-\theta c_1;\widehat{\pi}_\alpha\right) + v\left(\tau\right)\right] + q\left[V\left(1-\tau-\theta c_1+b;\widehat{\pi}_\beta\right) + v\left(\tau-b\right)\right] \tag{13}$$

The first-order condition for this problem can be written as

$$v'\left(\tau\right) = \mu_\alpha + \frac{q}{1-q}\mu_\beta\theta\frac{dc_1}{d\tau}. \tag{14}$$

If the probability of a crisis q were zero, the tax rate would be set to equate the marginal value of the public good in state α with the corresponding marginal value of private consumption. When q is positive, however, the policy maker must also take into account the fact that τ affects intermediaries' choice of c_1, which in turn affects the amount resources available in state β. This effect is captured by the second term on the right-hand side of (14).

Let $\mathbf{c}^D$ denote the allocation resulting from the best responses of intermediaries and the policy maker, which is characterized by equations (4) - (6), (10) - (11), and (14). To determine if there is an equilibrium where investors follow strategy profile (1), we must compare the level of consumption each patient investor would receive from $\mathbf{c}^D$ if she withdraws early to that she would receive if

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she waits. As in the previous section, it is straightforward to show that patient investors with $i > \theta$ will always prefer to wait and that investors with $i \leq \theta$ will prefer to wait in state α. Fragility thus depends again on the incentives faced in state β by patient investors with $i \leq \theta$, as established by the following proposition. The proof follows that of Proposition 1 closely and is omitted.

Proposition 3 *The financial system is fragile under the discretionary policy regime if and only if* $c_1^D \geq c_{2\beta}^D$ *holds.*

Let Φ^D denote the set of economies for which this condition holds and let W^D denote the level of welfare in the run equilibrium for such an economy.

4.4 Illiquidity and fragility

The distortion created by the bailout policy gives each intermediary an incentive to become more illiquid by offering a larger short-term return to its investors. To illustrate this effect, I define the *degree of illiquidity* in the financial system to be

$$\rho \equiv \frac{c_1}{1 - \tau}.$$

Since each investor has the option of withdrawing early, c_1 can be interpreted as the face value of the short-term liabilities of the financial system, measured in per-capita terms. The period-1 value of intermediaries' assets is equal to total deposits, or $1 - \tau$ per capita. Hence ρ represents the ratio of the short-term liabilities of the financial system to the short-run value of its assets. The degree of illiquidity in the planner-run financial system can be measured the same way by equating τ to g_α, the level of public consumption in the no-run state. The following proposition shows the effect of the incentive distortion in the discretionary policy regime: it leads to higher illiquidity.

Proposition 4 $\rho^D > \rho^*$ *holds for all* $q > 0$.

This higher degree of illiquidity increases the scope for financial fragility in the model, in the sense that the set of parameter values for which the financial system is fragile becomes strictly larger.

Proposition 5 $\Phi^D \supset \Phi^*$.

Consider an economy $e \notin \Phi^*$. In this case, a patient investor has no incentive to withdraw early under a planner-run financial system, even when she expects other investors to run. In the decen-

tralized environment, however, intermediaries become more illiquid (Proposition 4), which implies that investors would find themselves in a worse position in the event of a run. A patient investor with $i \leq \pi$ thus has a stronger incentive to withdraw early under the decentralized regime. For some economies, the increased incentive is strong enough to induce her to join the run in state β, in which case $e \in \Phi^D$ holds and the distortions created by the bailout policy introduce an equilibrium in which a self-fulfilling financial crisis can occur.

This set-theoretic approach to measuring fragility has a natural interpretation in terms of changes in the probability of a financial crisis. Suppose that at the beginning of period zero, the parameter values e are drawn at random from some probability distribution f. If the realized e is such that the economy is not fragile, investors do not run on the financial system in either state. If the economy is fragile, however, investors follow the strategy profile (1). The ex ante probability assigned to a crisis by this process will be strictly higher in the decentralized economy than under the planner-run financial system for *any* probability distribution f that has full support. In this sense, the likelihood of a financial crisis is inefficiently high in the decentralized economy.[12]

In the following sections, I analyze two policy measures that aim to mitigate the incentive problem and improve welfare – a no-bailouts policy and a tax on short-term liabilities – and I illustrate the results with a numerical example.

5 Committing to No Bailouts

Suppose now that the policy maker is required to set $b = 0$ in all states of nature. Note that a very limited form of commitment is being introduced: the policy maker is either committed to this simple rule or operates under discretion as in the previous section.[13] As described in the introduction, the idea that eliminating future bailouts is an effective way to discourage risky behavior and thereby promote financial stability has figured prominently in recent policy debates. In fact, restricting the policy maker to set $b = 0$ in all states resembles the requirement in Section 214

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[12] An alternative approach would be to attempt to resolve the multiplicity of equilibrium by introducing private information as in the literature on global games pioneered by Carlsson and van Damme (1993). However, this approach places rather strict requirements on the information structure of the model. Papers that have used the global games methodology in Diamond-Dybvig type models have done so by placing arbitrary restrictions on contracts between intermediaries and their investors (see, for example, Rochet and Vives, 2004, and Goldstein and Pauzner, 2005). These restrictions themselves are potential sources of financial fragility, quite separate from the issues related to bailouts under consideration here. The approach taken here captures the effects of changes in the incentives faced by investors in a reasonably clear and transparent way, and does not place any restrictions on financial arrangements other than those imposed by the physical environment.

[13] In particular, commitment to a more detailed plan of action, such as bailing out an intermediary if and only if it has set $c_1(j) = c_1^*$, is still infeasible.

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of the Dodd-Frank Act that "taxpayers shall bear no losses" in the resolution of a failed financial institution. In the environment studied here, a no-bailouts policy does indeed lead intermediaries to be more cautious. However, it is often undesirable and may actually increase financial fragility.

