Abraham H. Taub
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to accompany the Golden Oldie reprint of Taub's most famous article
[Empty Spacetimes Admitting a Three-Parameter Group of Motions,
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Abraham Haskel Taub was born in Chicago, Illinois, on February 1, 1911. He was educated in Chicago and received his undergraduate degree in mathematics at the University of Chicago in 1931. He then went to Princeton University for his graduate education. At Princeton, his intellectual development was deeply affected by his association with H. P. Robertson, O. Veblen, and J. von Neumann. These outstanding scientists became his mentors and profoundly influenced the direction of his scientific career.

He worked with Robertson on his doctoral thesis in mathematical cosmology ("Dirac Equations in Cosmological Spaces") and received his Ph.D. degree in mathematics in 1935. He then moved to the newly established Institute for Advanced Study in Princeton and worked on differential geometry with Veblen for a year as a postdoctoral fellow. Taub's first published paper was with Veblen in 1934 on the projective differentiation of spinors. Projective relativity theory had been developed by Veblen, Pauli, and others and was being intensively investigated at the time [1]. The projective theories are essentially equivalent to the five-dimensional Kaluza-Klein theories [2]. Six months later, the work of Veblen and Taub was followed by a paper of Taub, Veblen, and von Neumann on the Dirac equation in projective relativity.

In 1933, Robertson had summarized in the *Reviews of Modern Physics* his work on relativistic cosmology using a novel approach based on group theory. Robertson's work and independent studies by A.G. Walker have led to the spatially homogeneous and isotropic Robertson-Walker metric of standard cosmology. Futhermore, the general relativistic formulation of Dirac's equation had been developed by Weyl, Fock, Schrödinger, and Pauli among others. In lectures delivered at Princeton University in 1934, Schrödinger had discussed the Dirac equation in the Milne universe, while in 1935 Dirac had published an equation for the electron in de Sitter universe based on the embedding of de Sitter space in a flat five-dimensional space. Taub's third paper published in 1937 discussed the Dirac equation in the spatially homogeneous cosmological spaces of Robertson. In this significant work, Taub showed that his results for de Sitter space were different from Dirac's. By this time, Taub had joined the Mathematics Department at the University of Washington in Seattle; there he worked on various mathematical and physical aspects of spinors and published five further papers on this subject until 1940. He spent the academic year 1940–1941 on leave at the Institute for Advanced Study working on general spinor fields. In this area of research, Taub made basic contributions. He took great pride in this work and his interest in spinor fields persisted throughout his academic career. He lectured on this subject at Berkeley and returned fifty years later to active research on this topic after his retirement.

Abe Taub was professor of mathematics in Seattle until 1948; however, during this time the war intervened and he was called back to Princeton University to serve as a theoretical physicist during 1942-1945. He did research work on shock waves in connection with national defense; in fact, he was the theoretician in an experimental group led by Walker Bleakney of Princeton University. They worked on shock tubes, which provided a relatively simple means of studying blast waves. The theory of the shock tube was developed by Taub. The reflection and refraction of shock waves, unlike the familiar linear case of light waves, give rise to intrinsically nonlinear phenomena of Mach reflection and irregular refraction. The oblique reflection of shocks was discussed by John von Neumann in 1943. Taub worked on the interaction of shock waves, the oblique refraction of plane shock waves, and the theory of Mach reflection of a plane shock from a rigid wall, especially the phenomena associated with the Mach stem. In Mach reflection — in contradistinction to regular reflection — the reflected shock meets the incident shock at a triple point (or line) which is some distance from the wall and is joined to it by a third shock wave (usually curved) called the Mach shock or Mach stem [3]. The nonlinear theory of this phenomenon is extremely complicated, but crucial to the understanding of blast waves. Taub therefore continued to work on these problems after the war and returned in 1947 to the Institute for Advanced Study in Princeton as a Guggenheim post-service fellow on leave from the University of Washington.

During this stay in Princeton, Taub also worked on differential geometry and the groups of motions in Riemannian spaces. Among other things, he proved a theorem concerning the characterization of conformally flat spaces. Subsequently, he studied empty spacetimes that admit a three-parameter group of motions. Searching for a consistent formulation of Mach's principle in general relativity, he investigated, for the case of spatially homogeneous

Ricci-flat spacetimes, the general solutions of Killing's equation for each of the nine types of transitive three-parameter continuous groups discussed by Bianchi [4]. The three-dimensional Lie groups that are simply transitive on homogeneous 3-spaces had been classified by Bianchi in 1897. Taub recognized the significance of Bianchi's work for constructing cosmological models. He presented this major work at the International Congress of Mathematics in 1950, and it was subsequently published in the Annals of Mathematics in 1951 [Abraham Taub, "Empty Spacetimes Admitting a Three-Parameter Group of Motions," *Proceedings of the International Congress of Mathematicians (Cambridge, Mass., 1950), p.655*; *Annals of Mathematics*, **53**, pp. 472, 1951; reprinted in *Gen. Relativ. Grav.* **33**, (2001)].

This paper, which is reprinted below, has become a classic. It contains, among other things, an interesting Ricci-flat solution that is known as the Taub universe. It is interesting to note that at about the same time and independently Kurt Gödel constructed the first explicit spatially homogeneous expanding and rotating cosmological models with matter. The discoveries of Gödel and Taub have exerted a profound influence on the subsequent development of general relativity.

