Performance of the Bicycle Wheel
A Method for Analysis
by John Swanson

Abstract: My set of wheels is better than your set of wheels. I know why and I can prove it.

Introduction: As a competitive cyclist it's easy to succumb to bike lust. The latest and greatest parts are shiny, coated in carbon, and come with promises of EPO-like enhancements at prices per gram that rival gold. As an amateur cyclist who has only minor sponsorship (i.e., Zipp isn't firehosing the team with free product), it's reasonable to question how much benefit a component really gives and whether it justifies the cost.

It's my opinion that the wheel is one of those components where it's easy to consider a pair of top end Ksyrium's, but not really know why. Is it really that much better than a low end Velomax? How much better? Can I feel the difference? Will it be enough to impact my race performance?

So what are the qualities of a wheel?

- **Lateral Stiffness**: This is a measure of how much the rim will deflect under side load. The effect will be noticed mostly while cornering and during hard efforts out of the saddle. Stiffer is better. This is easy to measure, too. Simply clamp the hub, load the rim, and measure the deflection. Much has been said on this topic. Reference 1
- **Torsional Stiffness**: This is a measure of how much the hub will twist relative to the rim while under load. This would be noticed during accelerations such as sprints. The construction of bicycle wheels makes them incredible stiff in torsion, especially when the wheels are laced in a 2x or 3x pattern. Very little energy is dissipated here and although it is easy to measure, I doubt that there would be any measureable difference between wheels.
- **Radial Stiffness**: There's almost -no- vertical compliance in your wheel and people who insist that they can feel the vertical stiffness or "harshness" of a wheel are mistaken. The radial stiffness of a bicycle wheel is ~ 3-4000 N/mm. This equals a deflection of 0.1 mm under a 40 kg load. Sorry princess, but that gets obscured by the amount of deflection in the tires, fork, saddle, handlebar tape, frame, and even your gloves. Reference 2
- **Weight**: Weight is easily measured with a digital scale. The less weight the better, though the amount of performance gain is sometimes marginal and depends entirely on the course that's being ridden. This is easily calculated.
- **Moment of Inertia (MoI)**: Like mass, moment of inertia is a measure of how easy it is to accelerate a wheel. For you geek types, $I=\int_0^V r^2 \rho \, dV$. For the layman, it means that the more mass you have and the further out it is, the larger MoI will be. As you can see from the equation, MoI isn't trivial to calculate or measure.
- **Translational Aerodynamics**: This is the measure of how much drag is created as the wheel moves forward through the air. The less drag the better, though there are only large improvements for extreme shapes and designs that turn the tire into the leading edge of an airfoil. Drag is most often measured in a wind tunnel and is therefore an expensive and complex measurement. Reference 3
- **Rotational Aerodynamics**: This is a measure of how much drag, or torque, is created as the wheel rotates through the air. The less drag the better, and is largely a function of the number of spokes, the spoke shape and surface smoothness of the rim.

- **Rotational Friction**: The bearings will dissipate energy and higher quality bearings that are properly aligned will perform better. The higher the load on the wheel (i.e., the more you look like Jan Ullrich after a three week doughnut binge), the more important this becomes. An imbalance in the wheel will also exaggerate this effect. Measurement of this is not straightforward, but can be done...

You might notice that the various stiffness qualities are related to how the wheel will “feel” out on the road. It can be quite annoying to get brake rub during a sprint and downright scary to feel the wheel flex in a tight corner. Yikes! These are important parameters, but they are easily measured and evaluated out on the road. Generally, stiffer is better. All the other parameters listed above are related to the actual performance of the wheel rather than how it will feel. How much energy you will expend to accelerate the wheel and keep it moving.

So... why is it that when you go to buy a set of race wheels, the only numbers you get to evaluate are price and weight? Occassionally a manufacturer will wave their arms and declare that MoI is important, which is why their wheels are so light. Okay. But very, very few manufacturers quote actual numbers for MoI which makes it difficult to use as a means of comparison. Same goes for translational aerodynamics. And what about the bearing friction and rotational aerodynamics? What is needed is some cost effective way for manufacturers and owners alike to measure and compare these performance parameters.

Reference 4 – Apropos of nothing, take a break here and go sit at the feet of the master. Come back when you have acheived enlightenment. Shoo.

Analysis: I decided to start with the equations of motion for a wheel and see where it led me:

$$\tau = I\alpha = a + b\omega + c\omega^2$$

Where τ is the torque, which is equal to the moment of inertia, I, times the angular acceleration, α. This is a basic concept in rotational dynamics and is equivalent to the statement F = ma in kinematics. The rest of the equation as I've written it is an educated guess. The term, **a**, is related to the stiction associated with the bearings. In other words, you have to apply greater than **a** torque before the wheel will begin moving. The second term **bω** is mostly caused by bearing friction. Although the constant, **b**, is probably not constant at all and is more likely a function of load on the bearings. This is very similar to the force due to kinematic friction which is proportional to the normal load, coefficient of friction and speed. The last term, **cω²**, is almost solely due to aerodynamic interactions. The force due to air resistance is almost always proportional to speed squared (at low Reynold's numbers).

