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DIAGRAMMATIC REPRESENTATION AND REASONING

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Abstract. The rapidly developing field of diagrammatic knowledge representation and reasoning is surveyed. The origins and rationale of the field, basic principles and methodologies, as well as selected applications are discussed. Closely related areas, like visual languages, data presentation, and visualization are briefly introduced as well. Basic sources of material for further study are indicated.

Key words: diagrammatic representation, diagrammatic reasoning, visual languages, diagrams, visual programming, data presentation, visualization, knowledge representation, computer graphics, qualitative physics, geometry theorem proving.

1. Introduction

The field of *diagrammatic data and knowledge representation* and *diagrammatic reasoning* has recently become one of the most rapidly growing areas of research in artificial intelligence and related fields of computer science and cognitive science. It should not be surprising, since visual representation and thinking were for long considered to be a widespread and effective mode of human thinking and problem solving [2, 7, 8, 10, 12, 14]. What is more surprising is that it has grown into a respectable part of mainstream AI research so late [41].

The diagrammatic (visual) representation uses *diagrams* to represent data and knowledge, and diagrammatic reasoning uses direct manipulation and inspection of a diagram as primary means of inference. Diagrams are a kind of *analogical* (or *direct*) knowledge representation mechanism that is characterized by a parallel (though not necessarily isomorphic) correspondence between the structure of the representation and the structure of the represented. E.g., relative positions and distances of certain marks on a map are in direct correspondence to relative positions and distances of the cities they represent, whereas in a propositional representation, its parts or relationships between them need not correspond explicitly to any parts and relations within the thing denoted. The analogical representation can be said to *model* or *depict* the thing represented, whereas the propositional representation rather *describes* it. A similar distinction can be made regarding the method of retrieving information from the representation. The needed information can usually be simply *observed* (or *measured*) in the diagram, whereas it must be *inferred* from the descriptions of the facts and axioms comprising the propositional representation. Of course, in practice we are rarely confronted with pure cases, and the diagrammatic representation research recognizes the fact, postulating hybrid representation schemes [3, 4, 11, 54, 71, 73].

It should be added that analogical representations, including diagrams, do not provide any exceptional means for representing information that cannot be represented in other ways, e.g., by propositional schemes (say, logical formulas). They usually represent the same information, only differently organized—in the way facilitating its use for certain tasks (e.g., for reasoning). The main advantages of diagrammatic representations can be summarized as follows:

- They permit explicit representation and direct retrieval of information (especially structural and spatial relations) that can be represented only implicitly in other types of representations and then has to be computed (or inferred), sometimes at great cost, to make it explicit for use.
- They permit effective control of the reasoning process, facilitating search both in data and solution spaces, as it may be guided by explicit proximity or adjacency relations between appropriate entities on the diagram.
- The reasoning process and its results are more natural and understandable to humans (especially important in current man-machine interactive systems).

Moreover, as recent research has shown, it is quite possible to formalize diagrammatic reasoning so as to make it no less precise and formal than logical reasoning [54, 70, 71, 74].

Knowledge representation and reasoning schemes originating in logic (specifically, predicate calculus), called *propositional* [8, 10, 13] were for long predominant in AI, as opposed to *analogical* schemes [8, 10, 14] including diagrammatic representations. This was only partially due to limitations of early computers and software in handling visual (or graphical) data. Possibly even more important cause was that most of the researchers in computer science and AI were (and still are) of mathematical background. And mathematics has been generally ruled by an implicit dogma stating that propositional reasoning using logic is the ultimate tool of precise and formal thinking. Many mathematicians, especially logicians (but even researchers in geometry!), tend to use diagrams rarely, sometimes as heuristics to prompt certain trains of inference, but mostly only as informal aids to understanding for uninitiated. On occasion, some of them explicitly stated that the diagram has no proper place in the proof as such.

This state of affairs seems to change now. The number of important papers on diagrammatic representations and reasoning grows rapidly. Scientific journals and conferences are launched. The field undergoes broadening and diversification into a variety of specialized subfields. This survey is intended as a general introduction to this field of research. It covers the origins and rationale of the field, its basic principles and methodologies, as well as introduces briefly its selected applications. It is, however, not exhaustive, as it would require much more space to give proper credit to all diverse approaches and applications evolving in this rapidly broadening field.

2. Knowledge representation

One of the important early findings of artificial intelligence research has been the recognition of the importance of *problem representation* for finding its solution. Herbert Simon in his landmark book [17] summarized the finding as follows: *"...solving a problem simply means representing it so as to make the solution transparent"*. Therefore, it is the main task of a designer of an AI system to devise proper representation of the problem (i.e., the formulation of the problem and knowledge pertinent to its solution). The paper by Amarel [5] is considered to be the first important work explicitly addressing this issue. It is often thought of as a beginning of the *knowledge representation* subfield of AI research. Amarel devised there a series of representations for the extended version of the classic *"Missionaries & Cannibals"* problem, starting from a predicate calculus representation (augmented with a production system notation), and compared their effectiveness in solving the problem. Interestingly, the subsequent, more efficient representations designed by Amarel were increasingly *diagrammatic* in their nature. It is therefore quite surprising that further research on knowledge representation concentrated almost exclusively on logical (propositional) representations—the Amarel's diagrams became practically forgotten for a long time [41].

Since then, little has been accomplished in automation of the representation design—in most cases, it is the program creator that must devise the proper representation for the class of problems the program is supposed to solve. However, a promising step in this direction has been made recently by Van Baalen [68]. The automated representation design method proposed by him works with predicate calculus formulas and as yet has not been applied to diagrammatic representations.

An early result of knowledge representation research has been the discrimination of two main types of representations: so-called *analogical* (or *direct*) representations and *propositional* (or *Fregean*, or *sentential*) representations.

2.1. Analogical versus propositional representations

The following definition of the *analogical* knowledge representation is due to Sloman [10]:

> *"If R is an analogical representation of T, then there must be parts of R representing parts of T, [. . .] and it must be possible to specify some sort of correspondence, possibly context-dependent, between properties or relations of parts of R and properties or relations of parts of T."*

For example, size, shape, direction and distance of marks on a map may represent size, shape, direction and distance of towns. In analogical representations, relationships within T do not need to be explicitly *named* in R, i.e., there need not be a *part* of R corresponding to relations like *"above"* or *"intersects"* in T.

In contrast, in a sentential (textual) representation, like in the phrase *"The city 250 km south of Warsaw"*, its parts (e.g., the word *"Warsaw"*) or relationships between them (e.g., that *"Warsaw"* appears after *"south"* in the phrase) need not correspond to any parts and relations within the thing denoted. The city denoted by the phrase, namely Cracow, contains neither Warsaw nor south as a part, and Warsaw is not after south on the surface of the Earth. The relation between Cracow and Warsaw, explicitly named *"south of"* in the sentence does not appear explicitly, as an object pointable to, in the analogical representation of the fact (on a map). The same is true for more formal propositional representations, like predicate calculus formulas. The distinction may be also stated in another way (see also [14]):

- An *analogical* representation is a structure whose syntax parallels (models), to a significant extent, the semantics of the problem domain. Not surprisingly, analogical representations are usually *specialized* for some specific application domain.
- A *propositional* representation has a structure that has no bearing to the semantics of the problem domain. Therefore, propositional representations can be made easily, by their own nature, *universal*. However, it makes them much less effective as a result—they suffer from the lack of semantic (or heuristic) guidance necessary to effectively navigate through usually vast search spaces during information retrieval and reasoning.

