Equations of Lines &Planes

Academic Resource Center

Definition

A Line in the space is determined by **a point and a direction.**

Consider an line L and a point $P(x_0,y_0,z_0)$ on L.

Direction of this line is determined by a vector v that **is parallel to Line** L.

Let P(x,y,z) be any point on the Line

Let $r_0 \rightarrow$ is the Position vector of point P_0

$r \rightarrow$ is the Position vector of point P

Definition

Then **vector equation of line** is given by

$\mathbf{r=r_0+vt}$

Where t is a scalar

Let $v=<a,b,c>$

$r=<x_0,y_0, z_0>$

Hence the **parametric equation of a line** is:-

Parametric Equations of a line

$<x,y,z>=<x_0+at, y_0 +bt, z_0 +ct>$

Hence we get,

$\mathbf{x= x_0+at}$

$\mathbf{Y= y_0 +bt}$

$\mathbf{Z= z_0 +ct}$

Symmetric Equations of Line

- Symmetric Equations of line is given by:-

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Example 1

Question:-Find the vector equation and Parametric equations of the line through (1,2,3) and parallel to vector 3i+2j-k

Solution:-Given v= i+2j+3k

r_0=<1,2,3>

=i+2j+3k

Hence the **vector equation of line** is

r= i+2j+3k+t(3i+2j-k)

=(1+3t)i+(2+2t)j+(3-t)k

Example 1(Continued)

Since r=xi+yj+zk

Hence we get **parametric equation of line** is:-

$$x=1+3t$$

$$y=2+2t$$

$$z=3-t$$

Example2

Question:-Find the symmetric equation for line through point(1,-5,6) and is parallel to vector <-1,2,-3>

Solution:-We know that symmetric equation of line is given by:-

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$$

$$Here$$

$$a=-1, b=2, c=-3$$

Example 2(Continued)

- Hence we get

$$\frac{x-1}{-1}=\frac{y+5}{2}=\frac{z-6}{-3}$$

Hence the result

Planes

- The plane in the space is determined by a **point and a vector that is perpendicular to plane**.
- Let $P(x_0,y_0,z_0)$be given point and n is the orthogonal vector.
- Let $P(x,y,z)$ be any point in space and r,r_0 is the position vector of point P and P_0 respectively.
- Then **vector equation of plane** is given by:-

Planes

- $n.r=n.r0$
- Let $n=\langle a,b,c\rangle$
- $r=\langle x,y,z\rangle$
- $r0=\langle x_0,y_0,z_0\rangle$
- Hence the **vector equation becomes**:-
- $\langle a,b,c\rangle. \langle x,y,z\rangle=\langle a,b,c\rangle. \langle x_0,y_0,z_0\rangle$
- Hence the **scalar equation of plane** is given by

Planes

- $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
- We can also write above equation of plane as:-
- $ax+by+cz+d=0$
- Where $d=-(ax_0+by_0+cz_0)$

Example 3

Question:-Find the equation of plane through point (1,-1,1) and with normal vector i+j-k

Solution:-Given point is (1,-1,1)

Here a=1,b=1,c=-1

We know that equation of plane is given by:-

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$1(x-1)+1(y+1)-1(z-1)=0$

$x+y-z+1=0$

Example 4

- **Question**:-Find the line of intersection of two planes $x+y+z=1$ and $x+2y+2z=1$
- **Solution**:-Let L is the line of intersection of two planes.

We can find the point where Line L intersects xy plane by setting $z=0$ in above two equations, we get:-

$$x+y=1$$

$$x+2y=1$$

Example 4(Continued)

- By solving for x and y we get,
- $x=1$
- $y=0$
- Hence the Point on Line L is (1,0,0)
- Line L lies in both planes so it is perpendicular to both normal vectore
- a=i+j+k and b=i+2j+2k
- Hence vector v which is given by
- **v=axb** is parallel to L

Example 4(Continued)

- Hence
- v=0i-j+k
- We know **that equation of line is given by**

$$\mathbf{x= x_0+at}$$

$$\mathbf{y= y_0 +bt}$$

$$\mathbf{z= z_0 +ct}$$

Hence we get,

- x=1,y=-t,z=2t

Example 5

- **Question**:-Find the equation of plane through the points (0,1,1),(1,0,1) and(1,1,0)
- **Solution**:-Let p (0,1,1)),q(1,0,1) andr(1,1,0)

  denotes the given points.

  Let $a=<1-0,0-1,1-1>=<1,-1,0>$

  And $b=<1-1,1-0,0-1>=<0,1,-1>$

  n=axb is the orthogonal vector of the plane.

  Hence **n=i+j+k**

Example 5 (Continued)

- Hence **the equation of plane through (0,1,1) and perpendicular to vector i+j+k** is:-
- $1(x-0)+1(y-1)+1(z-1)=0$
- $x+y-1+z-1=0$
- $\mathbf{x+y+z=2}$
- Hence the result

Practice Problems

- 1)Find the parametric equation of line through point(1,-1,1) and parallel to line

  $x+2=y/2=z-3$.

- 2)Find the equation of plane through points(3,-1,2),(8,2,4) and (-1,-2,-3)

- 3)Find the symmetric equation of line of intersection of planes

Practice Problems

$5x-2y-2z=1,$

$4x+y+z=6$

- 4)Find the point at which line

  $x=3-t, y=2+t, z=5t$

  Intersects the plane $x-y+2z=9$

Answer To Practice Problems

1) $x=1+t,$

   $y=-1+2t,$

   $z=1+t$

2) $-13x+17y+7z=-42$

3) $x=1, y-2=-z$

4) Point is (2,3,5)

Important Tips for Practice Problem

- For **Question** 1,direction number of required line is given by(1,2,1),since two parallel lines has same direction numbers.
- For **question** 2,see solved example 5
- For **question 3,** see solved example 4
- **For Question** 4,put the value of x,y,z in the equation of plane and then solve for t. After getting value of t, put in the equations of line you get the required point.