5.1 Corrected incentives, but ...

Under a no-bailouts regime, an intermediary must use its own resources to provide consumption to its investors in both states. Suppose investors follow the strategy profile in (1). When the first investor arrives to withdraw in period 1, an intermediary will now choose c_1 to solve

$$\max_{\{c_1\}} \theta u\left(c_1\right) + (1 - q) V\left(1 - \tau - \theta c_1; \widehat{\pi}_\alpha\right) + q V\left(1 - \tau - \theta c_1; \widehat{\pi}_\beta\right).$$

Note that the quantity of resources available to the intermediary after θ withdrawals is the same in both states, but the allocation of these resources (c_{1s}, c_{2s}) will be different because the fraction of the remaining investors who are impatient, $\widehat{\pi}_s$, differs across states as shown in (6). The first-order condition for this problem is

$$u'\left(c_1\right) = (1 - q) \mu_\alpha + q\mu_\beta. \tag{15}$$

Comparing (15) with (11) shows how the no-bailout policy appears to correct the incentive problem that arises under discretion. An intermediary now equates the marginal value of resources before it observes the state to the expected future value of resources, exactly as the planner does in (7). The resulting choice of c_1 will, however, differ from the planner's because the no-bailouts policy affects the values of the multipliers μ_s.

The policy maker will again choose the tax rate in period 0 to maximize (13), but in this case with c_1 determined by (15) and b set to zero, which implies

$$g_\alpha = g_\beta = \tau. \tag{16}$$

The first-order condition can be written as

$$v'\left(\tau\right) = (1 - q) \mu_\alpha + q\mu_\beta. \tag{17}$$

Because all tax revenue now goes into the public good in both states, the policy maker sets the marginal value of public consumption equal to the expected marginal value of private consumption.

Let $\mathbf{c}^{NB}$ denote the allocation characterized by equations (4) - (6) and (15) - (17), which represents the best responses of intermediaries and the policy maker to the strategy profile (1) under

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a no-bailouts policy. The next result establishes the conditions for financial fragility under this regime. The proof, which follows those of Propositions 1 and 3 closely, is omitted.

Proposition 6 *The financial system is fragile under the no-bailouts regime if and only if $c_1^{NB} \geq c_{2\beta}^{NB}$ holds.*

Let Φ^{NB} denote the set of economies for which this condition holds and let W^{NB} denote the level of welfare in the run equilibrium for such an economy.

5.2 Competing effects on fragility

Compared to the outcome under the discretionary regime, a no-bailouts policy has two, competing effects on financial fragility. First, intermediaries become less illiquid.

Proposition 7 $\rho^{NB} < \rho^*$ *holds for all $q > 0$.*

Combined with Proposition 4, this result establishes that $\rho^{NB} < \rho^D$ holds, which captures the popular idea that by eliminating the moral hazard associated with the discretionary regime, a no-bailouts policy will lead intermediaries to be more liquid. The result is actually stronger, showing that intermediaries become even more liquid than the planner's allocation. This "overshooting" occurs because intermediaries must now completely self-insure against the possibility of a run, whereas the planner uses public resources to provide some insurance (Proposition 2). The fact that intermediaries are more liquid would, by itself, have a stabilizing effect on the financial system because it diminishes the incentive for patient investors to withdraw early.

However, the loss of the insurance associated with bailouts has another effect on investors' withdrawal incentives. Conditional on the level of c_1, a crisis now leads to lower consumption levels $(c_{1\beta}, c_{2\beta})$ for the remaining investors because no public funds are available to soften the blow. This second effect gives patient investors with $i \leq \theta$ a stronger incentive to withdraw early and would, by itself, tend to make the financial system more fragile.

In other words, the net effect of a no-bailouts policy on financial fragility is ambiguous: while it leads intermediaries to become more liquid, it also makes an investors' payoffs more sensitive to the withdrawal behavior of others. The next proposition shows that the latter effect always dominates when comparing a no-bailouts policy to the planner's allocation. Moreover, the latter effect sometimes dominates even when comparing a no-bailouts policy to the discretionary regime.

Proposition 8 $\Phi^{NB} \supset \Phi^*$. *Moreover, there exist economies in Φ^{NB} that are not in Φ^D.*

5.3 Welfare comparison

When the economy is fragile under one of the two policy regimes but not the other, the optimal policy is clear. In this case, selecting the regime under which the economy is not fragile ensures that the first-best allocation of resources will obtain in both states. If an economy is fragile under both regimes, however, the policy maker must compare the welfare levels W^D and W^{NB} associated with the run equilibrium in each case. In general, welfare may be higher under either regime, depending on parameter values. As the next proposition shows, however, a sharp comparison is possible when a crisis is sufficiently unlikely. In such situations, committing to a no-bailout policy never enhances financial stability and necessarily lowers welfare.

Proposition 9 *For any (R, π, θ, u, v), there exists $\overline{q} > 0$ such that $q < \overline{q}$ and $e \in \Phi^D$ implies both $e \in \Phi^{NB}$ and $W^D > W^{NB}$.*

The intuition behind this result is explained in the context of the example in Section 6 below.

5.4 Discussion

The results here demonstrate that committing to a no-bailouts policy will never generate an efficient allocation of resources and is often worse than a purely discretionary regime. They should not, however, be interpreted as implying that commitment in general is an ineffective policy tool. A policy maker with the ability to commit to *any* plan of action could always improve on the discretionary outcome. If, for example, the policy maker could commit to freeze all remaining deposits after π withdrawals have been made in period 1 – prohibiting any further withdrawals until period 2 – patient investors would have no incentive to withdraw early and a run would never occur. Such commitment is likely to be very difficult to achieve in practice, however (see Ennis and Keister, 2009a). A no-bailouts policy may be easier to implement through legislation that prohibits the use of public funds for certain activities, as some recent policy reform efforts aim to do. The main message of the above analysis is that commitment to this particular policy is often undesirable. In Section 7, I show how other types of policies aimed at influencing intermediaries' incentives are more effective than restrictions on the policy reaction to a crisis.

