ON THE OCCURRENCE OF SOME FAMILIAR PROCESSES REVERSED IN TIME

Section I

The course of everyday events reversed in time would contain many miracles. We have all seen movies run backwards: broken glasses resurrect themselves, newspapers emerge from roaring flames; chaos gives way to order without apparent cause. Though not so easily portrayed by the camera, in the reversed course of events there would be many uncaused coincidences among things distant in space and time. Consider the following sequence: two packs of cards are put in the same order, one pack removed to a distant place and both shuffled. Seen backwards, two packs of cards, shuffled in different places, would miraculously come into the same order. More generally, the loss of some resemblance between distant objects due to their decay or destruction would seem, reversed in time, like the miraculous emergence of that resemblance.

To say that an event reversed in time would seem miraculous is to say that the known laws of nature which bear on that event are asymmetrical in time. This is the case with most laws, especially those which are relatively particular, which bear on limited ranges of phenomena. And yet the laws of mechanics, which tie together so many particular laws and to which science largely owes its unity and power, are symmetrical in time. This suggests that time symmetry is the more fundamental condition of nature. In this paper I shall argue that it is also the more general condition; that the asymmetrical laws of the everyday world are special cases of symmetrical laws, and that their asymmetry may not be perfectly realized. Thus decay, destruction, learning, forgetting may sometimes occur reversed in time; I suggest that these occurrences explain the psychic phenomena.

In the present paper I hope to take a first step towards formulating a unified theory which includes present day physics and which removes the anomalous aspect of the psychic phenomena. The point of departure has been suggested: to find a law which will allow for certain familiar processes occurring reversed in time. The phenomena of precognition suggests that these should include something like memory, or more generally, the retention, transfer and destruction of information. Thus our first task will be to find a way of describing memory, so conceived, which remains intelligible reversed in time. Such a description must be quite general, and must bring out those laws which characterize a memory process as such.

Most laws of nature are formulated in terms of causes and effects. However, I believe it would be unwise to employ causal descriptions in the first stages of our inquiry. This point requires some expansion. Part of the meaning of "A causes B" is that A is always accompanied by B. Another and very important part is that we can imagine ourselves controlling A and independently observing B. An electric current in a light bulb is always accompanied by light, but so is light in a light bulb always accompanied by current. Yet we say that the current causes the light because we are accustomed to switching on the current and observing the light. There are situations where our habits are more symmetrical. Thus a certain current in the light bulb is always accompanied by a certain voltage and conversely. If we wish to discover the relation between voltage and current we could set up an experiment in which we control the current and measure the voltage, in which case we would say that the impressed current causes a certain voltage, or an experiment in which we control the voltage and measure the current, in which case we would say that the impressed voltage causes a certain current. What happens in the light bulb is the same in both cases; what has changed is in the relation between the light bulb and our will.

The concepts of cause and effect are of central importance in the applied and experimental sciences, but in the theoretical sciences, especially physics and astronomy, they have played a much smaller role. This is understandable, since in trying to find the most intelligible view of the way things are, we don't want to be encumbered by premature or excessively general commitments concerning what we can do. Thus in the present investigation I believe it is important that we first obtain a clear idea simply of what would happen if a process of memory were reversed in time, without attempting to decide what is cause and what is effect.

It must not be thought that I regard a causal analysis of reversed memory as meaningless or unimportant; quite the contrary. In particular it is of the first importance to know whether a cause can occur after its effect. I believe most people would agree that this had happened if the following conditions were realized:

1) An event A is always accompanied by a prior event B.

2) After the occurrence of B we can choose whether or not A occurs.

3) We can observe conclusive evidence of whether or not B occurs.

Our observation of the evidence must come after our choice, for if we observed that B did not occur, we would no longer be free to choose that A occur. However, the problem arises as to what would happen if we should try to observe before we choose, especially if the choice would falsify our observation.

Then I throw a ball, the force which I exert on the ball causes it to move. When I catch a ball, the succession of velocities and forces is precisely the time reversal of those in some instance of throwing the ball, exemplifying the time symmetry of the laws of mechanics. Yet there is nothing about catching a ball that suggests time reversed causality in the above sense; we would say that the velocity of the ball causes a force on my hand, or that my hand exerts a force which stops the ball. Similarly, when I remember an event in the past, the event certainly causes the memory, but we must not assume that this is also the case when the memory is of a future event. It may be that reversed memory is subject to a causal analysis just as "normal" as that of catching a ball. I don't believe that this matter can be settled on the basis of the results in the present paper.

Let us return to the problem of describing memory. Our experiences of our own memory, that is, our experiences of learning, bearing in mind, recalling, and forgetting, offer little help, for we have no way of identifying the time reversals of these experiences, if indeed there are such. However, our experience with writing, reading and erasing, i.e., with making and using records, seems more promising. Since records often function interchangeably with memories, there must be something essential in common. Most psychologists today postulate that our memories are associated with physical traces in our nervous system, which presumably are formed, transferred and erased like records on paper or "memory" traces in a computing machine; whatever the case, in the present paper we shall be concerned with only those features of memory that can be found in the behavior of physical traces.

For us the important thing about physical traces is that their behavior can be described in terms of motion, the motion of atoms, molecules, electrons, etc. Motion reversed in time is still motion; thus if we can discover those peculiarities of aggregate atomic motion which identify remembering and forgetting, the same peculiarities of the reversed motion should identify reversed remembering and reversed forgetting. We can then contemplate two possibilities: First, that there actually are aggregates of atoms whose motion displays reversed memory, second, that there are objects as yet unidentified whose aggregate behavior is compounded from their individual behavior in a way that embodies the same formal relation as that of reversed "physical" memory to atomic motion. I shall offer what seems to me strong evidence for the second possibility and equivocal evidence for the first; in particular, I doubt whether there is reversed memory in the atoms of the brain.

Let us further specialize our initial inquiry by considering only those traces in which information is represented by a spatial arrangement of atoms (this excludes the circulating and magnetic memory traces found in computing machines; these require a more complicated but not essentially different analysis). Such a trace will persist, i.e. its information will be remembered, as long as too many atoms do not move in a way that would break up the pattern. Thus the process of forgetting, which will concern us first, may be defined as the disappearance, of a pattern, which accompanies a suitable aggregate motion of the constituent atoms. We know that the atoms of any object are in constant uncoordinated motion whose energy constitutes the heat of that object; this thermal motion may lead to the forgetting of a suitably isolated trace. Let us consider a simple example where this happens in a fairly short time.

Suppose I take a fountain pen (the old fashioned kind, not a ball point) and write my name in a dish of water. There results a trace whose persistence is defined by the condition that in some regions of the water (the letters) there is a much greater concentration of ink molecules than in the rest. The trace slowly becomes more diffuse until it is illegible. Finally the ink is distributed uniformly throughout the dish; in this state we may regard the trace as completely forgotten.

The atomic explanation of this phenomena goes roughly as follows: Since water is a liquid, the ink molecules are free to move independently to any part of the dish, thus the motion of their heat causes them to wander in an uncoordinated way, each disregarding, as it were, the motion of the others. Thus more ink molecules go from regions of high concentration to regions of low concentration than go the other way, and the concentration of the whole tends towards uniformity.

Consider the disappearance of the trace reversed in time. Starting from a uniform distribution in the dish, the ink gradually concentrates into the letters of my name. First let us inquire: Is there anything in our common experience which resembles this, in the way that catching a ball resembles the time reversal of throwing a ball? Superficially the answer seems to be yes, it resembles the concentration of a liquid in a selective field of force; something like it would happen in a suitably shaped and polarized electric field if the ink molecules were ionized. When we examine the phenomena on the molecular level, however, the resemblance disappears entirely.

