# Relationships between comminution energy and product size for a magnetite ore

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## Synopsis

An extensive laboratory grinding study was carried out on a magnetite ore in order to assess the grinding behaviour of magnetic concentrate and tail from low intensity magnetic separation (LIMS). The test work involved Bond ball mill testing, rod milling, low intensity magnetic separation (LIMS), and batch ball milling down to product sizes of around $P_{80}$~25 microns. A total of 18 Bond tests and over 150 batch grinding tests and sieve sizing were carried out. Throughout the grinding tests, power draw was continuously monitored. The relationship between the grinding energy and product size was analysed using the conventional energy-size concepts. It was found that the Rittinger equation fits the experimental data well. However, Bond's equation does not fit the experimental data well, and therefore a modified Bond equation was developed. Differences in grinding properties between the magnetic and non-magnetic component were analysed and compared to the bulk ore. It was found that grinding properties differ significantly and therefore separate grinding test work may be required for each grinding step in the magnetite ore beneficiation flowsheet.

Keywords: grinding, iron ores, particle size, modelling.

## Introduction

The relationship between the comminution energy and the product size obtained for a given feed size has been a researched extensively over the last century. Theoretical and empirical energy-size reduction equations were proposed by Rittinger (1867), Kick (1885) and Bond (1952), known as the three theories of comminution; and their general formulation by Walker *et al.* (1937). Finally, Hukki (1962) proposed the revised form of the general form of comminution and suggested that the energy-size relation is a combined form of these three laws.

## Energy-size relationships

Walker *et al.* (1937) proposed the following equation, for a general form of comminution.

$$dE = -C\frac{dx}{x^n} \qquad [1]$$

Where $E$ is the net specific energy; $x$ is the characteristic dimension of the product; $n$ is the exponent; and $C$ is a constant related to the material. Equation [1] states that the required energy for a differential decrease in size is proportional to the size change ($dx$) and inversely proportional to the size to some power $n$.

If the exponent $n$ in Equation [1] is replaced by the values of 2, 1 and 1.5 and then integrated, the well-known equations of Rittinger, Kick and Bond, are obtained respectively.

Rittinger (1867) stated that the energy required for size reduction is proportional to the new surface area generated. Since the specific surface area is inversely proportional to the particle size, Rittinger's hypothesis can be written in the following form:

$$E = K_1\left(\frac{1}{x_p}-\frac{1}{x_f}\right) \qquad [2]$$

where $E$ is the net specific energy; $x_f$ and $x_p$ are the feed and product size indices, respectively; and $K_1$ is a constant.

Kick (1885) proposed the theory that the equivalent relative reductions in sizes require equal energy. Kick's equation is as follows:

$$E = K_2\ln\left(\frac{x_f}{x_p}\right) \qquad [3]$$

where $E$ is the net specific energy; $x_f$ and $x_p$ are the feed and product size indices, respectively, and $K_2$ is a constant.

Bond (1952) proposed the 'Third Law' of grinding. The Third Law states that the net energy required in comminution is proportional to the total length of the new cracks formed. The resulting equation is:

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$$E = K_3\left(\frac{1}{\sqrt{x_p}} - \frac{1}{\sqrt{x_f}}\right) \qquad [4]$$

where $E$ is the net specific energy; $x_f$ and $x_p$ are the feed and product size indices, respectively, and $K_3$ is a constant.

Hukki (1962) evaluated these energy-size relationships stating that each of Rittinger, Kick and Bond theories might be applicable for different narrow size ranges. Kick's equation is applicable for crushing, Rittinger's equation may be used for finer grinding, and Bond's equation is applicable in the conventional milling range. Hukki postulated that the exponent $n$ in the Equation [1] is not constant; it is dependent on the characteristic dimension of the particle. The revised energy-size equation has the following form:

$$dE = -C\frac{dx}{x^{f(x)}} \qquad [5]$$

Application of Kick's and Rittinger's theories has been met with varied success and are not realistic for designing size reduction circuits (Charles, 1957). However Bond's Third Law can be reasonably applied to the range in which ball/rod mills operate in. In spite of the empirical basis of Bond's theory, it is the most widely used method for the sizing of ball/rod mills and has become more likely a standard.

The general form of Bond's equation is as follows:

$$W = W_i\left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right) \qquad [6]$$

where $W$ is the work input (kWh/t); $W_i$ is the work index (kWh/t) which expresses the resistance of the material to crushing and grinding; and $F_{80}$ and $P_{80}$ are the 80% passing size of the feed and the product ( m), respectively.

Bond developed a standard laboratory test for the determination of the work index for ball and rod mills in 1952 and modified it in 1961. The standard Bond ball mill grindability test is a closed circuit dry grinding test with a 250% circulating load. The standard procedural outline of the test is stated in (Bond) 1961. The work index for a ball mill, $W_i$, is then calculated from the following equation.

$$W_i = \frac{49}{P^{0.23}G_{bP}^{0.82}\left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right)} \qquad [7]$$

where $P$ is the closing sieve size ( m), $G_{bp}$ is the grindability (net g/rev.), $F_{80}$ and $P_{80}$ are the 80% passing size of the feed and the product ( m), respectively.

Recently the Bond equation was revised to improve energy predictions for SAG/ball mill circuits (Morrell 2004b).

The proposed equations follows Hukki's solution with variable exponent, function of particle size:

$$W = M_i 4\left(x_2^{f(x_2)} - x_1^{f(x_1)}\right) \qquad [8]$$

where $M_i$ is is the work index related to the breakage property of an ore (kWh/t); for grinding from the product of the final stage of crushing to a $P_{80}$ of 750 lm (coarse particles) the index is labelled $M_{ia}$ and for size reduction from 750  m to the final product $P_{80}$ normally reached by conventional ball mills (fine particles) it is labelled $M_{ib}$, $W$ is the specific comminution energy at pinion (kWh/t), $x_2$ is the 80% passing size for the product ( m) and $x_1$ is the 80% passing size for the feed ( m).

$$f(x_i) = -\left(0.295 + x_j/1000000\right) \quad \text{(Morrell 2006)} \qquad [9]$$

The work index related to the breakage property of an ore for size reduction from 750  m to the final product $P_{80}$ normally reached by conventional ball mills $M_{ib}$, is calculated from the standard Bond test data using the following equation:

$$M_{ib} = \frac{18.18}{P_1^{0.295}G_{bp}\left(P_{80}^{\ f(P_{80})} - F_{80}^{\ f(F_{80})}\right)} \qquad [10]$$

where $P_1$ is the closing sieve size ( m), $G_{bp}$ is the grindability (net g/rev.), and $F_{80}$ and $P_{80}$ are the 80% passing size of the feed and the product ( m), respectively.

## Experimental

The test work programme was carried out on eighteen drill core samples obtained from a magnetite orebody. The outline of the experimental procedure is represented in Figure 1. Samples were prepared by crushing below 3.35 mm and grinding to a $P_{80}$ size of around 500  m using a laboratory-scale rod mill followed by low-intensity magnetic separation (LIMS). Assay on major constituents were carried out on LIMS products.

Standard Bond ball mill grindability and batch grinding tests were carried out. The Standard Bond ball mill grindability tests were carried out on crushed (-3.35 mm) ore samples (18) using a 106 micron closing screen.

Batch grinding tests were conducted on all magnetite concentrates and tails (36 samples total) and in addition three ore samples. A laboratory scale ball mill (D x L = 203 x 250 mm) with a charge of 19 and 25 mm steel balls

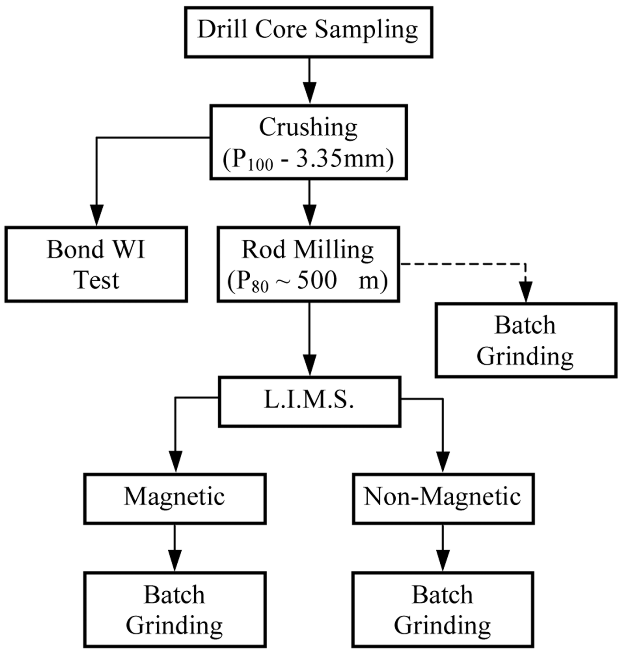

Figure 1—Outline of the experimental procedure

(50%–50%) was used for the test work. Samples were ground for 10, 30 and 45 minutes and the products were sized using wet and dry sieving down to 25  m. Prior to each test, the feed size distributions were determined as well. Throughout the batch grinding tests, the power draw of the mill was measured using a purpose built power meter to facilitate the calculation of specific grinding energy.

## Results

Table I shows the Bond ball mill work index obtained for the ore samples as well as the iron and silica content in concentrate and tail after LIMS. The concentrate weight recovery was in the order of 50–60%. Furthermore it may be observed that the Bond ball mill work indices for the ore ($W_i$) varies from 14.6 to 20.9 kWh/t.

The relationship between the ore hardness and iron content presented in Figure 2 indicates that the samples with a higher magnetite content (Fe%) are softer (lower bond work index), suggesting that the magnetite in the ore may be softer than the non-magnetic component. Based on that, one may conclude that LIMS concentrate (high % magnetite) may be softer and easier to grind than the LIMS tail (low % magnetite) and the original ore.

Conversely Figure 3, which illustrates the relationship between the product $P_{80}$ and the net specific grinding energy for all LIMS tail (non magnetic) and LIMS concentrate (magnetic) samples tested using the batch ball mill, indicates that at finer grind levels the LIMS tail (non magnetic) produces a finer $P_{80}$. This indicating that at a finer grind LIMS tail i.e. samples with low magnetite content, are in fact softer.

Figure 4 compares the batch ball mill test results for the hardest and softest ore samples and their LIMS products. It can be observed that for the 'soft' ore ($W_i$ = 15.2) samples (Figure 4 (a)), the non-magnetic (LIMS tail) fraction produces a significantly finer product than the bulk ore and the magnetic (LIMS concentrate) product at all grind sizes. For the 'hard' ore ($W_i$ =20.9) samples (Figure 4 (b)), this difference is significantly smaller. Also, ore and magnetic concentrate results are close for the hard and soft ore.

Figures 5 and 6 show the full sizing results from these tests and indicate that for the 'soft' ore, the non-magnetic fraction produce significantly more fines than the magnetic fraction and the bulk ore. Figure 7 shows that the magnetic component (LIMS concentrate) of the 'hard' and 'soft' ore have very similar product sizing, indicating similar grinding properties. On the other hand, it could be observed from Figure 8 that the non-magnetic component (LIMS tail) of the 'hard' ore is significantly coarser than the non-magnetic component of the 'soft' ore indicating more difficult (harder) material to grind.

The above analysis suggests that the ore hardness, expressed by the Bond ball mill work index, is controlled by the hardness of the non-magnetic (gangue) material in the

*Table I*

**Bond ball mill test results and composition of the LIMS product**

| | Ore | Concentrate | | Tail | |
|---|---|---|---|---|---|
| Sample # | Wi (kWh/t) | Fe% | $SiO_2$% | Fe% | $SiO_2$% |
| 1 | 16.0 | 49.1 | 27.5 | 23.6 | 57.0 |
| 2 | 15.2 | 47.3 | 28.2 | 10.7 | 72.7 |
| 3 | 16.3 | 48.4 | 28.1 | 21.0 | 59.8 |
| 4 | 15.3 | 49.4 | 27.4 | 24.0 | 57.4 |
| 5 | 18.3 | 41.3 | 34.7 | 20.2 | 54.4 |
| 6 | 17.1 | 49.8 | 26.8 | 18.9 | 63.8 |
| 7 | 16.0 | 51.0 | 25.0 | 18.5 | 64.0 |
| 8 | 16.0 | 47.3 | 26.9 | 13.9 | 62.8 |
| 9 | 15.2 | 44.5 | 31.9 | 17.8 | 63.9 |
| 10 | 15.3 | 47.9 | 28.0 | 17.9 | 59.6 |
| 11 | 20.9 | 42.5 | 33.6 | 15.1 | 67.8 |
| 12 | 19.8 | 42.0 | 35.3 | 13.2 | 71.0 |
| 13 | 14.6 | 45.1 | 31.1 | 17.1 | 64.6 |
| 14 | 19.1 | 46.2 | 29.2 | 17.2 | 64.1 |
| 15 | 18.5 | 48.7 | 26.9 | 13.4 | 67.3 |
| 16 | 18.7 | 48.5 | 27.4 | 18.8 | 59.3 |
| 17 | 19.9 | 45.4 | 29.2 | 14.5 | 67.7 |
| 18 | 17.8 | 45.7 | 29.2 | 20.1 | 61.1 |

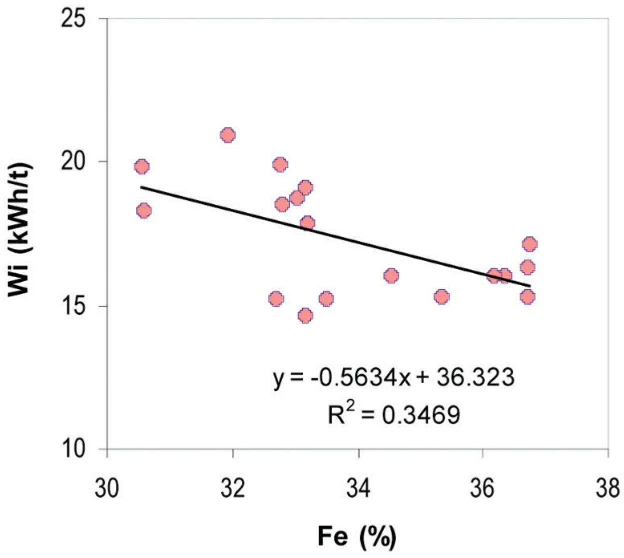

Figure 2—Relationship between iron (magnetite) content and ore hardness

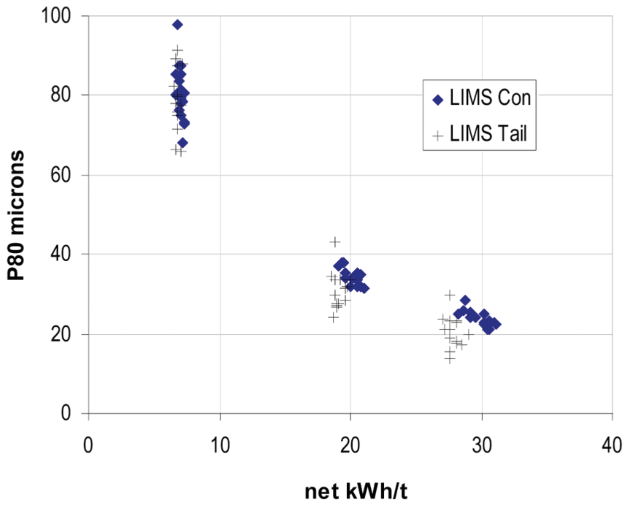

Figure 3—Relationship between $P_{80}$ and net specific grinding energy—batch ball mill tests on LIMS concentrate and tail

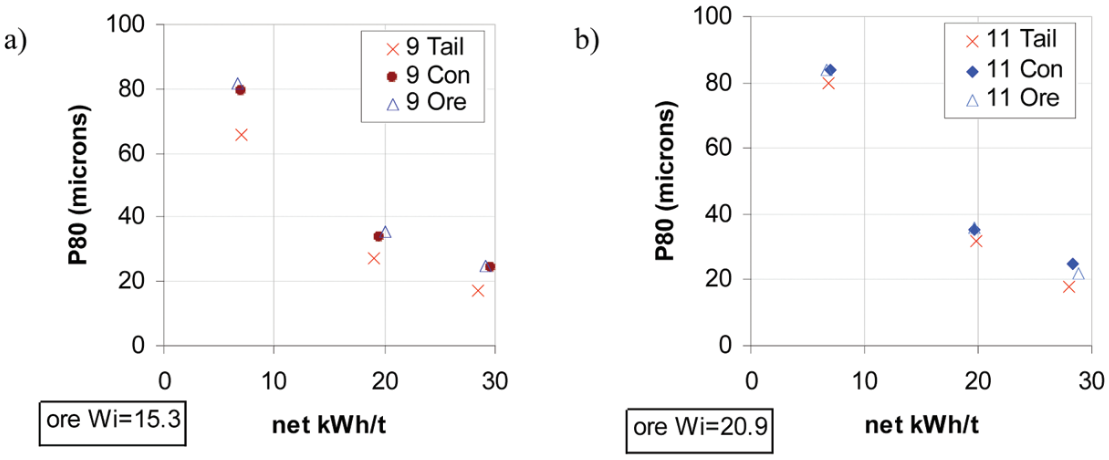

Figure 4—Relationship between $P_{80}$ and net specific grinding energy for (a) 'soft' and (b) 'hard' sample

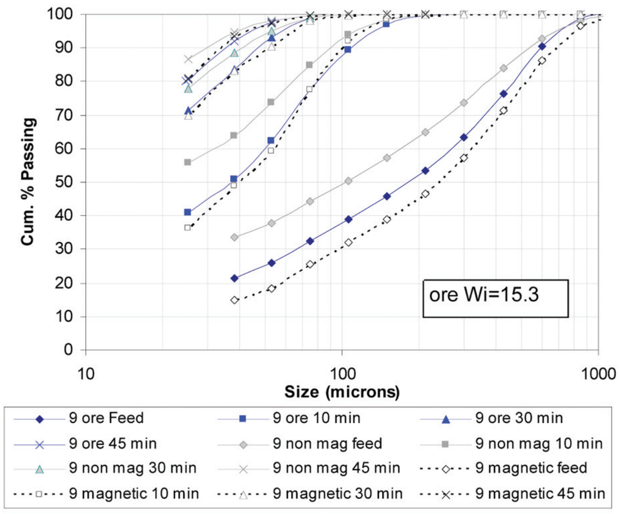

Figure 5—Product sizing comparison for 'soft' ore fractions, $W_i$ = 15.3 kWh/t

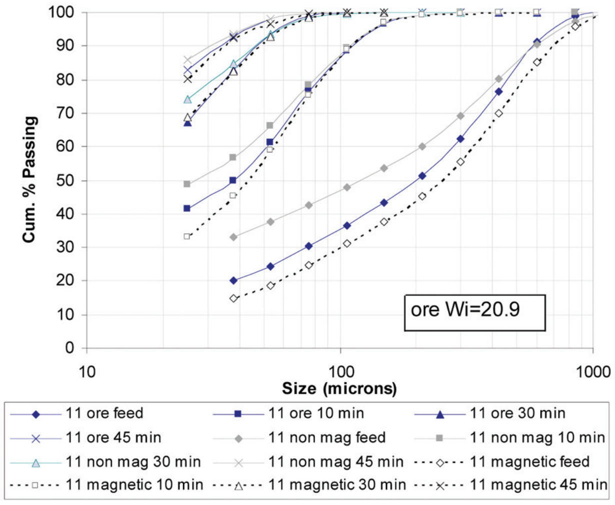

Figure 6—Product sizing comparison for 'hard' ore fractions, $W_i$ = 20.9 kWh/t

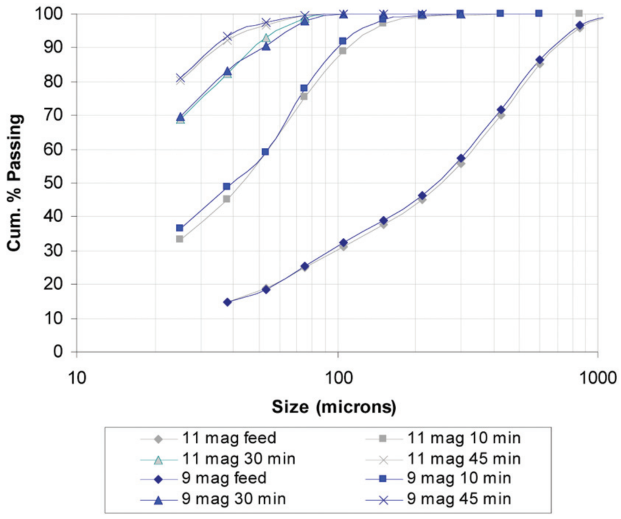

Figure 7—Product sizing for magnetic fractions from 'hard' (11) and 'soft' (9) ores

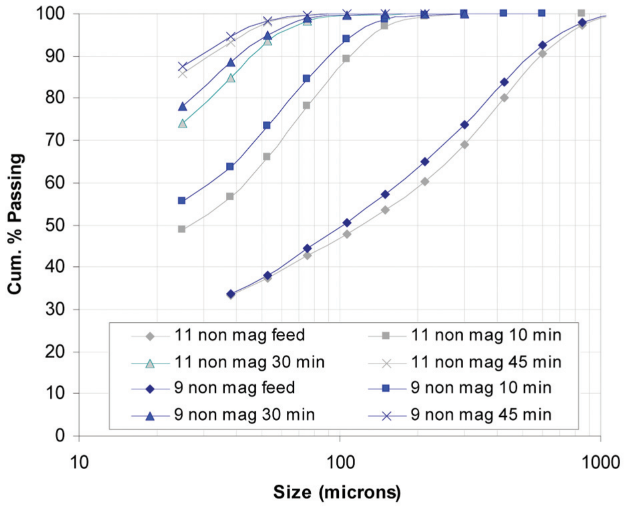

Figure 8—Product sizing for non magnetic fractions from 'hard' (11) and 'soft' (9) ores

ore. Ores with a higher magnetite content (%Fe) appear to be associated with softer gangue material, leading to a lower Bond ball mill work index $W_i$. At finer grinds, $P_{80}$<40 microns, magnetite appears to be harder than gangue.

The whole issue could be also influenced by increased 'heterogeneity' of the ore with increased magnetite content. There are more 'boundaries' between magnetic and non-magnetic phases that are weaker than bonds inside the phases.

Similar batch grinding results for the ore and the magnetic fraction obtained in this study (Figure 4) may suggest that the Bond ball mill work index determined for the ore samples could be used for design of a magnetic concentrate milling stage after the first magnetic concentration stage. This conclusion may be ore specific and needs to be confirmed for other magnetite ores.

## Batch grinding test results fitting

Experimental results were first predicted using the Bond equation and the ore Bond work index.. It can be observed from Figure 9 that using the Bond method, the required specific energy for around 80 microns grind size is consistently overpredicted, mostly overpredicted around 35 microns and mostly underpredicted around 20 microns product size. From this observation it appears that the exponent of the Bond equation (-0.5) is not accurate for prediction of the whole range of test work results. A significantly better fit was obtained using the Rittinger (exponent -1) formula. In the Rittinger equation the material constant was fitted. Figures 9 and 10 show the model fits for the magnetic and non-magnetic fractions using the Rittinger model. For the magnetic fraction the predicted values almost overlap the experimental data. For the non-magnetic fraction values were less accurate (see Figure 10). It may be noted that the spread of model predictions is wider than the experimental data; however, there is no indication of systematic error. Thus it may be concluded that the Rittinger model could be used to predict the grinding pattern of non-magnetic fractions as well, although with less accuracy. Some imperfections in experimental data may have contributed to apparent reduced model accuracy.

The methodology proposed by Morrell was also tested in the context of this work; however, its application was unsuccessful mainly do to equation exponent -0.3. As it was explained in the above discussion, the best fit of experimental data was obtained with exponents close or equal to -1. Therefore, Morrell's method is not applicable for batch grinding and may not extend beyond conventional SAG/ball mill grinding applications.

## Conclusion

Extensive comminution test work was carried out on eighteen drill core samples obtained from a magnetite orebody in order to assess the grinding behaviour of magnetic and non-magnetic components. The following observations were made:

- The relationship between the ore Bond work index and magnetite (or silica) content cannot be used to assess the relative grindability of the magnetic and non-magnetic fractions of the ore. This trend indicates a reduction in the Bond work index with an increase in magnetite content, suggesting that magnetite is the 'softer' component. In contrast, batch grinding tests with magnetic and non magnetic fractions showed that the magnetic fraction requires more energy for grinding to a particular grind size $P_{80}$ in the range of 80–25 microns. Therefore, to accurately assess the grinding energy requirement for the magnetic ore fraction after magnetic separation, additional grinding test work on the magnetic concentrate is required
- The Bond work index for the magnetite ore samples appears to be controlled by the properties of the gangue (non-magnetic) material. Samples with a higher magnetic content appeared to have a softer gangue
- The Bond 'law' and Morrell's method cannot be used to predict batch fine grinding results for the ore and ore components in fine grinding range. Rittinger's law predicts the batch fine grinding results for ore and ore components well
- Therefore, to assess the grinding energy requirement for fine grinding the magnetic ore fraction after magnetic separation, additional grinding test work on

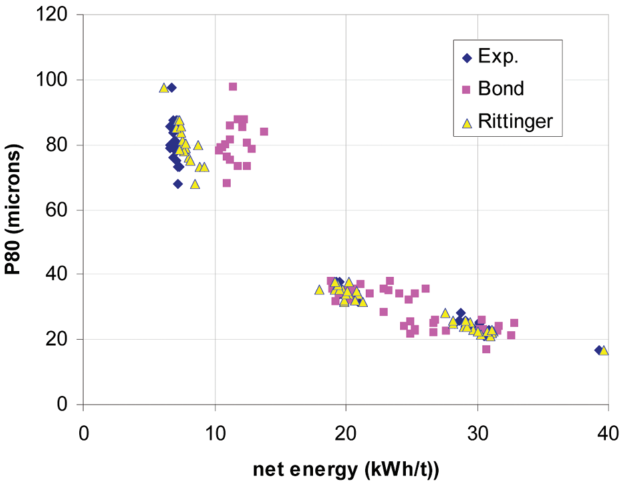

Figure 9—Bond model and Rittinger model fit—magnetic fraction

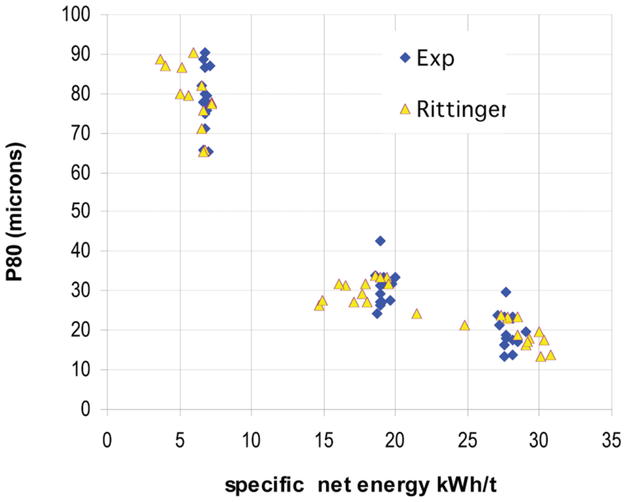

Figure 10—Rittinger model fit—non-magnetic fraction

magnetic concentrate is required. If the batch grinding results for the ore and the magnetic fraction are similar as it was the case in this work, the ore Bond work index may be used for design of a magnetic concentrate milling stage after the first magnetic concentration stage. This conclusion may be ore specific and needs to be confirmed for other magnetite ores.

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