6 An Example

To better understand the various forces at work in the model and the results presented above, it is useful to look at a numerical example. Suppose the utility functions are given by

$$u\left(c\right) = \frac{\left(c\right)^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v\left(g\right) = \delta\frac{\left(g\right)^{1-\gamma}}{1-\gamma},$$

and fix the parameter values $(R, \pi, \gamma, \delta) = (1.05, 1/2, 8, 1/2)$.

6.1 Fragility

Figure 1 presents a projection of the fragile sets Φ onto a two-dimensional diagram where the parameters q and θ are varied. Different shades are used to represent economies that are fragile under the different policy regimes.
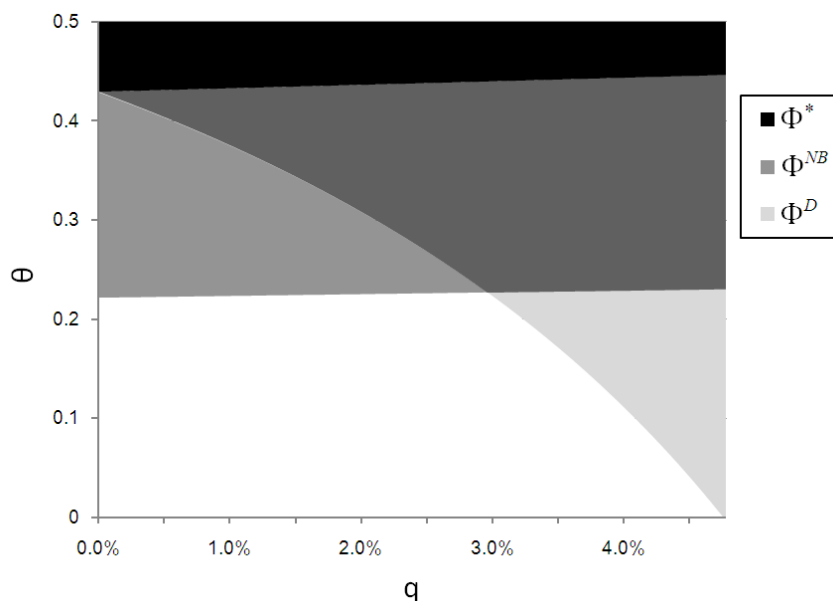

Figure 1: The sets Φ^*, Φ^D and Φ^{NB}. Darker areas indicate the intersection of sets.

The darkest area at the top of the figure represents the set of economies that are fragile under a planner-run financial system, Φ^*. From Propositions 5 and 8, we know these economies belong to all three sets. Notice that θ must be sufficiently large for a planner-run financial system to be fragile. When θ is small, the planner is able to react to an incipient run quickly by adjusting payments to a level that is appropriate given the high withdrawal demand. This quick reaction ensures that the losses created by the run are small and, as a result, the payoff to a patient investor from waiting to withdraw $(c_{2\beta})$ remains higher than the payoff from joining the run (c_1). The

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figure also shows that the threshold value of θ above which fragility arises is increasing q. When a crisis is more likely, the planner becomes more cautious and leaves the financial system more liquid, which raises the value of θ needed to generate fragility.

The set of economies that are fragile under the discretionary regime, Φ^D, is represented by the lightest shade, together with the two darkest areas where it overlaps with the other sets. The threshold value of θ decreases sharply with q in this case. As the probability of a crisis increases, the distortion of intermediaries' incentives becomes more pronounced and the financial system becomes more illiquid (that is, c_1 is set higher relative to total deposits $1-\tau$). Since more resources are consumed by the first θ investors to withdraw, the consumption of the remaining investors is lower in both states. As a result, withdrawing early and receiving c_1 becomes more attractive to a patient investor as q increases, which makes financial system fragile for lower values of θ. As q approaches the bound in (12), this threshold drops to zero: even if the the intermediary is able to react almost immediately to a run, it is so illiquid that the first few patient investors in the order will still have an incentive to withdraw early.

The fragile set associated with a no-bailouts policy, Φ^{NB}, is represented by the middle shade and occupies roughly the upper half of the figure. When q is small, the figure shows that the threshold value of θ is much lower in this case than under either of the previous regimes. If a crisis is very unlikely, the initial payment on early withdrawals (c_1) is similar under all three regimes. The payoff from staying invested in the event of a run $(c_{2\beta})$, however, is lower under a no-bailouts policy because intermediaries receive no transfer from the public sector. As a result, the incentive for an impatient investor to withdraw early is larger under the no-bailouts regime in this case, which translates into a lower threshold value for θ. As the probability of a crisis increases, however, intermediaries operating under a no-bailouts policy become more cautious; the moral hazard problem that causes the threshold value of θ to drop sharply under the discretionary regime does not arise.

In other words, Figure 1 illustrates the two competing effects on financial stability that arise when moving from a discretionary regime to a no-bailouts policy. First, the no-bailouts policy leads intermediaries to become more liquid, which by itself would tend to raise the threshold level of θ and make the financial system more stable. The magnitude of this effect is small when q is close to zero, but becomes large as q increases. Second, the no-bailouts policy makes investors more anxious to withdraw in state β because their payoffs are more sensitive to the withdrawal

behavior of others. The magnitude of this effect is largely independent of q, since withdrawal decisions are made after investors have observed realized state. The figure shows how this second effect dominates when q is small enough (in line with Proposition 9), while the first effect tends to dominate when q is larger (above $\sim 3\%$ in the figure).

6.2 Welfare

For economies that are fragile under both policy regimes, the information in Figure 1 does not determine which regime is preferable. Figure 2 plots the welfare gain from adopting a no-bailouts policy as a function of q, with θ set to $1/2$. When $q = 0$, the crisis state never occurs, so both policies deliver the first-best allocation and the benefit of switching regimes is zero. As q increases, the no-bailouts regime initially lowers welfare, as established in Proposition 9. For higher values of q, however, the effects of the incentive distortion in the discretionary case become more pronounced and the pattern begins to reverse. Beyond approximately $q = 3.5\%$, a no-bailouts policy generates higher welfare than the discretionary regime. It should be emphasized, however, welfare under both of these regimes is lower than under a planner-run financial system for all $q > 0$.