Whether it is possible to construct a significantly general, analogical knowledge representation seems to be still an open question. Even if it *is* possible in principle, it is probably not practical. The more practical approach seems to be to use mixed representation schemes, e.g., with separate domain-dependent analogical representations linked by a universal propositional superstructure [54, 71, 73]. Most working systems that use diagrammatic representations are hybrid, as were the early systems [3, 4, 11].

The distinction between analogical and propositional representations is often wrongly interpreted. This sometimes leads to misunderstandings in discussions between proponents of both approaches. The interested reader may find the list and discussion of the most common misrepresentations of the distinction in [10].

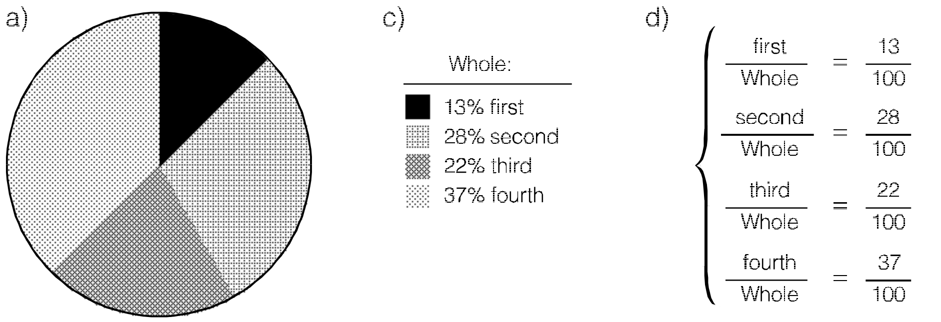
2.2. Diagrammatic representation terminology

The terms *analogical* and *propositional* were adequately discussed above. The term *Fregean* was introduced in [8] (in honor of Gottlob Frege, whose work laid foundations for the predicate logic notation), but recently is replaced by *propositional* (which term is used thorough this paper). Also, the term *sentential* was proposed [31], but it seems less fitting, as it may lead to confusion of propositional representations with descriptions using sentences in some (natural) language. The *analogical* representations are also called *direct* [14]. *Logical* representations are kinds of propositional representations using logical notations, usually predicate logic calculus and its numerous extensions, like modal logic or nonmonotonic logic [13]. The discussion and characterization of representations that might be propositional but not logical (or not wholly logical) lies outside the scope of this work, as is the place in this taxonomy of such well-known knowledge representation schemes as semantic nets and frames.

The next group of terms is headed by the term *visual*. It is supposed to mean, in this context, *perceivable by sight* (i.e., *"seeable"*). Indirectly, it also follows that the visual representation is necessarily at least two-dimensional (even when the concepts represented are one-dimensional, like time intervals, see [30]). Visual does *not* necessarily mean analogical—the *visual/non-visual* distinction is to large extent independent of the *analogical/propositional* distinction. It refers to sensual modality of the representation (in opposition to aural, tactile, etc.), not to the structure of the encoding involved in the *analogical/propositional* distinction. Subspecies of *visual* are: *graphic, pictorial, diagrammatic,* and, in a sense, *geometrical*.

Whether visual representation is analogical or not depends on the representation of *what* information it is, and *how* it represents this information. E.g., the set of logical formulas as written on paper (or computer screen) is a representation of the knowledge encoded with the formulas, but despite its visuality, it is not an *analogical* representation of this knowledge. However, it *is* an analogical representation *of the set of logical formulas*! The graphical (geometrical) relations between written graphical symbols *directly* correspond to the (syntactic) structure of the formulas, although rather not to the structure of the knowledge represented by them. Note, however, that visual and essentially two-dimensional representations that are nevertheless propositional (like various kinds of mathematical notation) gain certain advantages (hence their origination and widespread usage) in comparison to textual, purely one-dimensional representations (like linear notations for mathematical expressions used in traditional high-level programming languages). It stems from the fact that they *are* analogical with respect to the structure of the expressions, though not (or to a significantly lesser degree) with respect to the knowledge represented by the expressions.

An illustration of these distinctions is provided in Fig. 1. A knowledge about proportions of certain quantities is represented diagrammatically (Fig. 1a), then propositionally using "unadorned" predicate calculus (Fig. 1b). The "legend" (Fig. 1c) links corresponding symbols in both domains (being a part of the definition of the visual language used). The usual mathematical notation (Fig. 1d) is propositional with respect to the domain knowledge, but (partially) diagrammatic with respect to the structure of the formula in Fig. 1b.



b)

EQU(DIV(first, Whole), DIV(13,100)) & EQU(DIV(second, Whole), DIV(28,100)) &
EQU(DIV(third, Whole), DIV(22,100)) & EQU(DIV(fourth, Whole), DIV(37,100))

Fig. 1. Diagrammatic versus propositional representations (see text).

Pictorial is supposed to mean here *involving pictures*. *Pictures* are considered to be realistic (representational) images of the world objects and scenes, as opposed to more simplified, stylized or symbolic *graphical* images. *Graphical* is supposed to mean *involving artificial, symbolic or simplified visual depiction* of objects, notions and relations (as opposed to more realistic *pictures*). Historically, it also carried a connotation of restricted means of rendering (only black and white, with no intermediate tones or color, only line drawings, etc.), now mostly irrelevant due to proliferation of advanced graphics techniques in art, design, printing industry, and computer graphics. Distinction between pictorial and graphical is currently a matter of degree rather than principle. The distinction pictorial/graphical as accepted here is different, though related to that made in visual arts between *painting* and *graphics*, where it corresponds more to the technique of creation—the execution by hand producing only one original (painting) versus the use of more or less mechanical means of reproduction capable of producing multiple originals (graphics)—rather than to the specific visual features of the artifact or the intent of its creation.

Geometrical means *involving ideal mathematical objects as studied by geometry*, that is, points with no dimensions, lines with no thickness, ideally round circles, etc. Geometrical objects may be rendered (approximated) by graphical means on paper or screen. It should be kept in mind, however, that graphical attributes of these renderings like shading, various line thicknesses or styles (dashed, dotted), textual labels, etc., are *not* geometrical entities, although they may be used as elements of the visual language talking *about* geometrical elements (e.g., in geometry proofs). It is quite common practice, however, to identify geometrical objects with their renderings (drawings), what may sometimes lead to confusion.

The term *diagrammatic* is central here. *Diagram*, by a vocabulary definition, is a *visual means of representation* of (not necessarily visual) information. Considering this, the term *diagrammatic representation* translates somewhat awkwardly as "representation involving visual means of representation". Anyway, this term seems to be best suited to denote what we are most interested in here. Although, logically speaking, *visual* is the wider term than *diagrammatic*, it is hard to point out a *visual* representation that could not be called a *diagrammatic* representation. One candidate that comes to mind is a real-world picture, not artificially made with intent to represent some information. Thus, in the context of man-made or machine-made forms, *diagrammatic* can be considered practically synonymical with *visual* (as it is treated by many authors, e.g. [74]). *Diagrammatic* representations may use both *pictorial* and *graphical* elements, though usually graphical means are the most common in diagrams.

To be exact, we should add that *geometrical* objects cannot be parts of a diagram by themselves, although their graphical renderings (approximations) are common building blocks of diagrams. Geometrical concepts also are obvious conceptual tools for analysis, construction and discussion of diagrammatic representations.

3. Making it visible

3.1. Behold!

One might be tempted to start the presentation of origins of diagrammatic reasoning with the phrase *"Already ancient Greeks..."*. One cannot, however: the origins of diagrammatic (or, more broadly and less technically, pictorial) representation of information are older still. It seems certain from available evidence that the very first human attempts at communication with other means than oral language—the first writings—were pictorial. As Chinese writing still contains visible remnants of its pictorial origins (although, contrary to common impression, currently it is basically phonetic like other writings of the world), one should rather say *"Already ancient Chinese..."*.

It seems that the use of diagrams to aid more scientific thinking also originated in China. Not surprisingly, it was applied to explaining theorems of geometry. An old Chinese treatise, dated by some scholars at 600 B.C. [40], contains the visual proof of the Pythagoras Theorem, redrawn in Fig. 2a (with some inessential changes, like translating Chinese labels into Latin alphabet). A similar diagram appeared in a treatise by an ancient Hindu mathematician Bhaskara, augmented only by a single word: *"Behold!"*.

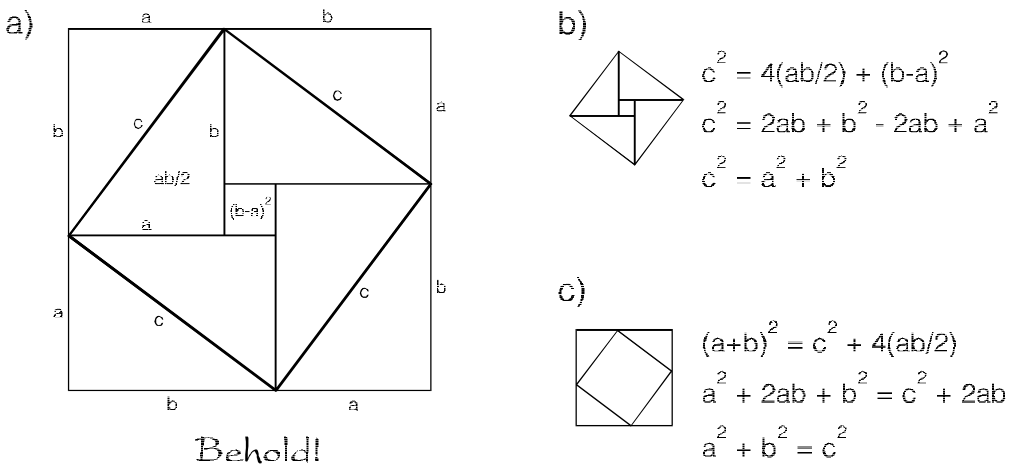


Fig. 2. A visual proof of the Pythagoras Theorem from an ancient Chinese treatise (a) and two algebraic derivations following directly from the diagram (b, c).

To be fair, some (easy) algebraic calculations seem to be necessary to definitely complete the proof. It may be done in two different ways, depending on which part of the diagram is focused upon. The part inside the tilted square leads to the derivation in Fig. 2b, whereas disregarding the interior of the square leads to the derivation of Fig. 2c. Thus, the diagram in fact contains two different proofs of the theorem! In various geometry texts usually only one of these versions is cited. Note, however, that the full diagram contains also a convincing visual demonstration of the formula for an area of a right triangle (needed in the proof), thus it is a more self-contained example of diagrammatic proof of the theorem than either of its two simplified versions.

Geometricians of ancient China and India used extensively this method of visual rather than propositional argument [7, 40]. Ancient Greeks, and western mathematics after them, followed another path. Euclid's *"Elements"* contain drawings, but they in most part play a role of mere illustrations, helping to understand the statement of the problem, whereas the whole argument is presented textually, in a formal, step-by-step sequence of logically connected propositions. This practice laid foundations for development of formal deductive scientific method, but the possible advantages of direct visual grasp were put aside and all but lost to generations of scholars in geometry and other sciences.

For example, Euclid's proof of the Pythagoras theorem is very intricate and nonintuitive. The accompanying diagram is very complicated and its major role is to help the reader in not becoming completely lost in the tangle of argument. An interesting attempt to "pictorialize" Euclid has been made by Byrne [1] in the middle of the 19th century (see [40] for excerpts of some interesting fragments). Instead of textual labels marking points, lines, and figures in the drawings, Byrne used colour to code the elements and areas in the drawing, and inserted corresponding coloured graphic symbols in appropriate places in the text. However, the structure and style of Euclid exposition remained unchanged. Not surprisingly, the attempt seems hardly successful—the intricate proofs remained intricate and the train of underlying reasoning obscure.

3.2. Visual languages

As the above example and numerous others indicate, propositional representations cannot be translated into visual ones "word for word". Proper visual representation requires its own, distinct *visual language*, comprising both appropriate visual vocabulary of graphical symbols and specifically "diagrammatic" rules of their composition into meaningful and effectively readable (or rather "seeable") representations. Not surprisingly, research on general aspects of visual languages appears to be the main line of research in the field, as titles of books, conferences and scientific journals signify [28, 51].

The fundamental knowledge on visual presentation methods has been accumulated by professionals of *graphics design* trade. It may be found in numerous old and recent books on the subject. The most useful ones (to the field of visual representation) are those concerned with presentation of (quantitative and qualitative) data and information, e.g. [6, 15, 19, 22, 25, 40]. As we humans are lacking a sufficiently effective *"picture effector"* [20], for a long time production of visual presentations was restricted to professionals of graphics design and visual arts, as the task required considerable skill and training. Recently presentation graphics is produced more and more frequently by non-professionals (and too often with visually disastrous results), due to wide accessibility of computer programs aiding creation of drawings, illustrations, graphical presentations and charts. The computer has become a kind of prosthesis for the lacking "picture effector".

More or less rigidly codified, specialized visual languages for various technical domains have been devised and used since long ago, e.g., in cartography, mechanical, construction and architectural drawing, electrical schemata, etc., as well as in computer programming [27]. Other visual language systems, like road signs, international information signs, or iconic interfaces with personal computers, can be found in everyday life as well.

However, more formal codification of visual data presentation rules, to make them suitable for computer implementation, has been attempted only recently [26]. The authors observed that some data representation languages, and especially visual languages, have the property that when some collections of facts are stated explicitly, additional facts are stated implicitly. E.g., to express a "part-of" relation between entities of some domain one may use an "inside" relation between geometrical figures. Then, one may represent the facts that a head is a part of an animal and an eye is a part of a head by placing the "head" figure inside the "animal" figure and the "eye" figure inside the "head" figure (see Fig. 3abc). As a result of this construction, the "eye" figure turns out to be inside the "animal" figure on the diagram, thus an additional *"implicit fact"* that an eye is a part of an animal is also stated.

The "implicit facts" phenomenon, known also as "emergent properties" feature of diagrams [65], may be a mixed blessing. On the one hand, it allows for more efficient coding of a set of facts and helps to use the diagram in reasoning—the implicit facts constitute a sort of automatic inferences provided by the diagrammatic representation at little or no cost (see Sec. 4.2 below). On the other hand, implicit facts may be not correct in the intended interpretation of the language statement. E.g., when one uses the same "inside" relation between geometrical figures to denote the "next-to" relation between countries on the surface of the Earth, stating the facts that Germany is next to France and Poland is next to Germany states implicitly that Poland is next to France—certainly a false fact of political geography (Fig. 3def).

