Advances in Engineering Software 42 (2011) 671–679

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Topology design of three-dimensional continuum structures using isosurfaces

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ARTICLE INFO

Article history:
Available online 14 June 2011

Keywords:
Topology design
Three-dimensional continuum
Isosurfaces
Evolutionary algorithm
Material redistribution
Fixed grid

ABSTRACT

Isolines Topology Design (ITD) is an iterative algorithm for the topological design of two-dimensional continuum structures using isolines. This paper presents an extension to this algorithm for topology design of three-dimensional continuum structures. The topology and the shape of the design depend on an iterative algorithm, which continually adds and removes material depending on the shape and distribution of the contour isosurfaces for the required structural behaviour. In this study the von Mises stress was investigated. Several examples are presented to show the effectiveness of the algorithm, which produces final designs with very detailed surfaces without the need for interpretation. The results demonstrate how the ITD algorithm can produce realistic designs by using the design criteria contour isosurface.

1. Introduction

In the design of some structures, a two-dimensional (2D) representation may be sufficient to capture the loading and supports and so, a 2D representation for the optimization is sufficient. There are instances, however, when a structure may need to be modelled and optimised in three-dimensions (3D) due to perhaps the loads not being symmetric, the supports not being regularly spaced or due to the complexity of the environment itself. Most examples presented in the literature on topology optimization are of 2D problems, with relatively few papers which have either extended topology optimization methods to 3D or which show 3D results.

The earliest published work which incorporated 3D effects to topology optimization were those of Bendsøe [1] and Diaz and Lipton [2] working with the Homogenisation method. Olhoff et al. [3] used optimum 3D microstructures for topology optimization of linearly elastic 3D continuum structures subject to a single case of static loading. In order to visualise the topology designs, a penalisation technique was applied. Beckers [4] used a dual method with discrete variables for topology optimization of continuous structures in static linear elasticity. The optimization consisted of minimising the compliance by distributing a given volume of material in a domain modelled by a fixed finite element mesh. Céa et al. [5] introduced a topological shape optimization algorithm based on a Fixed-Point method. The topological gradient concept [6] provides a mathematical justification of Céa's powerful method. The aim of the topological gradient is to compute the sensitivity of a cost function when a cavity is made in the domain. Borrvall and Petersson [7] considered large-scale topology optimization of elastic continua in 3D using the regularised intermediate density control. In order to deal with large-sized problems, parallel computing was used in combination with domain decomposition. Allaire et al. [8] proposed a new numerical method based on a combination of the classical shape derivative and of the Level Set Method (LSM) for front propagation. This method was implemented in 2D and 3D, linear and nonlinear elasticity. The cost of the numerical algorithm was moderate since the shape is captured on a fixed Eulerian mesh. Hsu and Hsu [9] presented an automated process for interpreting 3D topology optimization results into a smooth Computer Aided Design (CAD) model, using the Solid Isotropic Microstructure (or Material) with Penalization (SIMP) method of topology optimization. On each cross-section, a density redistribution algorithm transfers the black-and-white optimization result into a smooth density contour represented by B-splines curves. A 3D CAD model is obtained by sweeping through these cross-sections. Koguchi and Kikuchi [10] developed a surface reconstruction algorithm, which consisted of three parts: (1) an enclosed isosurface geometry from which the topologically optimised model was generated; (2) features detected; and (3) the parametric CAD solid model reconstructed as bi-quartic surfaces splines.

This paper is an extension of the Isolines Topology Design (ITD) algorithm (Victoria et al. [11]). The novelty of this work is in introducing into ITD the capability of designing 3D structures. The method of determining the isosurfaces is given, together with several examples to show the effectiveness of the algorithm. The results show the usefulness of ITD to provide quality solutions with very detailed contours, without the need to interpret the topology.

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doi:10.1016/j.advengsoft.2011.05.003

