# Differential Geometry, Strasbourg, 1953

*Michèle Audin*

The picture on the following page, taken in 1953, shows a group of mathematicians on the stairs of the historic Wilhelmian Building of the University of Strasbourg. The picture appeared in the local newspaper *les Dernières Nouvelles d'Alsace* to illustrate an article about a differential geometry conference organized by Charles Ehresmann (1905–1979), who was a professor at the University of Strasbourg, and André Lichnerowicz (1915–1998), who was a professor at the Collège de France. The photograph also appeared in the conference proceedings [6], which were published by the CNRS (Centre National de la Recherche Scientifique). The two organizers stand in the center of the front row. André Lichnerowicz is the taller of the two and easily recognizable by the pipe from which he was inseparable (notice the way the stairs are used!).

This photo inspired me to look back on some of the important results achieved by the participants in the years leading up to the conference and to consider the later impact of those results. The photo also led me to muse on the lives and work of three mathematicians who were students in the Strasbourg School of Ehresmann, namely, Jacques Feldbau, Georges Reeb, and Paulette Libermann.

The newspaper article was published in the Sunday May 31st/Monday June 1st edition, at the end of a very busy week (the French president was trying desperately to find a prime minister) and before another very busy week (the coronation of an English queen). We are lucky that there was nevertheless some space left for a photo and an article about a mathematical conference. The article is unsigned. However, it is clear that the journalist must have had some help in writing it, since he or she wrote a few rather pertinent statements on the difference between local and global geometry—the relation between global differential geometry and topology is also one of the highlights in the introduction of the conference proceedings.

Bernard Malgrange remembers that the participants had a good laugh at the declaration of Shiing-Shen Chern (1911–2004) to the journalist: "This conference is one of the main achievements of our time" (and this statement was quoted in the article!). It seems indeed to have been a great conference. The scientific program included André Weil (1906–1998) lecturing on nearby points on smooth manifolds (sketching, he said, ideas of his master Nicolas Bourbaki—despite a conflict between master and pupil, as the latter explains it in [12, p. 534]), Chern on infinite continuous pseudogroups, Jean-Louis Koszul on Lie groups of transformations (the very first result stated in his paper is the essential "slice theorem"), Nicolaas Kuiper (1920–1994) on locally affine surfaces, and Beno Eckmann on examples of compact complex Kähler manifolds. René Thom (1923–2002) spoke on cobordant manifolds, defining the so-called Thom spaces $MO(n)$, $MSO(n)$ and stating his results on the structure of the oriented and non-oriented cobordism rings $\Omega^*$ and $\mathcal{N}^*$, work that would bring him the Fields Medal in 1958. Laurent Schwartz (1915–2002), by then already a Fields Medalist, discussed the current associated with a meromorphic form on an analytic manifold. The two organizers also gave talks. Ehresmann explained the jet spaces he was inventing at that time, and two students of his, Paulette Libermann and Georges Reeb, spoke as well.

This conference was not an accident that came out of nothing. There was tremendous activity around Ehresmann in Strasbourg, including an ongoing seminar called "colloque de topologie de Strasbourg". There remain written reports of the seminar talks, given from 1951 to 1955, by speakers like Karl Stein,