Rearranging the equation a bit, we get:

$$\alpha = a/I + (b/I)\omega + (c/I)\omega^2$$

or

$\alpha = a' + b'\omega + c'\omega^2$ where $a' = a/I$, $b' = b/I$, and $c' = c/I$

Rewritten this becomes: $d\omega/dt = a' + b'\omega + c'\omega^2$ (Eq. 1) where the acceleration α is equal to dω/dt.

This is a nice and tidy little equation. It was at this point that I had a few beer and thought of my cycling computer. You know, the thing that measures speed... ω.... Hmmm. This could be useful.

If I had some way of measuring speed and acceleration then I could figure out the coefficients **a**, **b**, and **c** as well as the MoI by using linear regression techniques (i.e., curve fitting). The problem is that there are very few ways to directly measure either speed or aceleration. The most common method is to measure both distance and time to then calculate speed. For example, the common cycling computer uses a spoke magnet and reed switch to figure out when the wheel has completed one rotation. That is $\theta(t_2) - \theta(t_1) = 2\pi$ radians. If you know t_2 and t_1, then $\omega(t_2) = 2\pi/(t_2-t_1)$ radians per second.

Experimentally, I figured that the easiest measurement to take would be to spin up a wheel and record θ(t) as the wheel slows down and comes to a stop. For now let's assume that we can set something up that accurately measures θ(t), possibly using a setup similar to a cycling computer. We could then take Equation 1 from above, integrate a few times and get the theoretical model of θ(t). This would allow us to compare the data we measured to the theoretical model and obtain the various coefficients through the use of non-linear regression. Let's try it and see what happens. Rearranging Equation 1 we get:

Eq. 1 $\quad d\omega/dt = a' + b'\omega + c'\omega^2 \;\rightarrow\; d\omega/(a' + b'\omega + c'\omega^2) = dt \;\rightarrow$

$$\int d\omega/(a' + b'\omega + c'\omega^2) = \int dt \;\rightarrow\; t(\omega) = \int d\omega/(a' + b'\omega + c'\omega^2)$$

Using 'The Integrator' website we can get someone else to do the math:

$$t(\omega)=\frac{(2\arctan(b+2c\,\omega/\sqrt{4ac-b^2}))}{\sqrt{4ac-b^2}}$$

Ugh. Who wants to simplify this into the form ω(t)...? Not me. And what about integrating again to get θ(t)? Pffft. So let's abandon this method and do something else.

Let's start again with the equation $\alpha(t)=a'+b'\omega(t)+c'\omega(t)^2$. If we can directly measure θ(t), then why not numerically differentiate that data and obtain:

$\alpha(t)=d^2\theta(t)/d\,t^2$ and $\omega(t)=d\,\theta(t)/d\,t$

This is much, much easier. For those curious folk, a brief introduction to numerical differentiation can be found here.

So if we measure θ(t) and calculate α(t) and ω(t), then we can use very simple linear regression techniques to find the parameters **a'**, **b'**, and **c'**. But this only gets us part of the way. What we really want are the parameters **I**, **a**, **b**, and **c** where:

$a'=a/I$, $b'=b/I$, and $c'=c/I$ $\rightarrow$ which gives us: $a=a'I$; $b=b'I$; and $c=c'I$

So what we need to do is take a second set of data, $\theta_2(t)$. This second set is for the same wheel except that we've added a known amount of moment of inertia so that $I_2 = I + x$, where x is known. Such that:

$a_2'=a/(I+x)$, $b_2'=b/(I+x)$, and $c_2'=c/(I+x)$

We then take the data set $\theta_2(t)$, calculate $\omega_2(t)$ and $\alpha_2(t)$, and then use the same linear regression techniques to obtain $\mathbf{a_2'}$, $\mathbf{b_2'}$ and $\mathbf{c_2'}$. Just like we did above for θ(t).

Let's take both sets of coefficients and see if we can find the unknowns.

This leads to: $a'/a_2'=(I+x)/I$ $\rightarrow$ $I(a'/a_2')=I+x$ $\rightarrow$ $I=\frac{x}{(a_2'/a'-1)}$

similarly: $I=\frac{x}{(b_2'/b'-1)}$ and $I=\frac{x}{(c_2'/c'-1)}$

Aha! We now have a way to measure the moment of inertia as well as calculate the various coefficients associated with aerodynamic performance and bearing friction. Awesome.

Of course, there is a more accurate way to obtain (I+x)/I. Let's assume that:

$\alpha(\omega)=f(\omega)=a'+b'\omega(t)+c'\omega(t)^2$ and $\alpha_2(\omega)=f_2(\omega)=a_2'+b_2'\omega(t)+c_2'\omega(t)^2$

If both functions are of the same form as we assume they must be, then:

$$\alpha(\omega)/\alpha_2(\omega)=f(\omega)/f_2(\omega)=(I+x)/I$$

In other words, if we were to plot $\alpha(\omega)$ versus $\alpha_2(\omega)$ then the slope would be equal (I+x)/I. This would give the advantage of using a large set of data which would minimize the amount of error. The result is:

$$I=\frac{x}{(slope-1)}\,; \quad a=a'I\,; \quad b=b'I\,; \text{ and } \quad c=c'I$$

Victory! Now let's figure out a good way to measure $\theta(t)$.

