

Reduced feedback for capacity and fairness tradeoff in multiuser diversity

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Abstract: The network capacity of wireless systems can be increased drastically by extracting multiuser diversity gain. The multiuser diversity gain is obtained by granting scheduling priority to the mobile station (MS) with the best current channel condition. There are a couple of drawbacks in multiuser diversity schedulers such as the linearly increasing feedback load with the number of MSs and the degrading fairness. To balance the capacity request against the fairness request, the authors need a scheduler achieving capacity and fairness tradeoff. In this study, the authors propose a new scheduling scheme of selecting a single MS, which can achieve the trade-off between capacity and fairness with reduced feedback. The simulation result shows that the proposed scheduling scheme attains better fairness compared with the conventional method for achieving capacity and fairness trade-off.

1 Introduction

Multiuser diversity [1] exploits the independence of the channels among users and enables high throughput in wireless packet data networks by prioritising mobile stations (MSs) with good channel conditions. The base station (BS) grants channel access only to the MS having the highest signal-to-noise ratio (SNR). When there are a large number of MSs in a cell area, high multiuser diversity gain can be extracted. After selecting the MS having the best channel condition, the BS changes the modulation and coding scheme (MCS) according to the scheduled MS's channel condition. The MS located close to the BS can support a high data rate modulation scheme. However, the MS located far from the BS only supports a low data rate modulation scheme. Moreover, the BS favours the MS which can support high data rate. To fully extract multiuser diversity gain, the BS needs to know the channel quality indicator (CQI) information for every MS in the network. Accordingly, the feedback information in the network linearly increases as the number of MSs increases. This is one of the main problems in implementing the packet scheduler extracting multiuser diversity gain. Therefore the feedback information reduction schemes are required and many scheduling schemes [2–6] with feedback reduction are proposed. In [2], the feedback threshold is introduced and the MS having the higher SNR than the feedback threshold sends the CQI information to the BS. In [3], the quantised CQI is used to represent MS's channel state. Instead of sending the full values of channel state, MSs send the discrete value representing their channel state. Furthermore, in [4], only one bit feedback information with opportunistic feedback is used to achieve strict fairness, and

the authors show that the capacity of the scheduling scheme using one bit feedback information can be very close to the capacity of the proportional fair scheduling scheme [7]. In [5], multiple feedback thresholds and multiple access probabilities are introduced in the contention-based feedback protocol. In this protocol, MSs are divided into several classes according to their channel states. Then, different random access probabilities are assigned to each classes.

In [6], the feedback protocol using capture effect is proposed to reduce feedback load. However, the feedback information can be imperfect in a practical system. The multiuser diversity scheduling with imperfect feedback information is presented in [8]. Another issue in the multiuser diversity scheduler is a fairness problem. If the BS schedules MSs according to SNR, the MS located nearest to the BS actually monopolises resources and the MS located farthest to the BS hardly has the chance to access the BS. The proportional fair scheduling [7] achieves strict fairness while obtaining some multiuser diversity gain. It uses the instantaneous rate divided by the long-term throughput as a scheduling metric. In [9], however, the proportional fair scheduling and the conventional multiuser diversity scheduling are viewed as two extreme scheduling procedures. The former maximises system fairness without considering capacity and the latter maximises system capacity without considering system fairness. They propose a scheduling scheme of achieving the tradeoff between capacity and fairness by grouping MSs and then using the two-step selection process. However, the feedback reduction is not considered in that scheduling scheme. Moreover, it needs to classify MSs into several groups in advance. Deriving exact capacity of the multiuser diversity scheduler

is also an important problem. In [10], achievable rate of multiuser diversity scheduler considering delayed feedback is analysed. Another problem in the multiuser diversity scheduler is a large delay variation. Unlike the voice traffic, the packet data traffic does not request a stringent delay requirement. That is why the multiuser diversity is possible for a packet data traffic. Generally, multiuser diversity scheduler selects a single MS and allocates whole resources to it. However, the scheduling scheme of selecting multiple MSs with good channel condition can be considered [11]. In this paper, we propose a scheduling scheme of selecting a single MS based on the absolute SNR and the normalised SNR defined by the received SNR divided by the average SNR. Specifically, the feedback reduction scheme is also considered and grouping MSs is not required. Moreover, we show the comparison results among the proposed scheduling scheme, the conventional method of achieving capacity and fairness trade-off, and the scheduling scheme of selecting multiple MSs [11]. The analytical model is developed and the simulation result shows that the proposed scheduling scheme can achieve the flexible trade-off between capacity and fairness according to the network operator's demand.