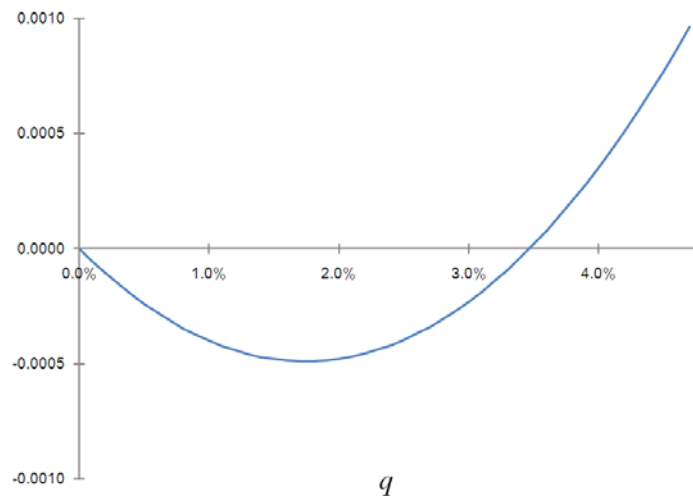

Figure 2: Welfare gain from a no-bailouts policy

7 Taxing Short-term Liabilities

Are there policies that would allow an economy in the decentralized environment to reach the same outcome as the planner-run financial system in Section 3? Doing so requires that investors

benefit from the insurance provided by the appropriate bailout payments in state β, but interme-diaries nevertheless choose the socially-efficient degree of illiquidity. In this section, I show how both of these objectives can be met if the policy maker is able to impose a tax on the payments c_1 made by intermediaries before the state is revealed. As discussed above, c_1 represents the value of an intermediary's short-term liabilities per investor and, hence, the policy can be interpreted as a tax on short-term liabilities.[14] This Pigouvian approach highlights the effectiveness of influencing intermediaries' choices directly through regulation rather than by restricting on the bailout policy.

7.1 Modified incentives

Each intermediary must now pay a fee that is proportional to the total value of its short-term liabilities per investor,

$$\text{fee}\,(j) \;=\; \eta c_1\,(j)\,.$$

For simplicity, I make the policy revenue neutral by giving each intermediary a lump-sum transfer equal to the average fee collected per investor, $N = \eta \overline{c}_1$, which an individual intermediary takes as given. The best responses of intermediaries and the policy maker to the strategy profile (1) are characterized by the same conditions as under the discretionary regime in Section 4 *except* for in-termediaries' choice of c_1. When the first investor arrives to withdraw in period 1, an intermediary will now solve

$$\max_{\{c_1\}} \;\theta u\,(c_1) + (1-q)\,V\,(1 - \tau - (\theta + \eta)\,c_1 + N; \widehat{\pi}_\alpha) + qV\,(1 - \tau - \theta\overline{c}_1 + b; \widehat{\pi}_\beta)\,.$$

The first-order condition of this modified problem is

$$u'\,(c_1) = \left(1 + \frac{\eta}{\theta}\right)(1-q)\,\mu_\alpha. \tag{18}$$

7.2 Choosing the Pigouvian tax rate

Comparing (18) with condition (7) shows that implementing the planner's choice of c_1 requires

$$\left(1 + \frac{\eta}{\theta}\right)(1-q)\,\mu_\alpha^* = (1-q)\,\mu_\alpha^* + q\mu_\beta^*.$$

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[14] Kocherlakota (2010) also advocates using taxes to offset the incentive distortions associated with bailouts and suggests a market-based mechanism for determining the appropriate tax rate.

Suppose the policy maker sets

$$\eta = \frac{\theta q \mu_\beta^*}{(1-q)\,\mu_\alpha^*} \equiv \eta^* \tag{19}$$

Let $\mathbf{c}^\eta$ denote the allocation that results from the best responses of intermediaries and the policy maker to the strategy profile (1) under the tax rate η^*, which is characterized by equations (4) – (6), (8) and (18). The following result shows that, as in the previous regimes, this allocation can be used to determine when the financial system is fragile; the proof is again omitted.

Proposition 10 *The financial system is fragile under the Pigouvian policy regime if and only if $c_1^\eta \geq c_{2\beta}^\eta$ holds.*

Let Φ^η denote set of economies that are fragile in the Pigouvian regime with tax rate η^*. The next result shows that this policy does indeed achieve the desired goal. The straightforward proof is omitted.

Proposition 11 *When the Pigouvian tax rate η is set according to (19), we have $\mathbf{c}^\eta = \mathbf{c}^*$ and $\Phi^\eta = \Phi^*$.*

The ratio of the two multipliers in (19) can be interpreted as the price of period-2 consumption in state β relative to state α. The tax rate η^* induces each intermediary to place an additional value on period-2 resources proportional to the marginal social value of resources in the event of a run, which exactly offsets the distortion created by the bailout policy. Note that the objective of this policy is *not* to make intermediaries pay for bailouts. In general, the revenue raised by η^* will not equal the size of the bailout package b^*, either in full or in expected value.[15] Instead, the objective is to impose a cost on each intermediary equal to the external effect its actions have in state β.

This result shows ex ante intervention to be a powerful policy tool in the environment studied here. An appropriately chosen tax rate allows the policy maker to provide investors with the optimal level of insurance against the losses associated with a financial crisis without leading intermediaries to choose excessively high levels of illiquidity. Not only does this policy improve the allocation of resources in normal times, it also conveys a macroprudential benefit, decreasing financial fragility relative to either the discretionary or the no-bailouts regime.

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[15] A stark way to see this point is to note that specifying the tax as $\eta^*\,(c_1\,(j) - c_1^*)$ would lead to the same outcome but yield zero revenue.