Addendum: Someone is going to notice that this does not measure translational aerodynamics. True. True. But I can't do everything. Maybe that'll be my next project. Stay tuned.

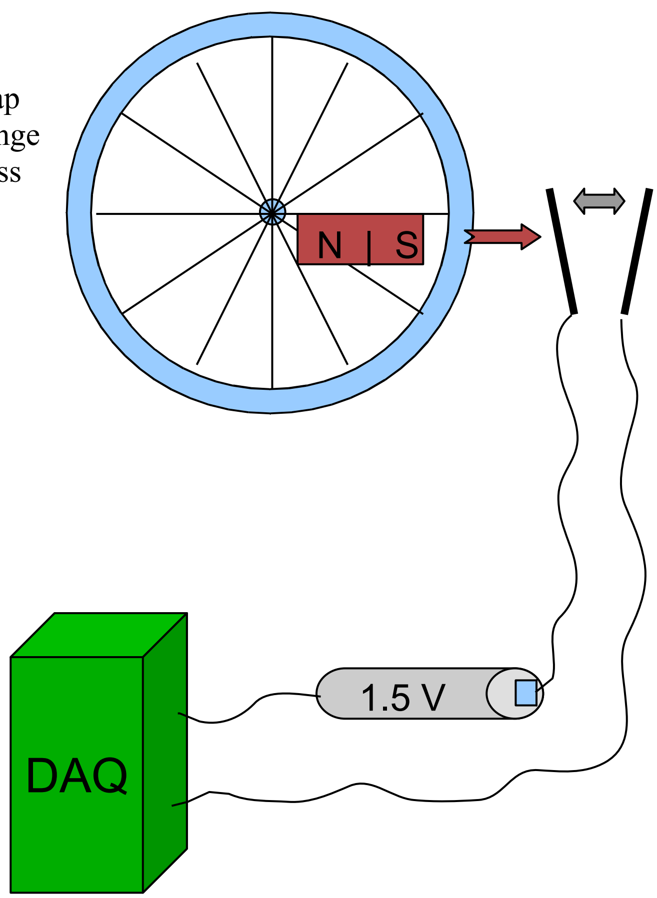
Experimental Setup:

I really like the simplicity of the way the a cycling computer measures θ. Every time the spoke magnet comes around, the contacts of a reed switch are pulled together to close the circuit. So the first thing I did was take an old cycling computer and cut off the reed switch, leaving the long leads attached.

The second thing I needed was some way to record the data using a computer. At first I thought I could use the printer port and hack together some kind of program. Being lazy, I got bored of that approach and bought a cheap DAQ instead. The one I chose is the DATAQ Instruments, model DI-194 RS. $24.95 + shipping. Awesome. Properly configured, it will record a single channel at 240 Hz, or roughly 4 ms between samples. I figure that if at top speed the wheel is rotating once every 250 ms, then a sampling error of +/- 2 ms gives a maximum error in velocity measurement of <1%. Given that hundreds of samples will be taken, this error rate is more than acceptable.

In a nutshell, I put a battery in series with the reed switch. Every time the magnet goes by the switch closes and the DAQ reads a voltage.

At first I thought that the DAQ might have a problem picking up the peak voltage from the switch closing and opening. It might have been necessary to add a small picofarad capacitor and a large resistor in series with the reed switch to extend the signal across several samples of the DAQ, but this was not the case. The input impedance to the DAQ is 200 Kohm and there was probably a bit of stray capacitance in the setup too.



It is quite easy to manually spin the wheel to ~30 km/hr, though I found it much easier to wrap dental floss around the hub between the hub flange and spokes. Then all I need to do is pull the floss and the wheel will accelerate up to ~35 km/hr. Nice.

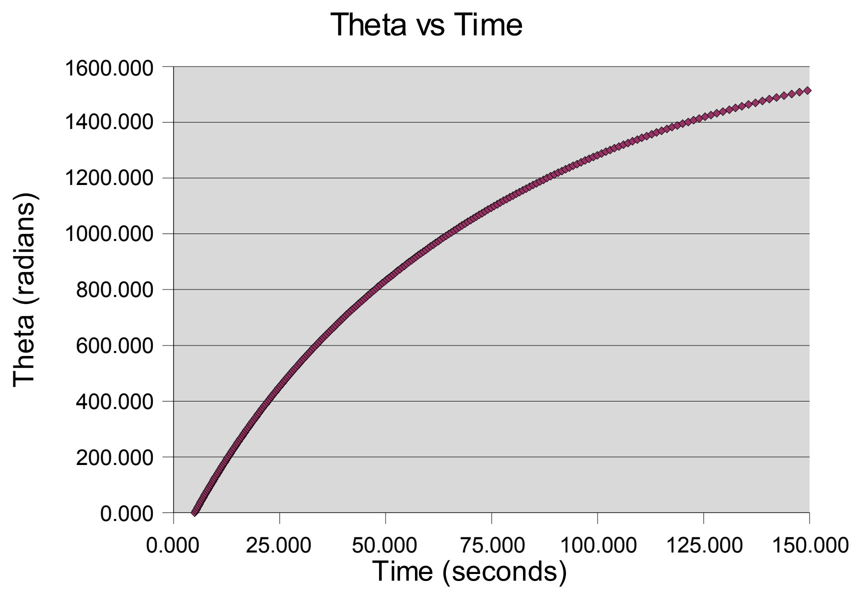
After the wheel slowed down and all the data had been recorded, I used the WinDaq software to directly export the data to a comma limited ASCII file. Something that could be easily read by OpenOffice or Microsoft Office. I use OpenOffice. You should too. You really, really should. Biggest reason? It's free and fully compatible with Microsoft Office.

