Appendix E

THE TWO-METER WATTMETER METHOD

A discussion on why the two-wattmeter method works on a three-phase, three-wire load regardless of how the load is internally connected

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Monitoring a three-phase, three-wire connected load requires a two-element wattmeter or transducer and not a three-element.

A three-element wattmeter or transducer may be used to monitor a three-wire connected load but a two-element will do the job just as accurately and at lower cost.

THE BASICS

KIRCHOFF'S LAWS

Current: The algebraic sum of the current in all lines of a circuit must at any given instant be zero.

Voltage: The algebraic sum of the voltages among the nodes of a circuit must at any given instant be zero.

Kirchoff's laws tell us the following about a three-wire circuit:

1) If two of the three currents are known, the third must be equal to the sum of the other two but opposite in direction or sign. Thus, if one measures the instantaneous current in two branches of a three-wire circuit, one can determine the instantaneous value of the third.

2) If two of the three voltages are known, the third must be equal to the sum of the other two but opposite in direction or sign. Thus, if one measures the instantaneous voltage between two pairs of lines, one can determine the instantaneous value of the third pair.

From these two laws one can infer that measuring two of the currents and two of the voltages in a three-wire circuit will be sufficient to measure the total power.

Multiconductor or polyphase power measurement is based on a paper delivered by Blondel at the International Electric Congress in Chicago, 1893.

Blondel's Theorem

"The total power delivered to a load system by means of *n* conductors is given by the algebraic sum of the indications of *n* wattmeters so inserted that each of the *n* wires contains one wattmeter current-coil, its potential coil being connected between that wire and some point of the system in common with all the other potential coils; if that common junction of all the potential leads is on one of the *n* wires, the total power is obtainable from the indications of *n*-1 wattmeter elements."

More simply stated, in any power system with n wires, n-1 wattmeters (or elements) are required to measure the total power. For a three-phase, four-wire system three elements are required. For a three-phase, three-wire system two elements are required.

The Bottom Line

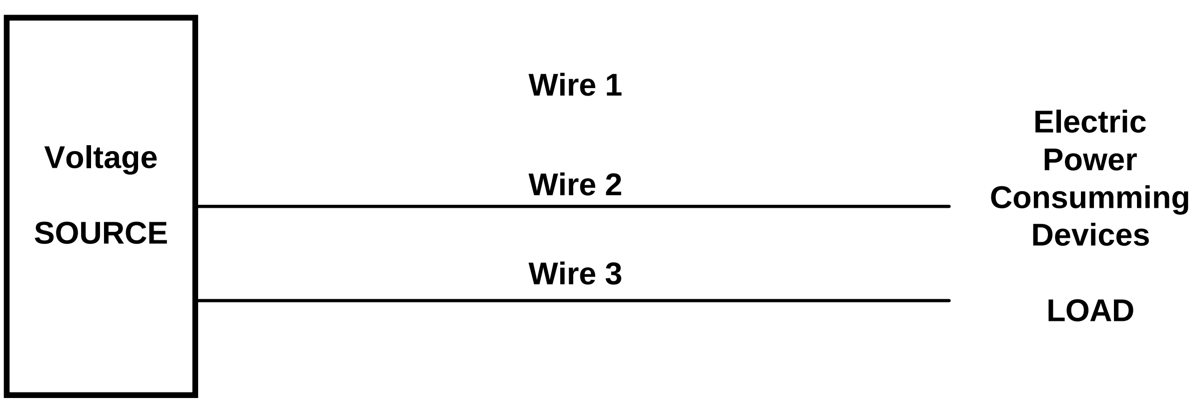


Figure 1

1) Given a voltage source of any internal complexity.
2) Given a load of any internal complexity.
3) Power drawn by the load from the source can be measured by placing a current measuring coil or device on two of the three wires connecting the voltage source and the loads and placing the voltage measuring coils or devices on those same two lines to the third line. In a three wire connected system, a two-element wattmeter properly measures the power transfer.