In the absence of a force field, a typical molecule of ink moves an approximately equal amount in every direction. In the presence of a force field, although the molecule's motion is still largely random, there is more motion in the direction of the force, with the result that it and the others drift in that direction. Since the first condition is obviously symmetrical in time, the tendency towards concentration in reversed forgetting cannot be the result of such a drift;* in fact it seems to result from an enormous number of coincidences in seemingly random changes of direction of the various molecules. There is no counterpart to such coincidences in our common experience, except possibly the psychic phenomena.

More generally, if we conceive of forgetting as the loss of order or structure due to the uncoordinated motion of parts, the time reversal of forgetting will contain many coincidences which we should have an awkward time explaining in terms of forces or other familiar agencies. What force could put Humpty Dumpty together again?

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*Of course the aggregate of molecules shows a surplus of motion towards the regions of higher concentration; the important point is that the drift of each molecule is within its normal range of fluctuation, while this is not the case with "forced" migration.

To return to our examples: Can we conceive of a process in the brain which resembles ink diffusing through water reversed in time, and which might explain precognition? With such a process, our coming to know of a future event would really be forgetting the future event reversed in time. Presumably this would occur at some unconscious level, along with ordinary forgetting, and the reversed trace would be brought into consciousness by the same kind of associative devices that bring up ordinary memories.

This simple model of precognition is open to a strong objection. We have been treating the time direction of diffusion as a property of the dish of water any ink in isolation, but how do we know that this direction was not established by the interaction of the dish with the pen? If we leave the dish in isolation forever traces will occasionally be formed by accident which will be forgotten backwards as well as forwards. If a trace is formed by the dish interacting with something else, such as a pen, it is reasonable to conjecture that it is the time direction of the free energy expended in the trace formation that establishes the time direction of forgetting. Applying this to precognition: whenever we learn of an event by sensory means, the information is conveyed to us by energy expended to stimulate our senses, and this energy is oriented forward in time. Thus it seems reasonable that this information could only be forgotten forward in time.

To answer this objection it is necessary to modify the ink example somewhat. Instead of writing my name with a pen, I will write it by letting a blob of ink diffuse through a template in which my name has been punched. In more detail: I take a thin flat piece of metal and punch many holes in it so as to spell out my name. Then I drop a blob of ink into one side of the dish of water. I place the template in the water on the other side of the dish and slide it over the blob. The ink diffuses through the holes writing my name above the template. I slide the template out and the remaining trace is forgotten as before.

It seems possible to conceive of the system of water and ink blob being reversed in time, while that of the template and myself remains forward-going. The template does not leave a trace by expending energy but merely by its passive presence. Even inserting and withdrawing it should do negligible work on the ink since this motion is at right angles to its expansion. Thus the time direction of the diffusion should be established by that of the energy which inserts the ink blobs and should be independent of the direction of the energy that forms and inserts the template. The ink blob may be regarded as a trace and its expansion as a form of forgetting; the important thing for our precognition model is that the information contained in the template is not contained in the blob. If our sense impressions form something like a template, they may leave a time reversed trace of an event provided our brain later acquires something like a time reversed ink blob to diffuse through the template. Perhaps the "ink blob" comes from our food like ordinary free energy; of course it is not the food that we have eaten but the food that we will eat.

In the above example we treated the forward and reversed processes as if they were isolated except for the intervention of the template. This seems quite implausible; if there are processes of reversed forgetting they are probably compounded with normal processes and if we hope to discover them we must discover the laws of the compounds. I shall propose one such law in the present section and another in section three; the latter is more general and seems more natural but demands more difficult mathematics, so most of the paper will be devoted to exploring the consequences of the former.

First we must take a somewhat more abstract approach to normal forgetting. Let us perform the following thought experiment. We take the dish filled with a mixture of water and ink and isolate it from the rest of the world forever. Or rather to be more accurate, since the actual dish is not an object of thought, let us take an idealized representation of the dish and imagine that it preserves certain attributes forever. We might choose any one of the following three representations:

1) The dish is idealized as a permanent impenetrable barrier (suppose it completely encloses the water). The molecules of water and ink are idealized as perfect Newtonian particles. The system retains its energy forever and its behavior is completely determined by its initial state.

2) The dish is ignored, the water is idealized as a permanent container of the ink molecules which are idealized as particles undergoing independent random motion specified by transition probabilities (e.g. random walk).

3) The system of water, ink and dish is idealized as being constrained to a fixed finite set of quantum states, the transitions among which are governed by a fixed set of probabilities.

Define the microstate of the system as the most complete description which our idealization will allow. Thus in 1) the microstate is the shape of the dish and the position and momentum of each of the molecules. In 2) it is the shape of the water, the position and perhaps the velocity of each of the ink molecules. In 3) it is the quantum state.

Define the macrostate of the system as the distribution of ink throughout the water, conceived as a continuous function. We know the macrostate if we know the density of ink at every point of the water.*

We have been regarding traces as properties of the macrostate of the system; we shall now formulate a general law of forgetting in terms of the evolution of the macrostate in our thought experiment.

If a macrostate occurs once in the thought experiment, it will recur in infinite number of times, though perhaps very infrequently. Furthermore, every sufficiently simple** macrostate will occur. Choose a particular macrostate M. Choose a time interval Δ. It can be shown that there exists a macrostate M' which almost always follows M after time Δ. Thus if M is the concentration of the ink in a quarter inch blob and Δ is one second, then M' might be the concentration of the ink in a half inch blob. In almost every instance where the ink becomes "accidently" concentrated into a quarter inch blob, it will be found in a half inch blob a second later. We shall say that M' is the expected state after one second, given state M. The expected state will always have a more uniform concentration, and in particular, the expected state after a sufficiently long time will always be the equilibrium state of completely uniform concentration, whatever the initial state. This justifies the name of the following empirical law, which holds of the actual dish, not the thought experiment.

Law of decay: The macrostate at a later time is always the expected state given the macrostate at an earlier time.

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* Macrostate, like differential, is a limit concept. Statements about macrostates become logically rigorous only when paraphrased as rather complicated statements about what happens when the number of elements in an ensemble (in the above case, molecules) becomes arbitrarily large, or (as in the next section) when certain statistical parameters approach zero. Such paraphrases seldom serve the purposes of communication.

** Which can be represented continuously by the limited number of molecules.

The purpose of the thought experiment is to enable us to define and calculate the expected state. Any of the three ways of idealizing the water-ink system, if carried out properly in detail, should lead to expected states such that the law of decay holds within the limits of experimental error.

Our thought experiment is an example of a very general procedure used more or less consciously in all the sciences. The scientist wants to investigate a system whose gross behavior is deterministic, that is, under suitably uniform conditions of environment, a given state is always followed by the same "expected" state. He has some knowledge of the microstructure of his system which leads him to imagine an ensemble of similar systems which are initially in the same macrostate but which may differ in their microstates. He makes certain assumptions about the distribution of the various microstates in the ensemble, and about the way that the microstates will change with time. If his assumptions are fortunate, he will find that most of the systems will evolve into the same macrostate, which is the expected macrostate in our sense. If his luck holds out, the resulting law of decay will predict the behavior of the actual system.

We used the successive recurrences of a macrostate in an isolated system to generate the ensemble. This is the method used in classical statistical mechanics, where it has the beauty that the distribution of microstates in the ensemble is completely determined by the laws of Newtonian mechanics.

It should be clear that the law of decay by itself is not a simple empirical law; one is tempted to regard it as without empirical content as merely a convention of scientific procedure. I believe that this would be going too far, for we can conceive of perfectly intelligible worlds which the law of decay would be of no help in understanding, in particular, our familiar world run backwards in time. Furthermore, it may be possible to use thought experiments of the kind discussed above to illuminate a much wider range of phenomena by generalizing the law of decay.