8 Concluding Remarks

The idea that bailouts can significantly distort the incentives of financial institutions and their investors has figured prominently in recent policy debates. One popular – and influential – view holds that the best way to deal with this problem is to restrict policy makers from engaging in future bailouts wherever possible. To evaluate this proposition, I have presented a model with three key features: (i) when there is aggregate risk in the economy, it is efficient for some of this risk to be borne by the public sector, (ii) the anticipation of being bailed out in the event of a crisis distorts the incentives of financial intermediaries, and (iii) investors are more likely to withdraw from the financial system when the potential loss they face in a crisis is larger. While the results above are derived in the context of this specific model, the underlying forces are quite general and will arise in a wide range of settings where these basic features are present.

It follows immediately from (i) that a strict no-bailouts policy cannot achieve an efficient allocation of resources in such a setting. If some risk should be borne by the public sector, achieving an efficient outcome requires allowing the policy maker to engage in bailouts if a crisis occurs. If bailouts are permitted, (ii) requires that the policy maker use prudential policy measures to offset the resulting distortion in incentives. In Section 7, I showed how placing a tax on intermediaries' short-term liabilities – and no restrictions on the bailout policy – leads to the same outcome as when the financial system is operated by a benevolent planner. This outcome is strictly better than what occurs under a no-bailouts policy; it generates higher welfare and a more stable financial system.

Proponents of the no-bailouts view may argue, however, that this result relies too heavily on the ability of policy makers and regulators to design and maintain an effective prudential policy regime. In practice, using taxes or other regulatory tools to precisely set incentives can be extremely difficult. Among the many problems that arise, regulation may create an incentive for intermediaries to devise new types of liabilities or funding structures for which the appropriate regulatory treatment is not immediately clear. In a world where effectively controlling incentives through taxes/regulation is infeasible, one might be tempted to view a no-bailouts policy as at least representing a step in the right direction.

The primary message of the paper is that this view is often incorrect. The analysis highlights two important costs that limit the attractiveness of a no-bailouts policy. The first is a consequence of (i): by inefficiently concentrating all risk in the private sector, eliminating bailouts makes in-

termediaries too cautious from a social point of view and leads to an underprovision of financial services. Moreover, (iii) implies that removing the insurance provided by bailouts will tend to increase the susceptibility of the financial system to a crisis by making investors more prone to run at the first sign of trouble. In Sections 5 and 6, I showed how a no-bailouts policy can be strictly worse than pure discretion, lowering equilibrium welfare and making the financial system more fragile. In other words, the costs that arise from a no-bailouts policy will, in many cases, outweigh the benefits.

It should be emphasized that the bailout policies studied here are efficient; they do not lead to rent-seeking behavior, nor are they motivated by outside political considerations. In reality, these types of distortions are important concerns. The message of the paper is *not* that any type of bailout policy is acceptable as long as the ex ante effects on intermediaries' incentives are somehow offset. Limits on the ability of policy makers to undertake some types of redistribution during a crisis may well be desirable. Rather, the message is that restrictions on bailouts alone cannot ensure that intermediaries and investors face the correct ex ante incentives, which highlights the importance of efforts to develop improved regulatory and other prudential policy tools.

Appendix A. Proofs of Selected Propositions

Proposition 1: *A planner-run financial system is fragile if and only if* (9) *holds.*

Proof: The discussion in the text establishes that (9) is a sufficient condition for the financial system to be fragile, since it guarantees there exists an equilibrium in which investors follow strategy profile (1). What remains to be proven is that this condition is also necessary for fragility to arise.

To begin, note that an equilibrium strategy profile must satisfy two basic properties. First, all impatient investors must withdraw in period 1, since they receive no utility from consuming in period 2. Second, because (5) implies that the planner's best response to *any* strategy profile satisfies $c_{1s} < c_{2s}$ in both states, all period-1 withdrawals that occur after the planner has observed the state must be made by impatient investors. For any strategy profile satisfying these two properties, define $\varepsilon_s \in [0, 1 - \pi]$ to be the fraction of the first θ withdrawals in state s that are made by patient investors. (Note that (1) corresponds to the profile with $\varepsilon_\alpha = 0$ and $\varepsilon_\beta = 1 - \pi$.) Definition 1 states that the financial system is fragile if there exists an equilibrium in which investors follow some strategy profile with $\varepsilon_\beta > 0$. I will show that whenever such an equilibrium exists, (9) holds.

Let $\widetilde{y}$ be any such strategy profile and let $\widetilde{\mathbf{c}}$ denote the allocation generated by the planner's best response to this profile. This allocation is characterized by equations (4), (5), (7) and (8), where the fraction of remaining investors who are impatient after θ withdrawals have been made, $\widehat{\pi}_s$, is now given by

$$\widehat{\pi}\left(\varepsilon_s\right) = \frac{\pi - \left(1 - \varepsilon_s\right)\theta}{1 - \theta} \quad \text{for } s = \alpha, \beta. \tag{20}$$

If there is an equilibrium in which investors follow $\widetilde{y}$, we must have

$$\widetilde{c}_1 \geq \widetilde{c}_{2\beta}, \tag{21}$$

that is, the patient investors who withdraw early in state β would not benefit by deviating and withdrawing in period 2. To establish that (9) is a necessary condition for fragility, therefore, it suffices to show that whenever (21) holds for some $\widetilde{y}$, (9) also holds.