THE TWO WATTMETER METHOD

Many users of power monitoring equipment believe that it is necessary to measure all three pairs of line to line voltages and all three-phase currents to properly measure power consumed by a three-wire load. This is not necessary. By measuring the current in two of the lines and the two voltages from those two lines with respect to the third line, one can accurately measure total power regardless of unbalance of voltage, current, or load and regardless of variations in power factor. The method of measuring two of the currents and two of the voltages is known as the **two-wattmeter method**.

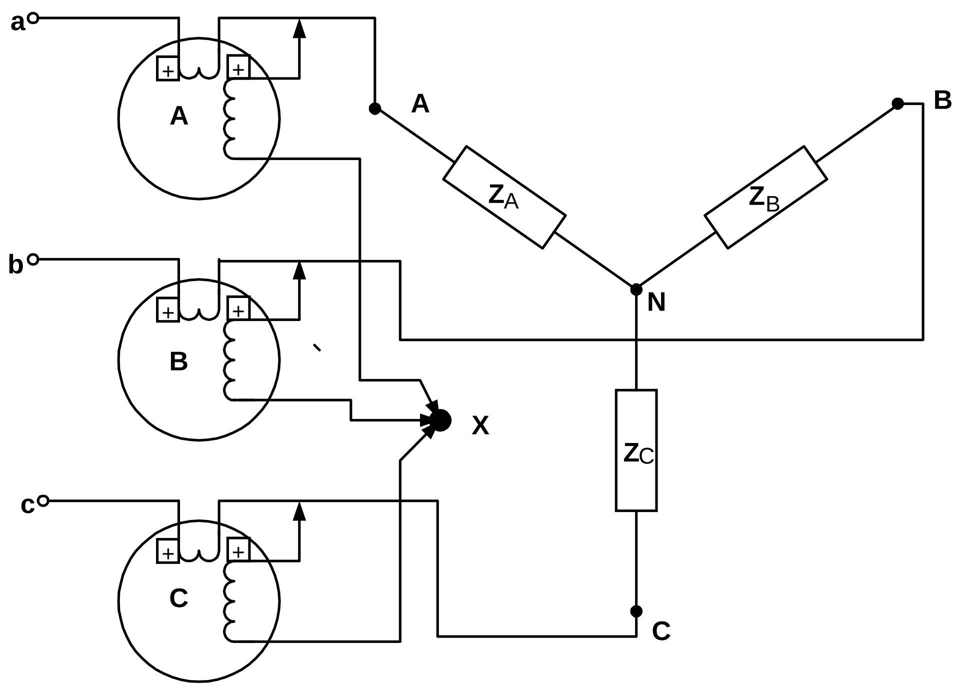


Figure 2 Three wattmeters connected to monitor power taken by a three-phase load.

The following is a mathematical proof of why the two-wattmeter method works on a three-wire system.

Figure 2 shows three wattmeters connected to a three-phase load with a voltage coil in each connected to a phase line and the other side of the voltage coils connected to a common point X as illustrated above. The point X is an unspecified point in the three-phase system or it may be an arbitrary point in space. The average power indicated by wattmeter A is:

$$P_A := \frac{1}{T} \cdot \int_0^T v_{AX} \cdot i_A \, dT$$

T is the period of all the source voltages. The readings for the other two wattmeters are given by similar expressions. The total power measured by all three wattmeters is:

$P = P_A + P_B + P_C$

$$P := \frac{1}{T} \cdot \int_0^T \left(v_{AX} \cdot i_A + v_{BX} \cdot i_B + v_{CX} \cdot i_{CX} \right) dt$$

Each of the three voltages in the above expression may be written in terms of a phase voltage and the voltage displacement between point X and the neutral N.

$v_{AX} = v_{AN} + v_{NX}$
$v_{BX} = v_{BN} + v_{NX}$
$v_{CX} = v_{CN} + v_{NX}$

Thus total power can be expressed as:

$$P := \frac{1}{T} \cdot \int_0^T \left(v_{AX} \cdot i_A + v_{BX} \cdot i_B + v_{CX} \cdot i_{CX} \right) dt + \left(\frac{1}{T} \right) \cdot \int_0^T v_{NX} \cdot \left(i_A + i_B + i_C \right) dt$$

But Kirchhoff's Law of current tells us that the sum of the currents in a circuit must be zero (0) so:

$$i_A + i_B + i_C := 0$$

The second integral reduces to zero and therefore we end up with the expression we expect if the point X were placed on the neutral:

$$P := \frac{1}{T} \cdot \left[\int_0^T \left(v_{AX} \cdot i_A + v_{BX} \cdot i_B + v_{CX} \cdot i_{CX} \right) dt \right]$$

Recall that " The point X is an unspecified point in the three-phase system or it may be an arbitrary point in space."