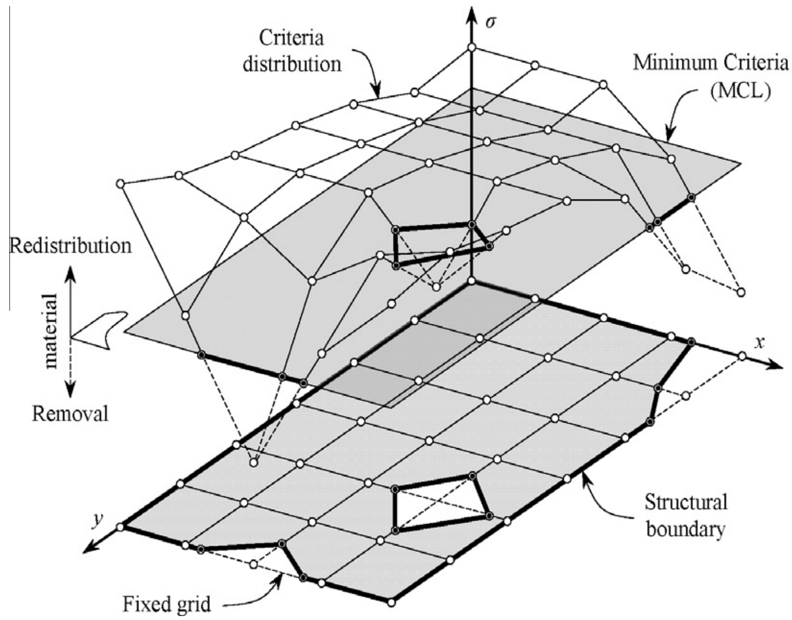

Fig. 1. Structural boundary is defined by the intersection of the MCL with the criteria distribution.

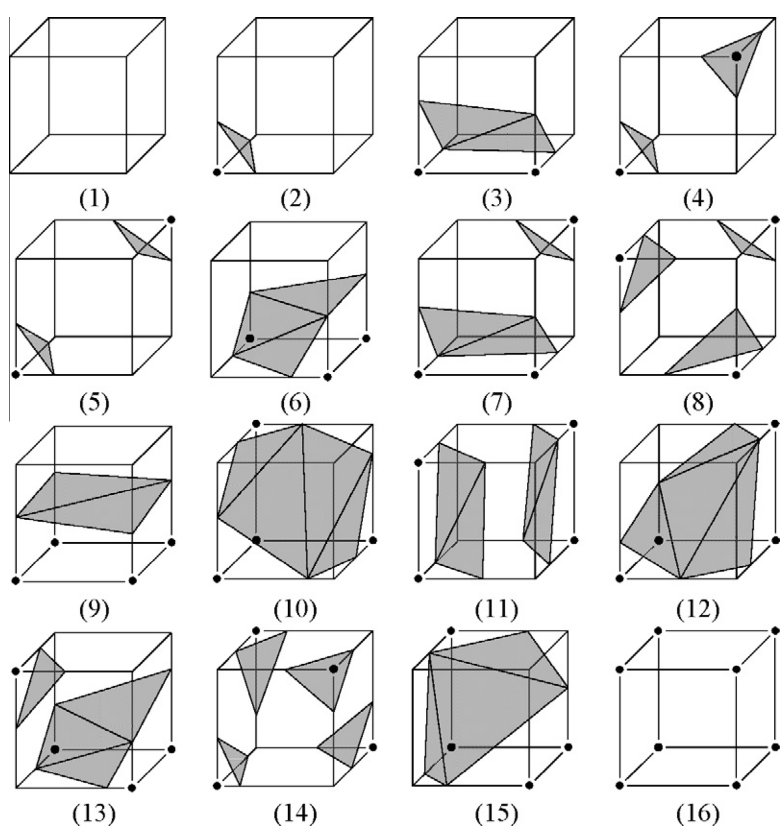

Fig. 2. Look-up table for the MC algorithm showing the 16 different topologic states.

2. Design with isosurfaces

The ITD is an iterative algorithm which redistributes (adds and removes) material inside a design domain until a desired volume fraction is reached. The redistribution process consists of four steps: (1) obtain the design criteria distribution within the design domain; (2) determine the Minimum Criteria Level (MCL), where its intersection with the design criteria distribution produces the new structural boundary, Fig. 1; (3) eliminate all regions from the design domain where the criteria distribution is lower than the MCL; (4) re-evaluate the remaining structure in order to recalculate the design criteria distribution.

The criterion used by ITD, [11] is loosely based on that of the ESO method [12–16]. In the ESO method, a structure evolves