The law of decay seems to formalize our intuitive understanding of physical forgetting; uncoordinated motion can occur in many more ways than coordinated motions and the former always leads to the expected state. Reversed forgetting should therefore be formalized by the

Law of Reversed Decay: The macrostate at a given time is always the state which would be expected from a later state, were that later state earlier by the same length of time.

We now return to the question: Is there a law which might formalize the behavior of a system containing both forward and reversed forgetting? I shall propose a law which has both the law of decay and the law of reversed decay as special cases and which may apply to some such systems.

Let M and M' be two particular macrostates of the ink system. Consider in the thought experiment the class of all instances in which M' follows M after time Δ. Even though M' is not the expected state it may occasionally occur "accidentally"; if it occurs at all it will occur an infinite number of times. What can be said about the state which occurs at an intermediate time in these instances? If this intermediate time is fixed relative to the occurrence of M (say $\Delta/2$ later) the intermediate state will almost always be the same, hence we may refer to it as the expected state given M and M'.* We can now formulate

The first symmetrical law: The macrostate at a given time is the expected state given the macrostates at both earlier and later time.

The intuitive meaning of this law may become clearer from an example. Let M and M' be the same state, that where the ink is in a quarter inch blob. Suppose this state occurs and then recurs after ten seconds. Of course this would be impossible under the law of decay, but in the thought experiment it will occur occasionally. The first symmetrical law (which will henceforth be referred to as the first law) says that during the intervening ten seconds the ink will behave as "probably" as it can; thus the wanderings of the ink molecules will not be constrained by any condition other than that they be back in the blob after ten seconds. The wanderings cannot be completely uncoordinated, but they will be as uncoordinated as possible, thus the blob will begin to expand, then contract again. It will not expand as rapidly as it would when decaying, for then the molecules would have to do an excessive amount of coordinating to get back together; better that they spend some of that in staying together.

* There will be an expected intermediate state for any system in which the macrostates are defined as distributions in an ensemble of independent systems.

The first law clearly has the law of decay as a special case. To see that it has the law of reversed decay as a special case we note that the statistics of macrostates in the thought experiment is symmetrical in time, that is, the states occur before a given state with the frequency that they occur after it. This symmetry is characteristic of the thought experiments in statistical mechanics, and derives from the time symmetry of the fundamental laws of mechanics.

Let us briefly consider the relevance of the first law to psychic phenomena. Define the <u>constraint</u> on a future state of a system as the way in which that state differs from the expected state conditioned by a past state. In our last example, the recurrence of the blob was constrained in that the blob was too small; it might have been constrained to be too large, or too irregular, or in the wrong place, etc. It must not be thought that constraint has anything to do with destiny or miracles or extraphysical guiding forces or what-not. It merely refers to the failure of a given thought experiment to provide an appropriate law of decay. It may be that with a new thought experiment, which takes into account new forces or finer micro-structure, the state would not seem constrained. The important point at present is this: If, given the old thought experiment, the course of events leading to the constrained state satisfies the first law, these events will exhibit the same peculiar kind of coincidences that we noted in pure reversed forgetting. These coincidences will seemingly disregard space and time, thus discouraging the attempt to find new micro-entities or forces which could save the law of decay (I shall argue in the next section that we may be able to regard forces as resulting from constraints satisfying something like the first laws but not conversely).

What sort of events might be constrained so as to lead to the psychic phenomena? The first law says that any constraint must be such as we would expect from a future constraint; this limits the possibilities somewhat, for it seems to imply that nothing can be constrained which is completely forgotten, which leaves no trace or effect. This does not tell us much about what can be constrained, however. We distinguish a spectrum of possibilities: On the one hand we might have a completely amorphous constraint, which has very little to do with the particulars of a person's life; something in his brain might be constrained in the way that the ink blob was constrained to be too small. If this "blob" were to leave a reversed trace which would result in an earlier precognition, then some of the person's particular actions would also be constrained, e.g., his telling someone of his precognition. This leads us to the other end of the spectrum, where some peculiarities of a human situation are constrained. For instance, the end state of an experiment in extrasensory perception might be constrained such that the experimenters believe the experiment a success. If the experiment was well designed, the most likely course of events leading to this state might include the percipient actually guessing the targets correctly. If psychic researchers become obliged to study constraints such as this, they will clearly have to drastically revise their concepts of experimental method.

Section II

In the first section we contemplated a law which, if it held in familiar domains of nature, would result in coincidences among events separated by space and time which would be very hard to explain by more familiar laws. Since we know essentially nothing more about the psychic phenomena than that they exhibit such coincidences, it is at least possible that they result from the operation of this law. Yet this is a very weak conclusion, for the first law asserts much more than the occurrence of strange coincidences. If we are to separate it from its competitors, we must find phenomena which not only might be explained by it, but which clearly exhibit its lawful character. Since human events are so complicated and hard to repeat, it would seem wise to look first at the behavior of the lower animals, or at inorganic nature. In the present section we shall do the latter, and we shall see that the first law, with a suitable thought experiment, leads to the laws of Newtonian mechanics.

It was remarked in the first section that the laws of Newtonian mechanics are symmetrical in time. This sharply distinguishes them from all other known laws of change which apply to the macroscopic world. In the last century this caused no embarrassment since the Newtonian laws are deterministic, while the others were assumed to be ultimately statistical, their apparent determinism deriving from the large number of atoms involved. Since all statistical changes were analysed by the law of decay, the asymmetry of these laws seemed guaranteed.

In the present century physicists have discovered that the Newtonian laws do not apply to sufficiently small particles, whose motion is subject to irrepressible fluctuations, given by the Heisenberg uncertainty principle. This strongly suggests that the Newtonian laws are also statistical, or more precisely, that those objects which obey Newtonian laws seem deterministic because their motion is an aspect of the behavior of a very large number of elementary processes.

Let x be the position of the center of gravity of a Newtonian object moving in a conservative force field (to simplify the discussion we will suppose that the object can move in only one direction). The trajectory of x will satisfy the equation:

$$F = ma$$

where F is the force, m the mass and a the acceleration. We can calculate this trajectory if we know the mass, the force field (i.e., F as a function of x) and boundary conditions, i.e. either the value of x and of dx/dt at a single time or the value of x at two times.

If x is a large-number variable, that is, if x bears the kind of relation to the changes in some large aggregate of elementary processes that temperature bears to the motion of atoms, it should be possible to derive the law F = ma from a statistical analysis of these elementary processes, just as the laws of heat flow have been derived from the statistics of atomic motions. The procedure should follow that which we used in analysing the ink-water system. We construct a suitable idealization of the aggregate of elementary processes which we can imagine to be in permanent isolation. This idealization should embody the mass and force field as permanent features, while allowing that all possible boundary conditions occur by accident. The class of expected or most probable trajectories, given the accidental occurrences of the boundary conditions, should then be the class of trajectories satisfying F = ma.

Consider the conventional way of carrying this out, i.e., so as to use the law of decay. The macrostate of the system would be defined by the position and velocity of the object; the two would fluctuate to some extent independently so that by chance they would occasionally come to any pair of values. In the vast majority of cases, after the successive recurrences of a given pair of values the system would evolve according to F = ma, with this pair as boundary conditions.