Since X is an arbitrary point in space, we can place it anywhere including one of the points A B or C. **If we place point X on point C of figure 2 then wattmeter C reads zero and the sum of wattmeter A and wattmeter B will be total power.**

EXAMPLES

The graphs on the following pages illustrate the use of both a three-element watt transducer and a two-element watt transducer measuring the power used by the same load.

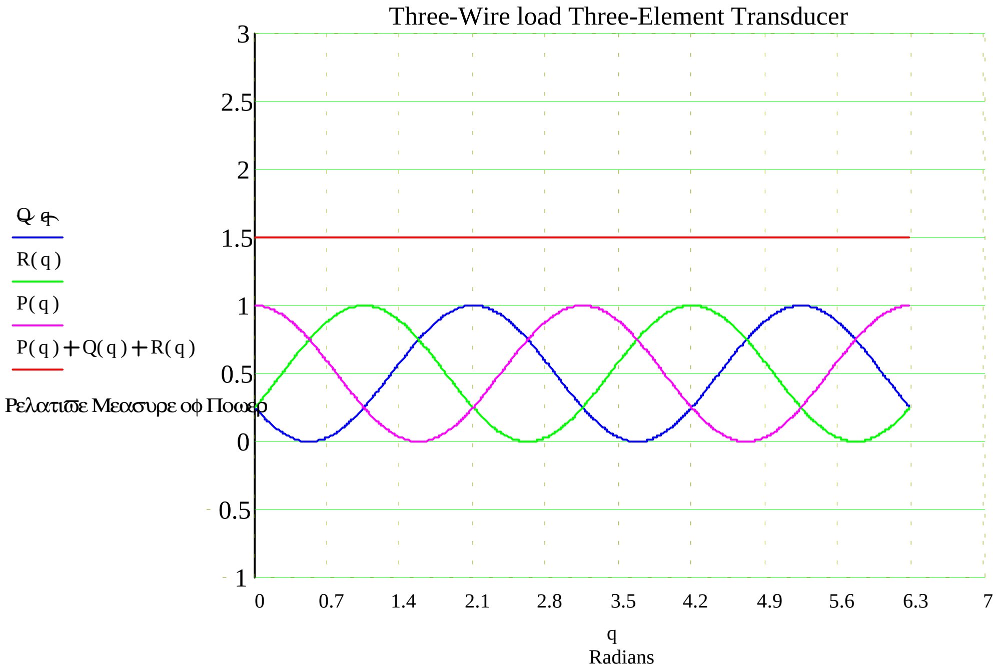
These graphs show the instantaneous outputs from each element and the sum of the elements. Note that in all cases the sum of the individual elements is the same for both the three-element and two-element watt transducers.