In 1946, Abe Taub received the Presidential Certificate of Merit for his defense-related work. The main scientific results of this work were published in a series of papers in 1946-1951. The theory was confronted with experiment in two papers with Bleakney (who originated the Mach-Zehnder interferometric method of studying shock diffraction) and Fletcher in the Reviews of Modern Physics; the excellent agreement of theory with experiment in most cases must have been a source of immense satisfaction for Taub, while the few cases of disagreement — such as the Mach reflection of *weak* shocks — pointed to the vast complexity of nonlinear dynamics. In his investigation of these nonlinear phenomena, Taub recognized the significance of numerical analysis. Inspired by von Neumann, Taub became a pioneer in computational hydrodynamics and computer science.

After the war, Taub made very significant contributions to the relativistic theory of continua. Among his achievements, mention must be made of the first development of Hamilton's principle for a perfect fluid and other variational principles in general relativistic hydrodynamics, the circulation theorem, the relativistic Rankine-Hugoniot equations, and the stability of fluid motions in general relativity. As an applied mathematician, he was the leading authority in relativistic hydrodynamics and his work is indispensable in relativistic astrophysics.

In 1948, Abe Taub joined a project at the University of Illinois (as a research professor of applied mathematics) to build a computer based on von Neumann's plans. Taub's association with computer science had its roots in his research on hydrodynamics and his collaboration with John von Neumann. He admired von Neumann's scientific genius and was the general editor of his collected works published in six volumes in 1961-1963. Taub was the chief mathematician associated with this project. The computer, called ORDVAC, was completed in 1952 and was delivered to the Aberdeen Proving Grounds. Then a second computer, called ILLIAC, was built; it remained at Illinois and was the prototype of several other computers. Taub was the head of the Digital Computer Laboratory at Illinois from 1961 until 1964, when he became the director of the Computer Center of the University of California-Berkeley from 1964 until 1968. He was a professor of mathematics at Berkeley from 1964 until his retirement in 1978. A collection of essays was published in 1980 to honor Abraham Taub on the occasion of his retirement. As professor emeritus of mathematics, he remained very active in research until a few years before his death. He died on August 9, 1999, after a long illness. He was survived by his wife of 66 years, Cecilia Vaslow Taub, and their two daughters and one son.

During his distinguished career, Abe Taub had many research students and postdoctoral associates who have made significant contributions to computer science, applied mathematics, and general relativity theory. The Berkeley relativity seminars, organized by Taub, provided a lively environment for discussions of mathematical relativity and Lorentzian geometry. Taub was a member of a number of important scientific societies and served on various advisory panels for applied mathematics. He edited a book on "Studies in Applied Mathematics" (1971) and coedited with S. Fernbach a book on "Computers and Their Role in the Physical Sciences" (1970).

Taub's theoretical contributions to general relativity cover a wide range of important topics including studies in singularities, scale invariance, and gravitational radiation. Taub's approach to certain singularity problems in general relativity was based on the theory of distributions. In this connection, it is interesting to note that black hole singularities have also been studied using distributional geometry. Moreover, starting from a theorem of Laurent Schwartz that a distribution with a single point as support is a linear combination of Dirac's delta function together with a finite number of its derivatives, Taub developed the theory of the motion of test bodies with multipole moments in general relativity. In particular, he showed that for a "pole-dipole" particle his approach led to the Mathisson-Papapetrou equations with the Pirani supplementary condition.

In the field of general relativity, Abraham Taub is best known at present for pioneering studies of the spatially homogeneous and anisotropic cosmological models and the discovery of the Taub universe, which is a special singularity-free and Ricci-flat model given by equations (7.3) and (7.4) of his famous 1951 paper reprinted below. This two-parameter solution is now usually represented as [1]

$$ds^2 = -U^{-1}d\tau^2 + (2\ell)^2U(d\psi + \cos\theta d\phi)^2 + (\tau^2 + \ell^2)(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $U > 0$ is given by

$$U(\tau) = -1 + 2\frac{m\tau + \ell^2}{\tau^2 + \ell^2}, \quad (2)$$

with constant parameters $m > 0$ and $\ell > 0$. Here $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$ and $0 \leq \psi < 4\pi$. This metric can be extended across the horizon $U = 0$ to a spacetime discovered for unrelated reasons by E.T. Newman, T. Unti, and L. Tamburino ("NUT") in 1963 in a form that properly interpreted reduces to the Schwarzschild metric for $\ell \to 0$. The remarkable properties of the Taub-NUT spacetime were studied by Misner and Taub [5]. The source of the Taub-NUT solution is a gravitational dyon that consists of a gravitoelectric monopole (characterized by the mass parameter m) and a gravitomagnetic monopole (characterized by the Taub-NUT parameter ℓ) [6]. The Taub-NUT solution can be extended to include a cosmological constant[6], or to include an infinite set of multipole moments pertaining to axisymmetric deformations of a rotating source [7]. The Taub-NUT spacetime has had significant applications in theoretical studies of the spacetime structure in general relativity and, more recently, in quantum gravity. Its Euclidean extension is important for the study of monopoles in gauge theories. Embedding the Taub-NUT gravitational instanton into five-dimensional Kaluza-Klein theory leads to a Kaluza-Klein monopole [8]. Euclidean Taub-NUT spacetimes have been discussed by a number of authors in connection with monopoles in supersymmetric gauge theories [9,10]. Further generalizations and extensions of the Taub-NUT spaces are topics of current research.

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$$\tau = m + (m^2 + \ell^2)^{\frac{1}{2}}\tanh(\frac{kt + \beta}{2})$$

is the new temporal coordinate, and $(x^1, x^2, x^3) = (\theta, \phi, \psi)$ be the Euler angles. Then, with $\sinh(\alpha - \beta) = m/\ell$ and $k = 4\ell(m^2 + \ell^2)^{\frac{1}{2}}$, one recovers the usual form for the metric of the Taub universe.

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