Principles of Seismic Tomography

Methods to construct images of an object from "projections" of x-rays, ultrasound or electromagnetic waves have found wide applications in electron microscopy, diagnostic medicine and radio astronomy. Projections are measurable quantities that are a functional – usually involving a line integral – of physical properties of an object. Convolutional methods or iterative algorithms to solve large systems of linear equations are used to reconstruct the object. In principle, there is no reason why similar image reconstructions can not be made with seismic waves.

The idea of seismic tomogrphy study is that the travel-time anomalies observed for many ray paths, criss-crossing the Earth between various points near the Earth's surface and reaching different depths in its interior, could be resolved formally into a 3-D model. The motivation for studying 3-D structure of the Earth's interior is that it may offer the best information on the dynamic processes in the deep interior of the Earth.

Basic Theory

Tomography is a type of inverse problem. That is, measurements are first made of some energy that has propagated through a medium. The received character of this energy (e.g., amplitude, travel-time) is then used to infer the values (e.g., velocity, density, permittivity) of the medium through which it has propagated.

The tomographic problem can be stated as: "From projections (sums of some interior value) measured outside of an object find the interior distribution of values inside the object." A projection is the sum of an object's parameters along a given linear energy transit path. A sum or integral of this type is also known as a Radon transform.

The Radon transform is the forward part of the tomographic problem. Then, in the tomographic procedure, we must take these projections and create an image from them. There are two broad categories of techniques used to infer the medium's internal values from the projections. They are "transform" and "series expansion" methods.

Transform techniques

Transform methods start with the motion of an object being described by a continuous function, with a continuous set of projections. There are two main transform methods: the Fourier techniques and the filtered backprojection method.

Fourier transform technique

The Fourier projection theorem states that the 2-D (3-D) Fourier transform of an image or medium can be obtained form the 1-D (2-D) Fourier transforms of the projections. Thus, by measuring the projection of the object and constructing the 2-D (3-D) transform space accordingly, then inverse 2-D (3-D) Fourier transforming, an image of the object may be reconstructed. A major difficulty with this reconstruction is that it requires a complete (all the way around the objects) set of projections.

Backprojection method

Backprojection is an operation which sums projected values (Radon transforms) together. The basic idea is that each point that is traversed by the ray from the source to receiver is given the value of the total projection. The image is built up by summing all the values of the points determined for every ray.

This backprojection method can be used to create images, but it is one that leads to blurring of the final reconstructed image. To attain a better image, it is reasonable to attempt a spatial deconvolution. This method is called "filtered backprojection", the notion arising of filtering the backprojection to provide a clearer image.

In the above discussion, it has been assumed that energy propagates as a ray (i.e. by infinitely thin beams in straight lines). However, elastic or acoustic waves have well known properties of divergence and diffraction in accordance with the wave equation. It is nonetheless possible to build wave equation propagation into a tomographic framework. This method is called "diffraction tomography" (DT). Examples of DT inversion methods are "filtered backpropagation algorithms".

Series Expansion Methods

The series expansion methods start by considering the object or area of interest to be comprised of boxes or pixels. Energy is considered to propagate through the various pixels to provide a sum or projection of the pixel values. The pixel values are now related to the sum. This is often related to solving large linear equations. A stable but approximate solution, as discussed previously, is known as backprojection. In the matrix formulation backprojection corresponds to using the transpose of matrix instead of the inverse. Two other more accurate but iterative methods are known as ART (Algebraic Reconstruction Technique) and SIRT (Simultaneous Iterative Reconstruction Technique).