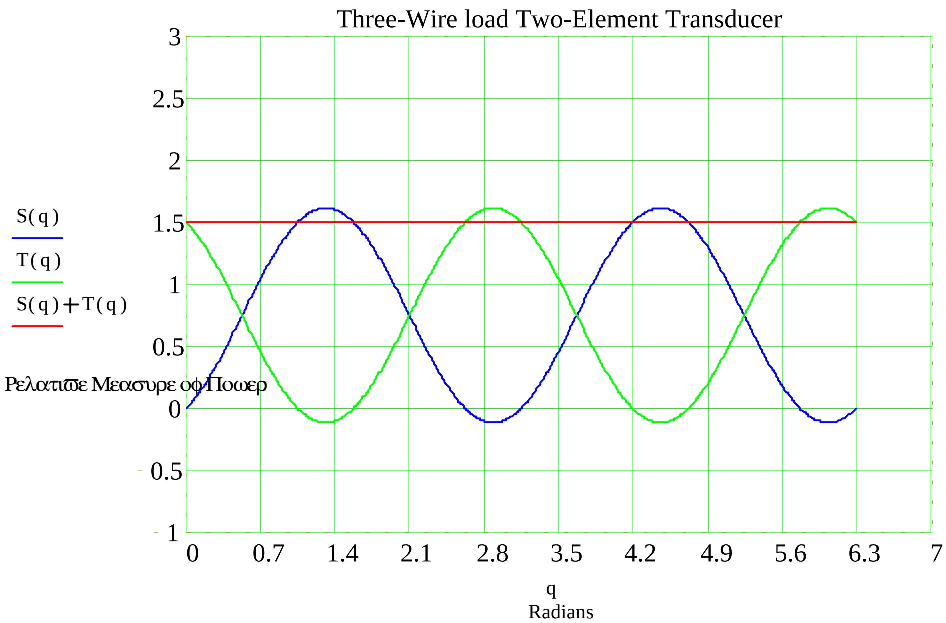
The graphs depict power as measured over one complete cycle of the voltage applied to the load. To keep the illustrations simple I used radian measure. $2\pi = 360°$ (2π rounds to 6.3 radians.)

The five sets of graphs illustrate:

1) Unity power factor. The voltage and current are in phase.
2) 0.866 power factor. The current lags the voltage by 30°.
3) 0.707 power factor. The current lags the voltage by 45°.
4) 0.500 power factor. The current lags the voltage by 60°.
5) 0 power factor. The current lags the voltage by 90°.

Graph Set 1 — Unity Power Factor





For all of the graphs representing the Three-Element Transducer:

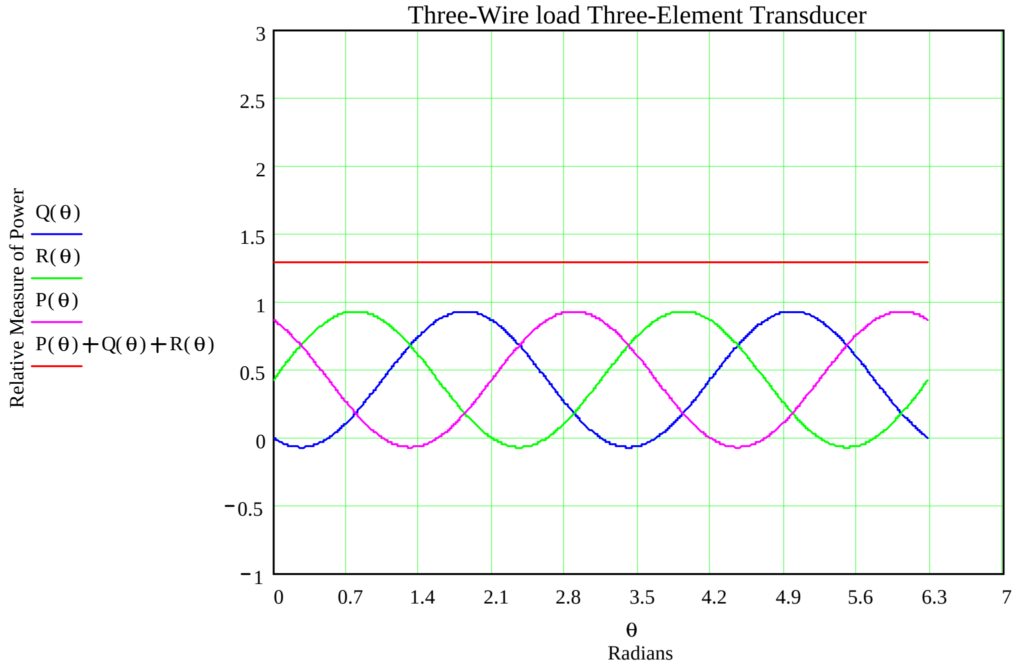
P(θ) is the result of *v*(AN) X *i*(A)
Q(θ) is the result of *v*(BN) X *i*(B)
R(θ) is the result of *v*(CN) X *i*(C)