2 System model

We consider a single-cell downlink channel with K MSs. The downlink channel model can be described as follows

$$r_i(t) = h_i(t)x_i(t) + n_i(t), \quad i = 1, 2, \dots, K \quad (1)$$

where $x_i(t)$ is the transmitted symbol, $h_i(t)$ is the channel gain, and $n_i(t)$ is the independent and identically distributed zero-mean complex Gaussian noise with unit variance. The received SNR of MS i is assumed to be Rayleigh distributed with the following probability density function (pdf)

$$f_{\gamma_i}(\gamma) = \frac{1}{\bar{\gamma}_i} \exp\left(-\frac{\gamma}{\bar{\gamma}_i}\right) \quad (2)$$

where γ is the instantaneous SNR and $\bar{\gamma}_i$ is the average SNR of MS i . We propose a new scheduling scheme which selects only one MS per scheduling interval. For the comparison purpose, we introduce the scheduling scheme [11] which selects multiple MSs per scheduling interval and the scheduling scheme [9] which classifies MSs into several groups and the two-step selection process is applied.

2.1 Scheduling scheme of selecting a single MS

The metric of each MS is as follows

$$Z_i(t) = \frac{\gamma_i(t)}{\bar{\gamma}_i} \quad (3)$$

where $\gamma_i(t)$ is the instantaneous received SNR, $\bar{\gamma}_i$ is the average received SNR of MS i and $Z_i(t)$ is a normalised SNR for MS i at time t . To reduce feedback information, opportunistic feedback is applied. If an MS has a higher metric value than the predetermined feedback threshold η_{th} , the MS sends feedback information to the BS. At the first round, the scheduler in the BS selects at most L MSs as candidates for the MS to be selected finally. When the total number of MSs sent feedback information is higher than L ,

the scheduler selects L MSs according to the normalised SNR. When the total number of MSs sent feedback information is below or equal to L , the scheduler selects all the MSs sent feedback as candidates. Then, the scheduler sorts the candidate MSs according to the absolute SNR, then it selects the MS having the highest absolute SNR. When the total number of feedback MS is zero, the scheduler randomly selects one MS. Compared with the scheduling scheme in [9], this scheduling scheme considers the feedback reduction. Moreover, it does not need to classify MSs into several groups in advance. In this scheduling scheme, the parameter L plays a key role in adjusting tradeoff between capacity and fairness. The BS selects the optimal one MS among K MSs. The parameter L in the first round is an intermediate bottleneck of this selection process. When the parameter L is small compared with the total number of MS, the first round selection becomes very competitive. Hence, the first round selection becomes a dominant factor in this selection process. To the contrary, when the parameter L is large, it is easy to be selected at the first round. However, the second round selection becomes very competitive. Hence, the second round selection becomes a dominant factor in this selection process. When the parameter L is 1, this scheduling scheme becomes the proportional fair scheduling. However, when $L = K$, it becomes the conventional multiuser diversity scheduler which maximises the system capacity.