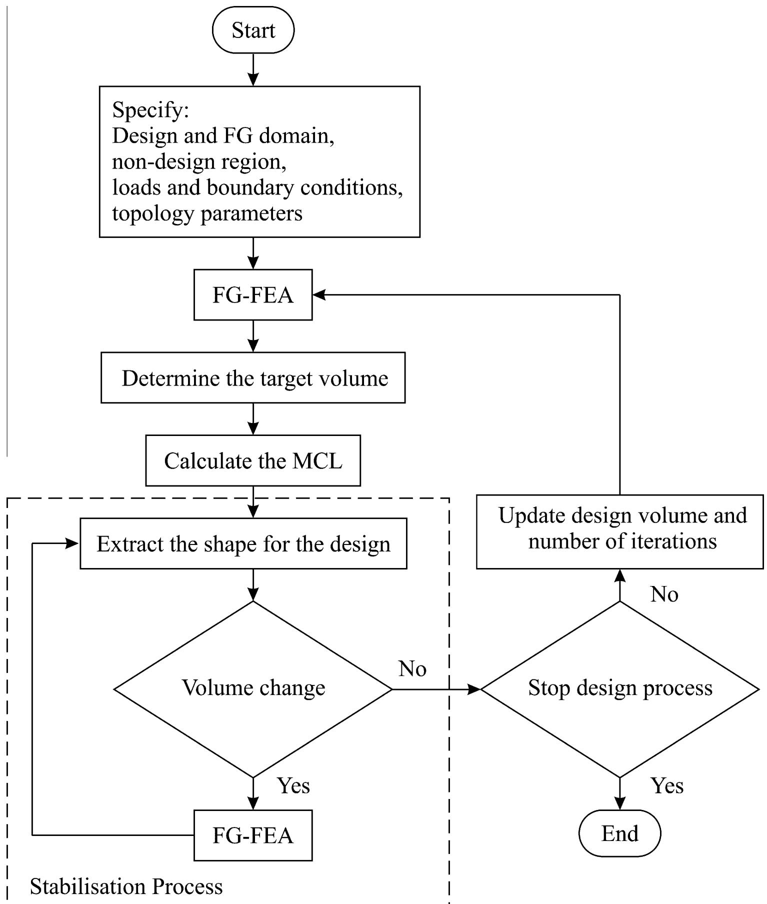

Fig. 3. Flow chart showing the ITD process.

Table 1
ITD parameters.

Example	n_i	$\frac{V_f}{V_0}$	$\Delta V(\%)$
Cube with roller supports	50	0.05	1.0
Embedded beam	50	0.05	0.1
Electric mast	50	0.05	1.0
Sphere	200	0.05	0.1

towards a fully stressed design by slowly removing the lightly stressed elements. In ITD, the isolines (or isosurface for 3D structures) which represents the minimum stress (or driving criteria) of the structure are generated, where all material with a lower stress is removed. The boundary formed by the isolines (or isosurface) then defines the new boundary of the structure.

The Level Set Method (LSM) [17–22] appears to be similar to ITD in that it uses a curve or surface. However, whereas in ITD, the isolines/isosurfaces represent the stress (or driving criterion) of the structure; in LSM, the curve/surface represent the structural boundary in an implicit form, as the zero level set of a high-dimensional function. The method then traces the deformation of the curve/surface by means of the deformation of this embedding function [22], which is used to both to represent and evolve the interface or boundary. For structure topology optimization, the curves/surfaces which depict the structure boundaries are deformed to, for example, minimize the elastic deformation energy.

The objective of this work was to extend the ITD algorithm [11] to allow the topology design of three-dimensional continuum structures using the isosurfaces of the structural behaviour desired of that structure.

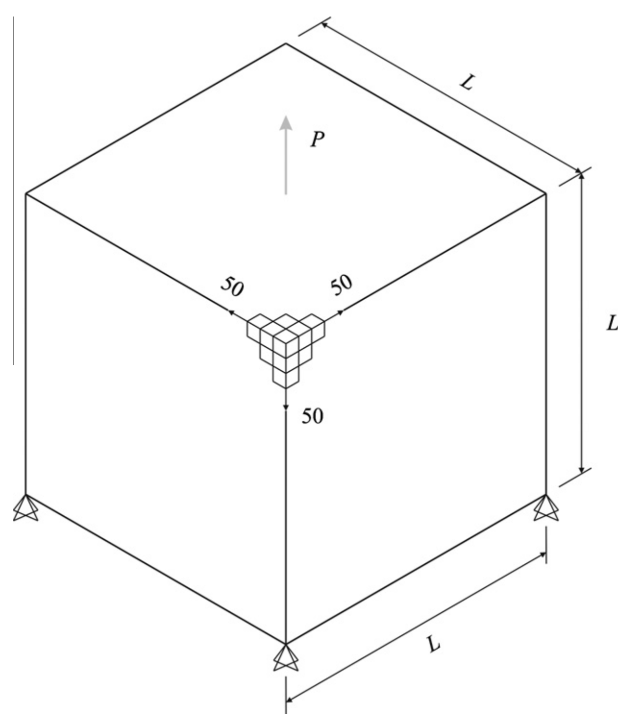

Fig. 4. Cube with roller supports.

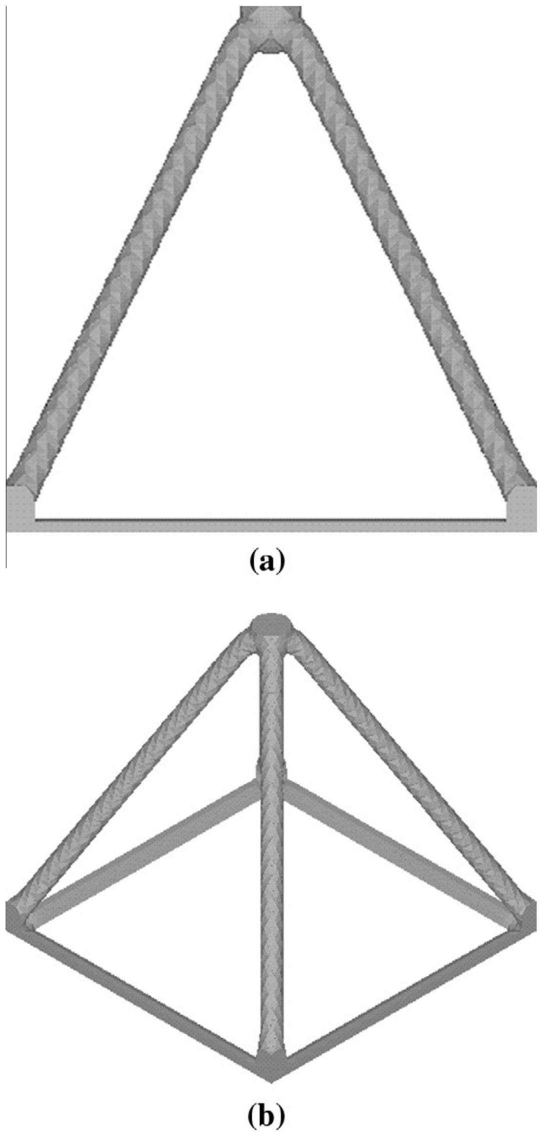

Fig. 5. Cube with roller supports. Final design: (a) Front view. (b) Isometric view.

2.1. Criterion selection

The design criterion used in this work was the von Mises stress, which for a three-dimensional continuum is given by (1).

$$\sigma_{VM} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \quad (1)$$

where, σ_1, σ_2, and σ_3 are the principal stresses.

2.2. Determination of the criteria distribution

When a new structural boundary has been generated by the ITD algorithm, the new structural domain needs evaluation. An accurate, but time consuming way to do this, would involve generating an unstructured finite element mesh (FEM) every time the boundary changed, followed by a finite element analysis (FEA). The use of an approximate method of FEA with an unchanging regular sized FEM where only the material properties of each FE are modified significantly reduces the computational cost of obtaining a design.

The Fictitious Domain Method (FDM), Hyman [23] and Saul'ev [24], has been applied to problems requiring the solution of partial differential equations, where the domain of the problem is complex. FDM is capable of simplifying the domain of the problem by using a regular fixed domain. This method has been in existence since the early 1950s and has been extensively used to solve problems such as: particle flow, heat transfer, fluid flow, shape and topology optimization, to name but a few [25–33]. In 1999 Garcia and Steven [34], were amongst the first to applied FDM for structural shape optimization, calling the method Fixed-Grid Finite Elements Analysis (FG-FEA). The approximate method of FEA used in this work was FG-FEA as it had been previously researched by one of the authors [35,36]. However any other approximate method can be used.

2.3. Definition of minimum criteria level

The MCL is calculated in each iteration and depends on both the distribution of the design criterion and the volume of the design domain in that iteration, given by (2).

$$V_i = V_0\left(\frac{n_i - i}{n_i}\right) + V_f\frac{i}{n_i} \quad (2)$$

where i is the ith iteration; V_0 is the initial volume of the design domain; V_f is the final volume desired for the design; n_i is the total number of iterations to use for the ITD to design the structure.

Once the criterion has been calculated for each element in the design domain, these are arranged in decreasing order of criterion value. An element by element volume summation of the ordered list is carried out until a volume is reached which is as close as possible to the target volume given by (2), where the level of error between the summed and target volume depends on the size of the elements. The criteria value of the next element in the ordered list is then used as the value for the MCL.

2.4. Minimum criteria level extraction

There are several approaches to the generation of a 3D surface, e.g. Keppel [37], Herman and Udupa [38], Farrell [39], Shen and Jhonson [40], Koguchi and Kikuchi [10].

The procedure to generate the structural boundary in 3D designs depends on the determination of the MCL isosurface. In order to determine the line segments which produce the profile of the boundary, the contouring algorithm called Marching Cubes (MC) [41] was implemented.

The MC method uses a divide and conquer approach, treating each finite element independently as a cube cell. The basic assumption of this algorithm is that a contour (MCL isosurface) can only pass through a finite element in a limited number of ways. This algorithm requires the value of the MCL as well as the value of the criteria at each node, and consists of two basic steps:

1. Identify from Fig. 2 the topological state of each element;
2. Determine the shape of the contour of the MCL isosurface through each element.

The interaction of an isosurface through a cubic element can have a maximum of 256 different topological states. Since a cube has double symmetry, the maximum number of states can be reduced to 16, Fig. 2. This shows all different states where a black circle at a node means that the value of the criteria at that node is less than the MCL (i.e. outside the design).