Avoiding such errors requires proper choice of the (visual) language to express the facts in the given domain. E.g., to express the "next-to" relation between countries, one should rather resort to the "adjacent" relation between geometrical figures (Fig. 3dgh).

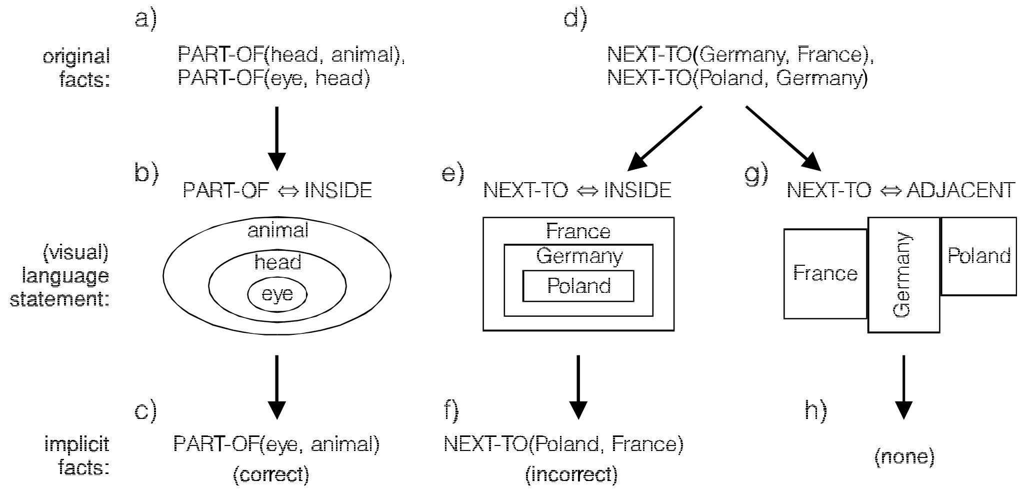


Fig. 3. Implicit facts (c, f, h) resulting from stating given explicit facts (a, d) in different visual languages (b, e, g).

The above considerations led to distinguishing two basic criteria for choosing proper visual language for a presentation of a given data, called *expressiveness* and *effectiveness* of the language [26].

The visual language must be able to *express* (1) all the facts and (2) only the facts we want to visualize. The first requirement is obvious; the second takes into account the "implicit facts" feature—the visual representation should not, as a by-product of representing the wanted facts, represent implicitly unwanted (false, incorrect) facts (see Fig. 3).

The *effectiveness* of a visual language is determined by how easy it is to state the facts in the language and how easy it is to perceive the facts from the representation. Effectiveness is relative with respect to the user of the representation—

other rules apply when it is to be used by humans, other when by computers. Human visual perception criteria are quite well known and codified in extensive human factors and graphic design literature [15, 19, 22, 25, 40]. E.g., it is well known that color and shading are far less effective in representing accurately numerical quantities than position on a scale or length of a line. Much less can be said about the preferences of computers, although some results, both negative and positive, are emerging [11, 18, 28, 44, 59, 74], see also Sec. 5.

Other authors [55, 59, 61] added to the above two criteria the *presentation goal* criterion. It takes into account what kind of operations the user wants to perform with the representation, or what kind of tasks the representation will be used to aid. Examples of different goals might be, e.g., accurate value lookup (then representing the values as marks on a numerical scale is advisable) versus general magnitude comparison (then representing the values by size of graphical entities might be better).

Combining the above considerations and the traditional data presentation knowledge (esp. Cleveland's work [25]), several researchers implemented experimental computer programs for automatic presentation design [29, 55, 61]. Interesting earlier work going in similar direction was done by Reilly and Roach [23]. They attempted, with some success, to use the then developing expert system technology to build a graphic design advisor helping to design good graphical interfaces. Later on, Myers [34] used also a knowledge-based module in his system for interactive design of graphical user interfaces, and an interesting graphic design advisor has been implemented by MacNeil [39, 50]. As, in general, there are no computationally tractable algorithms (i.e., of less than exponential complexity) for finding "optimal" layout of complex diagrams (see, e.g. [21, 42]), using heuristic, or knowledge-based, approaches to automating graphic design is in fact a necessity.

Many other approaches to formalization of visual languages have been published over the years. One of the most influential was that by Harel [33], further developed into an exact diagrammatic reasoning formalism by Hammer [70, 71]. For other approaches, see especially [28, 44, 46] and other general sources on visual languages (see the last Section).

3.3. Uses of diagrams

Once created, the diagrammatic representation may be used in various ways. These ways can be classified along several dimensions. Main distinctions seem to be: *who created* the diagram (a human, or a computer?); *who is to use* it (its creator or someone else, and again, human or computer?); and in *what kind of information* *activity* it will be involved (e.g., information storage, communication, or processing). Several other features can be distinguished, complicating the classification even further. Without going into details, some most common uses of diagrams are:

- *Data presentation* means encoding of some (usually non-visual) data in visual format, intended to *communicate* it to *other* party. If both an originator and recipient are (different) humans, it is the traditional *graphic design* field (possibly computer-aided). In case of computer originator and human recipient, it is called either *data (knowledge) visualization* (when human controls the process to some extent), or else (automatic) *presentation design* (when computer controls the process, using some *design knowledge*). Human originator and computer recipient constitute the case of *diagrammatic (graphical) data input*. The man-machine *graphical interfaces* usually combine several of the above cases [66].
- *Diagrammatic representation* means in general encoding any data in diagrammatic form, mostly for *storage* purposes, to be later used as a data or knowledge base, by the same or other party, be it human or computer.
- *Diagrammatic reasoning* involves use of diagrammatic representations to aid information *processing*, especially extracting new facts from the ones encoded in the representation. The reasoning may be either made only by a human (as both the originator and user of the diagram), or only by a computer, or in other combinations (in man-machine interactive systems). Useful new dimension in this case involves distinguishing *internal* use of a diagram (*visual imagery*—human or computer) and *external* use of a diagram (*diagram drawing* as an aid to reasoning—usually specific to humans).

4. Why a diagram is worth 10000 words?

The phrase *"picture is worth ten thousand words"* is widely known and considered to be a Chinese proverb (though, as noted in [31], native Chinese seem not to have heard of it). Chinese or not, the saying is widely believed. But the question arises: what are the advantages of pictures that make them worth so many words?

The first use of diagrams in computer reasoning was the Gelernter's "geometry-theorem proving machine", implemented as early as in late fifties [3, 4] as one of the first successful AI programs. To prove theorems in elementary Euclidean geometry, it employed the problem reduction technique, i.e., a backward chaining inference working from the goal to be proved to the premises and axioms. It used diagrams of geometric configurations in two ways—"negative" and "positive". The "negative" one was to prune search in the space of possible subgoals: the program *rejected* any hypothesis (subgoal) that was not true in the diagram (it was estimated that about 99.5% of the possible subgoals were rejected in this way!). That is, the diagram was used to filter out hypotheses inconsistent with the data stored in the representation, in compensation for the inefficient, "blind" hypothesis generation algorithm. The "positive" use of a diagram allowed to shorten inference paths by *assuming as true* various facts (reliably) obvious from the diagram, but tiresome to prove from fundamental axioms, like that a certain point on a line lies between two others, or two angles share common arms and are therefore equal, etc. The latter use of a diagram obviously utilizes the "implicit facts" (or "emergent properties") feature of diagrammatic representations (see Sec. 3.2 and 4.2), the former is also to a large extent dependent on this feature. Like several other early attempts to implement diagrammatic reasoning in computers, the Gelernter's work has been practically forgotten, and it did not start any serious research on the possible advantages of diagrammatic representations.