2.2 Scheduling scheme of selecting multiple MSs

As shown in [11], the scheduling scheme of selecting multiple MSs are as follows: the metric of each MS is same as (3) and the opportunistic feedback is also applied. Hence, MSs with higher metric values than the predetermined feedback threshold η_{th} send the feedback information. The scheduler in the BS schedules $L = L_1 + L_2$ MSs at each scheduling instant. At the first round, the scheduler schedules the upper L_1 MSs according to the normalised SNR. At the second round, the scheduler sorts the remaining MSs in descending order of the absolute SNR. Then, it selects the upper L_2 MSs. When the total number of feedback is below L , the scheduler selects all the MSs who sent feedback information and randomly selects the necessary number of MSs from the remaining MSs who did not send feedback information. Even with the parameter $L = 1$, this scheduling scheme is quite different from the newly proposed scheduling scheme. If we set $L = 1$, ($L_1 = 1, L_2 = 0$) or ($L_1 = 0, L_2 = 1$). When ($L_1 = 1, L_2 = 0$) is used, the scheduler selects a single user having the best normalised SNR. Hence, this scheme becomes the conventional proportional fair scheduler maximising system fairness. When ($L_1 = 0, L_2 = 1$) is used, the scheduler selects a single user having the best absolute SNR. Hence, this scheme becomes the conventional multiuser diversity scheduler maximising system capacity. However, the scheduling scheme in this paper achieves the trade-off between the capacity and fairness by changing the number of candidate MSs.

2.3 Conventional trade-off scheduling scheme of selecting a single MS

As shown in [9], the metric for this scheduling scheme is same as (3). In this scheduling scheme, the feedback reduction is not considered. Hence, the opportunistic feedback is not applied. The scheduler in the BS divides all MSs in the cell area into L groups. Then, at the first round,

the scheduler selects one MS per group based on either the absolute SNR or the normalised SNR. At the second round, the scheduler chooses one MS from all L selected MSs based on either the normalised SNR or the absolute SNR.

3 Capacity and fairness analysis

Assume that we have the Rayleigh fading wireless channel and the pdf of SNR is given in (2). By changing variable, the pdf of the normalised SNR for all the MSs is same as $f_i(t) = e^{-x}$, $i = 1, 2, 3, \dots, K$. Let η_{th} be the predetermined feedback threshold. The probability that the normalised SNR of an MS is higher than η_{th} is $\pi_g = \int_{\eta_{th}}^{\infty} f(x)dx = e^{-\eta_{th}}$, and the probability that the normalised SNR of an MS is lower than η_{th} is $\pi_b = 1 - \pi_g = 1 - e^{-\eta_{th}}$. We tag an arbitrary MS i for convenience. MS i can be scheduled in three mutually exclusive cases.

- *Case 1:* MS i feeds back and L or more than L remaining MSs feed back. Moreover, MS i is in the upper L candidates and finally selected in the absolute SNR criterion.
- *Case 2:* MS i feeds back and less than L remaining MSs feed back. MS i is finally selected according to the absolute SNR criterion.
- *Case 3:* None feeds back and MS i is finally selected through a random selection process.

The probability that k MSs out of $(K - 1)$ MSs have the normalised SNR higher than η_{th} is as follows

$$\delta_k = \binom{K-1}{k} \pi_g^k \pi_b^{K-1-k}, \quad 0 \leq k \leq K-1 \quad (4)$$

The conditional pdf of an MS having the normalised SNR higher than η_{th} is given as $g(x) = e^{-x}/e^{-\eta_{th}}$, $x \geq \eta_{th}$. The conditional cumulative distribution function (cdf) that the normalised SNR of an MS is less than x when the MS has sent feedback information is as follows

$$G(x) = \int_{\eta_{th}}^x \frac{e^{-t}}{e^{-\eta_{th}}} dt = 1 - \frac{e^{-x}}{e^{-\eta_{th}}}, \quad x \geq \eta_{th} \quad (5)$$

The cdf that the normalised SNR of an MS is higher than x when the MS has sent feedback information is as follows

$$H(x) = 1 - G(x) = \frac{e^{-x}}{e^{-\eta_{th}}}, \quad x \geq \eta_{th} \quad (6)$$

In Case 1, the total number of feedback MSs including MS i is above L . MS i should be included in the upper L candidate MSs. The number of MSs having higher normalised SNR than MS i should be $(L - 1)$ or below $(L - 1)$. Let k be the number of feedback from untagged MSs. There are $\binom{k}{L-1}$ cases in selecting the L candidate MSs including MS i from $(k + 1)$ MSs. All the cases are equally probable because the pdfs for all the MSs are same. The probability of selecting the specific L MSs as candidates is $P = 1/\binom{k}{L-1}$. Hence, the conditional cdf for MS i being included in the specific L candidate MSs is calculated

as follows

$$Q_{i,k}^{(L)}(x) = \binom{k+1}{L} \sum_{m=0}^{L-1} \binom{k}{m} \int_{\eta_{th}}^x H(t)^m g(t) G(t)^{k-m} dt, \quad k \geq L \quad (7)$$

where m is the number of feedback MSs having higher SNR than MS i . By differentiating (7), we calculate the conditional pdf for MS i as follows

$$q_{i,k}^{(L)}(x) = \binom{k+1}{L} \sum_{m=0}^{L-1} \binom{k}{m} \left(\frac{e^{-x}}{e^{-\eta_{th}}} \right)^{m+1} \times \left(1 - \frac{e^{-x}}{e^{-\eta_{th}}} \right)^{k-m}, \quad k \geq L \quad (8)$$

From the specific L candidate MSs, one MS is finally selected in the absolute SNR criterion. Strictly speaking, the normalised SNR values of MSs in the L candidate MSs are not independent of each other because the upper L MSs are selected. However, the correlation among MSs in the L candidates are weak and the final selection criterion is changed to the absolute SNR. Hence, we approximate that MSs in the L candidates are independent of each other. Let S be the index set of the L candidate MSs. Then, the conditional pdf for MS i being finally selected according to the absolute SNR is as follows

$$f_{i,k}^{(L)}(x|S) = \left(\prod_{s \in S - \{i\}} Q_{i,k}^{(L)}\left(\frac{\bar{y}_i}{\bar{y}_s} x\right) \right) q_{i,k}^{(L)}(x) \quad (9)$$

Consider the set $C(S, k)$ whose element is a k -combination from S , that is

$$C(S, k) = \{S_s | S_s \subset S \text{ and } |S_s| = k \leq |S|\} \quad (10)$$

Let U be the index set of total MSs, and $\Omega = U - \{i\}$. Then, the capacity for Case 1 can be calculated as follows

$$C_{i,1} = \sum_{k=L}^{K-1} \pi_g^{k+1} \pi_b^{K-1-k} \sum_{S \in C(\Omega, L-1)} \frac{1}{\binom{k+1}{L}} \times \int_{\eta_{th}}^{\infty} \log_2(1 + \bar{y}_i x) f_{i,k}^{(L)}(x|S) dx \quad (11)$$

In Case 2, the total number of feedback is $k + 1 \leq L$. There are $\binom{K-1}{k}$ cases in selecting $(k + 1)$ candidate MSs including MS i from K MSs. The conditional pdf and cdf for the specific $(k + 1)$ MSs are as follows

$$f_{i,k}^{(L)}(x) = \binom{K-1}{k} e^{-x}/e^{-\eta_{th}}, \quad x \geq \eta_{th} \quad (12)$$

$$F_{i,k}^{(L)}(x) = \binom{K-1}{k} \left(1 - \frac{e^{-x}}{e^{-\eta_{th}}} \right), \quad x \geq \eta_{th} \quad (13)$$

Then, MS i is finally selected according to the absolute SNR criterion. Hence, MS i should have the highest absolute SNR compared with k MSs. Let S be the index set for k untagged

feedback MSs. Then, the conditional pdf for MS i is as follows

$$q_{i,k}^{(L)}(x|S) = \left(\prod_{s \in S} F_{i,k}^{(L)}\left(\frac{\bar{\gamma}_i}{\bar{\gamma}_s} x\right) \right) f_{i,k}^{(L)}(x), \quad x \geq \eta_{th} \quad (14)$$

Therefore the capacity for this case is calculated as follows

$$C_{i,2} = \sum_{k=0}^{L-1} \pi_g^{k+1} \pi_b^{K-1-k} \sum_{S \in C(\Omega, k)} \frac{1}{\binom{K-1}{k}} \times \int_{\eta_{th}}^{\infty} \log_2(1 + \bar{\gamma}_i x) q_{i,k}^{(L)}(x|S) dx \quad (15)$$