When only one of the nodes in an edge of an element is marked with a black circle it indicates that the MCL isosurface intersects that edge, which is the case for topological states 2–15. To find that intersection point, linear interpolation can be used. To construct facets from the intersection points, Delanauy triangulation [42]

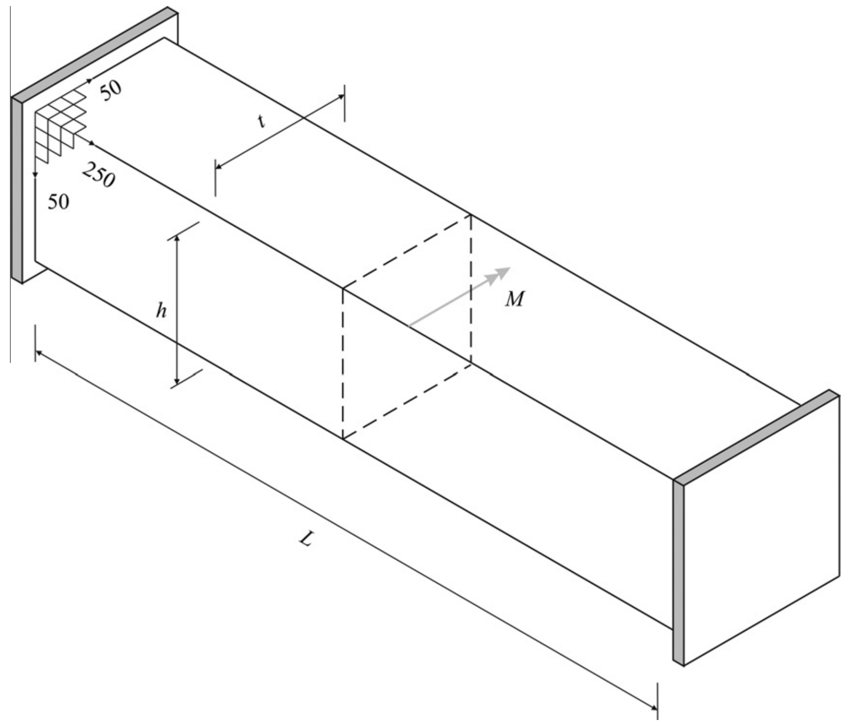

Fig. 6. Embedded cantilever beam.

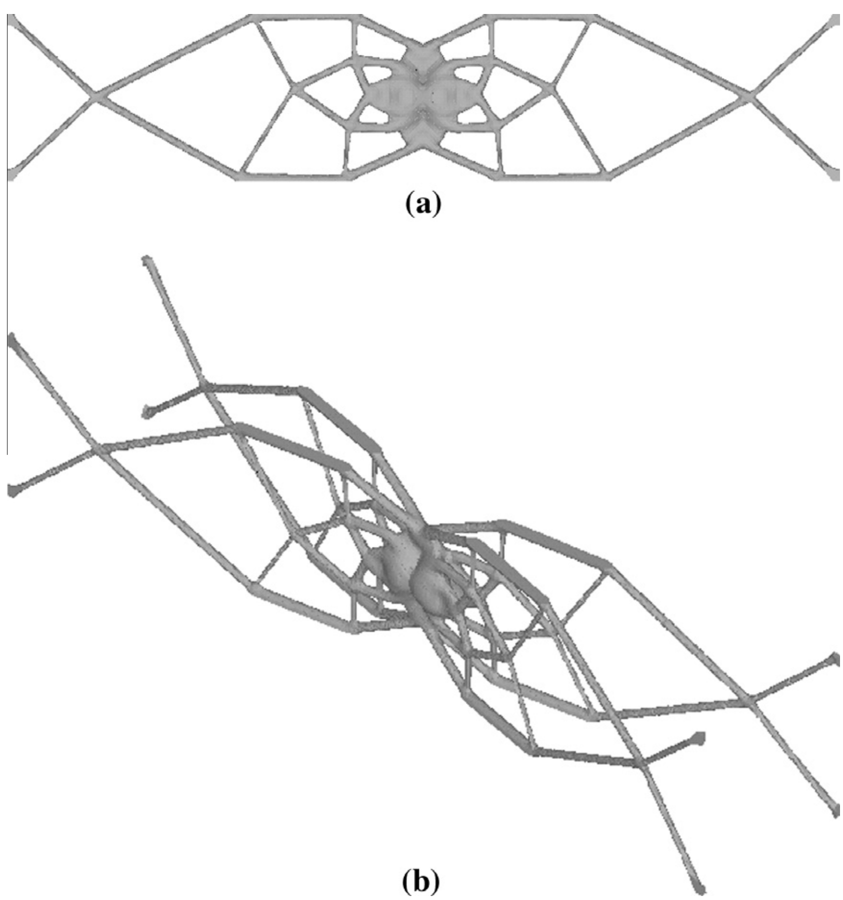

Fig. 7. Embedded beam. Final design: (a) Front view. (b) Isometric view.

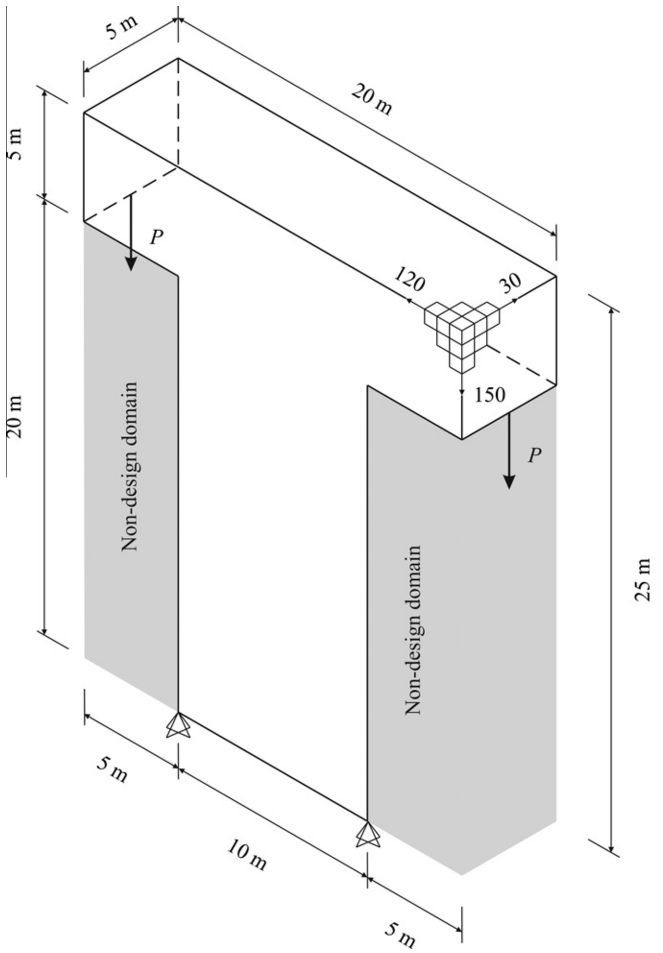

Fig. 8. Electric mast.

can be used. The shape of the MCL isosurface through the element is then obtained by connecting these facets as shown in Fig. 2.

2.5. Structural boundary stabilisation

When the MCL is modified, the structural boundary changes and this affects the criterion distribution. Therefore, before the new iteration is started, an iterative process of reanalysis and material redistribution is carried out until the change in the domain volume between successive boundary adjustments is less than a minimum volume change limit (ΔV). Typical value are $\Delta V(\%) = 0.1$–1%.

$$\Delta V(\%) = \frac{V_i - V_{i-1}}{V_{i-1}} \times 100 \tag{3}$$

This iterative process only requires a few iterations, although the exact number depends on the value of the volume of design domain at the ith iteration (V_i) determined by (2).

3. The isosurface topology design algorithm

The procedure for extending the ITD method for three-dimensional designs, using the FG-FEA as the approximate method of structural analysis is as follows:

1. Define the design and non-design domains, supports, loads and material properties.
2. Specify the size of fixed-grid mesh.

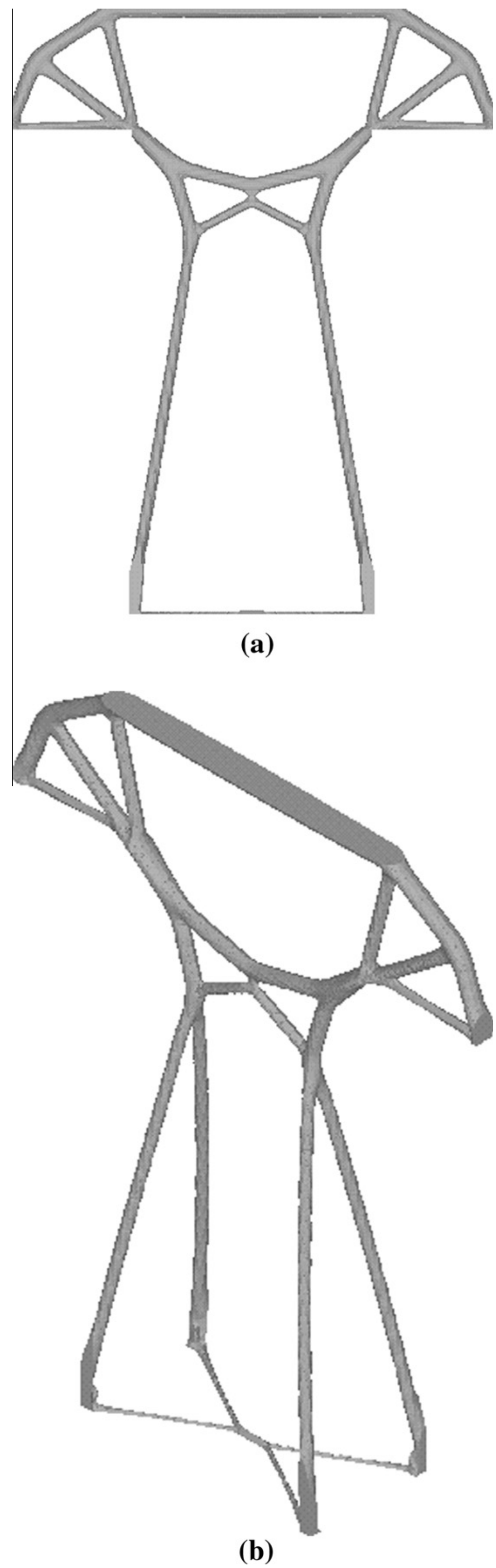

Fig. 9. Electric mast. Final design: (a) Front view. (b) Isometric view.

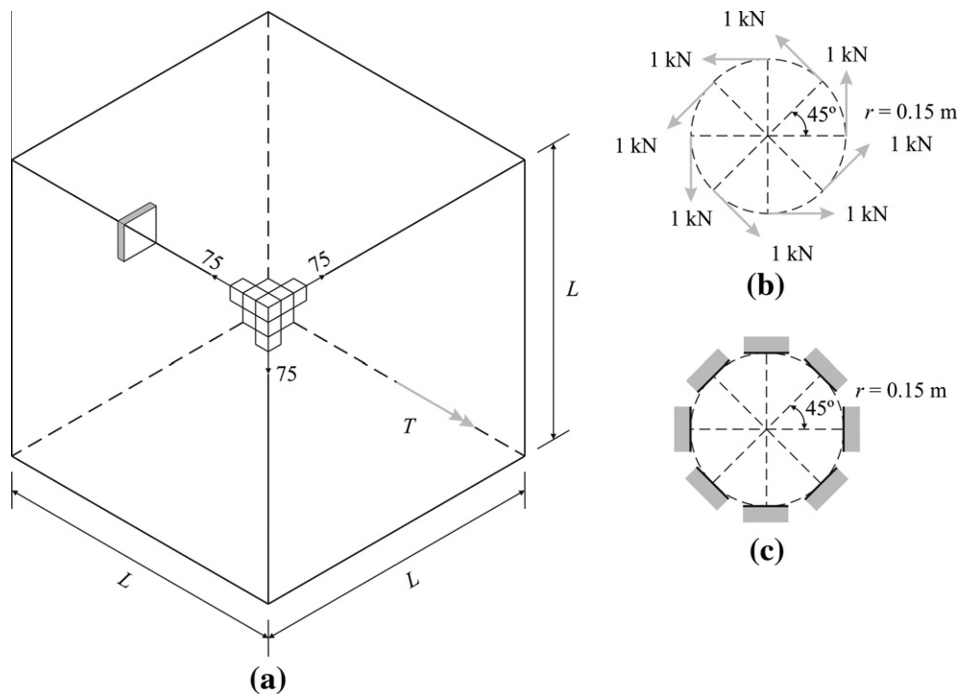

Fig. 10. Sphere. (a) Design domain. (b) Loads. (c) Boundary conditions.

3. Specify the final design volume V_f, the total number of iterations n_i and the minimum volume change limit $\Delta V\%$.
4. Specify the design criterion to use: von Mises stress, etc.
5. Carry out a FG-FEA of the design domain.
6. Determine the target volume and calculate the MCL.
7. Extract the shape for the design.
8. If the percentage volume change is greater than the minimum volume change limit ($\Delta V\%$), go to step 9, otherwise go to step 10.
9. Carry out a FG-FEA of the design domain. Go to step 7.
10. If the total number of iterations n_i has been reached, go to step 11, otherwise, update design volume and increment the iteration number i by 1 and go to step 5.
11. Stop the design process.

This process can be viewed in the flow chart of Fig. 3.

4. Examples

To illustrate the ITD algorithm, four structures were studied and are presented here: (1) a cube with roller supports, (2) the embedded beam, (3) an electric mast, and (4) Michell's sphere.

For all the examples the elastic modulus is 210 GPa and the Poisson's ratio is 0.3. Since FG-FEA was used to analyse the structure, the elastic modulus of out elements was set to 0.021 MPa. The FE used for the examples is the eight-node isoparametric element with eight Gauss integration points [43]. The design criterion used was the von Mises stress. Table 1 shows the ITD parameters used.

4.1. Cube with roller supports

The design domain is a cube. The length of each side of the cube is $L = 1$ m. The domain was subdivided into $50 \times 50 \times 50$ elements, of which only a quarter was used for the design process. Fig. 4 shows a cubic design domain which is subjected to a point load $P = 1$ kN on the centre of the upper surface. The four corners of the lower surface can slide in the horizontal plane.

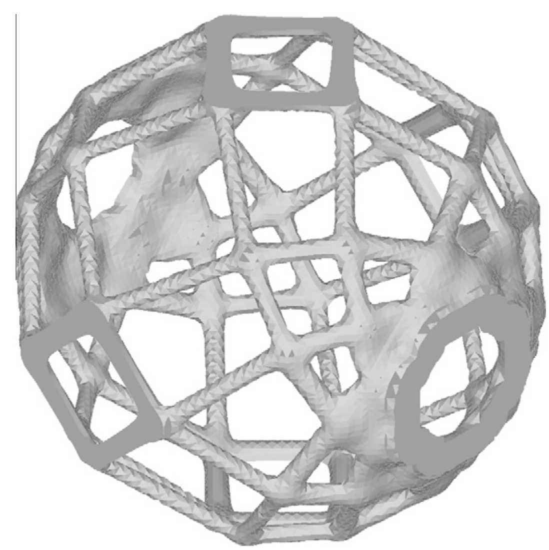

Fig. 11. Sphere. Final design: Isometric view.

Fig. 5 illustrates the resulting design to this first problem which is a quadropod consisting of four legs [6]. If the four supports are clamped, the horizontal lattice located between the four supports is not required. The solution required 50 iterations with a total of 161 FEA. The time to carry out step 7 of the ITD algorithm was approximately equal to 10% of a FEA. So the total solution time could be estimated as being equivalent to 166 FEA.

4.2. Embedded beam

The length of the embedded beam is $L = 5$ m, height $h = 1$ m, and thickness $t = 1$ m. The domain was subdivided into $50 \times 50 \times 250$ elements, of which only a quarter was used for the design process, see Fig. 6. The oblong hexahedral design domain is clamped at the two vertical sides. A concentrated bending moment $M = 1$ kNm is applied at the centre of the domain.

The resulting final design (Fig. 7) agrees with the solutions obtained by other authors [3,4,44]. In general, when the final volume fraction is small, the topology design process produces truss-like structures (although, in truss models, bars cannot have bending). In fact, if we observe Fig. 7 the design bears a resemblance to shapes found in nature. The solution required 50 iterations with a total of 382 FEA. So the total solution time could be estimated as being equivalent to 387 FEA.

4.3. Electric mast

The design domain is the T-shaped box of Fig. 8. Two symmetric vertical loads $P = 10$ kN are applied in the middle of the lower edges of the horizontal part of the T-section and represent the loads exerted by the wires on the mast. Simply supported boundary conditions are imposed at the corners of the base of the T-shape box. Due to symmetry conditions only a quarter of the design domain was used to generate the design.

Fig. 9 shows the design obtained. The ITD algorithm produces a truss-like design that evolves the design of real electric masts. The number of truss elements which emerge depends on the mesh density. A full-scale real industrial application would require a much finer mesh, and a larger design domain in the vertical direction. The solution required 50 iterations with a total of 160 FEA. So the total solution time could be estimated as being equivalent to 165 FEA.

4.4. Michell's sphere

In this last example, the ITD algorithm is used to study Michell's sphere which is the only known-well formulated and solved example of a spatial Michell structure [45]. The design domain is a cube. The length of each side of the cube is $L = 1$ m. The domain design is subdivided using $75 \times 75 \times 75$ elements (Fig. 10a). Boundary conditions and load applied are shown in Fig. 10b and c.

Fig. 11 presents the final design for Michell's sphere, compromising two families of 45° truss-like structures around the spherical surface for the transmission of the torque moment. The solution required 184 iterations with a total of 187 FEA. So the total solution time could be estimated as being equivalent to 206 FEA.

5. Conclusions

This paper presents an enhancement to the ITD algorithm which allows it to obtain three-dimensional designs. The ITD is an iterative process, where the generation of new contours allow the removal and redistribution of material. It allows for important topology changes during the design process.

The use of the isosurfaces of the desired structural performance has a number of major benefits: (1) although the design criteria can be local (such as the von Mises stress), by using the MCL isosurface to define the shape/topology of the domain, the process works globally; (2) the generated designs have smooth boundaries and need no further interpretation, enhancement or processing.

Four examples of topology design of 3D continuum structures were presented to demonstrate the applicability and effectiveness of the ITD algorithm. The main conclusion of this work is that the ITD algorithm is a useful design method for 2D/3D structures.

Acknowledgements

This work has been partially supported by the CARM of Murcia and the Technical University of Cartagena. Their support is greatly appreciated. Travelling funds for the second named author were provided by the School of Mechanical Engineering at the University of Leeds.

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