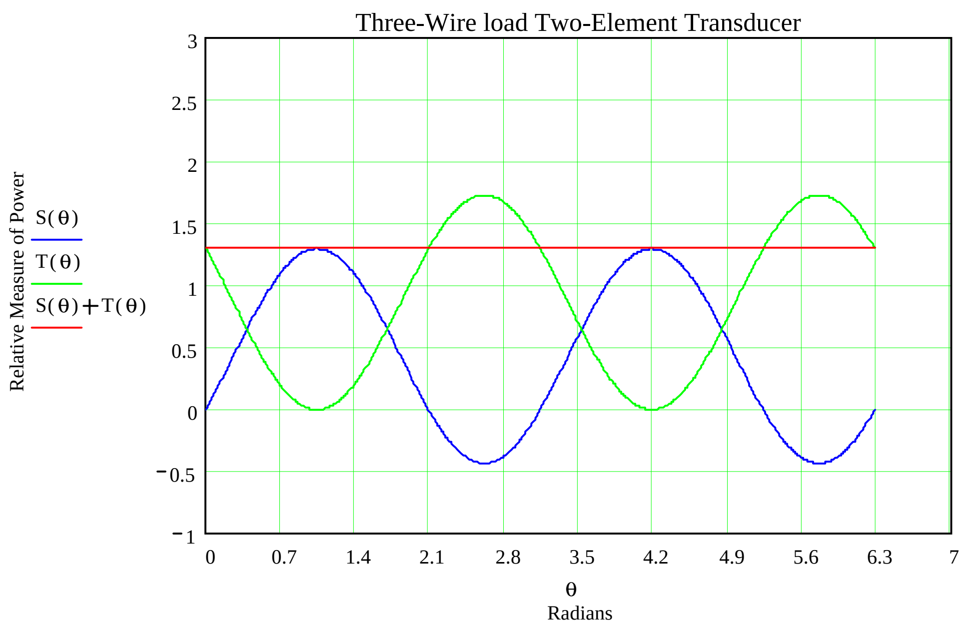
For all of the graphs representing the Two-Element Transducer:

S(θ) is the result of *v*(AC) X *i*(A)
T(θ) is the result of *v*(BC) X *i*(B)

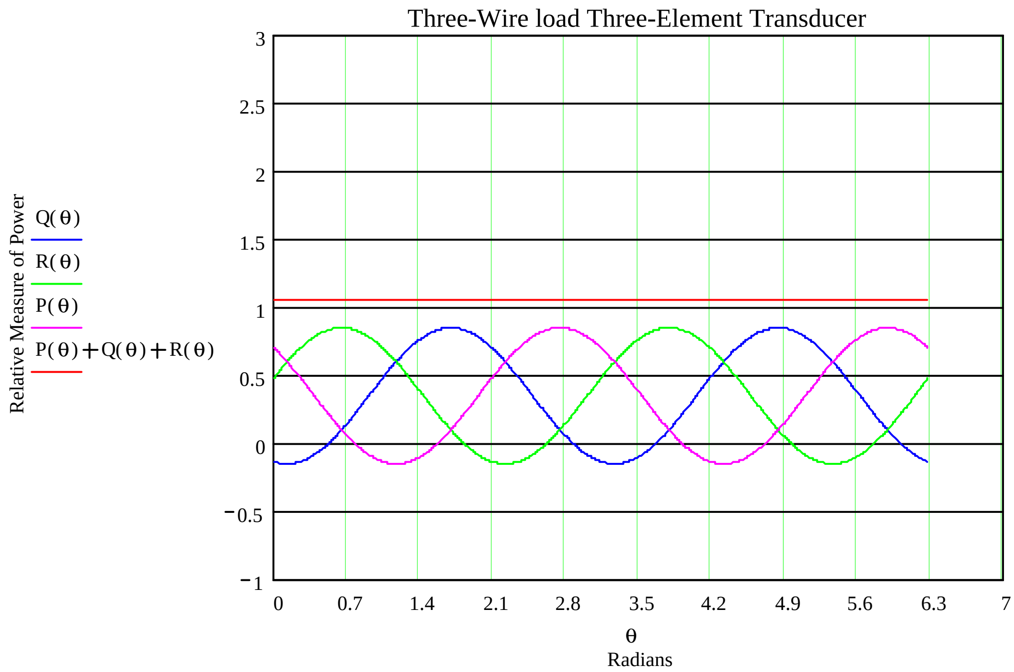
Lower case *i* and *v* represent instantaneous values of current and voltage.