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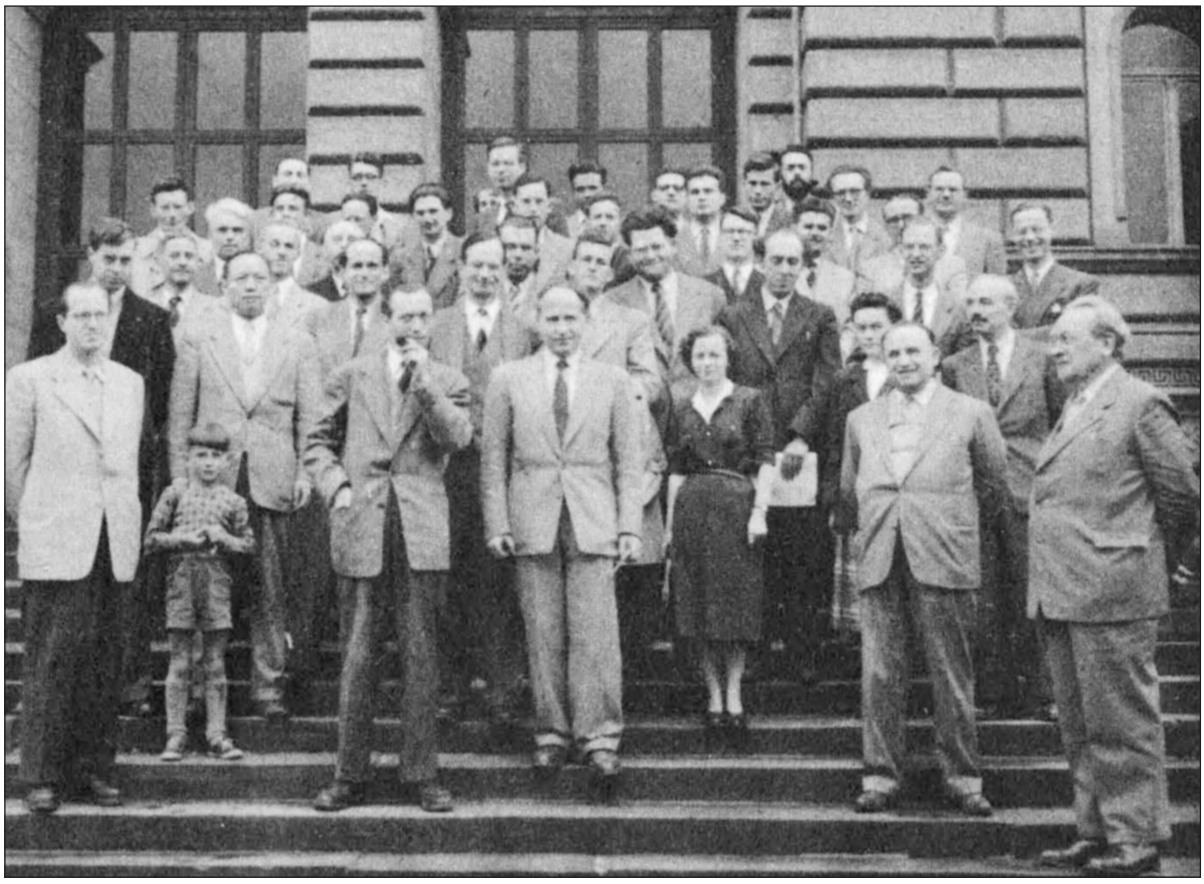


Photograph: Dernières Nouvelles d'Alsace.

**Differential geometry conference, University of Strasbourg, 1953. See photo key, page 369.**

Shoshichi Kobayashi, Kazumi Nomizu, Robert Hermann, Thom, Reinhold Remmert, André Haefliger, Reeb, Georges de Rham, Lawrence Markus, Libermann, and of course Ehresmann himself.

Charles Ehresmann had been a student of Élie Cartan (1869–1951), who would have been seen by the conference attendees as a kind of grandfather of the field. As our journalist wrote, a huge number of new problems were investigated or sketched at the conference, and many of them originated in the work of Élie Cartan. Ehresmann, who defended his thesis in 1934, was also a member of the Bourbaki group. He was very active and was one of the lecturers in the so-called "Julia seminar" in 1935–37, the seminar that gave birth to the Bourbaki seminar after the war. From 1939, Ehresmann taught at the University of Strasbourg, where he began to lead an outstanding school of geometry and topology. He is estimated to have directed more than seventy theses, including those of Haefliger and of Wu Wen Tsun. Let us now focus more closely on three of Ehresmann's students in the Strasbourg school, Jacques Feldbau, Georges Reeb, and Paulette Libermann.

## The One Who Is Missing

The very first student of Ehresmann was Jacques Feldbau. Born in 1914 in Strasbourg, Feldbau was one of the founders of the theory of fiber bundles. He is the one who first proved that a bundle over a simplex is trivializable and who used this to classify bundles over spheres. This work appeared in a *Comptes-Rendus* note [5] in 1939. Two years later, together with Ehresmann, he wrote two more *Compte-Rendus* notes, in which the two authors introduced the notion of an associated bundle and proved results known today as the exact homotopy sequence of a fibration.

Two years later means 1941. And in the meantime, there was the war (in which Feldbau served in the French air force), the evacuation of Strasbourg University to Clermont-Ferrand, the occupation of France by Nazi Germany, the *Anschluss* of Alsace (which had long been claimed as a part of Germany) by the Third Reich, and the installation of the Pétain government, which very soon (October 1940) promulgated a set of antisemitic laws…So Feldbau (who, as a Jew, was forbidden by the new laws to teach), Ehresmann, and most of the other professors and students of Strasbourg met again, at the end of 1940, in Clermont-Ferrand, in the center of France. And the first of the two joint notes by Ehresmann and Feldbau [4] appeared in June 1941. The second one [3] was presented by Élie Cartan on October 27th but, at that time, it was already impossible for the Académie des

Photograph courtesy of the Feldbau-Debré Family.

**Jacques Feldbau.**

Sciences to publish the name of a Jew…So the note appeared under the name of Ehresmann alone. It must have been very painful for Élie Cartan to remove Feldbau's name from the paper before publication on December 1st and to reduce the credit given to Feldbau to a mention that the results were obtained by the author "in collaboration with one of his students". See [13, 1].

Feldbau could publish two short notes in the *Bulletin de la Société mathématique de France* (which was edited by Henri Cartan) under the pseudonym of Jacques Laboureur (*Feldbau* in German means agriculture, and *Laboureur* is French for plowman). Feldbau was also one of the inventors of what is called the Whitehead product between homotopy groups (because it was invented and published by Whitehead at the same time). However, he could not publish his results because he was caught in a Gestapo roundup in June 1943 and sent to Auschwitz in October of that year. He survived Auschwitz and even the deadly evacuation of the camp in January 1945 but eventually died of exhaustion the following April, two weeks before the end of the war, in the Bavarian concentration camp of Ganacker. See [2].

This is why Jacques Feldbau—a handsome young man with a very friendly and likeable personality, who practiced all kinds of sports (he was a champion of the butterfly-stroke) and played the piano, and who would have been a great topologist—does not show up in the picture.

## The One from Saverne

The second student of Ehresmann, also an Alsatian studying in Clermont-Ferrand, was Georges Reeb. Born in Saverne (40 kilometers west of Strasbourg) in 1920, he is one of the inventors of the theory of foliations. He invented what is now called the "Reeb foliation", a foliation of the 3-sphere, all the leaves of which are diffeomorphic to $\mathbf{R}^2$, except one, which is a (compact!) 2-torus. Another useful tool in topology was named after him, the "Reeb vector field" associated with a contact form. And Reeb's theorem is the one that tells you that, if a compact manifold has a function with only two critical points, this manifold is homeomorphic to a sphere. This is the way you prove that the Milnor spheres, although not diffeomorphic, are homeomorphic to the sphere $S^7$, a result that came in 1956 [10]—by the way, did you notice the young man above everybody in the back row? Yes, John Milnor, who would be a Fields Medalist in 1962,

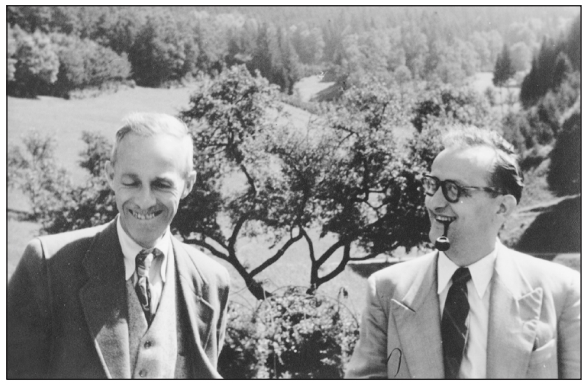


Photo courtesy of the Mathematisches Forschungsinstitut Oberwolfach.

**Wilhelm Süss (left) and Georges Reeb in Oberwolfach, 1953.**

was participating in the conference too. Reeb's talk at the conference was about Finsler (and Cartan) spaces. This paper contains an intrinsic definition of the Liouville form.

Georges Reeb fit all the prejudices French people might have against Alsatians: he was massive and slow, very slow. Slowness is not a quality French mathematicians usually praise very much. But Reeb did not care. He was even proud of it. He used to say that, when he was a professor at Grenoble University (namely, at the time of our picture), the students attending his course suggested recording his lectures on tape, so that they could listen to them at double speed. When I arrived in Strasbourg in 1987, he used to come to the department and go from one office to another talking with people, which had always been his way of working. Since he reckoned I was always in a hurry, he used to tell me the "story of the hurrying Savernois" (both a paradox and a never-ending story). He also told many jokes about his complete baldness—a state he had not reached by 1953, which is why it took me some time to recognize him in the picture. After a number of achievements in geometry, topology, differential equations, and nonstandard analysis, Georges Reeb died in 1993.