In Case 3, no MS sends feedback information to the BS, and MS i is selected through a random selection process. The probability that no MS sends feedback information is π_b^K . Moreover, the probability that MS i is selected through random selection is $1/K$. The conditional pdf for MS i who does not send feedback information is as follows

$$f_i(x) = \frac{e^{-x}}{1 - e^{-\eta_{th}}}, \quad 0 \leq x \leq \eta_{th} \quad (16)$$

Therefore the capacity for Case 3 is calculated as follows

$$C_{i,3} = \frac{\pi_b^K}{K} \int_0^{\eta_{th}} \log_2(1 + \bar{\gamma}_i x) \frac{e^{-x}}{1 - e^{-\eta_{th}}} dx \quad (17)$$

The capacity for MS i is $C_i = C_{i,1} + C_{i,2} + C_{i,3}$. In assessing the fairness, we use the fairness measure defined in [12] as a self-fairness

$$F_i = \frac{-\log(P_i)}{\log(K)} \quad (18)$$

where P_i is the access probability of MS i . We can calculate the access probability P_i by integrating the pdfs for the above mentioned three cases. Moreover, we can calculate the average system fairness as

$$\bar{F} = -\sum_{i=1}^K P_i \frac{\log(P_i)}{\log(K)} \quad (19)$$

4 Simulation

4.1 Scheduling scheme of selecting a single MS

In this simulation, we classify the average SNR of MSs into five different groups. The average SNR of each group is 10, 15, 20, 25, and 30 dB. We increase the number of MSs in each group by one to make the average SNR remain the same regardless of the total number of MSs. We increase the number of MSs from 5 to 45. We change the parameter L from 1 to 5 and feedback threshold $\eta_{th} = 0.7$ is used. The simulation result is depicted in Fig. 1. As the parameter L increases, capacity increases and fairness decreases. As we mentioned above, when $L = 1$, this scheduling scheme is equivalent to the proportional fair scheduling scheme. To the contrary, when $L = K$, it is equivalent to the capacity maximising scheduling scheme. When L is increased from

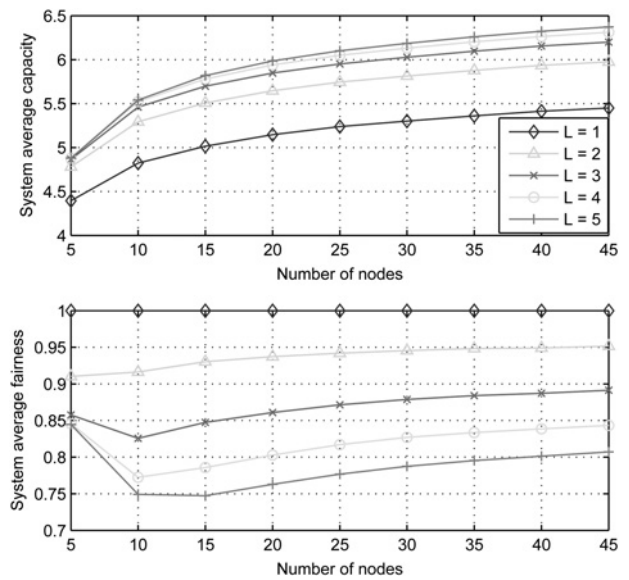


Fig. 1 Average system capacity and fairness according to L

1 to 2, capacity is increased fairly high. However, the fairness drop is not severe. In Fig. 1, the fairness drops until the number of MSs is 10 and it increases again. Note that when the number of MSs is small like 5, the random selection dominates. When the number of MSs is increased further, the number of feedback is increased. Since some MSs become candidate MSs and the absolute SNR is the main criterion in selecting a single MS, the fairness decreases in this case. When the number of MSs is further increased, the normalised SNR is the main bottleneck in selecting an MS. Since all the MSs have the same pdf of the normalised SNR, the fairness increases as the number of MSs increases. In Fig. 2, average system capacity and average system fairness are depicted according to feedback threshold η_{th} ranging from 0.7 to 1.5. As the feedback threshold increases, the number of feedback MSs decreases. Therefore as shown in Fig. 2, the system capacity decreases as the feedback threshold increases. As we can see in Fig. 2, the fairness remains down when the feedback threshold is low. However, fairness approaches to 1 as the feedback threshold increases because the random selection process dominates. In Fig. 3, the delay variance for a target

