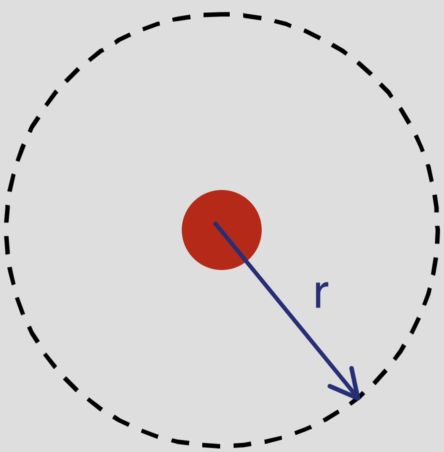
The Eddington Limit

Radiation is important:

- *inside* stars, as a source of **energy transport**
- *outside* stars and other sources, from its effect on surrounding gas

Consider force which photons exert on surrounding gas when a fraction of them are absorbed:



Source of radiation:

- Luminosity L
- Spherically symmetric emission
- Energy flux at distance r: $L / 4\pi r^2$
- Each photon has momentum p=E/c
- Momentum flux: $L / 4\pi c r^2$

If the source is surrounded by gas with opacity κ, then in traveling a distance ds the fraction of radiation absorbed is:

$$\frac{dI}{I} = -\kappa \rho\, ds$$

column density of gas

Can therefore interpret κ as being the **fraction** of radiation absorbed by unit column density of gas. Force exerted by radiation on that gas is then:

$$f_{rad} = \frac{\kappa L}{4\pi c r^2}$$

outward force

Force due to gravity on that gas (unit mass):

$$f_{grav} = \frac{GM}{r^2}$$

inward, toward star of mass M

Radiation pressure balances gravity when:

$$f_{rad} = f_{grav}$$

$$\frac{\kappa L}{4\pi c r^2} = \frac{GM}{r^2}$$

$$L = \frac{4\pi c GM}{\kappa}$$

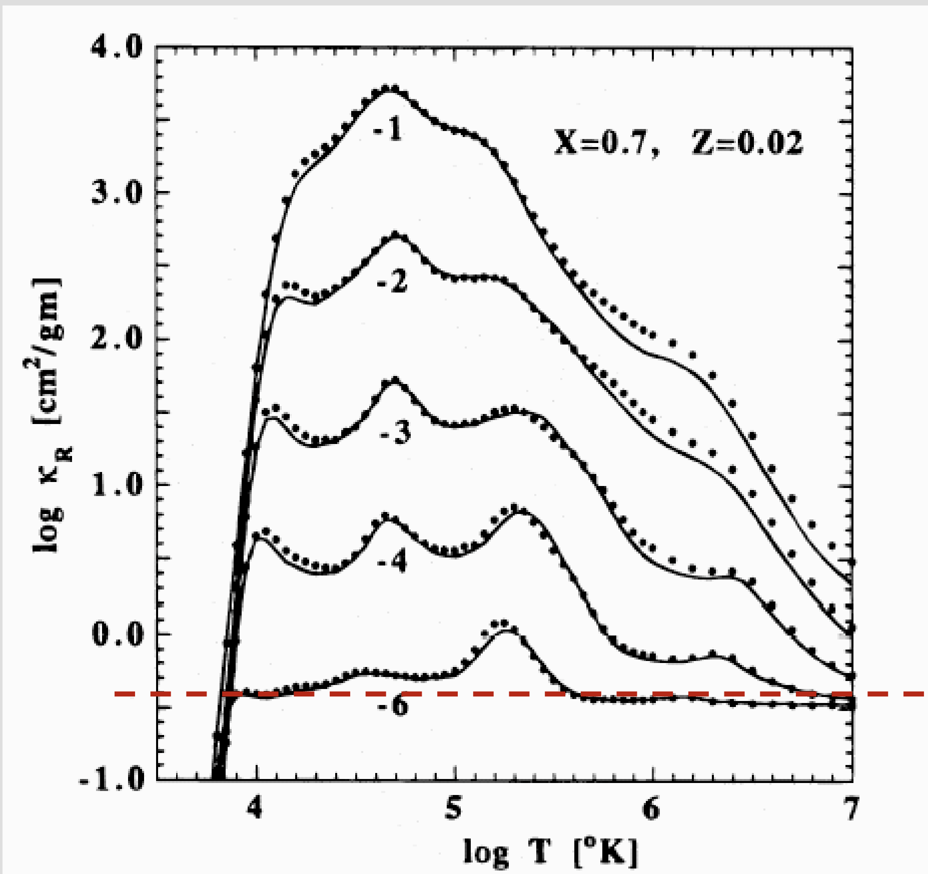
If L is larger than this value the pressure due to radiation exceeds the gravitational force at all radii, and gas will be blown away.