## The Woman

It was much easier to find Paulette Libermann in the picture, the shorter of the two women in the photo. It was from her that I heard the names of Ehresmann and Feldbau for the first time. This was in the spring of 1975. At that time I was a student in Paris, and I had to choose a few courses from a long list. I remember I took *cohomologie de de Rham des variétés différentielles*, taught by Karoubi, and I found it very fancy, because I did not understand a single word in the title (by the way, Georges de Rham (1903–1990) is the man between Lichnerowicz and Ehresmann in the picture). I chose also *géométrie différentielle*, given by Paulette Libermann, because it seemed to fit. The very first thing I learned in her lectures was the definition of a bundle "in the sense of Ehresmann", which seemed to consist of

an intricate list of symbols (a total space, a base space, a map from one to the other, a fiber space, a group…) and Feldbau's theorem.

Paulette Libermann ("Mademoiselle Libermann", as people used to call her) was very helpful to young mathematicians. I think the very first seminar in Paris in which I was invited to give a talk was hers. Quite a few geometers can probably say the same thing. This might have been because she remembered having been helped herself when she was a student.

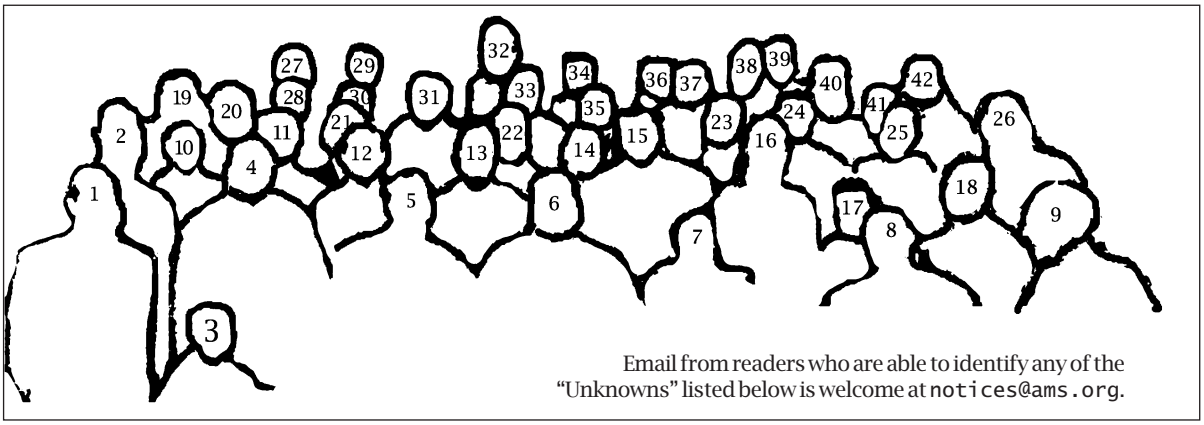
Paulette Libermann also had a difficult story. Born in 1919, she entered the so-called "École normale supérieure de jeunes filles" in 1938. Like the other young ladies of this school, she was supposed to pass a few exams and, after two years, to prepare the concourse of the (feminine) *agrégation* to become a teacher in a secondary school (for girls, of course). Except that two years later meant the fall of 1940, so that she was supposed to begin to prepare for the concourse at the precise time the French legislation I mentioned earlier forbade Jews from taking certain kinds of jobs, among them teaching. So she was not allowed to pass the *agrégation*. With a kind of black humor, Paulette Libermann used to say that the antisemitic laws had been lucky for her, since Élie Cartan, who was teaching these young ladies, suggested that she start research instead.

But life was not that easy. Paulette Libermann and her family had to move to Lyon in 1942, where they lived a half-clandestine life under a fake name. She gave private lessons for a living. But she was luckier than Jacques Feldbau and, at the *Libération* in 1944, she eventually passed the *agrégation*, became a teacher in a secondary school…and, once again at Élie Cartan's suggestion, started her thesis on equivalence problems under the supervision of Ehresmann in Strasbourg. She then got a position as a professor at the University of Rennes, and then at Paris 7.

Photograph courtesy of Corinne Mounier-Veil.

**Paulette Libermann.**

In her thesis, which she defended in 1953 (the year of the picture), can be found a number of things which lie now at the bases of symplectic geometry—which was not as fashionable then as it is today. For instance, she investigated foliations of symplectic manifolds and the local structure of the manifold together with certain foliations (a generalization of Darboux's theorem); she defined what Alan Weinstein would twenty years later call the affine structure of a Lagrangian foliation (this is at the basis of the theory of integrable systems); she investigated the almost-complex structures on a symplectic manifold that would turn out to be very useful, thirty years later, when Mikhael Gromov introduced and made such beautiful use




Email from readers who are able to identify any of the "Unknowns" listed below is welcome at notices@ams.org.

Key to photo on page 367.
1. Unknown
2. R. Debever
3. Ehresmann's son
4. Shiing-Shen Chern
5. André Lichnerowicz
6. Charles Ehresmann
7. Paulette Libermann
8. Mario Villa
9. Lucien Godeaux
10. Heinz Hopf
11. Wilhelm Süss
12. Laurent Schwartz
13. Georges de Rham
14. Unknown
15. H. Guggenheimer
16. Thomas Willmore
17. Simone Lemoine
18. B. H. Neumann
19. René Thiry
20. E. T. Davies
21. Unknown
22. Unknown
23. Unknown
24. Unknown
25. Nicolaas Kuiper
26. Beno Eckmann
27. Unknown
28. Jean-Louis Koszul
29. Unknown
30. André Weil
31. René Thom
32. John Milnor
33. Marcel Berger
34. Unknown
35. Bernard Malgrange
36. Daniel Bernard
37. André Aragnol
38. G. Legrand
39. Jean-Marie Souriau
40. Unknown
41. Georges Reeb
42. Unknown

of his pseudo-holomorphic curves, creating symplectic topology. Her name is attached to one of the very first textbooks on symplectic geometry (which she wrote jointly with Charles-Michel Marle) [9].

Like Georges Reeb, Paulette Libermann used to tell stories. Unlike him, she was very quick. She was also very lively. The short and tiny young lady of the picture became a short and tiny old (but ever young) lady, and she still was very active, very energetic, traveling all over the world to participate in conferences. The last time I met her, in April 2007, just before I left for a conference in Vietnam, she was sorry not to be able to participate in that conference. This is too far for me, she told me, too tiring, I am getting old. She was eighty-seven. She died in July 2007—and one of the motivations I had in writing this article was to make a tribute to her memory.

## Some Others

Two other women participated in the conference, Marie-Hélène Schwartz, who would create a version of Chern classes for singular analytic manifolds (she does not seem to be in the picture), and Simone Lemoine, the one standing near Paulette Libermann, a differential geometer (who would become Simone Dolbeault-Lemoine). Most of the people in the picture were very hard to recognize. As usual, the list of participants does not fit. For instance, Georges Cerf (1888–1979), at that time the director of the mathematics institute in Strasbourg, was there but does not show up in the picture. On the other hand, Bernhard Neumann (1909–2002) is not in the list, but was definitely recognized as the man with the moustache, on the right, by two of his sons.

Heinz Hopf (1894–1971) is the one on the right of Chern, just above. He told the journalist that he was very sorry not to have been able to attend all the talks, because those he listened to were indeed excellent. Bernard Malgrange, who helped me to identify some of the people in the picture, is the second one above Ehresmann, in the same row as Thom (the man between them is Marcel Berger). The man in dark between the two ladies is the English topologist Thomas Willmore (1919–2005).

The man standing in front of Koszul, between Chern and Schwartz, is Wilhelm Süss (1895–1958), the founder of the mathematical institute at Oberwolfach in the Black Forest (hence a neighbor to those in Strasbourg), whose personality was much less controversial in 1953 than it is today (see [7, 11]).

**Acknowledgments:** I thank Daniel Bernard, Jean Cerf, Jean-Pierre Jouanolou, Bernard Malgrange, Charles-Michel Marle, John Milnor, and Walter Neumann for their help in identifying individuals in the picture.

## References

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