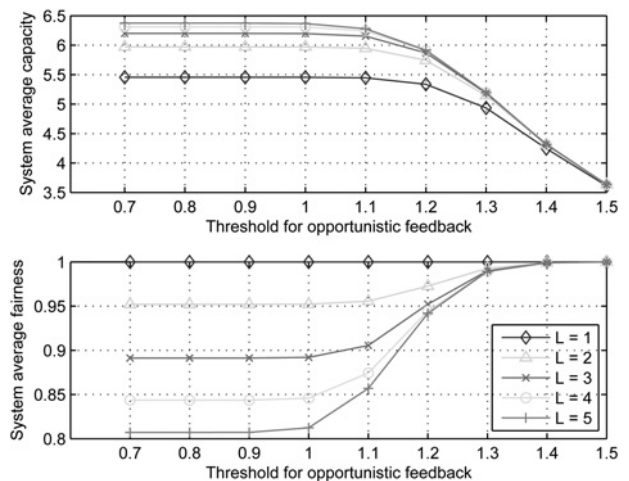


Fig. 2 Average system capacity and fairness against feedback threshold η_{th}

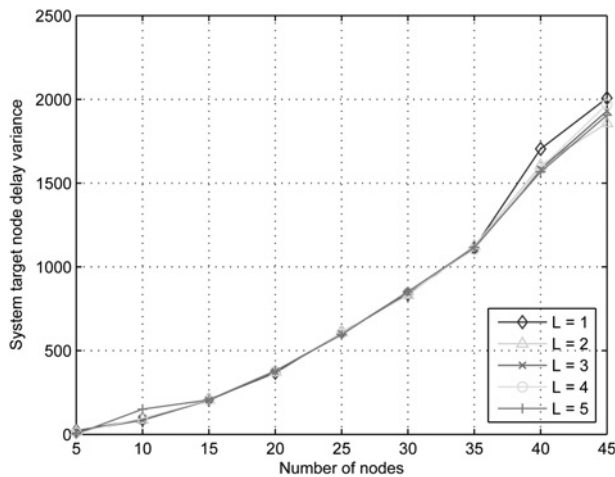


Fig. 3 Target node delay variance

node having 30 dB SNR is depicted. The delay variance increases sharply as the number of MSs increases.

4.2 Scheduling scheme of selecting multiple MSs

The same number of MSs and η_{th} with the scheduling scheme of selecting a single MS is used. The parameter L is set to 5, and we change the parameter L_1 and L_2 . In Fig. 4, the capacity increases as the parameter L_2 increases; however, the fairness decreases. When $L_1 = 5$ and $L_2 = 0$, this scheduling scheme is similar to the proportional fair scheduling scheme. To the contrary, when $L_1 = 0$ and $L_2 = 5$, it is similar to the capacity maximising scheduling scheme. When L_2 is shifted up from 0 to 1, the capacity increases considerably. As L_2 further increases, the capacity increases. However, the amount of capacity gain decreases and the fairness degrades more severely. As pointed out in [9], the proportional fair scheduling sacrifices much capacity to achieve strict fairness and the absolute SNR scheduling degrades fairness severely with little gain in capacity. In case of the parameters $L_1 = 3$ and $L_2 = 2$, the average system capacity is about 94.3% of the absolute SNR scheduling and still it achieves the fairness which is higher than 0.95. In Fig. 5,

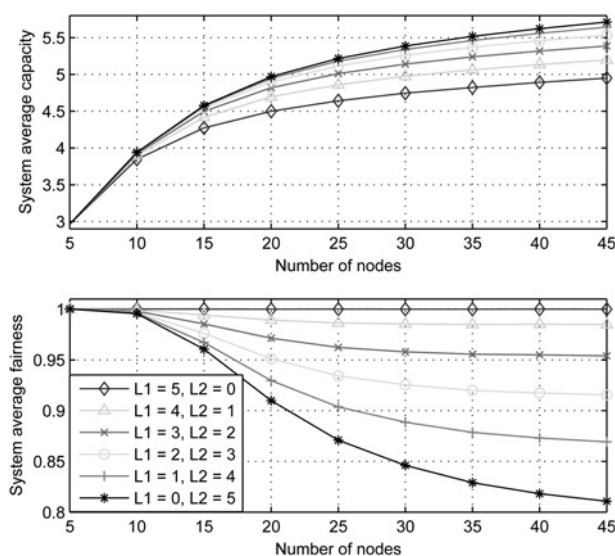


Fig. 4 Average system capacity and fairness according to L_1 and L_2 [11]

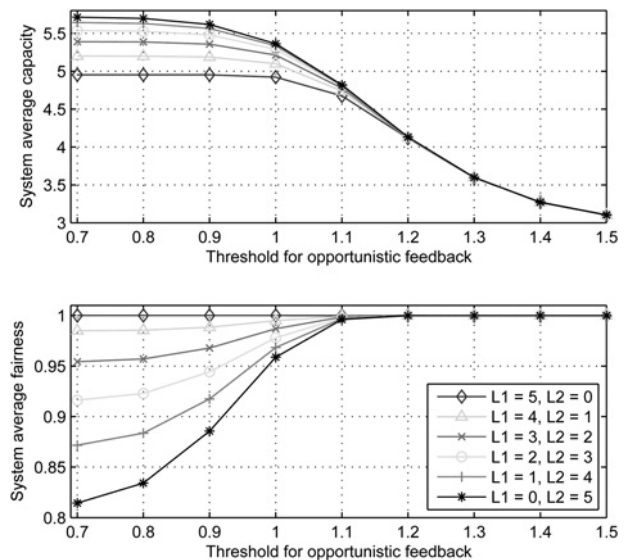


Fig. 5 Average system capacity and fairness against feedback threshold η_{th} [11]

the average system capacity and average system fairness is presented along with varying feedback threshold η_{th} . The parameter η_{th} changes from 0.7 to 1.5 and the total number of MSs is fixed at 45. If we increase η_{th} , the system capacity decreases moderately until η_{th} reaches 0.9. Hence, the unnecessarily low η_{th} degrades system fairness with little gain in system capacity. Moreover, it causes the heavy feedback load. When η_{th} is higher than 0.9, the system capacity drops clearly and system fairness increases sharply. Moreover, the service differentiation according to different L_1 and L_2 disappears because the random selection process dominates. In Fig. 6, the delay variance for a target node having 30 dB SNR is depicted. Compared with the scheduling method of selecting a single MS, the delay variance is reduced. Though the scheme of selecting multiple MSs has the advantage in reducing delay variance, the scheme of selecting a single MS achieves higher capacity than the scheme of selecting multiple MSs. The scheduling scheme of selecting a single MS presents more flexible tradeoff than the scheduling scheme of selecting multiple MSs because it can increase the parameter L up to the number of MSs. However, we cannot increase $L_1 + L_2$ in the scheme of selecting multiple MSs easily. If we