Larkin and Simon in their seminal paper [31] tried to give a more precise answer to the question, from the point of view of cognitive science. Their main conclusion has been that diagrams allow effective control of the reasoning process, facilitating search for the elements needed to make the next inference step, due to explicit proximity or adjacency relations between appropriate entities on the diagram. With propositional (e.g., predicate calculus) representation, to find an information needed to apply some inference rule one must search the entire data base to find the relevant entities and their properties. On the diagram, however, the relevant entities are usually placed just in the (topological or metrical) vicinity of the current "attention point" that has resulted from the previous reasoning step. The diagram, as an analogical representation of the structure of the problem, organizes the problem description in a way corresponding to the problem's internal structure that should be followed in order to find the path to the solution. The authors support their claim with appropriate calculation of the number of search steps needed to solve two example problems (from elementary mechanics and from geometry) using both representations.

4.1. Diagrammatic reasoning: an example

A reformulated version of the first of Larkin and Simon examples [31] illustrates the point. Fig. 4 shows a predicate calculus formulation of some simple mechanical problem. First, the list of the entities involved, their properties and relations holding between them is given. It comprises the set of (initial) facts. Then, the solution goal is stated, i.e., a statement to be proven true with appropriately instantiated variable (denoted *?n*). The "physics" of the system is given by a set of rules for derivation of new facts from the existing (known) facts. The set of (initial) facts and rules together comprises the *knowledge base* of the problem. The reader is kindly asked not to try to guess what kind of device is described here, nor to look forward to the next page of text, before he/she tries to imagine how solving of the problem using only this representation might look like.

FACTS: structural description and parameters

WT(P), WT(Q)
RP(p), RP(q), RP(s), RP(x), RP(y), RP(z), RP(t),
PL(A), PL(B), PL(C),
CL(T)

PLS(p, A, q), PLS(x, B, y), PLS(y, C, z),
HNG(P, p), HNG(Q, q), HNG(Q, s), HNG(A, x), HNG(B, t),
HNG(s, C), HNG(t, T), HNG(z, T)

VAL(P, 1)

GOAL:

VAL(Q, ?n)

RULES:

Rule 1: **if** WT(w_1) & RP(r_1) & HNG(w_1, r_1) & ~HNG(w_1, r_2) & VAL(w_1, n_1)
then VAL(r_1, n_1)

Rule 2: **if** PL(p_1) & RP(r_1) & RP(r_2) & PLS(r_1, p_1, r_2) & VAL(r_1, n_1)
then VAL(r_2, n_1)

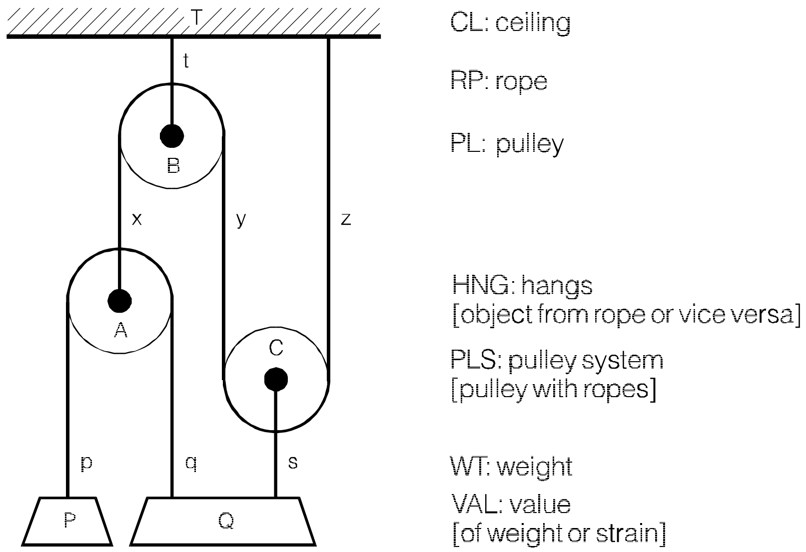
Rule 3: **if** PL(p_1) & RP(r_1) & RP(r_2) & RP(r_3) & PLS(r_1, p_1, r_2) &
{HNG(r_3, p_1) $\vee$ HNG(p_1, r_3)} & VAL(r_1, n_1) & VAL(r_2, n_2)
then VAL(r_3, n_1+n_2)

Rule 4: **if** WT(w_1) & RP(r_1) & RP(r_2) &
HNG(w_1, r_1) & HNG(w_1, r_2) & ~HNG(w_1, r_3) &
VAL(r_1, n_1) & VAL(r_2, n_2)
then VAL(w_1, n_1+n_2)

Fig. 4. A simple mechanical problem: predicate calculus formulation.

Without the guidance of any mental model of the mechanical *system* described by the above set of formulas, one must make extensive search of the whole set of facts any time he/she tries to match a candidate rule with appropriate facts in the knowledge base, trying various possible variable instantiations on the way. E.g., in *Rule 3* there are $7 \cdot 7 \cdot 7 \cdot 3$ = 1029 possible instantiations of the variables p_1, r_1, r_2 and r_3, with only three of them consistent with the whole store of facts. And real problems that must be solved by expert systems involve usually thousands of facts and rules. To cope with the task, one must spend a large amount of computational effort or appropriately organize the knowledge base and reasoning procedure. Van Baalen's [68] representation design system in effect does just that—reorganizes the knowledge base, reformulating it by using new predicates and functions constructed to fit regularities detected in the original set of formulas.

Another organizing approach would be to use a diagram. Look at the Fig. 5: this is how our mechanical problem would be usually stated in a textbook. It is far easier (at least for a human) to understand it, and to solve it.



PROBLEM: What is the ratio Q/P of the weights if the system is in equilibrium?

Fig. 5. The diagram of the "pulleys" example (adapted from Larkin and Simon [31]); the legend explains meanings of predicate symbols used in the logical representation.

In this representation, the solution is straightforward, and is guided directly by the diagram. That is, one may obtain it using *diagrammatic reasoning* (see Fig. 6). One moves with a finger (real or imaginary) along the lines (representing ropes) in the diagram, applying along the way appropriate rules that match current local diagram configuration, without need to search blindly through all the data for the facts matching the rule. The names (textual labels) of ropes and pulleys are unnecessary, as is the explicit naming of relations (like "hangs" or "pulley system"). This is how a human would solve the problem; how a computer may implement this kind of reasoning is explained in Sec. 5.

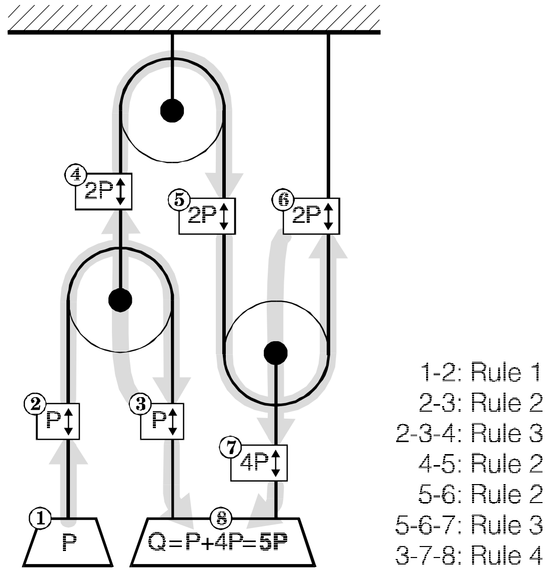


Fig. 6. Diagrammatic reasoning leading to the solution of the "pulleys" example; reasoning steps are numbered consecutively, and the legend lists rules used at every step.

4.2. Advantages of diagrammatic reasoning

The main sources of advantages of diagrammatic representations for reasoning, found by Larkin and Simon [31], have been further elaborated and extended by Koedinger [65]. Their findings can be summarized as follows:

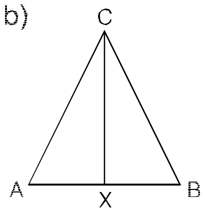
– *Locality aids knowledge and problem search*: explicit adjacency or proximity (topological or metrical) of related problem entities on a diagram reduces search for the data needed to make the next inference step, and search for the proper path through the problem space (e.g., a proof tree). This feature has been illustrated by the "pulleys" example above, and it was used by Gelernter program [3, 4] to very efficiently prune the subgoals tree.

- *Less necessity for symbolic labels*: this is in fact a consequence of the previous feature—related elements are usually grouped in proximity, avoiding the need to mark them with symbolic labels (like letter labels of weights, pulleys and ropes in Figs. 4 and 5) and then search through the knowledge base for a match. However, diagrams do not totally eliminate the use of symbolic labels—there are cases when they are still needed or even quite useful [65].
- *Easy realization of perceptual inferences*: humans possess a well-developed apparatus for making easy perceptual inferences on a diagram, but are far less proficient in processing logical formulas or other propositional representations (compare the ease of following the route to the solution in Fig. 6 with search for matching predicates among the formulas of Fig. 4). It is probably true for humans, although one may argue that humans may become equally proficient in processing formulas after appropriate practice. Unfortunately, this advantage hardly applies to computers, as they are better suited for symbolic rather than diagrammatic processing.
- *Certain inferences are already present in a diagram*: this feature is called an *"emergent properties"* phenomenon [65] and is discussed in more details below. The second use of diagrams in Gelernter's program [3, 4] was based on it, and the feature was also extensively used by the DC program of Koedinger and Anderson [65].

Let us illustrate the *"emergent properties"* feature of diagrams with a simple geometrical example. The predicate-calculus definition of some geometrical configuration is given in Fig. 7a. Using the definition and assuming appropriate coordinates for the points A, B and C, the diagram is constructed (Fig. 7b). It fulfils all the conditions listed in Fig. 7a. However, it fixes also certain other properties of and relations between the elements of the diagram that can be directly "read" from the diagram without need to make sometimes lengthy and tedious inferences from the stated premises and geometry laws—they are "perceptually obvious" [31]. Let us note that the emergent properties are nothing other than "implicit facts" [26], discussed in Sec. 3.2 as affecting both expressiveness and effectiveness of visual languages in an essential way. Koedinger [65] groups them in two classes: *topological* and *geometrical* properties.

The topological properties are guaranteed to be true, as they follow from the general structure of the diagram, independently of its metrical properties (coordinates of points, lengths of segments, etc.). In the example (Fig. 7c, stated in traditional geometrical notation), they include detection of certain new geometrical entities (in this case, angles and triangles) and relations between them (here, equality of some angles due to sharing of the same arms).

a)
POINT(A), POINT(B), POINT(C)
SEG(a, A, B), SEG(b, B, C), SEG(c, C, A),
TRIANGLE(A, B, C),
POINT(X), ONSEG(X, a),
SEG(x, C, X),
EQU(length(b), length(c)),
PERPEND(a, x)

b)


COORD(A, 0, 0),
COORD(B, 2, 0),
COORD(C, 1, 2),
COORD(X, 1, 0)

c) topological properties:
$\angle$ACX, $\angle$BCX, $\angle$ACB, $\angle$CXA, $\angle$CXB,
$\angle$CAB, $\angle$CAX, $\angle$CBA, $\angle$CBX,
$\angle$CAB = $\angle$CAX,
$\angle$CBA = $\angle$CBX,
$\overline{AX}$, $\overline{BX}$, ΔACX, ΔBCX

d) geometrical properties:
$\angle$ACX = $\angle$BCX,
$\angle$CAX = $\angle$CBX,
$\angle$CXA = 90°,
$\angle$CXB = 90°,
$\overline{AX} = \overline{BX}$

Fig. 7. Emergent properties: propositional definition (a), the corresponding diagram (b), and the topological (c) and geometrical (d) properties that can be directly "read" from the diagram.

The geometrical properties (Fig. 7d) are based on metrical data following from the particular point coordinates assumed during construction of the diagram. Thus, they may be not true in general, either due to "spurious coincidences" generated unwillingly when constructing the diagram, or measurement imperfections in calculating, e.g., lengths of segments. Therefore, properties of this class cannot be used as steps in the reasoning. However, they can be still used to guide problem search.

Note that "spurious coincidences" can lead to serious problems with diagrammatic reasoning when not taken properly into account. More detailed discussion of this subject lies outside the scope of this work, however.

5. Implementing diagrams

It is easy to say to a human: *"Behold!"*. Humans are born with an excellent apparatus for extracting knowledge from pictures (including diagrams) and reasoning with the help of them. It consists of perceptual part (the sense of *vision*) and internal processing part (visual *imagery*). The visual perception mechanisms are comparatively well understood, at least in their overall structure and some low-level technical details. The working of "visual thinking" [2, 7] is also understood in general terms, though far less than perception, but details of its "implementation" in the brain are still a subject of hot debate among cognitive scientists [62]. One school, lead by Kosslyn [12] claims we reason visually by generation and examination of images on an "internal retina", using for reasoning essentially the same brain apparatus as for perception. The opposing school, lead by Pylyshyn [16] claims the "pictorial" images we sometimes see in introspection during reasoning are purely epiphenomenal and the "real" reasoning is essentially propositional. Recently, interesting models attempting a reconciliation of the two positions were proposed [43]. Be it as it may, we certainly are able to reason with diagrams, even if we may not clearly know how we do it.

It is quite another matter with computers. They must be endowed by us (humans) with appropriate programs able to model the visual perception and reasoning abilities. Computer vision systems are still far behind humans in their perceptual abilities. "Intelligent" computer image interpretation of adequately complex and arbitrary pictures still requires too much computing power and processing time to be practical as an input of diagrammatic representations to a computer reasoner. Our knowledge about human visual imagery does not yet allow for using it as a direct model for computer programs. Luckily, advances in computer vision and computer graphics do provide certain tools for implementation of diagrammatic reasoning in machines.

Below, first the ways of diagram input to the machine are summarized. Then two essentially different ways the diagrams may be represented in a computer (specifically for use in reasoning) are discussed. The two methods parallel in a way the distinction between raster and vector graphics known from computer graphics and image processing domains.

5.1. Putting diagrams inside

The main method of inputting diagrams to computers is to use interactive graphic editors. In this way, the diagram is built from predefined primitives, either geometrical (lines, circles, rectangles, etc.) or specific to the domain (images of pulleys, ropes and weights; electronic component symbols, etc.). The computer does not need to process the (usually noisy) input picture to extract and recognize appropriate meaningful objects in it. This method is used in majority of practical systems, e.g., in probably the first commercially available visual programming language Prograph [37]. For input of more complicated diagrams, systems of such kind are often knowledge-aided like the interesting "designer's apprentice" named TYRO [39, 50].

It is instructive to recall here the historical development of computer-aided design (CAD) systems. At the beginning, they were nothing more than drafting aids, to produce technical drawings from geometrical primitives, store them in computer data banks, and plot them on paper. Growing computerization of drawing storage created a necessity to change into computer files the vast libraries of hand-made paper drawings already accumulated by design and production companies. Redrawing them with CAD programs was impractical for various reasons, therefore, a demand for automated "vectorization" systems (as they were called) ensued. Thanks to sufficiently regular structure of technical drawings, the combination of powerful computer image processing and interactive human supervision led to successful vectorization systems. The image processing research and the experience gained with such systems bring closer the day of diagram input systems based on direct scanning of pictures or tracing the handwriting (or hand-waving) actions of humans. The work is under way since some time [18], and a device like Apple Computer's *Newton*, though as yet not very reliable and possibly a little ahead of its time, marks the days of computers capable to really communicate with pictures.

In the meantime, a new direction in computer-aided design emerged, called *feature-based design*. Here, instead of purely geometrical primitives, the designs are assembled from (technologically) meaningful fragments, called "features", like slots, holes, shafts, steps, grooves, etc. [47]. Soon the problem appeared—how to translate the huge libraries of CAD drawings into feature-based format? The research on automatic feature recognition and extraction from drawings has been born. The systems for doing this are usually knowledge-based, and have to resort to essentially diagrammatic reasoning [47].

5.2. Diagrams on a raster

The "raster-graphics" method represents diagrams as a raster image, i.e., as regular, discrete, two-dimensional assembly of picture elements (pixels). Inspection of the diagram to extract needed information, and, possibly, writing down the intermediate results of reasoning, is done with appropriate image processing operators. As was mentioned above, the "pictorial" school of visual imagery [12] postulates that humans use just this method for diagrammatic representation and reasoning. However, taking into account the previous discussion of the limitations of contemporary computers, it is not surprising that there were very few attempts at using this method for diagrammatic representation in computers.

The first one was the system called WHISPER, implemented by Funt [11] in late seventies. It is a mixed, vector/raster/propositional system. The overall structure of WHISPER is shown in Fig. 8. The High Level Reasoner is a procedural problem-solving program embedding in its structure the qualitative physical knowledge about motion and stability of rigid bodies acted upon by gravity (see Fig. 9 for an example configuration). To conduct the reasoning, the reasoner asks its "perceptual system" to "look at" the (vector) diagram with its (raster) "retina". Using the retina to locate objects and their supports, the reasoner checks the stability of each object shown in a diagram. Unstable objects may rotate or slide.

In cases where one is unstable, the reasoner asks the retina to "visualize" it rotating (or sliding) and thereby determine at what point it will hit some other object. Then the program updates the diagram with the object moved into its new position and restarts the problem-solving process from the beginning. Fig. 9 shows a typical problem solved by WHISPER. It shows the initial configuration and subsequent reasoning steps, with the object moved at each step highlighted.

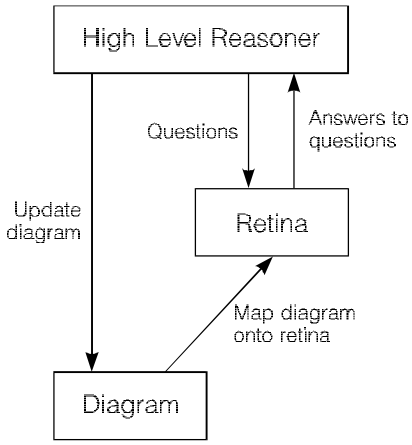


Fig. 8. The general structure of Funt's WHISPER system (adapted from Funt [11]).

What is interesting, the WHISPER's retina was not rectangular but polar—individual receptors (pixels) were arranged in concentric circles (with 36 receptors in each), so that its resolution was nonhomogeneous, decreasing from the center to the periphery. The reasoner moved and "fixated" the retina at different points of the diagram. The retina conducted a couple of picture-processing operators, like detecting contact points between bodies, and was able to simulate ("imagine") motions of individual objects.

Recently, purely diagrammatic reasoning method has been proposed by Furnas [45, 64]. His BITPICT system is based on the supposition that a local, pixel-neighbourhood rewriting operator on a raster image can be, if appropriately formulated, considered as an inference rule, and the resulting picture as a result of a (single) reasoning step. Thus, image processing is considered as a sort of inference process, just like formula manipulation in propositional representations. A favourite Furnas' example is a reasoning system that is able to count linear connected components on a binary image, and write the result, in Roman numerals, on the resulting image!

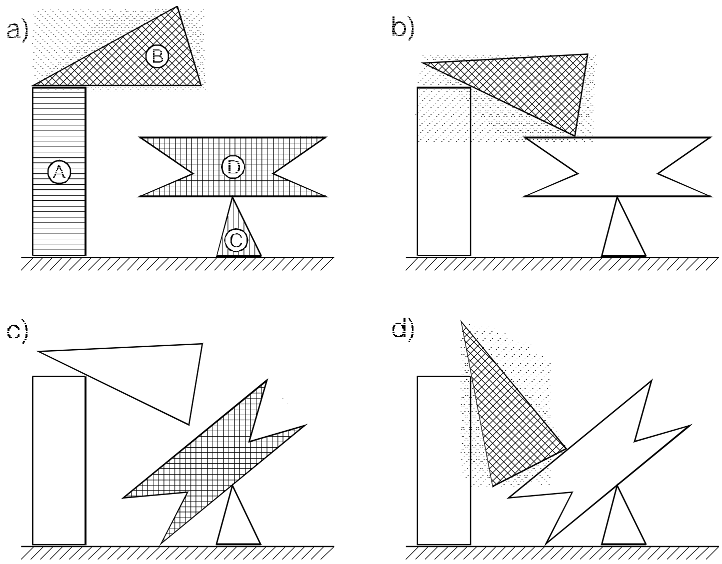


Fig. 9. An example block configuration processed by Funt's WHISPER system and "snapshots" of principal steps of its diagrammatic simulation.

5.3. Diagrams as graphs

The "vector-graphics-like" method of representing diagrams in a computer is more popular and universal. Diagrams are here represented, generally, as various types of structural or relational graphs. Methods of computer representation and processing of graphs are fairly well developed and their adaptation to diagrammatic reasoning is usually easy and straightforward. Nodes of the structural graphs represent appropriate elements (of the diagram or underlying semantic domain), and edges represent various relations between them. A version of this method was employed in the first program that used diagrams to augment the computer reasoning—the Gelernter's "geometry-theorem proving machine" [3, 4].

In more complex cases, several structural graphs can be used simultaneously for a given problem, each one modelling different aspect of it. E.g., one graph may represent the structure of the problem situation, either only topological or else augmented with metrical (position) information, another the structure of its formal model (a set of equations or a system of constraints linking the state variables).

For example, the structural graph of the "pulleys" example (cf. Fig. 5) might look like that in Fig. 10a. Graphs of this type can be easily implemented in a computer as linked lists of records. Reasoning with such a representation may be formulated as a set of graph rewriting rules (see Fig. 10b), directly corresponding to the predicate calculus implications of Fig. 4, and controlled by appropriate domain heuristics. A full set of rewriting rules for the domain of pulley systems should be augmented by some more rules and additional notation to capture more complicated configurations and applicability conditions, omitted in the example for simplicity. E.g., the rule (1) has a version usable in the opposite direction; the rules (1) and (4) should explicitly state that no more ropes are attached to the weight; the rules (3) and (4) should require that the ropes are parallel (otherwise, vector rather that scalar summation of forces would be necessary).

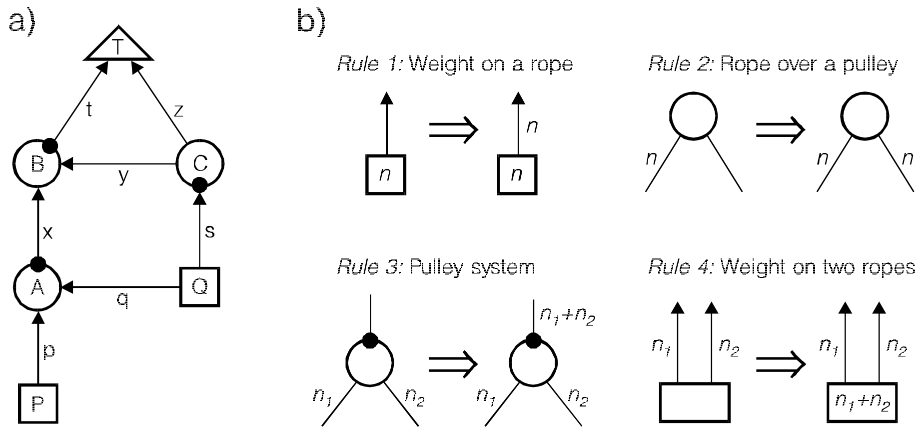


Fig. 10. The structural graph of the "pulleys" example, easy to implement within a computer (a) and a set of graph rewriting rules suitable for diagrammatic reasoning with it (b).

For graphical communication with human users, the internal representations of structural graphs can be visualized in various ways, either using traditional graph notation (i.e., as sets of nodes linked with edges [59]), or after translation into the semantic domain using appropriate visual language fitting the domain. In the "pulleys" example, the output translation of the structural graph might look just like in Fig. 5, possibly, if required, augmented with the visualization of the reasoning process itself (see Fig. 6).

Note that with this kind of representation one is not restricted to diagrams of only two dimensions. Moreover, one may represent arbitrary (analogical) representation schemes. It leads to a generalization of diagrammatic reasoning, called *model-based reasoning*, which reasons by direct manipulation and inspection of an arbitrary *model* (in logical model theory sense) of the problem [36, 57].

6. Application areas

Diagrammatic representation and reasoning already find numerous applications in various fields of AI and other theoretical and applied research. In fact, the research in this field was fuelled mostly by developments in and demands of various application areas. The main application areas are briefly surveyed below:

– *Theorem proving*: primarily, geometry theorem proving, with the first successful computer implementation by Gelernter [3, 4]; it is still a favourite field of study [65]. Other early applications were in logic—let us recall Euler circles and Venn diagrams. The first full system of formal diagrammatic reasoning in logic was proposed by the philosopher and logician C.S. Peirce at the end of 19th century [9, 24], and was later developed by Sowa [24] into a general scheme of knowledge representation. Another exact logical formalism of diagrammatic reasoning, based on Harel's "higraphs" [33], has been developed by Barwise, Etchemendy, and Hammer [54, 70, 71].

– *Visual programming*: already a wide field of research, with many published papers and books [32, 35, 44, 46]. Some authors classify it as a part of the wider area called *software visualization* [51, 72] which includes also *algorithm visualization* (static visualization and *algorithm animation* [32]) and *program visualization*, including *data* and *code* visualization and animation. This field originated from various diagramming techniques (starting from the well-known flow-charts) devised as aids for programmers [27]. Further developments, closely linked with visual languages research, led to elaborate systems of algorithm animation [32] and large number of *visual programming languages* [35, 44, 46], some of them exploring simultaneously novel programming paradigms, like constraint satisfaction [49, 52], or combination of data flow and object orientation (the commercially available Prograph language [37]).

– *Qualitative physics*: also a considerably new area of AI research attempting to model the everyday, qualitative, non-numerical reasoning humans use to estimate (the range of) possible solutions to some real-world problems, especially in the case of inexact or incomplete data [38]. The qualitative reasoning is also a necessary foundation of the quantitative knowledge the engineer or designer use in their understanding and construction of physical systems or structures. Traditional mechanical or construction design used extensively drawings for documentation purposes, as informal aids for reasoning, and even to solve certain problems graphically. With the advent of computers, however, the purely numerical methods started to predominate. Similarly, the qualitative physics research has largely been dominated by formal algebraic methods, despite the fact that one of the early working systems of diagrammatic reasoning, namely WHISPER [11], was designated to solve qualitative physics problems. Recently, however, diagrammatic reasoning is again becoming an important ingredient of qualitative physics approaches. The most obvious application of diagrams is the so-called *spatial reasoning* [56, 58, 60] which uses them to reason about spatial configurations, relations, and interactions. In computational kinematics, for example, local interactions of mechanical components are represented in a *configuration space* defined by component degrees of freedom. The regions of allowed component states are then geometrically subdivided into subregions containing qualitatively equivalent states and the partitioning graph is then used directly to control the reasoning about possible behaviours of the whole mechanical assembly [56, 58]. Other examples of diagrammatic reasoning are an analysis of nonlinear systems dynamics using their classic graphical representation, namely phase portraits [63] and analysis of frame structures [73]. The popular algebraic method of *qualitative simulation* [38] may benefit from the use of diagrams to represent and control the solution of constraint networks.

– *Data presentation*: closely related to diagrammatic representation and reasoning domain, though usually considered as a separate field. Data presentation research provides much of the knowledge and insight needed to construct proper diagrammatic representations [19, 26, 53], and diagrammatic reasoning is often indispensable for automatic presentation design [55, 59, 61]. The field has already been discussed in Sec. 3.2 and 3.3.

7. Suggestions for further study

Readers interested in further study of the subject are advised to consult the provided references. Some general sources are briefly introduced below:

– *Journal of Visual Languages and Computing*: a quarterly journal published by Academic Press (London), launched in 1990; concentrates on general issues of visual languages and their applications in computing.

– *IEEE Workshops on Visual Languages*: a series of annual conferences organized by IEEE Computer Society; launched in 1987; concentrate on general issues and applications of visual languages.

– *IEEE Conferences on Visualization*: a series of conferences organized by IEEE Computer Society; launched in 1990 [48]; concentrate on methods and examples of (scientific) data visualization.

– *AAAI Spring Symposium on Reasoning with Diagrammatic Representations—Working Notes*: edited by N. Hari Narayanan in 1992 [67]; materials of the first conference devoted to the subject of diagrammatic representations and reasoning; the comprehensive review of the symposium scope and results can be found in [69].
– *Visual Programming Environments*: a large, two-volume collection of reprints compiled by E.P. Glinert in 1990 [46]; concentrates mostly on visual programming, but contains also many papers of general significance, together with extensive source bibliography.

Also, many papers on the subject may be found scattered in journals and conference proceedings on artificial intelligence (esp. knowledge representation), computer graphics (esp. graphical interfaces, business and scientific data visualization), software engineering (esp. visual programming), and cognitive science.

After the AAAI Symposium listed above, a diagrams discussion list has been established on the Internet network. To subscribe, send an e-mail request to: < `diagrams-request@csli.stanford.edu` >.

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