I went through the data manually and recorded the time of each switch closing. This was determined by the rising edge of the voltage. The first peak voltage point was considered to be the time the switch closed.

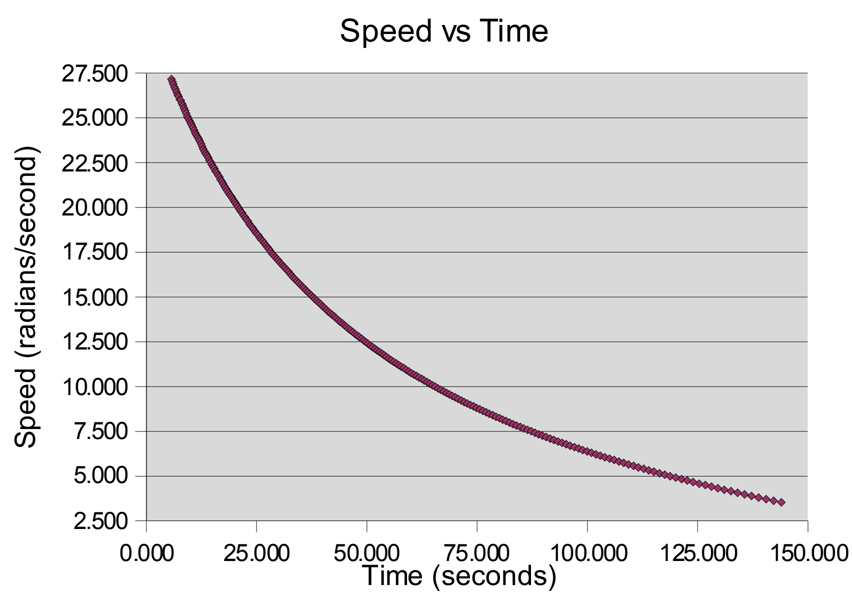
Results:

The following data is for a Kult, model ProMotion front wheel. It has 18 aero spokes radially laced into a light rim. Five sets of data were taken.

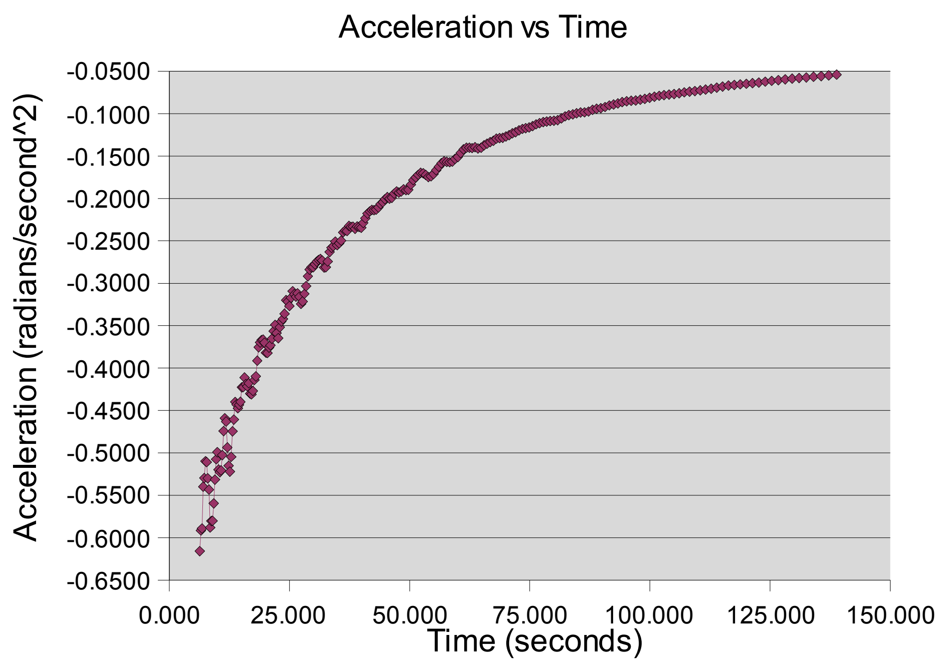
From the first set of data:



From there I used numerical differentiation to obtain the speed. However, instead of using the nearest neighbours technique I calculated the slope of the line passing through seven data points. The central data point and the three on either side. The reason I did it that way is because the time between data points was exceptionally small and therefore the error in using the nearest neighbours method was large. Here is the result from processing the data in the above graph.

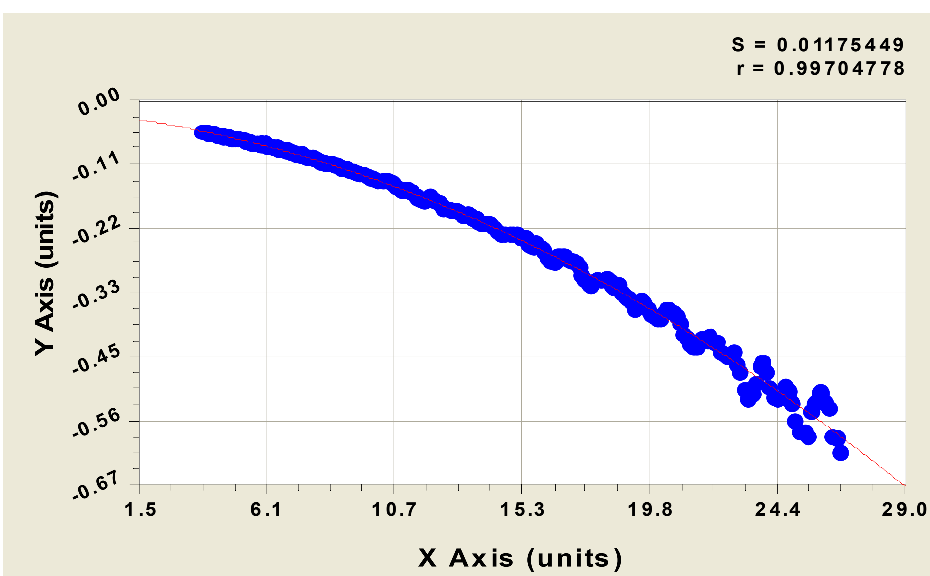


The same method was then used to calculate the acceleration, using the speed data above.



You can see that there is some noise in the data, especially at high accelerations. At first I thought it was strictly random noise. After taking a closer look through all the data sets it appears to be a high frequency oscillation. This could be an artifact of the measurement method and/or the numerical differentiation. Or it could be a real physical effect. My guess is that it's real. This will require all my physics-fu to figure out. Rrr. RRRrrrr. Nope. Nothing. Instead I will invoke the unwanted interference of aliens. Aliens caused weird high frequency stuff to appear in my data. You have a better explanation?

Next, I used the free curve fitting program [CurveExpert version 1.3] to plot $\alpha(t)$ vs $\omega(t)$.



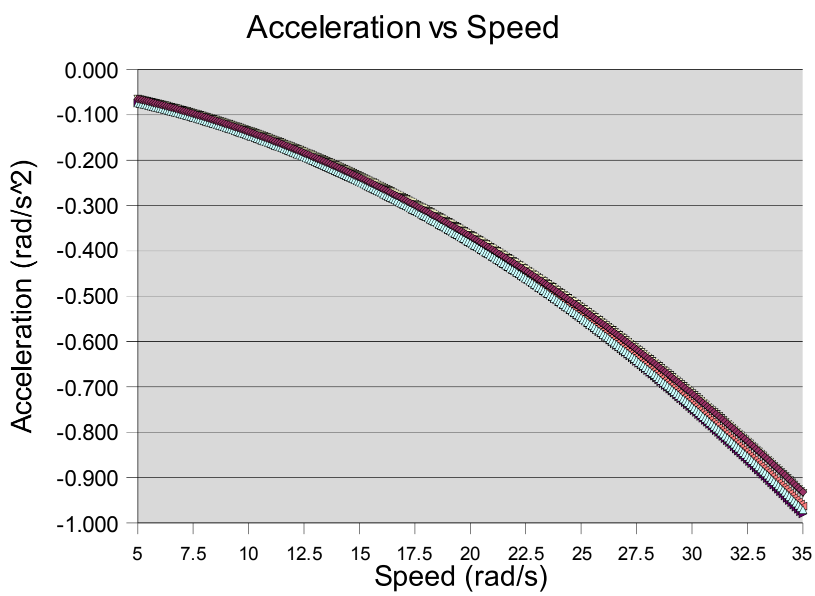
Since calculating the coefficients of regression is tedious, I used CurveExpert to do it for me. This is the result.

Quadratic Fit: y=a+bx+cx^2
Coefficient Data:
a = -0.021706314
b = -0.0057418747
c = -0.00057868065

Could this get any simpler? I love it. This gives us what we need to calculate $\alpha(\omega)$. Doing the exact same thing for all five data sets we get:

Coefficient Data:	1	2	3	4	5	Avg	Std Dev
a' =	-0.0217	-0.0250	-0.0327	-0.0368	-0.0221	-0.0277	0.0068
b' =	-0.00574	-0.00509	-0.00499	-0.00418	-0.00589	-0.00518	0.00068
c '=	-0.000579	-0.000597	-0.000623	-0.000651	-0.000600	-0.000610	0.000028

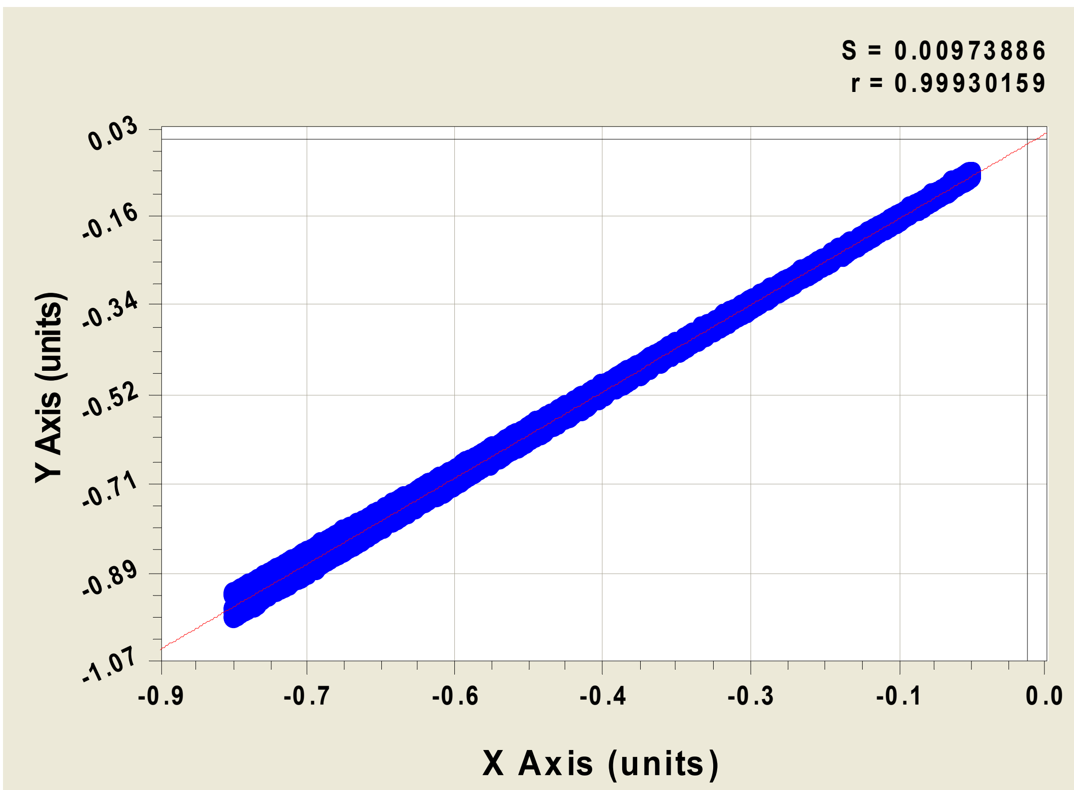
The plot shows all five data sets.



Next, I added four Schwalbe plastic rim strips to the wheel and took five more sets of data. The added mass was 84.0 ± 0.5 grams (as weighed on a digital scale, Salter model 2001, uncalibrated) placed at 0.312 ± 0.0005 meters. This works out to an added MoI of x = 0.00807 ± 0.00005 kgm^2. Five more sets of data were taken. After crunching the numbers as above, we get these results.

Coefficient Data:	Average	Std Dev
a_2' =	-0.0246	0.0026
b_2' =	-0.00370	0.00039
c_2' =	-0.000522	0.000012

Here comes the exciting part. Plotting $\alpha(\omega)$ versus $\alpha_2(\omega)$ for all five data sets we get:



The slope of this, as calculated by CurveExpert is:

Linear Fit: y=a+bx
Coefficient Data:
a = -0.004271159
b = 1.2027327

Note that the standard error and coefficient of regression are excellent. Now the moment we've all been waiting for:

$$I=\frac{x}{(slope-1)}\,; \quad a=a'I\,; \quad b=b'I\,; \text{ and } \quad c=c'I$$

where slope = 1.2027, and x = 0.00807

Therefore:

I = 0.0398 kgm^2
a_{avg} = -1.10 x 10-3
b_{avg} = -2.06 x 10-4
c_{avg} = -2.43 x 10-5

Real numbers!

Discussion:

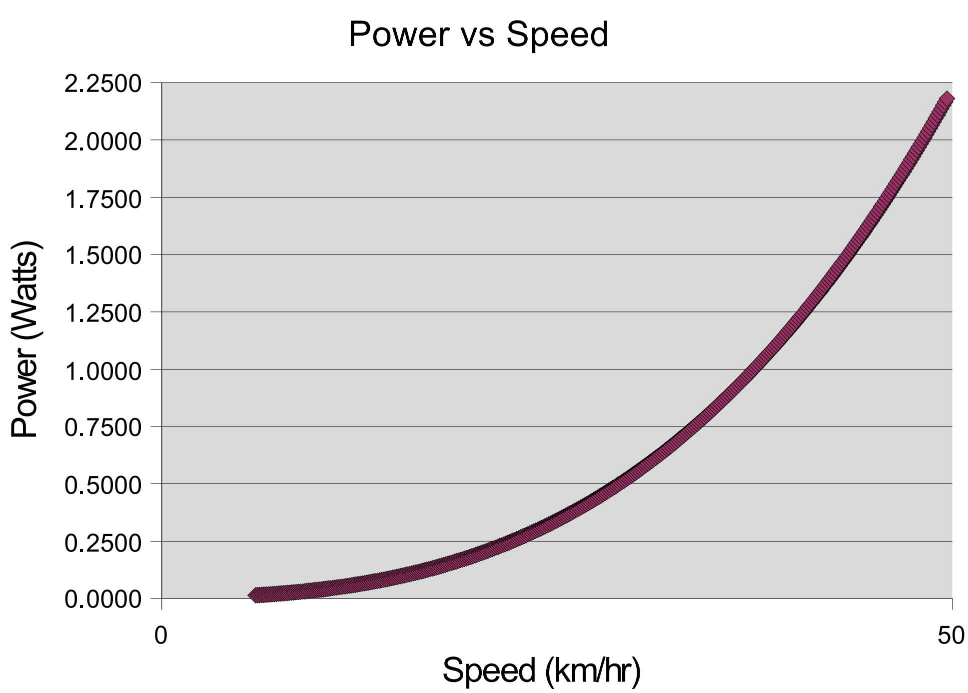
There are several limitations to the approach laid out in this paper. It does not account for bearing load due to rider weight. It does not compensate for the effects of wind. Additionally, I have glossed over any rigorous calculation of accumulated errors. I believe the total error to be small, but have not put a number to it. Perhaps I will investigate this at some later time. Despite these limitations, I do believe that the results adequately predict real world performance and give a credible basis of comparison between wheels.

As a bonus, I saved something until the end. Did you know that the amount of power you expend is equal to the force you exert times the speed you're travelling? Stated another way: Power = $\tau \times \omega$. Well, we just finished measuring torque:

$$\tau = I\alpha = a + b\omega + c\omega^2$$

And we know the speed, so $P = \tau\omega = a\omega + b\omega^2 + c\omega^3$

Which means we can calculate how much power it will take to keep the wheel at a constant speed. I'll graph this for the Kult ProMotion front wheel we just analyzed.



As another bonus, we can also use the MoI to calculate the energy expended is spinning up a wheel. The rotational kinetic energy of a wheel is equal to:

$KE_{wheel} = \frac{1}{2} I \omega^2$ This is how much energy you will have to put out to accelerate your wheel, say after coming out of a corner in a crit. Using the Kult wheel as an example, it would take KE_{wheel} = 15.5 J to go from 30 km/hr to 45 km/hr. If it takes you 5 seconds to make that acceleration, then you will have to put out about 3 Watts of power.

From our single measurment of θ(t), we now have six different parameters that we can use as a basis of comparison. They are:

a, **b**: These parameters are a measure of the bearing performance. As these numbers increase, so will the drag due to bearing friction. Conceivably, these numbers could be lowered for any wheel by using high quality bearings and ensuring that they are well aligned and adjusted.

c: This component is solely due to aerodynamic drag and is a product of the wheel design. Since the power lost due to aerodynamic drag increases with ω^3, it's important to get this right.

I: The moment of inertia is really only important during accelerations. This parameter has absolutely no effect when travelling at a constant speed, even up hill.

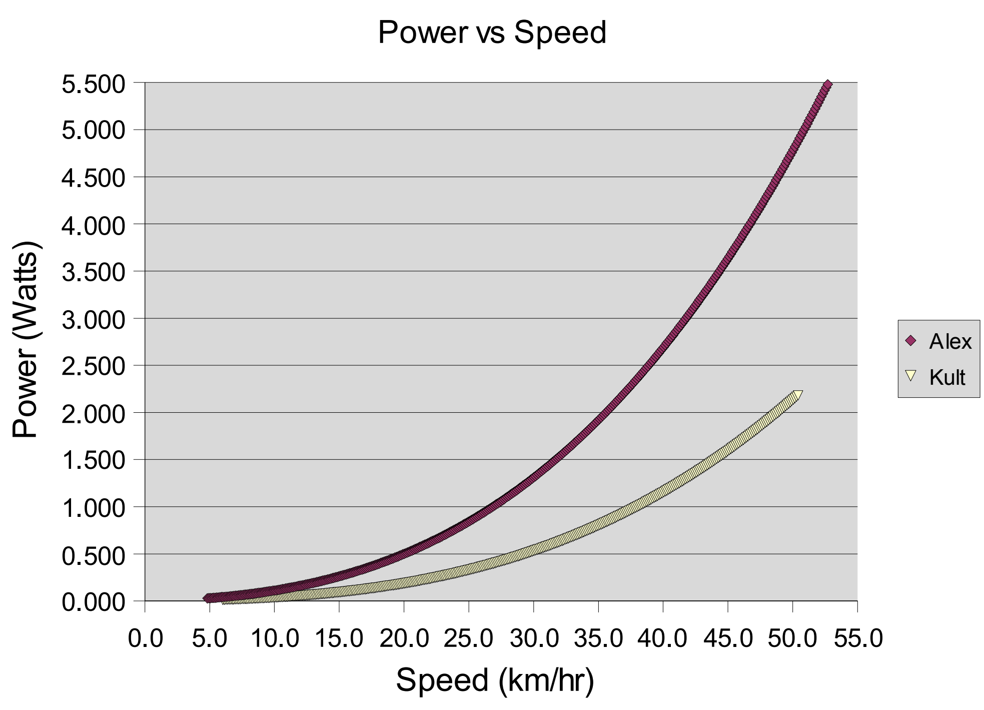
P: The power required to keep a wheel moving at a constant speed is equal to the losses due to parameters a, b, and c. This is a very important performance characteristic.