No doubt systems can be imagined which carry out this procedure. But the procedure does not recommend itself, for several reasons. First, there is no reason to think that velocity has any meaning apart from change of position, thus the definition of macrostate is artificial. If the two can fluctuate independently, we may sometimes have the velocity going one way while the position goes the other, which is absurd. More serious is the consideration of time symmetry. The statement that the law of decay leads to asymmetrical laws needs some qualification, since particular variables of a decaying system may exhibit time symmetry, e.g. the voltage and current in an electrical oscillator. However, the oscillator is an open system; it maintains its oscillation by taking in free energy and losing heat, which is quite asymmetrical. If we wish to idealize the oscillator as an isolated system, we must introduce an asymmetry into its microstatistics which just counterbalances the tendency of its voltage and current to decay. The same is presumably true of our Newtonian model; the microstatistics would need some sort of "rejuvenating" asymmetry, which seems completely artificial.

Now consider the experiment carried out to use the first law. The macrostate will be defined simply as the position, which will fluctuate so as to occasionally take any pair* of values at the beginning and end of a fixed time interval. In the vast majority of cases the position will change during this interval according to F = ma, with the initial and final positions as boundary conditions.

It has long been known that the motion of a Newtonian system can be regarded as determined by a variational principle. The system is assumed to be initially in one configuration** and finally in another, its evolution being governed by the condition that a certain integral of the motion is minimized. The first law, as applied above, gives a simple statistical interpretation of the variational method; that which is minimized is the improbability, in the thought experiment, of the system's behavior. Given the initial configuration, it may be very improbable that the system be in the final configuration at the allotted time, but if we are also given that it goes to the final configuration, its most probable behavior in getting there will satisfy the Newtonian laws of motion. That the probability*** of the most probable behavior can be expressed by an integral of motion follows from what seems to be the characteristic feature of large-number variables: the fluctuations in consecutive time intervals are statistically independent. It appears that the theory of arbitrary systems of large-number variables under the first law is almost formally identical with the theory of Newtonian motion. The assumptions we shall make will be such as to simplify our cases; dropping them would mean that mass is no longer constant, that the coordinates are no longer rectangular, etc., but would not destroy the essentially Newtonian character of the motion.

Before going on to mathematical details it seems in order to pause for a moment and take stock -- to view in a general way the nature of our enterprise. In the first section we considered a thought experiment which provides a statistical means for finding the most probable succession of states in those processes which we intuitively recognized to embody forgetting

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* Or any pair not too distant. We shall not be concerned with very high velocities, which belong in the province of relativity theory.

** The configurations are only of positions, not velocities.

*** Or more accurately, its logarithm.

or decay. We saw that the same thought experiment provides a more general statistics which would give the most probable succession of states in a process embodying both decay and time-reversed decay. After remarking its relevance to the psychic phenomena, we inquired whether there is anything in nature which exhibits the lawful character of these statistics, preferrably in a simple quantitative way. We found (with details still to come) that if any collection of things whose collective behavior is characterized by large-number variables exhibits these statistics, the large-number variables will behave according to Newton's laws. Now, where does this leave us? It seems that there are two hypotheses up for test:

1. There are some processes in nature satisfying the first law, for a suitable thought experiment.

2. There exist elementary things whose collective behavior, satisfying the first law, makes up the motion of large objects.

If these elementary things do not exist our derivation of Newton's laws is a mere mathematical curiosity, which does not further the case for the first law. It would be very helpful if we had some independent knowledge of the elementary things. Is it possible that they are to be found among the known micro-entities -- atoms, electrons, etc.? I very much doubt it, for these things satisfy laws which are sufficiently close to Newtonian to suggest that they are individually subject to something like first law constraints, and that the motion of a large object is merely the sum of the motion of its atoms, in the way that the heat in a pail of sand is merely the sum of the heat in the grains of sand, and has nothing to do with the motion of the grains. Furthermore, the statistics of atomic and particle notion is to some extent covered by statistical mechanics, and in the simple cases we know of seems to be pure decay. If the elementary things exist, the known micro-entities must be composed of relatively small collections of them. This could explain why the laws of microscopic motion are more complicated than the Newtonian laws (ten musicians playing different tunes make a complicated polyphony, a million musicians merely make a hiss).

Our knowledge of elementary things will probably have to come from very indirect pieces of evidence which will tend to confirm or modify our theory as a whole rather than adding to it separately. In this respect it resembles our knowledge of atoms rather than e.g. our knowledge of cells in biology. The history of atomic theory shows certain parallels to our present inquiry of which we may profitably take note. In the eighteenth and nineteenth centuries it was found that the atomic hypothesis, together with certain statistical assumptions, led to the laws governing a wide range of macroscopic phenomena. Yet no one had ever seen, heard, measured or otherwise encountered a single atom, and the phenomena were all amenable to alternative explanations in terms of continuous "fluids" subject to deterministic laws. As late as 1905 W. Ostrand contended that the atomic hypothesis was the sort of speculation that had no place in science. The laws explained by atoms were all stated in deterministic form; their statistical interpretation was of the expected behavior of large-number variables or fluid macrostates. It was apparently Einstein who first realized that, due to the finite number of atoms involved certain phenomena should exhibit measurable fluctuations from their expected behavior; the magnitude of the former should be predictable from the same statistics that give the latter. His successful analysis of Brownian motion on this basis effectively ended opposition to the atomic hypothesis, for there was no longer a reasonable continuous alternative.

It seems to me that the Heisenberg uncertainty principle provides an argument for the existence of elementary things, something like that which the laws of Brownian motion provide for the existence of atoms. We can derive the uncertainty principle from the assumption that the center of mass is a large-number variable, whose fluctuations are related to the mass in what seems to be the only natural way. Let x be the center of mass of, say, a water molecule. Suppose the fluctuations of x in successive small time intervals are independent. Then the probability distribution for the value of x after a sufficiently long time will be normal* with the expected position as mean, and, assuming the fluctuation rate constant, with dispersion proportional to time. Let Dw be the dispersion in unit time. If two water molecules do not interact at all their fluctuations will be independent, while if they interact strongly their fluctuations should be correlated. It seems a safe guess that if their energy of interaction is small compared with their energy of mass, the correlation should be negligible. This is certainly true of the molecules in a glass of water, which we shall regard as independent. Let Dg be the dispersion after unit time of the center of gravity of all the water molecules in the glass. Since the sum of independent

* By the central limit theorem.

dispersions is the dispersion of the sum, it is easily seen that:

$$Dg = Dw/n$$

where n is the number of water molecules. Let M be the mass of the water; M will be proportional to n, hence for some constant k:

1) $Dg = k/M$

Thus it is clear that if the dispersion is a function of mass alone, this function is given by 1), where k is a universal constant.

Suppose that a body starts in a position x and after moving uniformly for a very long time T ends at y. In the Newtonian idealization the instantaneous velocity is always $v = x\text{-}y/T$, but if we take into account fluctuations the instantaneous velocity will be unrelated to v. If we want to estimate v at the initial time we could take the average velocity over an interval t from the beginning; the longer is t the better the estimate. However, the longer we take t the more will the position at the end of t err from the ideal position of the Newtonian trajectory which starts at x and travels at v. It is easily seen from 1) that the product of this error times the error in momentum based on the velocity estimate is the constant k; taking k as Planck's constant, this is the uncertainty principle.

The above argument for the existence of elementary things is of course independent of the argument from the first law. However, the two strongly reenforce each other because of the following coincidence: The relationship between mass and dispersion given by 1) is precisely that in the thought experiment when the first law is interpreted as giving Newton's laws. Thus there is a parallel with the case for atoms. A certain thought experiment with atoms leads to expected states which, with the law of decay, give the laws of thermodynamics, and exhibits fluctuations matching those of Brownian motion, etc. A certain thought experiment with elementary things leads to expected states which, with the first law, gives Newton's laws of mechanics, and exhibits fluctuations which may be interpreted as the uncertainty principle.

It may seem strange to consider arguments for the existence of things about which we have assumed absolutely nothing except that they are very numerous -- which as yet have no individual features. In that we are trying to build up a plausible theory on the basis of very indirect evidence, it is important that we not start with overly detailed hypotheses, for the erroneous details may be hard to weed out, especially when we become used to them. On the other hand, it seems to me a great mistake to regard the elementary things as a mere convenience, to be judged by their usefulness in physics. It may be that the laws of physics express as little of what really happens to these things as the diffusion equation would express of the scattering of a company of soldiers under bombardment. Our ignorance should stimulate our curiosity. Are we dealing with little billiard balls or cavorting angels? Are the things totally exotic or are they perhaps already known to us in another context? Are they uniform or diverse in their qualities? Are they small or large or are they perhaps not in space at all?

Let us return to the derivation of Newton's laws. Since we will often have occasion to contrast the expected states of the first law and the law of decay, it will be useful to have a distinguishing terminology. An expected state conditioned by both a past and future state will be called a <u>constrained</u> state, while <u>expected</u> state, without further qualification, will mean expected state conditioned by a past state alone. The same distinction will be made of trajectories, velocities, moments, accelerations, etc., thus a constrained trajectory will be a succession of constrained states conditioned by the initial and final states, an expected trajectory a succession of expected states conditioned by the initial state.

First, let us consider an isolated system with only one position variable, such as that discussed at the beginning of the section. This system will be characterized by several assumptions about x.

Axiom 1. x defines the macrostate of the system, and the system provides both expected and constrained states.

We must now pay a slight penalty for our cavalier logic in treating macrostates. Since x is always undergoing small fluctuations, clearly we do not intend by axiom 1 that the exact same value of x almost always recurs as the expected state, but rather that the conditioned values of x are almost always close to the expected value which is their average or expectation. With this understood, we can retain our old way of speech with "macrostate" regarded as the approximate value of x; in the limit as the fluctuations approach zero this becomes the exact value.

Axiom 2. The fluctuations in x during a sufficiently long time interval are independent of the fluctuations before or after that interval. As we remarked before, this seems to be the characteristic property of a large-number variable, and says, in effect, that the elementary processes are to some extent random and independent.

Axiom 3. The expected trajectories are continuous.

Axiom 4. The fluctuations of x during a fixed time interval are independent of time and of the initial value of x.

Axiom 5. The statistics of x are symmetrical in time.

The last two axioms are conditions of symmetry which simplify our case.

Our problem is now this: given an initial position x_1 and a final position x_2 after time T, what is the constrained trajectory between these positions? This means: if we observe those instances in the thought experiment where x_1 occurs followed T seconds later by x_2* what, in the vast majority of instances, will be the succession of intermediate states?

As we observed before, axiom 2 implies that after a sufficiently long time the fluctuations will be normal. In the thought experiment this means that if we observe the successive recurrences of the position x_1, the positions after t seconds will be normally distributed about the expected position; the dispersion is proportional to t. Due to the enormous number of atoms in a macroscopic body, t may be extremely short for our purposes, of infinitesimal length. Let us define mass by equation 1), for convenience taking k as unity:

$$2) \quad M = 1/D$$

where D is the dispersion after one second. Our unit of mass is extremely small; if k is Planck's constant and distance is measured in centimeters unit mass is about 7×10^{-27} grams. This gives us some idea of how small the fluctuations must be; the expected deviation from the mean of a one gram object after one second is only about 10^{-13} centimeters.

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* The pair x_1 and x_2 may never occur exactly; since we are only concerned with the approximate position, it suffices that the system come arbitrarily close.

We will evaluate the constrained trajectory by regarding it as a very large (and of course very improbable) fluctuation from the expected trajectory. By axiom 2 the probability P of any trajectory (not necessarily the most probable) joining x_1 and x_2 conditioned by x_1 is the product of the probabilities of the successive fluctuations in time intervals dt, conditioned by the initial positions in these intervals. Let us evaluate the probability p of the fluctuation in an interval dt, assuming the trajectory is continuous. During dt the motion may be regarded as uniform; furthermore, this must be the most probable motion leading from the initial to the final point during dt.* Thus p may be identified with the probability of the deviation of the final point from the expected final point. Let the initial point be x. By axiom 3 there will be an expected velocity at x; call it V; call the actual velocity v. At the end of dt the expected position will be x + Vdt, the actual position x + vdt, the deviation (V-v)dt. The distribution is normal with dispersion Ddt = 1/Mdt, hence we can express p approximately:

$$3)\quad p = (\tfrac{1}{2} M(V-v)^2 dt)^{-\frac{1}{2}} * e^{\frac{1}{2} M(V-v)^2 dt} .$$

Taking the negative logarithm of both sides:

$$4)\quad -\log p = \tfrac{1}{2} M(V-v)^2 dt \text{ plus negligible term.}$$

The negative logarithm of the probability of the entire trajectory can be written:

$$5)\quad -\log p = \int_{t_1}^{t_2} \tfrac{1}{2} M(V-v)^2 dt$$

where t_1 and t_2 are the times of occurrence of x_1 and x_2 respectively. The constrained trajectory will be that for which this expression is a minimum.

We can repeat the above argument regarding the trajectory as a large fluctuation conditioned by the final state x_2. Since the statistics are symmetrical in time, the expected velocities conditioned by the final state will be the negatives of those conditioned by the initial state and we can write for the overall probability P'

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* If the most probable overall trajectory is continuous, which it must certainly be.

6) $-\log p = \int_{t_1}^{t_2} \frac{1}{2} M(V+v)^2 \, dt$.

Since the quantities under the integral signs in both 5) and 6) are always positive and each is a minimum for the constrained trajectory their sum must be a minimum. Let A be ½ the sum. Then

7) $A = \int_{t_1}^{t_2} (\frac{1}{2} Mv^2 + \frac{1}{2} MV^2) \, dt$

The expression $\frac{1}{2} Mv^2 + MV^2$ may be regarded as the Lagrangian of a Newtonian object of mass M in potential energy field given by

8) $U = -\frac{1}{2} MV^2$

Thus when the expression 7) is a minimum the trajectory will satisfy

9) $F = Ma$

where the force is defined as usual as the negative gradient of the potential field:

10) $F = -\partial U/\partial x$.

The formal resemblance between the terms for kinetic and potential energy is striking, but it becomes more so when we examine the meaning of equation 5). The negative logarithm of a probability has become familiar in recent times as a measure of information; in equation 5) this seems not inappropriate. Suppose that the basic things were people who in the case of pure decay are completely out of communication; each makes a series of decisions disregarding the others, and x measures some aspect of the totality of what is decided. The lack of coordination among the decisions ensures that x follows the expected trajectory. Suppose that we wished to steer the trajectory of x to something other than the expected by instructing the people what to decide, a separate instruction being required for each decision. Then the least possible number of words in all the instructions should be proportional to -logP of equation 5). To move a mass of one gram a distance of one centimeter in one second when it is expected to remain at rest would require a hundred thousand billion times as many words as are in all the books in the library of congress, which is a hint of the enormous number of things which must be involved in macroscopic motion.

Referring to equation 5) we see that the kinetic energy gives the amount of information required per second to sustain the object in uniform motion if it is not in a potential energy field. In constrained motion this information would all be "remembered" from the future. Symmetrically, the negative potential energy gives the time rate of information required to maintain the object at rest in a potential field. I suspect that this relationship between energy and information is fundamental, and will be retained in the generalizations of the first law necessary for a more complete physics.