Since the function u exhibits constant relative risk aversion, expected utility preferences over pairs (c_{1s}, c_{2s}) are homothetic and the solution to (3) satisfies the linear relationship

$$c_{2s} = \lambda c_{1s} \tag{22}$$

for some scalar λ. Using the fact that R and the coefficient of relative risk aversion in u are both

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greater than unity, the first-order condition (5) implies that λ satisfies

$$1 < \lambda < R. \tag{23}$$

Using (22), we can write the resource constraint (4) for state s as

$$\left[(1 - \theta) \left(\widehat{\pi} \left(\varepsilon_s \right) \frac{1}{\lambda} + (1 - \widehat{\pi} \left(\varepsilon_s \right)) \frac{1}{R} \right) \right] c_{2s} = \psi_s.$$

Condition (23) implies the term in square brackets is strictly increasing in $\widehat{\pi} \left(\varepsilon_s \right)$. In other words, holding the level of resources ψ_s fixed, the consumption given to each remaining investor is lower when more of these investors are impatient. Using the bounds $\varepsilon_\alpha \geq 0$ and $\varepsilon_\beta \leq 1$ and the fact that (20) is increasing in that ε_s, we have

$$\frac{\widetilde{c}_{2\beta}}{\widetilde{c}_{2\alpha}} = \frac{\widehat{\pi} \left(\varepsilon_\alpha \right) \frac{1}{\lambda} + (1 - \widehat{\pi} \left(\varepsilon_\alpha \right)) \frac{1}{R}}{\widehat{\pi} \left(\varepsilon_\beta \right) \frac{1}{\lambda} + (1 - \widehat{\pi} \left(\varepsilon_\beta \right)) \frac{1}{R}} \geq \frac{\widehat{\pi} \left(0 \right) \frac{1}{\lambda} + (1 - \widehat{\pi} \left(0 \right)) \frac{1}{R}}{\widehat{\pi} \left(1 \right) \frac{1}{\lambda} + (1 - \widehat{\pi} \left(1 \right)) \frac{1}{R}} = \frac{c_{2\beta}^*}{c_{2\alpha}^*}. \tag{24}$$

This expression shows that the strategy profile (1) generates the lowest value of the ratio $c_{2\beta}/c_{2\alpha}$, because it represents the (extreme) case where no patient investor withdraws early in state α and all patient investors with $i \leq \theta$ withdraw early in state β. Using the homotheticity of preferences, condition (24) implies

$$\frac{u' \left(\widetilde{c}_{2\alpha} \right)}{u' \left(\widetilde{c}_{2\beta} \right)} \geq \frac{u' \left(c_{2\alpha}^* \right)}{u' \left(c_{2\beta}^* \right)} \quad \text{or} \quad \frac{\widetilde{\mu}_\alpha}{\widetilde{\mu}_\beta} \geq \frac{\mu_\alpha^*}{\mu_\beta^*}.$$

We then clearly have

$$(1 - q) \frac{\widetilde{\mu}_\alpha}{\widetilde{\mu}_\beta} + q \geq (1 - q) + q \frac{\mu_\alpha^*}{\mu_\beta^*},$$

or

$$\frac{(1 - q) \widetilde{\mu}_\alpha + q \widetilde{\mu}_\beta}{R^{-1} \widetilde{\mu}_\beta} \geq \frac{(1 - q) \mu_\alpha^* + q \mu_\beta^*}{R^{-1} \mu_\beta^*}$$

Using (5) and (7), this inequality implies

$$\frac{u' \left(\widetilde{c}_1 \right)}{u' \left(\widetilde{c}_{2\beta} \right)} \geq \frac{u' \left(c_1^* \right)}{u' \left(c_{2\beta}^* \right)}.$$

Again using the homotheticity of preferences, we have

$$\frac{\widetilde{c}_1}{\widetilde{c}_{2\beta}} \leq \frac{c_1^*}{c_{2\beta}^*}.$$

It follows immediately from this inequality that (21) implies (9), as desired. ∎

Proposition 2: $g_\beta^* < g_\alpha^*$ holds for all $q \geq 0$.

Proof: Using the relationship (22) and the definition of ψ_s in (2), we can write the resource constraint (4) for state s as

$$\left[(1-\theta) \left(\widehat{\pi}_s \frac{1}{\lambda} + (1-\widehat{\pi}_s) \frac{1}{R} \right) \right] c_{2s} + g_s = 1 - \theta c_1. \tag{25}$$

From (6), we have

$$\widehat{\pi}_\beta = \pi > \frac{\pi - \theta}{1 - \theta} = \widehat{\pi}_\alpha,$$

which together with the second inequality in (23) implies that the term in square brackets in (25) is strictly larger in state β than in state α. Therefore, at least one of the inequalities

$$c_{2\beta} < c_{2\alpha} \qquad \text{and} \qquad g_\beta < g_\alpha$$

must hold. The first-order conditions (5) and (8) then imply that *both* of these inequalities hold. In fact, these conditions imply

$$(c_{1\beta}, c_{2\beta}, g_\beta) \ll (c_{1\alpha}, c_{2\alpha}, g_\alpha) \qquad \text{and} \qquad \mu_\beta^* > \mu_\alpha^*.$$

Intuitively, these conditions simply establish that state β is a negative outcome. When some patient depositors withdraw early, the fraction of the remaining investors who will need to consume early $(\widehat{\pi})$ is larger. Providing early consumption is more expensive and, hence, the planner assigns a higher shadow value of resources μ_β in this state. The optimal response to this outcome is for the planner to lower both remaining private consumption $(c_{1\beta}, c_{2\beta})$ and public consumption (g_β) ∎

Proposition 4: $\rho^D > \rho^*$ holds for all $q > 0$.

Proof: First, since the multipliers μ_α^* and μ_β^* are always strictly positive, we clearly have

$$(1-q) + q \frac{\mu_\beta^*}{\mu_\alpha^*} > (1-q).$$

This inequality can be rewritten as

$$\frac{(1-q) \mu_\alpha^* + q \mu_\beta^*}{\mu_\alpha^*} > \frac{(1-q) \mu_\alpha^D}{\mu_\alpha^D},$$

which implies

$$\frac{u'(c_1^*)}{u'(c_{2\alpha}^*)} > \frac{u'(c_1^D)}{u'(c_{2\alpha}^D)}.$$

Using the homotheticity of preferences over pairs $(c_1, c_{2\alpha})$, the above inequality implies

$$\frac{c_1^*}{c_{2\alpha}^*} < \frac{c_1^D}{c_{2\alpha}^D}. \tag{26}$$

The resource constraint (25) for state α can be written as

$$\rho^{-1} = \frac{1 - g_a}{c_1} = \theta + (1 - \theta)\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right)\left(\frac{c_{2\alpha}}{c_1}\right). \tag{27}$$

Using this equation, the inequality in (26) immediately implies implies $\rho^* < \rho^D$, as desired. ∎

Proposition 5: $\Phi^D \supset \Phi^*$.