KE: The kinetic energy required to spin up your wheel. This is what you're really interested in when you consider moment of inertia. This is also an important performance characteristic.

As a point of comparison, I quickly measured a second wheel from my rain bike. It is a 32 round-spoke front wheel with an old 105 SC hub and an Alex DA22 rim. Also, the bearings needed repacking 3000 km and seven rain storms ago. The results show just how significant the differences can be:

	a	b	c	I	Power at 50 km/hr
Kult	1.10×10^{-3}	2.06×10^{-4}	2.43×10^{-5}	0.040	2.2 W
Alex	2.46×10^{-3}	9.64×10^{-4}	4.23×10^{-5}	0.060	5.5 W
% Difference	224	468	174	150	250

The Alex wheel is an absolute loser by every measure. The measurements of **a**, **b** confirm that the bearings are toast. The aerodynamics, **c**, suck. The weight and hence, MoI is quite awful. And after putting it all together, you get a wheel that sucks up two and a half times more power to keep it spinning at 50 km/hr.



This also serves as a handy confirmation that the techniques described in this paper can discern real differences between wheels. Comparison between wheels is possible and gives an accurate reflection of real world performance.

Now for a bit of perspective. The total mass of a light rider (60 kg) plus bike (8kg) plus associated gear like helmets and water bottles (2 kg) is roughly 70 kg. Of that mass, the wheels (including tires and tubes) account for roughly 2 kg of that number. The total kinetic energy of a cyclist is therefore:

$$KE_{total} = KE_{wheels} + KE_{bike} = \left(\frac{mv^2}{2} + \frac{Iv^2}{2r^2}\right)_{wheels} + \left(\frac{mv^2}{2}\right)_{bike}$$

Where **v** is the linear velocity, **I** is the moment of inertia and **r** is the radius of the wheels. Now let's say that we're moving at 10 m/s (36 km/hr).

KE_{wheels} = (100 J + 20 J) = 120 J.
KE_{bike} = 3400 J.

So you can see that the wheels account for about ~3% of the total kinetic energy of a cyclist. The rest, 97%, is dedicated to overcoming the inertia of you and the rest of your gear. Therefore, a large change in your wheels is going to have a relatively small effect. However, a moderate change in body mass is going to have a huge effect. That said, small changes can add up to big ones.

My point is that changing your wheels will not dramatically change how fast you go. It will make a small, but significant change. You should also beware anyone who tells you that brand X wheels give you a 3 km/hr advantage. They probably don't. As shown above, a 10% change in wheels gives less than 1% change in total mass and kinetic energy. I doubt anyone is so sensitive that they can tell when they are putting out 404 W versus 400 W while accelerating out of a corner. Well, I know they can't because PowerTap is still in business. Still, a 1% performance gain can be justified by someone who is racing competitively. Just make sure you put it in perspective.

I have one other public service announcement. I just showed that the moment of inertia of a front wheel is about 0.04 kgm^2. Now let's consider a tire of average radius 0.335 meters. From Weight Weenies the average weight of a Michelin Megamium 2 is 265 grams. The average weight of a Veloflex Corsa is 160 grams. The difference between stock tubes (107 grams) and light ones (60 grams) is about 50 grams.

So... the difference between "heavy" tires and tubes and "light" tires and tubes is about 150 grams. This works out to a difference in moment of inertia of 0.017 kgm^2. Compare that to the wheel we just analyzed. Could you imagine the sales coup if someone could claim their wheels had 40% less MoI than their competitors?! Riders would line up to give testimonials about how much faster they accelerate and how much energy they save in a long race. That doesn't happen for tires though. Makes you think, doesn't it?

This just goes to show that nice tires and tubes are a great and inexpensive way to get a significant performance boost. The benefits are even greater if you decide to switch to tubulars since you'll save another 50-100 grams. Also, higher quality tires generally have a better ride quality and lower rolling resistance. There really is no downside and no reason to ride crappy tires.

Conclusion:

I don't get paid for doing this, but I should. Send money.