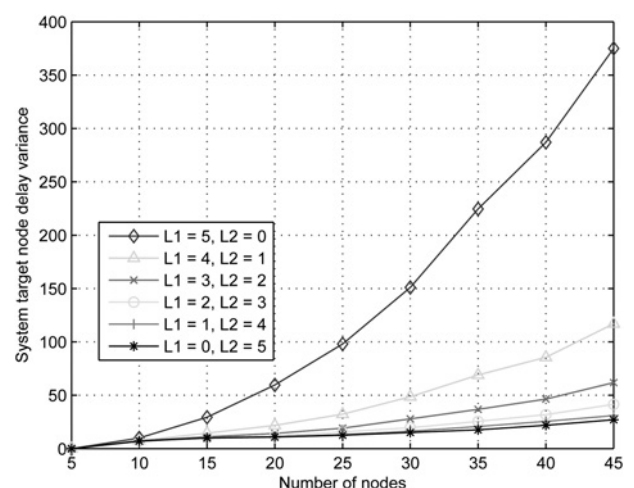


Fig. 6 Target node delay variance

increase $L_1 + L_2$ to a high value, the average capacity degrades because we select many suboptimal MSs instead of the optimal MS.

4.3 Conventional tradeoff scheduling scheme of selecting a single MS

In this scheduling scheme [9], the feedback reduction is not considered. Hence, we adopt the opportunistic feedback in this scheduling scheme for the comparison. We modify this scheduling as follows: if an MS has a higher metric value than the predetermined feedback threshold η_{th} , the MS sends feedback information to the BS. When the total number of feedback MS is zero, the scheduler randomly selects one MS.

The same number of MSs, η_{th} and average SNR with the above scheduling schemes are used, and the number of groups is 5. We apply the normalised SNR criterion in the first round and absolute SNR criterion in the second round. We increase the number of MS from 5 to 45. In this scheduling scheme, grouping method plays an important role in the scheduler performance. In this simulation, two different groupings are used. The first method is grouping by the average SNR, and the second one is random grouping. We classify MSs into five groups according to the average SNR or by random selection. In Fig. 7, the average system capacity and fairness for the modified scheduling scheme [9] with two grouping methods is depicted. In this scheduling scheme with SNR grouping, the average system capacity is similar with the newly proposed scheduling scheme shown in Fig. 1. However, the system average fairness are severely degraded. In Fig. 7, the average system fairness with SNR grouping remains below 0.72 when the number of MSs is bigger than 10. Hence, the average system fairness is severely degraded with very small gain in average system capacity. As we can see in Figs. 1 and 7, the average system capacity with random grouping is similar with the newly proposed scheduling scheme with parameter $L = 3$ or $L = 4$. However, the

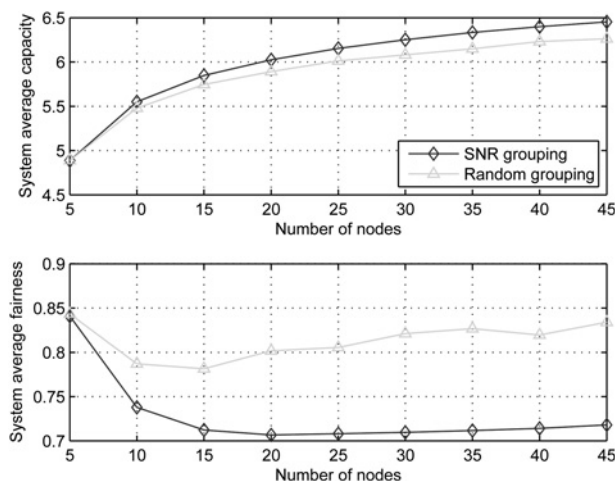


Fig. 7 Average system capacity and fairness according to grouping method

average fairness is worse than the newly proposed scheduling scheme.

5 Conclusion

In this paper, we propose a new scheduling scheme considering multiuser diversity gain, capacity and fairness trade-off, and feedback reduction. This scheduling scheme selects a single MS according to the normalised SNR and the absolute SNR. We add the opportunistic feedback reduction scheme based on the threshold η_{th} . Moreover, we introduce the parameter L to achieve the tradeoff between capacity and fairness. As L increases, this scheduling scheme approaches from the proportional fair scheduling scheme to the capacity maximising scheme. The demand for the capacity–fairness tradeoff can be satisfied by adjusting the parameter L . From the comparison results, we conclude that the proposed scheme is better for high capacity compared with the scheme selecting multiple MSs and the proposed scheme achieves better fairness compared with the conventional tradeoff scheduling scheme.

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