Proof: The proof is divided into three steps.

Step 1: Derive bounds for the derivative $dc_1^D/d\tau$. First, differentiating the resource constraint

$$\theta c_1^D + (1 - \theta)\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right) c_{2\alpha}^D = 1 - \tau$$

with respect to τ yields

$$\theta \frac{dc_1^D}{d\tau} + (1 - \theta)\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right)\frac{dc_{2\alpha}^D}{d\tau} = -1.$$

Next, combining (5) and (11), then differentiating with respect to τ yields

$$\frac{dc_{2\alpha}^D}{d\tau} = \frac{1}{1 - q}\frac{u''\left(c_1^D\right)}{Ru''\left(c_{2\alpha}^D\right)}\frac{dc_1^D}{d\tau}.$$

Substituting this relationship into the previous equation and solving yields

$$\frac{dc_1^D}{d\tau} = -\frac{1}{\theta + \chi}$$

where

$$\chi \equiv \frac{1 - \theta}{1 - q}\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right)\frac{u''\left(c_1^D\right)}{Ru''\left(c_{2\alpha}^D\right)} > 0.$$

Note that $\chi > 0$ implies the following bounds

$$-1 < \theta\frac{dc_1^D}{d\tau} < 0.$$

Combined with (14), the first of these inequalities implies that for any $q > 0$, we have

$$v'\left(g_\alpha^D\right) > \mu_\alpha^D - \frac{q}{1 - q}\mu_\beta^D. \tag{28}$$

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Step 2: Show that $c_1^D > c_1^*$ holds for all $q > 0$. Suppose, to the contrary, that $c_1^D \leq c_1^*$ held for some $q > 0$. Then the first-order conditions (7) and (11) would imply

$$(1 - q)\,\mu_\alpha^D \geq (1 - q)\,\mu_\alpha^* + q\mu_\beta^*.$$

In addition, $1 - \theta c_1^D \geq 1 - \theta c_1^*$ would imply $\mu_\beta^D < \mu_\beta^*$. Combining these two inequalities yields

$$\mu_\alpha^D - \frac{q}{1 - q}\mu_\beta^D > \mu_\alpha^*. \tag{29}$$

The inequality (28) and condition (8) would then imply

$$g_\alpha^D < g_\alpha^*.$$

Note that (29) would also immediately imply $\mu_\alpha^D > \mu_\alpha^*$, which in turn would imply $c_{2\alpha}^D < c_{2\alpha}^*$. But then we would have

$$\theta c_1^D + (1 - \theta)\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right) c_{2\alpha}^D + g_\alpha^D < \theta c_1^* + (1 - \theta)\left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha)\frac{1}{R}\right) c_{2\alpha}^* + g_\alpha^*.$$

In other words, if c_1^D were smaller than c_1^*, then all components of the resource constraint in state α would be smaller under the discretionary regime than under the planner-run financial system, which contradicts the fact that the resource constraint (25) holds with equality in all cases.

Step 3: Show $\Phi^D \supset \Phi^*$. Consider any economy in Φ^*, that is, any economy for which $c_1^* \geq c_{2\beta}^*$ holds. Using the result from Step 2, we then have $c_1^D > c_{2\beta}^*$ and the first-order conditions (5) and (11) imply

$$(1 - q)\,\mu_\alpha^D < \frac{1}{R}\mu_\beta^*.$$

Furthermore, $1 - \theta c_1^D < 1 - \theta c_1^*$ implies $\mu_\beta^D > \mu_\beta^*$. We thus have

$$(1 - q)\,\mu_\alpha^D < \frac{1}{R}\mu_\beta^D, \tag{30}$$

which implies $c_1^D > c_{2\beta}^D$ and, hence, the economy is also in Φ^D. Moreover, the fact that the inequality in (30) is strict implies that the inclusion relationship is also strict: there exist economies for which c_1^* is slightly smaller than $c_{2\beta}^*$, but (30) still holds. Alternatively, it is easy to find examples of economies that belong to Φ^D but not to Φ^*; see Figure 1. ∎

Proposition 7: $\rho^{NB} < \rho^*$ holds for all $q > 0$.

Proof: First, recall that Proposition 2 establishes $g_\alpha^* > g_\beta^*$. Using the resource constraint (25) for each state, this relationship implies

$$\left(\widehat{\pi}_\beta \frac{1}{\lambda} + (1 - \widehat{\pi}_\beta) \frac{1}{R}\right) c_{2\beta}^* > \left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha) \frac{1}{R}\right) c_{2\alpha}^*.$$

Under a no-bailout policy, $g_\beta^{NB} = g_\alpha^{NB}$ holds by definition and the resource constraints imply

$$\left(\widehat{\pi}_\beta \frac{1}{\lambda} + (1 - \widehat{\pi}_\beta) \frac{1}{R}\right) c_{2\beta}^{NB} = \left(\widehat{\pi}_\alpha \frac{1}{\lambda} + (1 - \widehat{\pi}_\alpha) \frac{1}{R}\right) c_{2\alpha}^{NB}.$$

Combining these two relationships yields

$$\frac{c_{2\beta}^*}{c_{2\alpha}^*} > \frac{c_{2\beta}^{NB}}{c_{2\alpha}^{NB}},$$

which, by the homotheticity of preferences, implies

$$\frac{u'\left(c_{2\beta}^*\right)}{u'\left(c_{2\alpha}^*\right)} < \frac{u'\left(c_{2\beta}^{NB}\right)}{u'\left(c_{2\alpha}^{NB}\right)} \quad \text{or} \quad \frac{\mu_\beta^*}{\mu_\alpha^*} < \frac{\mu_\beta^{NB}}{\mu_\alpha^{NB}}. \tag{31}$$

