Technical Note: Inequality measures and their decompositions

Inequality measures have been introduced in the main text. This note provides mathematical expressions for the three main measures: the Gini, Theil, and Atkinson indices. Each index can be generalized in order to put more weight on selected parts of the distribution of income or consumption. As is the case for poverty measures, some inequality measures can be decomposed, and this note presents decomposition formulas for the GE class which includes the Theil index.

Inequality measures

The standard **Gini index** measures twice the surface between the Lorenz curve, which maps the cumulative income share on the vertical axis against the distribution of the population on the vertical axis, and the line of equal distribution. A large number of mathematical expressions have been proposed for the Gini index, but the easiest to manipulate is based on the covariance between the income Y of an individual or household and the F rank that the individual or household occupies in the distribution of income (this rank takes a value between zero for the poorest and one for the richest). Denoting by $\bar{y}$ the mean income, the standard Gini index is defined as:

$$Gini = 2\ cov\ (Y, F) / \bar{y}$$

The Gini has attractive theoretical and statistical properties which other inequality measures do not have, which explains why it is used by most researchers. The extended Gini uses a parameter ν to emphasize various parts of the distribution. The higher the weight, the more emphasis is placed on the bottom part of the distribution (ν=2 for the standard Gini index):

$$Gini(\nu) = \frac{-\nu \operatorname{cov}(y, [1-F]^{\nu-1})}{\bar{y}}$$

Another family of inequality measures is the **General Entropy measure**, defined as:

$$GE(\alpha) = \frac{1}{\alpha^2 - \alpha}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\bar{y}}\right)^{\alpha} - 1\right]$$

With $GE(0) = \frac{1}{n}\sum_{i=1}^{n}\log\frac{\bar{y}}{y_i}$, $GE(1) = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\bar{y}}\log\frac{y_i}{\bar{y}}$ and $GE(2) = \frac{1}{2n\bar{y}^2}\sum_{i=1}^{n}(y_i - \bar{y})^2$

Measures from the GE class are sensitive to changes at the lower end of the distribution for α close to zero, equally sensitive to changes across the distribution for α equal to one (which is the Theil index), and sensitive to changes at the higher end of the distribution for higher values.

A third class of inequality measures was proposed by Atkinson. This class also has a weighting parameter ε (which measures aversion to inequality) and some of its theoretical properties are similar to those of the extended Gini index. The Atkinson class is defined as follows:

$$A_{\varepsilon} = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\bar{y}}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$$

Decomposition of inequality measures: Illustrations for the GE class

Inequality is often decomposed by population groups to assess the contribution to total inequality of inequality within and between groups, for instance within and between individuals in urban and rural areas. Inequality measures can also be decomposed according to consumption or income sources in order to identify which component contributes most to overall inequality. Finally, decompositions can be used to analyze changes in income inequality over time. Below, decompositions are provided for the GE class.

Decompositions at one point in time

Total inequality I can be decomposed into a component of inequality between the population groups I_b and the remaining within-group inequality I_w. The decomposition by population subgroups of the GE class is defined as:

$$I = I_w + I_b = \sum_{j=1}^{k} v_j^{\alpha} f_j^{1-\alpha} GE(\alpha)_j + \frac{1}{\alpha^2 - \alpha}\left[\sum_{j=1}^{k} f_j \left(\frac{y_n}{y}\right)^{\alpha} - 1\right]$$

where f_j is the population share of group j ($j=1,2,..k$); v_j is the income share of group j ; and y_j is the average income in groups j.

Inequality measures can also be decomposed by source of consumption or income. The decomposition for the GE measure with α=2 is as follows:

$$I = \sum_f S_f = \sum_f \rho_f \frac{\mu_f}{\mu} \sqrt{GE(2).GE(2)_f}$$

where S_f is the contribution of income source f; ρ_f is the correlation between component f and total income; and μ_f / μ is the share of component f in total income. If Sf is large, then component f is an important source of inequality.

Decompositions for changes in inequality over time

Using sub-group decompositions, changes in inequality can be decomposed into: 1) changes in the numbers of people in various groups or "allocation" effects; 2) changes in the relative incomes of various groups or "income" effects; and 3) changes in inequality within groups or "pure inequality" effects. Because the arithmetic can be complex for some inequality measures, this decomposition is usually applied only to Generalized Entropy index GE(0) as follows:

$$\Delta GE(0) = \sum_{j=1}^{k} \bar{f}_j \Delta GE(0)_j + \sum_{j=1}^{k} \overline{GE(0)_j \Delta f} + \sum_{j=1}^{k} \left[\overline{\lambda_j} - \overline{\log(\lambda_j)}\right]\Delta f_i + \sum_{j=1}^{k} \left(\overline{v_j} - \overline{f_i}\right) \Delta \log(\mu(y))_j$$

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Pure inequality effects   Allocation effects   Income effects

where Δ is the difference operator, λj is the mean income of group j relative to the overall mean (i.e., λj = μ(yj)/μ(y)) and the over-bar represents averages. The first term captures the pure inequality effects, the second and third terms, the allocation effects, and the fourth term, the income effects.

Using source decompositions, changes can be decomposed by income source. This allows to see whether an income source *f* has a large influence on changes in total inequality over time. For the General Entropy index with α=2, defining S_t as above, the decomposition is:

$$\Delta GE(2) = \sum_f \Delta S_f$$