The Newtonian laws for any number of objects in three dimensions* can be derived by a straightforward extension of the above analysis, which will not be carried out here. Axioms 1 through 5 are simply generalized, and we add a sixth axiom saying that the objects fluctuate independently.

I shall conclude this section with a brief discussion of a more specialized thought experiment which seems to throw some light on the relation of energy and mass, and which may provide a small clue as to the nature of the elementary things. Suppose that in an aggregate of elementary things there occur two kinds of instantaneous events, call them left events and right events. The two kinds of events occur at completely random times, but, during unconstrained motion, at constant and equal rates. If R is the number of right events per second then R is constant in time and equal to the number of left events per second, and the probability of a right (or of a left) event in a sufficiently small time dt is always Rdt. The sequences of left and right events form independent Poisson processes.

Assume that the center of mass x is related to the left and right events in any way so that over a period of uniform motion the approximate velocity of x is proportional to the excess of right over left events divided by the total number of events. It is convenient to have an exact definition of x even though this be artificial and without a counterpart in nature.* Suppose that x always travels with the speed of light but in alternate directions, increasing after a right event and decreasing after a left event; this clearly satisfies the above assumption. It also satisfies axioms 1 through 5. The expected

* In rectilinear coordinates. There may be new problems in general coordinates.

* There is no reason to think that the center of mass of an actual object has more than an approximate value.

velocity is always zero so there is no potential field and the constrained motions are uniform.

Define mass as before, $M = 1/D$ where D is the dispersion after one second. It is easily seen that at low speeds the mass equals the total number of events per second divided by the square of the speed of light, $M = 2R/c^2$. We shall also be concerned with high speeds, beyond that at which Newton's laws hold; we don't yet know the meaning of mass at these speeds.

The energy is purely kinetic; at low speeds this is $\frac{1}{2} Mv^2$, and is equal to the information rate -logP due to the constraint, where P is the probability of v occurring "accidentally" for one second. Let us assume that the energy at high speeds is also given by the information rate. Suppose x were constrained to increase at the speed light. This means that there would be no left events. Since the left events form a Poisson process, the probability of there being none for one second is e^{-R}, thus the information rate is simply R. This constraint should not affect the probability of the right events, hence the "light packet" contains R right events per second.

The combined center of mass of two objects given as above would ordinarily be the average of their separate centers weighted by their masses. The approximate value of the resulting variable changes correctly, but its exact value does not always change at the speed of light. We can also obtain the correct approximate center of mass of the combined object by simply adding the right and left events of the two objects and defining the exact center as above. This procedure seems more natural, and we shall assume it extends to objects at high velocities whose mass may not be defined.

Suppose we have two light packets each of energy R but traveling in opposite directions. One packet consists of R right events, the other of left events, so their combined center of mass is at rest and has mass $2R/c^2$. This means that

$$E = Mc^2$$

for this system, where E is the energy in the two packets.

In view of the symmetry between kinetic and potential energy, the above analysis suggests a picture of the equivalence of energy and rest mass: A solid object at rest, were it to behave in the expected way conditioned only by the past state of the elementary things, would fly apart in all directions at the speed of light. It remains together due to a constraint, the information in which is equal to the negative potential energy of the mass,

This picture suggests a mechanism for force, or at least for those forces between gross objects which can be thought to occupy distinct regions of space: The stability of an object is a dynamic stability, like that of a wave; the elementary things may come and go, for what persists is a pattern. Thus it generally happens that the information required to maintain stability will change in the proximity of another object because of the flux of elementary things to and from the other object. The object will travel in such a path as to minimize the information required to get to its ultimate destination, thus it will deviate from a straight course to be near the other object if the stability there is easy, and to be away from it if the stability is hard; the result is a force.

More precisely: Suppose that in a moving object, as well as in an object at rest, the mass is related to the information rate required to hole the object together by

$$-E = Mc^2$$

where -E is the information rate, regarded as a negative potential energy. Of course mass retains its old meaning, at low velocities, as the reciprocal of the dispersion rate. Suppose the quantity -E is a function of position. If the object is subject to something like first law constraint, it will move so that the total information required both to move it and to hold it together is a minimum. Thus if we identify the total information rate with the Lagrangian, the object's motion will be Newtonian at low velocities, with potential energy simply equal to E; the forces are contained in the way E varies with distance, and there is no potential energy other than that of mass.

This may also explain the mass of kinetic energy. If the object is traveling in a fixed conservative force field, an increase K of the kinetic energy will be accompanied by an equal decrease of the potential energy in the Lagrangian; since the latter is the potential energy of mass and since it is negative, there will result an increase in the mass of K/c^2. Thus the kinetic energy in itself does not actually contribute to the mass, but in order for the object to gain kinetic energy it must also gain mass.

There is a seeming anomaly in the above model of force; it seems as if stability should be easier in the vicinity of an attractive object, harder in the vicinity of a repulsive object, and yet the sign of the energy of mass shows that it must be the other way around. Suppose the stability of A is easy in the vicinity of B. Then A should want to spend as much time as possible in that vicinity. In human terms this is attraction, but in physical terms it is repulsion, for it means that A decelerates while approaching B, accelerates while leaving B.

Section III

In view of the suggestive results obtained in the last section concerning the relation of energy and mass, the question arises whether it is possible to extend the first law analysis to high velocities and derive the special theory of relativity. We must find a model which exhibits the correct relation between the information required to keep an object together and the fluctuations of its position, at any velocity. The stability of an object is a much more complicated concept than the motion of its center of mass, and it seems very doubtful that it can be formulated in terms of large number variables. There is more hope for an analysis in terms of spatial distribution and flow. The first law does not seem well suited to such an analysis.

Not that the first law cannot be applied to distributions; we have seen such an application in section 1. Rather, the first law is not general enough. Consider the glass of water and ink undergoing the following two events: First, it starts in the equilibrium state of uniform distribution and remains in it, the molecules following independent random courses, and second, it starts in the uniform state, remains in it, but the molecules follow sufficiently coordinated courses so that the ink as a whole slowly rotates. Clearly the macrostates cannot be constrained by the first law to bring about the second event; even if the first and last macrostates were rotating, the intermediate states would not be. And yet the second event is quite distinct from the first when regarded as a flow of ink. We shall undoubtedly have to take into account flow in this sense in an adequate field theory of matter.

In order to obtain a natural description of flow, I believe that we must give up the familiar scientific device of regarding the course of events as a succession of states. This is not such a radical step, for we seldom employ this device in our non-scientific thought. We tend to think of change in terms of events. Consider the event of picking up my pencil. As a scientist I might regard a particular occurrence of this event as a succession of states described by the position and velocity of the hand and pencil. However, as pencil user I am concerned with something more macroscopic; picking up the pencil means something common to many events which differ in details of position and velocity. Yet what these events have in common cannot be understood by a more general concept of state; there is no way of defining macrostate as a class of positions and velocities of the hand and pencil such that the concept of picking up the pencil can be described as a succession of these macrostates. We understand a complex event by breaking it into smaller events which form a familiar or intelligible pattern. In science this generally means breaking it into a succession of very short events, or states, and also into a collection of things which persist through the event and, so to speak, tie the states together. The latter decomposition is usually contained in our way of describing the former; the description of a state contains a description of the permanent things which are identified by means of distinguishing marks or labels. To understand a flow we must know how its states are tied together, that is, which part of the fluid goes where. If the fluid is homogeneous its parts have no identifying marks so that its states, conceived as what we could observe at successive times, do not characterize the flow. Of course, we can imagine labels attached to the various parts of the fluid; if the labels remain attached the succession of label states, i.e. distributions of the labels through space, will describe the flow. Or more exactly, since the labels were attached arbitrarily, the flow is that which is common to all label flows which differ only in their initial distributions of labels.

We wish to find a decomposition of the complex event of a flow into component events whose pattern will contain simple regularities which render the flow as a whole intelligible. The decomposition of the label flow into label states seems very unsatisfactory, for one suspects that regularities in their succession have as much to do with labeling as with flow. We can dispense with the labels, however, if we decompose the flow, not into states, but into changes.

A change, conceived as something connected with an earlier and a later time* is a relation between an object at the earlier time and the same object at the later time. Sometimes a change may be described by properties of the state of the object at the two times: "He died" means "He was alive, now he is dead." But this is not usually the case; consider "He grew an inch." Nor is it the case with flow.

We know the flow of a fluid if, given any two regions of space and any two times, we know how much of the fluid which was in the first region at the first time is in the second region at the second time. This knowledge, for all regions and a given pair of times, specifies a change in the fluid from the first time to the second, the most detailed change we can describe if the fluid elements are indistinguishable. I suggest that these changes are the natural time components of flow, as states are

* We sometimes think of changes as occurring throughout a period of time.

the natural time components of the motion of distinguishable objects. The second symmetrical law will be a principle for deriving regularities in the pattern of these changes.

We know the state of the fluid if we know how much fluid is in every region of space. But it suffices to know the amounts of fluid in the infinitesimal regions, thus the state may be specified by a distribution function. Similarly a change* may be specified by a six dimensional distribution function which gives, for any pair of points, the amount of fluid in the infinitesimal region of the first point at the first time which is also in the infinitesimal region of the second point at the second time.**

Let us perform as thought experiment as before: Consider a container of fluid, idealized as a collection of identical particles moving to some extent at random, brought into a state of permanent isolation. We shall now be concerned with the recurrence of changes in the fluid. Let C be a change occurring between times T_1 and T_2. If C' is another change occurring within the time interval T_1 to T_2, we shall say that C <u>contains</u> C', or C' is <u>within</u> C. If the fluid is sufficiently random, not only all possible states but all possible changes will occur by accident an infinite number of times. Consider the successive recurrences of C. In the vast majority of these, the changes within C will be the same. That is, for any pair of times between and fixed with respect to the times at which C recurs, in the vast majority of cases the change will be the same. We shall refer to this as the expected change conditioned by C, or, to distinguish it from the expected change conditioned by a past state, as the change <u>constrained</u> by C. As before, the thought experiment is used to generate concepts with which we can formulate a law which may hold of an actual fluid:

<u>The second symmetrical law for closed systems</u>: The changes within a given change C are constrained by C.

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* The change in a fluid will always mean the most detailed change.

** If the flow is such that adjacent elements of fluid always remain adjacent, a change may be represented by a transformation of the space of the fluid. However, if the fluid motion is to some extent random, the fluid elements will tend to spread, so that such a transformation would have to be discontinuous; if the fluid idealizes the behavior of a finite collection of particles, this would be extremely artificial. The distribution function is always continuous, for anything which we could reasonably describe as a fluid.

By a system being closed we mean that no fluid is entering or leaving the system. The second law is easily extended to open systems, in which form it seems more relevant to physics. The flow over a period of time may be specified by an eight dimensional distribution function which combines all the distribution functions of the changes: call this the flow function. Let R be an arbitrary region of space-time within that occupied by the flow. The boundary of R is three dimensional, hence the flow function confined to that boundary is only six dimensional. We shall regard this confined flow function as the <u>change</u> in R. Let R and R' be two regions of space-time in the thought experiment, where R' is contained in R. Define a <u>recurrence</u> of the pair R and R' as a new pair obtained through a translation in time. Suppose this is accompanied by a recurrence of the change in R. Then the change in the majority of cases in the recurrence of R' is the same, the change <u>constrained</u> by the change in R.

<u>The second symmetrical law for open systems</u>: If a region of spacetime R includes a region R', the change in R' is constrained by the change in R. Or equivalently, the flow in a region of space-time is constrained by the flow on its boundary.

The second law for open systems holds of the fluid in a closed system which satisfies the second law for closed systems, but it can also hold of fluid not in a closed system, even fluid distributed throughout all of space. We now come to the fundamental questions: Is it possible that some aspect of the activity of the elementary things can be idealized as a changing fluid, an ether, distributed through space and time and satisfying the second law? And would the resulting field theory be a sufficient basis for mechanics?

We should not submit to the temptation to identify the ether with the elementary things. It may be as violent an abstraction to regard the things as a fluid as it would be to regard a crowd of people as a fluid. Thus even if an ether theory, under the second law or any other, leads to all of modern physics, it may be put a peripheral fragment compared with the understanding to be gained by a more concrete knowledge of the things: like Boyle's law set beside modern chemistry. Nevertheless, the problem of finding a simple and intelligible basis for modern physics is a very pressing one, so for the present the etherial aspect of the things should perhaps be their chief interest.

If the fluctuations in non-overlapping regions of space-time are independent, the information* in the changes is an extensive quantity, and thus the flow can be derived from a variational principle. It should be possible to characterize the flow in the infinitesimal region of a point-instant by a finite set of quantities (e.g. a tensor) and thus the flow as a whole by a set of differential equations in these quantities together with the boundary change. Unfortunately I do not know how to carry out this analysis, nor do I know of an adequate model for the ether.

Aside from the possibility of its leading to a unified field theory, the second law is relevant to our present purposes for two reasons:

1.) It is the simplest conceivable law connecting the changes in a random fluid, which lends a certain a-priori plausibility to the idea of a fluid being constrained to other than its decay behavior. As we remarked earlier, the second law generalizes the first; this means that for any succession of states of a fluid satisfying the first law there is a flow satisfying the second which exhibits this succession. Thus the random decisions in a second law flow will exhibit the same kind of "miraculous" coincidences that we noted in the ink flow, though they only seem miraculous when we regard the flow in terms of states rather than changes.

2.) Suppose that the first law analysis of the last section applies to a large collection of identical particles regarded as a dynamical system (the state of the collection is specified by the position of every particle). We might also regard the particles as composing a fluid. If the interactions among the particles are sufficiently weak the flow of this fluid will satisfy the second law. The molecules of a gas are sufficiently large so that at normal temperature their motion is approximately Newtonian. Thus if this motion results from first law constraint we should expect gas flow to satisfy the second law; in fact it seems to satisfy a much more restrictive law. This weighs against the present basis for mechanics unless we can account for the additional restrictions on that basis.

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* Defined as -log P, where P is the probability of the change conditioned by the distribution of the ingoing fluid. Presumably the Lagrangian of the field is the average of forward and reverse information in the infinitesimal region.

Section IV

In the last two sections we have been contemplating the statistics of things as yet undiscovered, whose existence is still highly conjectural. Large parts of present day science are based on the statistics of things quite well known, both from direct observation and from many independent large-number effects. In all cases where large-number laws of change have been discovered these have conformed to the law of decay. In fields like biology and psychology, where our ignorance is still very great, it may be that only decay laws have been discovered because no one has thought of looking for any other kind. However, in the field of thermodynamics our knowledge is much more comprehensive, and in the normal range of temperatures, pressures, etc., can all be derived from a very natural thought experiment with the law of decay. It will be necessary to account for this, and more generally, to understand the relationship between the statistics of the elementary things and of familiar things. At present this understanding seems remote; the problems appear to be very complicated. In this section we shall briefly consider some of the concepts of statistical mechanics, such as entropy and heat, as they apply to systems exhibiting reversed decay.

First we must clearly distinguish between thermal thought experiments and those of the last two sections which will be referred to as ethereal thought experiments. In the former the elementary objects are known micro-entities such as molecules, atoms, electrons, etc., while in the latter they are unknown. The states of the former depend on the velocities of the particles; the microstates are specified by the position and velocity of every particle, the macrostates by the distribution of the particles in the six dimensional phase space of position and momentum. The microstates of the latter are unknown, but the macrostates depend only on position or spatial distribution. Finally, the condition of isolation of the former the energy is assumed constant, while in the latter energy is the result of constraint. Decay has a very different meaning for the two experiments; in the latter it means that the total energy is zero.

The ethereal thought experiment is more basic -- we should be able to derive the expected thermal states from it as well as from the thermal thought experiment. Furthermore it is more general, in the sense that the ethereal macrostates and changes may contain more information about a system than the thermal macrostate. We can conceive of systems for which a knowledge of the hidden ethereal structure is necessary to understand the orderly evolution of even the thermal macrostates. The special case where the thermal experiment renders a system intelligible is probably a degenerate case in the same sense that equilibrium is a degenerate case of decay. Such systems as have been thoroughly analyzed in statistical mechanics are highly simplified, such that even the thermal macrostate is specified by a few continuously changing variables. The question naturally arises whether such systems invariably decay, or whether they may occasionally exhibit first law constraints. A general answer to this question is beyond our present means, but we shall consider a theoretical argument against the possibility of finding such systems under normal terrestrial conditions.

For a thermal system under first law constraint the change in entropy bears a very simple relation to the information in the constraint, as defined in section 2. Let M and N be the macrostates of the system at the beginning and end of the time interval Δt. The forward information ΔIf is given by

1) $\Delta If = -\log P(M,N)$

where P(M,N) is the probability of M conditioned by N at time Δt later.

Similarly the reverse information ΔIr is given by

2) $\Delta Ir = -\log P(N,M)$.

Then the change of entropy ΔS over Δt is proportional to their difference:

3) $\Delta S = k\ (\Delta Ir - \Delta If)$*

where k is Boltzman's constant.

The relation 3) obtains not only under first law constraint but at any two times in the thought experiment and thus for any freakish behavior of the system. However, it is only under first law constraint that the terms ΔIf and ΔIr measure what is forgotten of the initial and final states during

* This follows from the elementary formula P(M and N) = P(M)P(N,M) = P(N)P(M,N) from which we derive P(N)/P(M) = P(N,M)/P(M,N); taking the logarithm of both sides leads to 3).

Δt, which justifies calling them forward and reverse information.

During a first law history the quantities ΔIf and ΔIr are (separately) additive in time. This suggests that we regard them as components of the entropy change. More precisely, define the change of forward entropy ΔSf

4) $\Delta Sf = k\Delta If$

and the change of reverse entropy ΔSr

5) $\Delta Sr = -k\Delta Ir$

Then

6) $\Delta S = \Delta Sf + \Delta Sr$

In a system at temperature T an entropy increase ΔS means that a quantity Δq of available energy has degenerated into heat, where

7) $\Delta q = T\Delta S$.

Let us define forward and reversed heat by

8) $\Delta qf = T\Delta Sf$, $\Delta qr = T\Delta Sr$.

Δqr is the amount of energy which has "miraculously" come from the random motion of the molecules to perform work in changing the macrostate.

At normal temperatures (300°K) a very small amount of work represents a large amount of information:

9) $\Delta If \approx 5x10^{13}\ \Delta qf$

where Δqf is measured in ergs. This is relevant to psychokinesis. It has often been argued by critics of Rhine's experiments that the energy required to influence dice would be so large that it could be applied to produce more definite physical effects such as moving a suspended needle. If the dice are influenced by time-reversed energy resulting from a thermal constraint this criticism is unfounded. Suppose a die is thrown 100 times consecutively, and each throw is a hit. The probability of this run conditioned by a prior thermal macrostate is 6^{100}, hence the forward information given by 1) is about 50. By 9) this represents about 10^{-12} ergs which, were it expended by a force, could not move anything macroscopic.

It is precisely because of the large information content of work that we should not expect to find isolated systems in which the usual thermodynamic variables exhibit first law constraints. The most probable behavior of a system leading to a constrained future state is that which minimizes the forward information. The reversed work required to steer variables such as temperature pressure etc. an appreciable amount from their decay courses would represent an enormous amount of information which could be much more efficiently spent in "triggering" normal agencies to intervene in the system to bring about the constrained end state, i.e., the system could not remain isolated.

This analysis presupposes that the agencies which could be easily triggered (e.g. living beings) are not committed elsewhere, i.e. that they are not required to bring about an even less probable state in some other system. Thus what we have shown is that if thermodynamic variables are for the most part unconstrained, as the experimental evidence strongly indicates, we should not expect to find occasional systems in which they are constrained.

Nevertheless, we may find other aspects of the course of events which depart radically from thermal decay behavior. We can now contemplate two conditions which might lead to such departures first, the thermal macrostate of a system is constrained from the future (such a constraint being realized by the conspiracy of isolated "chance" events) and second, the relevant structure needed to understand the lawful changes of the system is not completely described by the thermal macrostate. It seems very likely that the latter condition obtains in living beings; the crucial question for our present inquiry is whether a more complete description of the relevant structure of living beings can be formulated in terms of an ethereal thought experiment.

In a system which is partly governed by hidden ethereal variables the thermal macrostate may be regarded as deviating from its decay course on the basis of information injected from the ethereal substratum. This information might coordinate apparently chance events similarly to that derived from a future constraint. However, unlike the latter, the injected information will not necessarily be the minimum which would bring about the future thermal macrostate and hence is not related to its conditional probability. Thus there is not any simple relationship between the injected information and the entropy of the system.

Nevertheless the concepts of reversed entropy heat etc. may still be relevant to this information. If the ether is subject to first or second law analysis, its structures will contain traces of both past and future. Injected information based on an ethereal trace of the future may behave much like that due to thermal constraint, in fact it might even place a constraint on the future of a purely thermal system. Suppose a man, while playing dice, has a precognition of his wife being hit by a car. He has a strong impulse to telephone her and tell her not to leave the house. But just then he begins to win heavily. He dare not interrupt his winning streak which lasts long enough for his wife to get out of the house; when he calls her it is too late. The information concerning his wife's accident places a constraint on the purely thermal system of rolling dice, which win for him as the most probable way to keep him from falsifying his precognition.

Whether this example is at all realistic is impossible to say; we still have no idea of the relationship between injected ethereal information and conscious knowledge or will. More generally it is impossible to say whether this information, in conjunction with or by lending to thermal constraints, explains the psychic phenomena. As we remarked, all we know of the psychic phenomena is that they involve strange conditions over space and time. In the absence of more definite identifying marks, the most that our theory, or any general theory, can do at present is to allow for their possibility.

I believe that the first line of experimental inquiry should be the search for simple phenomena in inorganic nature which are intelligible within the present theory and which directly exhibit the peculiarities of the psychic phenomena. Certain anomalies have appeared in electrical conduction and fluid flow at low temperatures which are very suggestive. Some metals appear to lose all electrical resistance at a few degrees above absolute zero. Since the conducting electrons interact with the metal lattice this phenomena is completely inconceivable in classical statistical mechanics -- it flagrantly violates the law of decay. The present explanation, as far as I can make out, is that although the electrons move individually at random, there are certain pairs of electrons whose wanderings are correlated so that their net contribution to the current does not change. Could this be an intrusion of ethereal information?

I am afraid this paper has raised many more questions than it has answered. At least I hope I have established two points: that it makes sense to inquire whether memory-like processes occur reversed in time, and that the laws of mechanics may be large-number laws in spite of their time symmetry. Whether the proposed symmetrical laws lead to present day physics or explain the psychic phenomena only a much more extensive investigation will reveal.