From the latter inequality, we clearly have

$$(1 - q) + q\frac{\mu_\beta^*}{\mu_\alpha^*} < (1 - q) + q\frac{\mu_\beta^{NB}}{\mu_\alpha^{NB}},$$

which can be rewritten as

$$\frac{(1 - q)\mu_\alpha^* + q\mu_\beta^*}{R^{-1}\mu_\alpha^*} < \frac{(1 - q)\mu_\alpha^{NB} + q\mu_\beta^{NB}}{R^{-1}\mu_\alpha^{NB}},$$

or

$$\frac{u'\left(c_1^*\right)}{u'\left(c_{2\alpha}^*\right)} < \frac{u'\left(c_1^{NB}\right)}{u'\left(c_{2\alpha}^{NB}\right)}.$$

Again using the homotheticity of preferences, this last inequality implies

$$\frac{c_1^*}{c_{2\alpha}^*} > \frac{c_1^{NB}}{c_{2\alpha}^{NB}}.$$

Using (27), this inequality yields $\rho^* > \rho^{NB}$, as desired. ∎

Proposition 8: $\Phi^{NB} \supset \Phi^*$. Moreover, there exists economies in Φ^{NB} that are not in Φ^D.

Proof: For any $e \in \Phi^*$, we know that (9) holds. Using (5) and (7), this condition can be rewritten as

$$\frac{\mu_\alpha^*}{\mu_\beta^*} \leq \frac{R^{-1} - q}{1 - q}.$$

The second inequality in (31) then implies

$$\frac{\mu_\alpha^{NB}}{\mu_\beta^{NB}} < \frac{R^{-1} - q}{1 - q}.$$

Straightforward algebra, using (5) and (15), shows that this latter inequality is equivalent to $c_1^{NB} > c_{2\beta}^{NB}$, meaning that $e \in \Phi^{NB}$ also holds. Moreover, the fact that this inequality is strict implies that the inclusion relationship is also strict: there exist economies for which (9) is violated by a small amount, but $c_1^{NB} \geq c_{2\beta}^{NB}$ still holds. Alternatively, it is easy to find examples of economies that belong to Φ^{NB} but not to Φ^*; see Figure 1. Figure 1 also presents examples of economies that are in Φ^{NB} but not in Φ^D. ∎

Proposition 9: For any (R, π, θ, u, v), there exists $\overline{q} > 0$ such that $q < \overline{q}$ and $e \in \Phi^D$ implies both $e \in \Phi^{NB}$ and $W^D > W^{NB}$.

Proof: For any (R, π, θ, u, v), in the limit as q goes to zero, the effects of events in state β on the choice of c_1 disappear and the same value obtains in all three cases,

$$\lim_{q \to 0} c_E^{NB}(q) = \lim_{q \to 0} c_E^D(q) = \lim_{q \to 0} c_E^*(q).$$

However, it follows from Proposition 2 and the resource constraint (25) that $c_{2\beta}$ will be lower in the no-bailouts regime,

$$\lim_{q \to 0} c_{2\beta}^{NB}(q) < \lim_{q \to 0} c_{2\beta}^D(q) = \lim_{q \to 0} c_{2\beta}^*(q).$$

Therefore, there exists some $\overline{q} > 0$ such that

$$\frac{c_1^{NB}(q)}{c_{2\beta}^{NB}(q)} > \frac{c_1^D(q)}{c_{2\beta}^D(q)} \quad \text{for all } q < \overline{q}.$$

If $e \in \Phi^D$ for any $q < \overline{q}$, then $c_1^D(q) \geq c_{2\beta}^D(q)$ holds by definition. The inequality above then implies $c_1^{NB}(q) > c_{2\beta}^{NB}(q)$ and, hence, $e \in \Phi^{NB}$ also holds, establishing the first part of the proposition.

For the second part of the proposition, note that when the economy is fragile under both policy regimes, equilibrium welfare in the limit as the probability of a crisis goes to zero is the same,

$$\lim_{q \to 0} W^{NB}(q) = \lim_{q \to 0} W^D(q).$$

When q is close to zero, there is almost no distortion of ex ante incentives and both policy regimes deliver approximately the first-best allocation of resources. The proposition will, therefore, be established if we can show that welfare initially falls faster under the no-bailouts regime as q rises, that is, if we can show

$$\lim_{q \to 0} \frac{dW^{NB}(q)}{dq} < \lim_{q \to 0} \frac{dW^D(q)}{dq}. \tag{32}$$

The derivative for the discretionary regime can be written as

$$\lim_{q \to 0} \frac{dW^D(q)}{dq} = -V_\alpha \left(1 - g_\alpha^* - \theta c_1^*\right) - v\left(g_\alpha^*\right) + V_\beta \left(1 - g_\beta^* - \theta c_1^*\right) + v\left(g_\beta^*\right).$$

This expression uses the fact that the equilibrium allocation $\mathbf{c}^D$ converges to the efficient allocation $\mathbf{c}^*$ as q goes to zero, so that c_1^D can be replaced by c_1^*, g_α^D by g_α^*, etc. To evaluate the derivative under the no-bailouts regime, note that c_1 and the state-α components of the allocation $(c_{1\alpha}, c_{2\alpha}, g_\alpha)$ converge to those in $\mathbf{c}^*$ as q goes to zero, but the state-β components $(c_{1\beta}, c_{2\beta}, g_\beta)$ do not. This happens precisely because no bailout takes place, so that g_β^{NB} equals g_α^{NB} and both values converge to g_α^*. In this case, the limit of the derivative can be written as

$$\lim_{q \to 0} \frac{dW^{NB}(q)}{dq} = -V_\alpha \left(1 - g_\alpha^* - \theta c_1^*\right) - v\left(g_\alpha^*\right) + V_\beta \left(1 - g_\alpha^* - \theta c_1^*\right) + v\left(g_\alpha^*\right).$$

Notice that the first two terms in these derivatives are the same, but the last two terms differ because the level of public consumption is kept at g_α^* by the no-bailouts policy. Since g_β^* is chosen to maximize continuation utility in state β (and g_α^* is different from g_β^*), it follows that (32) holds, which establishes the result. ∎

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