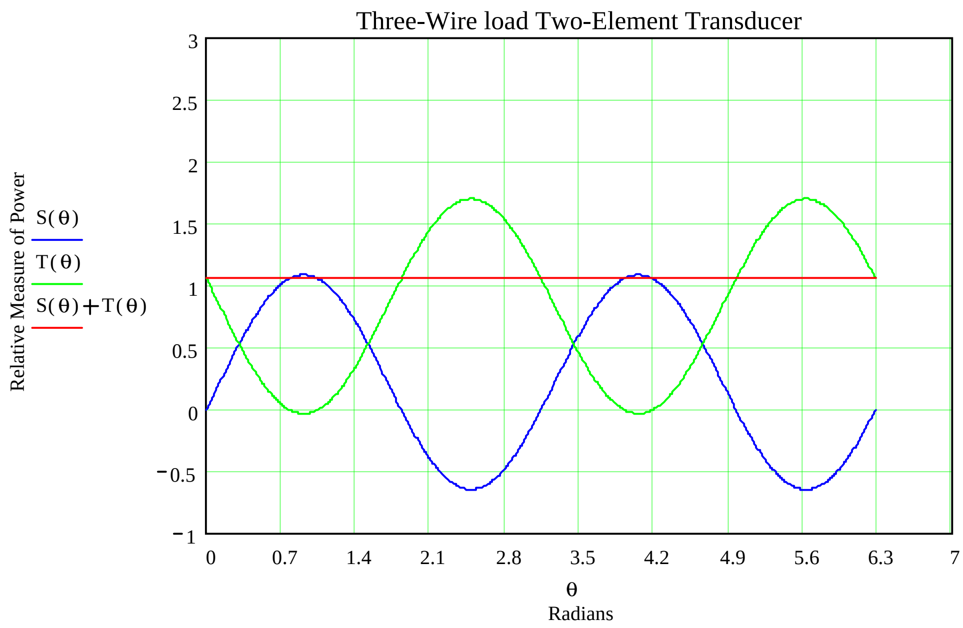
Graph Set 2 — 0.866 Power Factor. Typical for a fully loaded Induction Motor.



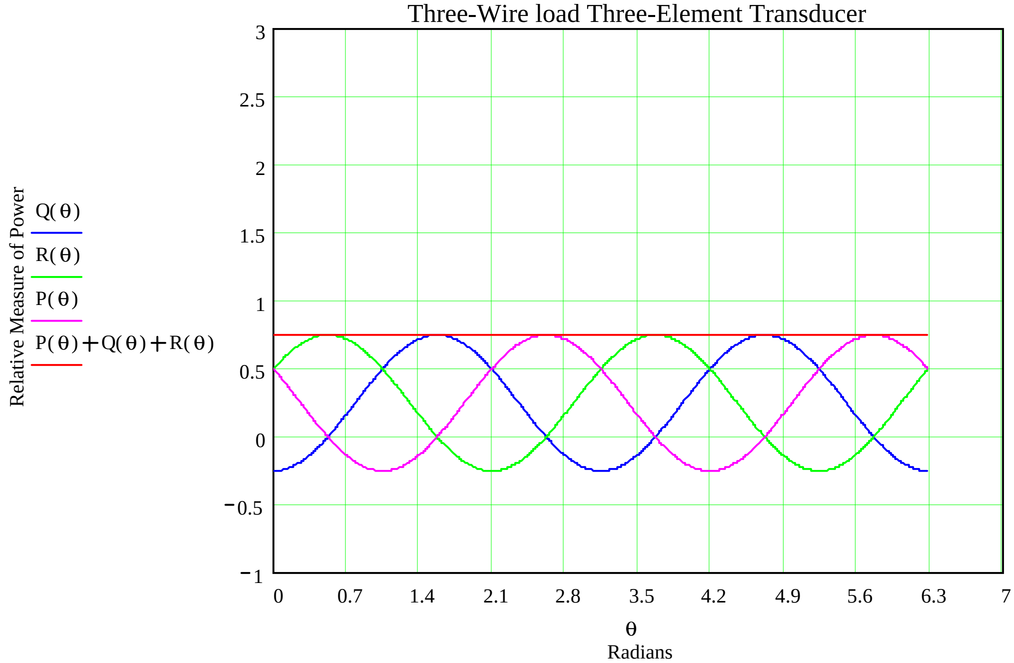


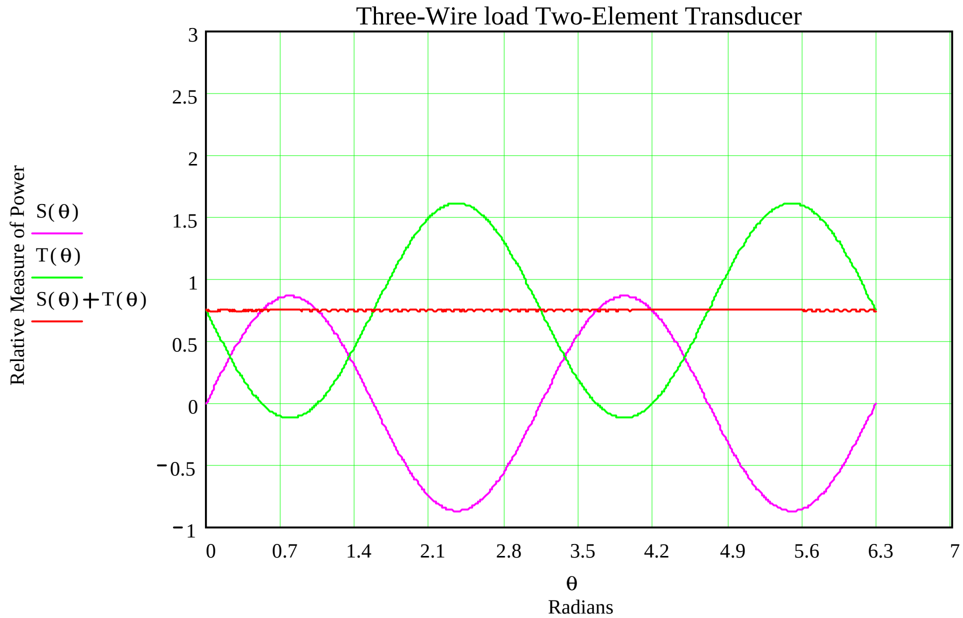
Graph Set 3 — 0.707 Power Factor.



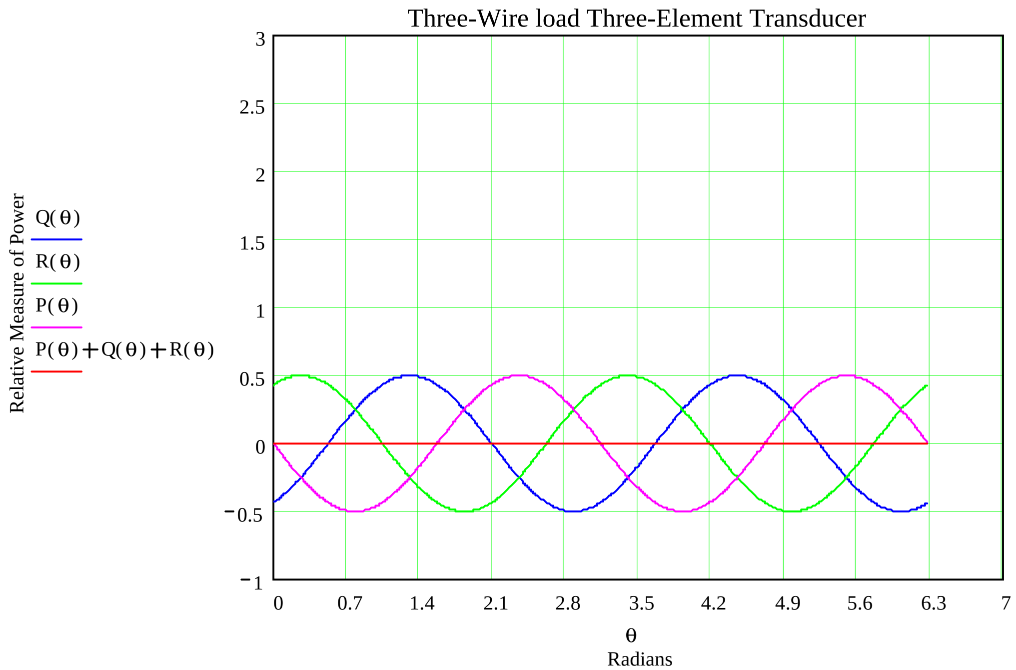


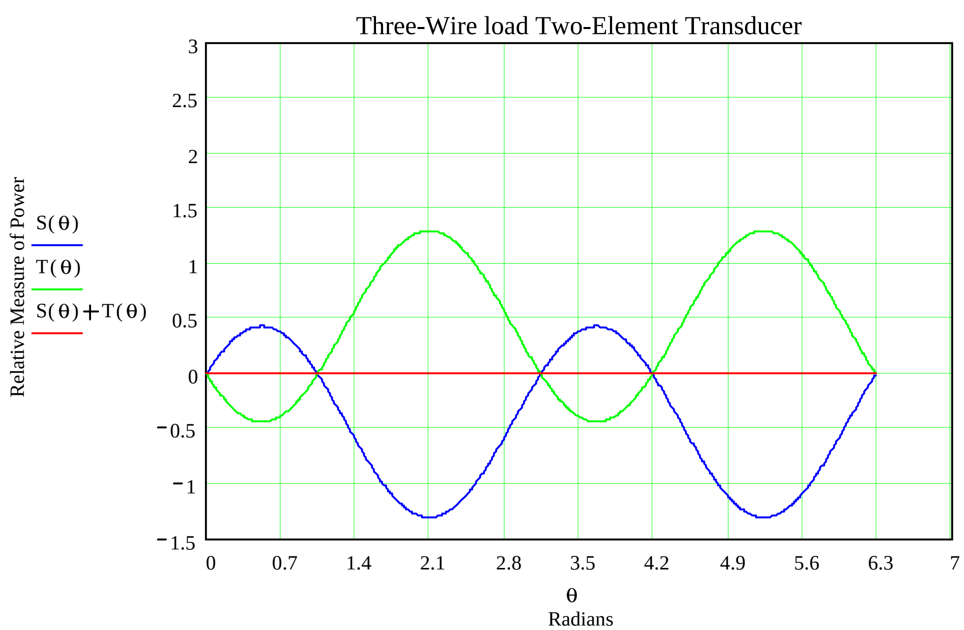
Graph Set 4 — 0.500 Power Factor.





Graph Set 5 — 0 Power Factor. No power is used by the load even though it is drawing current and is supplied with voltage.





This paper on the two-wattmeter method was prepared by William D. Walden at Ohio Semitronics, Inc. using Mathcad 8 for the mathematical formulas and graphs for the customers of Ohio Semitronics, Inc.

Other important facts that can be read from the graphs:

- Note that the total power drawn by the load is always constant even though the phase power varies with time or angular displacement. This is a very important reason for using a three-phase voltage source for electric motors — the power drawn by a three-phase motor is constant; therefore, the power output from that motor is constant.

- When using the two-wattmeter method note that the power drops below zero. This is because the voltage leads the current on one element by 30° and lags the current by 30° on the other element. Because of this one cannot use a single element watt transducer or meter to obtain a useful measure of power. Two must be used. The sum will be correct.

- When the power factor is less than unity (1), each of the three individual phase power readings will drop below zero. They are in fact feeding power back into the supply line. Low power factor causes higher effective current than the same load at unity (1) power factor. This increase in current results in unnecessary losses in the transmission lines and transformer substations. For this reason electric utility companies often charge a penalty in dollars for customers who operate below some power factor such as 0.8.

- At zero power factor the total power consumed by the load is zero. While current is drawn and voltage is applied, no power is actually consumed. Since power is a measure of the rate at which work is being done, the load is doing no useful work. Think about this when measuring the power used by an unloaded motor — **it is not doing any work** — the power consumed by that motor only represents work done to over come bearing and air friction and that lost to heat. An unloaded induction motor will have a power factor that is close to zero.

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