Critical luminosity is called the **Eddington limit**. Depends upon:

- the mass of the star
- the opacity of the gas surrounding the star / source

How large is the opacity?

Opacity depends upon temperature, density, and composition. For a fully ionized gas scattering of photons by free electrons (**Thomson scattering**) provides an opacity of $\kappa = 0.4\ \mathrm{cm^2\ g^{-1}}$, independent of frequency.



Good estimate at high T, at lower T opacity is usually **higher** because of bound-free and bound-bound atomic processes…

electron scattering opacity value

Using the value for the opacity appropriate for Thomson scattering:

$$L_{Edd} = \frac{4\pi cGM}{\kappa}$$

$$= 6.3 \times 10^{4} \left(\frac{M}{\mathrm{g}}\right) \mathrm{erg\ s^{-1}}$$

$$= 1.25 \times 10^{38} \left(\frac{M}{M_{sun}}\right) \mathrm{erg\ s^{-1}}$$

$$= 3.2 \times 10^{4} \left(\frac{M}{M_{sun}}\right) L_{sun}$$

Only important for sources that are much more luminous that the Sun.

So… why is there a Solar wind?

Applications: 1) Massive stars

A rough formula for the luminosity of very massive stars immediately after formation (`zero-age main sequence') is:

$$\frac{L}{L_{sun}} \approx 1.2 \times 10^{5} \left(\frac{M}{30\ M_{sun}} \right)^{2.4}$$

Using M_{sun}=1.989 x 10^{33} g and L_{sun}=3.9 x 10^{33} erg s^{-1}:

$$L = 1.6 \times 10^{-45} M^{2.4} \text{ erg s}^{-1}$$ (with M in grams)

Compare with formula for Eddington limit:

$$L_{Edd} = 6.3 \times 10^{4} M \text{ erg s}^{-1}$$

L = L_{Edd} for M = 2.6 x 10^{35} g ~ 130 Solar masses

Radiation pressure is an important effect for massive stars.

Implications / speculations

a) The most massive stars known have masses of around 100 Solar masses - perhaps radiation pressure sets the limit to how massive a star can form?

   Problems: stars have a range of masses, with massive stars being rare (therefore distant). Observation is rather uncertain.

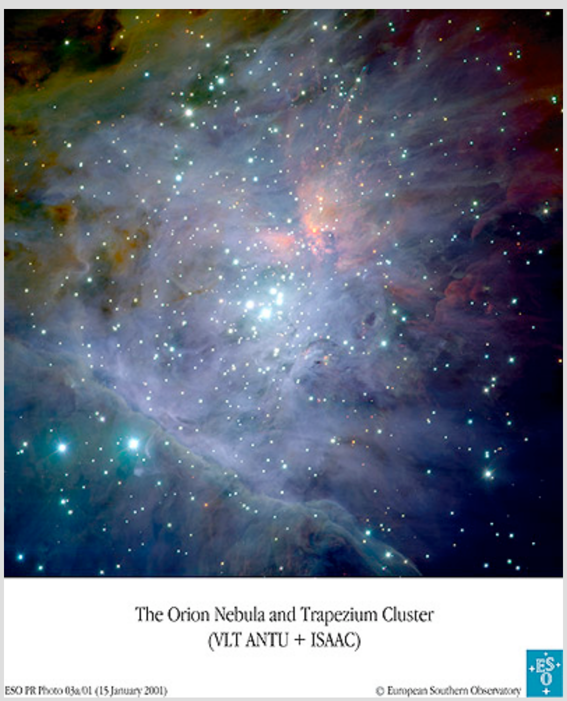
b) Stars today form out of gas that also contains dust, so the opacity is larger than the Thomson value. Radiation pressure is therefore important for less luminous (less massive) stars too.

c) Stars don’t form from spherically symmetric collapse, so the `limit’ can be evaded.

d) Perhaps massive stars don't form from one collapsing cloud, but instead form from **collisions** of smaller stars?

Idea: stars today never collide, but collisions would be more frequent:

- In young clusters where stars form, which are much denser than the Galaxy in the Solar neighborhood.
- Young stars have disks, so they present a larger cross-section for collisions.



e.g. the core of the Orion nebula cluster

Speculative idea which has not yet been tested…

Applications: 2) Feeding black holes

Gas flowing toward black holes produces radiation as the gravitational potential energy is released. Write this as:

$$L_{accretion} = \epsilon \dot{M} c^2$$

accretion luminosity

radiative efficiency of the accretion process = the *fraction* of the rest mass energy of the gas that is radiated

gas inflow rate (the accretion rate): units g s^{-1}

For a black hole accreting matter through a disk, the radiative efficiency $\epsilon = 0.1$ or thereabouts.

Setting the accretion luminosity equal to the Eddington limit gives us the **maximum** rate at which a black hole can accrete gas:

$$L_{Edd} = L_{accretion}$$

$$\frac{4\pi cGM}{\kappa} = \varepsilon \dot{M} c^2$$

$$\dot{M} = \frac{4\pi G}{\varepsilon \kappa c} M = 1.4 \times 10^{18} \left(\frac{M}{M_{sun}} \right) \text{g s}^{-1}$$

assume:

- $\kappa = 0.4$ cm^2 g^{-1}
- $\varepsilon = 0.1$

In alternative units, can write this as:

$$\dot{M} = kM = 2.2 \times 10^{-8} \left(\frac{M}{M_{sun}} \right) M_{sun} \text{ yr}^{-1}$$

a constant

How fast can a black hole grow?

Assume that a black hole grows as fast as it can - always at exactly the Eddington limit. Then:

$$\dot{M} = kM$$

$$\frac{dM}{dt} = kM$$

$$\int \frac{dM}{M} = \int k\,dt \Rightarrow M = M_0 e^{kt}$$

M_0 is the mass of the hole at the initial time t = 0

Substituting the value we derived previously for k:

k = 7 x 10^{-16} s^{-1}

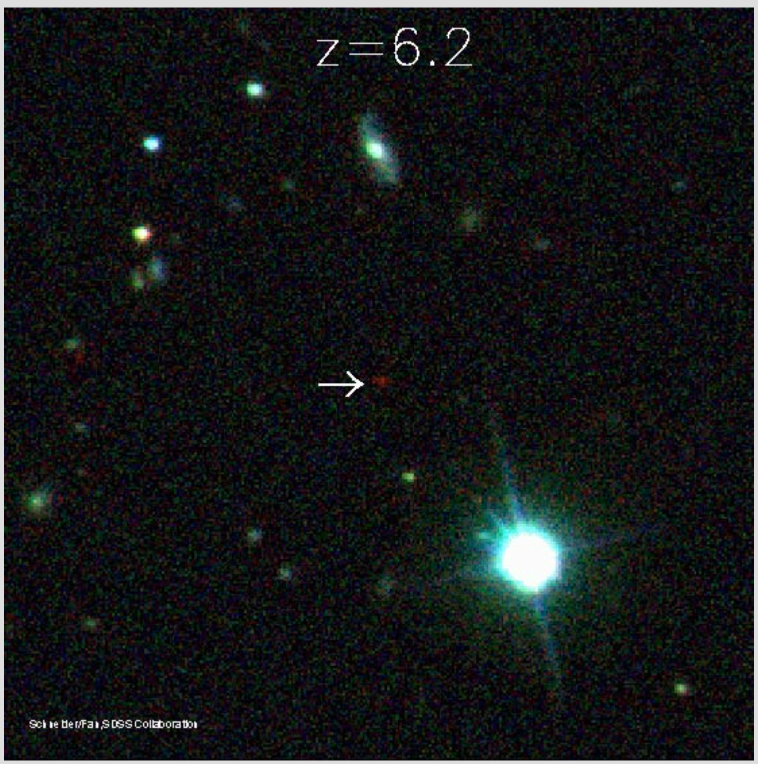
$$\Rightarrow M = M_0 e^{t/\tau}$$

where τ, the time scale for the black hole to grow by a factor of e, is 4.5 x 10^7 years

Example: how large can a black hole formed from the collapse of a massive star grow in 1 Gyr?

Initial mass M_0 = 10 Solar masses

Final mass $M = 10 \times \exp(10^9 / 4.5 \times 10^7)$
$= 4 \times 10^{10}$ Solar masses



Conclude: if the record breaking quasars at $z > 6$ have black hole masses of $\sim 10^{10}$ Solar masses, and are being seen when the Universe was about 1 Gyr old, **just** enough time for them to have grown from small seed black holes…

Exceeding the Eddington limit

What happens if we try to `feed' a neutron star or a black hole with gas at a rate that exceeds the Eddington limit?

Unknown… but possibly results in most of the mass being ejected. The Galactic X-ray source SS433 may be an example: