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AN OPTIMUM METHOD FOR TWO-LEVEL RENDITION OF CONTINUOUS-TONE PICTURES

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Summary

A rule is established for designing optimum dither patterns for two-level rendition of continuous-tone pictures. In such a rendition, apparent brightness is controlled by the presence or absence of fixed-sized dots on a regular picture lattice. The dither pattern specifies the order in which these dots are added to the lattice as brightness is varied, and therefore dictates picture quality. The visibility of unwanted artificial texture can be measured usefully in terms of a Fourier analysis of the dot patterns at different brightness levels. When the dot pattern for a uniform patch has components at several different wavelengths, that component or components with the longest finite wavelength should usually be the most visible. If this longest wavelength is used as a measure of quality, a necessary and sufficient rule can be derived for the optimum order of adding dots to the lattice. The rule applies to the construction of dither patterns having a basic subcell of either 2^m by 2^m or 2^m by 2^{m-1} lattice points. The 4 by 4, square, ordered-dither pattern described by Limb[3] and also by Lippel and Kurland[4] satisfies this rule. Use of the rule ensures that picture detail will be rendered well. Pictures are included whereby the optimum arrangement can be compared visually with a random arrangement and with a simulated halftone print.

Introduction

An old printing problem is to render a continuous-tone picture in two light levels. The halftone process is suited to conventional printing and uses an array of dots that vary in size. A different approach, however, has sometimes been considered for pictures that are encoded and processed digitally. At each point in a regular array, only the presence or absence of a fixed-size dot is specified. This paper is addressed to the theoretical problem of best arranging fixed-size dots on a regular picture lattice of given dimensions.

The making of pictures in this way has been discussed before in relation to graphic arts and digital picture transmission. Klensch, Meyerhofer, and Walsh[1] have described an experimental graphic arts printer that uses the method. Roberts[2] has discussed the use of random dither for reducing the number of light levels for picture transmission. Adding a dither pattern to a regular array of continuous light values and then quantizing to two light levels is equivalent to specifying an arrangement of fixed-size dots on a regular lattice. Limb[3] has considered nonrandom dither, using 2 by 2 or 4 by 4 repeat patterns that correspond exactly to the arrangement of dots that we shall show to be theoretically optimum. Lippel and Kurland[4] conclude that this kind of dither achieves, theoretically, the highest possible capacity for information transfer on a Cartesian picture array.

In this paper, we shall treat in a different way the problem of optimizing picture quality. We shall derive a set of conditions that the dot pattern must meet to minimize unwanted texture in uniform areas. For that purpose, regular arrays of light values can be analyzed in terms of visual response to sinusoidal components of the image. Accordingly, we shall review that spectral analysis, choose an index of texture, and use that index to derive the optimum sequence of positions for adding dots to a picture lattice as gray level is increased. We shall finally show, in a different way than did Lippel and Kurland, that the dot pattern derived provides nearly ideal rendition of picture detail.

Types of Patterns for a Lattice of Fixed-Size Dots

We shall first illustrate three different ways of arranging fixed-size dots on a regular picture lattice. We shall show (1) a random pattern, (2) an imitation halftone pattern, and (3) a pattern that meets the conditions that we shall derive.

The prints reproduced here were made by means of a K. S. Paul graphic arts scanner, specially modified so that photographs could be recorded, processed, and played back digitally. The original continuous-tone picture, a 4- by 4-inch transparency, was scanned to produce a 2000 by 2000 array of light values on a scale of integers from 0 to 255. Then a Digital Equipment Corporation PDP-10 computer was used to produce a two-level array for each of the three methods. Finally, each two-level picture was played back through the scanner at twice size to make a print. Only a 750- by 750-element section is shown in Figs. 1, 2, and 4.

The first of these prints, Fig. 1, has a random arrangement of fixed-size dots on a regular picture lattice. To make the print, the original picture array was compared element by element with a corresponding array of random threshold values. Each threshold value was drawn at random from a distribution of 32 different absorptance values, equally spaced from 0 to 1. For the final print, a black dot was added to the lattice wherever the original picture was darker than the threshold value. Using a threshold pattern is equivalent to using a dither pattern. Either one dictates how the picture lattice is filled with dots as gray level is increased.

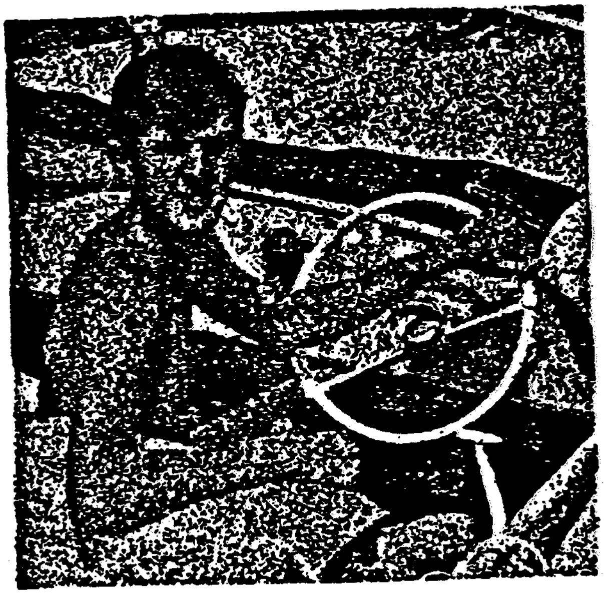

Fig. 1. Random dot pattern.

Figure 2 shows how the same picture lattice of fixed-size dots may be used to imitate a conventional halftone print. The picture was made with a threshold pattern of 32 equally spaced absorptance values, repeated both ways in a regular manner. Figure 3 shows a basic subsection of the threshold pattern, with the threshold values numbered from 0 to 31. These numbers also show the order in which dots are added to the lattice as gray level is increased. The appearance of a variable-size dot is achieved by a growing cluster of fixed-size dots. Other methods of generating halftone prints electronically have been discussed by Hallows and Klensch.[5]

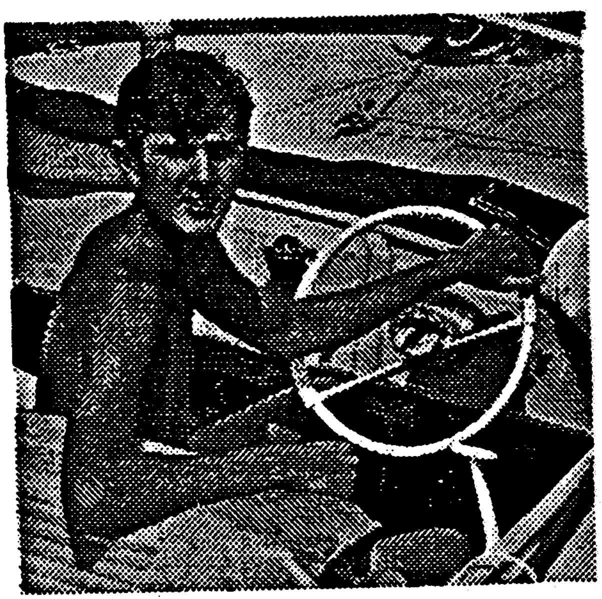

Fig. 2. Imitation halftone pattern.

6	5	4	15	16	17	18	19
7	0	3	14	27	28	29	20
8	1	2	13	26	31	30	21
9	10	11	12	25	24	23	22
16	17	18	19	6	5	4	15
27	28	29	20	7	0	3	14
26	31	30	21	8	1	2	13
25	24	23	22	9	10	11	12

Fig. 3. Section of threshold pattern for Fig. 2.

The central result of this paper is illustrated by Fig. 4. The picture lattice is the same as in Fig. 1 and Fig. 2, but the 32 threshold values are arranged as in Fig. 5. This threshold pattern meets the conditions that we shall derive for optimum picture quality.

Fig. 4. Example of optimum dot pattern.

0	16	4	20	1	17	5	21
24	8	28	12	25	9	29	13
6	22	2	18	7	23	3	19
30	14	26	10	31	15	27	11
1	17	5	21	0	16	4	20
25	9	29	13	24	8	28	12
7	23	3	19	6	22	2	18
31	15	27	11	30	14	26	10

Fig. 5. Section of threshold pattern for Fig. 4.

It is not fair to conclude from these pictures that Fig. 4 represents a better way to use conventional printing materials than Fig. 2. Ink and paper may be such that an ordinary halftone with a finer screen cannot be substantially improved upon. That question, however, is not within the scope of this paper. The figures merely illustrate different ways of arranging fixed-size dots on a lattice of given dimensions.

An Optimization Criterion

In this section, we shall define optimum picture quality in terms of the texture of uniformly gray areas of a picture. We shall first review how a partially filled, regular lattice of fixed-size dots can be analyzed in terms of sinusoidal components.

Let us suppose that a uniform gray area is represented by repeating both horizontally and vertically a 2^m by 2^m subarray of black and white elements. That is, let f_{xy} be the absorptance value, 0 or 1, at column x and row y of the picture lattice, and let

$$f_{x+2^m,\, y} = f_{x,y+2^m} = f_{xy}. \tag{1}$$

The basic subarray can be specified by all combinations of integers x and y such that $0 \leq x < 2^m$ and $0 \leq y < 2^m$. In that case, the function f_{xy} can be represented by the Fourier series

$$f_{xy} = \sum_{u=-\ell+1}^{\ell} \sum_{v=-\ell+1}^{\ell} F_{uv} e^{-2\pi i\left(\frac{ux+vy}{2^m}\right)}, \tag{2}$$

where $\ell = 2^{m-1}$ and x and y are any integers. The Fourier coefficients F_{uv} are given by

$$F_{uv} = \frac{1}{2^{2m}} \sum_{x=0}^{2^m-1} \sum_{y=0}^{2^m-1} f_{xy} e^{2\pi i\left(\frac{ux+vy}{2^m}\right)}, \tag{3}$$

where u and v are any integers. The real part of each term in Eq. (2) is a sinusoidal plane wave with amplitude

$$A_{uv} = \sqrt{F_{uv}F_{-u,-v}} \tag{4}$$

and a wavelength, measured at right angles to the wavefront, given by

$$\lambda_{uv} = \frac{2^m}{\sqrt{u^2 + v^2}}. \tag{5}$$

Clearly, only those sinusoidal components with nonzero amplitude A_{uv} and finite wavelength λ_{uv} will contribute to the appearance of texture.

We can use such an analysis to define an index of texture. We first observe that under reasonable conditions for making and viewing two-level pictures, the texture of a uniform area will be almost entirely accounted for by its low frequency, nonzero components. Consider, for example, either Fig. 2 or Fig. 4 viewed at a distance equal to five times picture height. The basic dot pattern in any uniform area is repeated diagonally about 12 times per degree of visual angle. Yet it is widely observed[6] that visual response to sinusoidal patterns diminishes rapidly with spatial frequency beyond about 10 cycles per degree.

Accordingly, a reasonable index of texture in a uniform area is the longest finite wavelength of the nonzero sinusoidal components of the dot pattern. Let us therefore define texture by the quantity Λ, given by

$$\Lambda = \mathrm{Max} \left\{ \lambda_{uv} \mid A_{uv} \neq 0,\ \lambda_{uv} < \infty \right\}. \tag{6}$$

We want to choose a dot pattern so that Λ is as small as possible. This ranking will sometimes disagree with visual ranking because relative amplitudes A_{uv} are not taken into account. However, visual response decreases so rapidly with wavelength that upsets should be uncommon. Furthermore, if we use Λ to measure texture, we can derive mathematically the best way to add dots to a picture lattice.

To complete our definition of optimum picture quality, we need to take into account, not just a single gray level, but simultaneously all of the gray levels from which the picture is made. Let us consider the dot pattern in a 2^m by 2^m subsection within an area of uniform gray level. Each time we increase that gray level by one step, one more level in the threshold pattern will be exceeded. The dot pattern will be changed only by adding one black dot to an empty position of the lattice. Our problem therefore consists, not of choosing independently dot patterns with 0, 1, ..., 2^{2m} black dots, but of choosing a sequence of 2^{2m} positions within a 2^m by 2^m lattice.

Our choice of definition is based on the premise that if we view an entire sequence of gray levels at one time, we shall base our first impression of overall texture on those patterns having the largest value of Λ. We want that value of Λ to be as small as possible, and we want as few patterns as possible to share it. Once that criterion has been met, we shall look at those patterns with smaller Λ and apply a similar criterion. In formal terms, let $L_1 > L_2 > \ldots > L_r$ include all possible values of Λ for a 2^m by 2^m dot pattern. Then for the sequence of patterns with 0, 1, ..., 2^{2m} black dots, we wish to choose the order of dot positions such that as many patterns as possible satisfy $\Lambda < L_1$. Of those that do we wish to make as many as possible satisfy $\Lambda < L_2$, and so on in a similar way. With the definition just made, we can proceed to derive an optimum order of adding dots to a picture lattice.

A Set of Conditions for an Optimum Dot Arrangement

In this section, we shall present a set

of conditions that a sequence of lattice positions must meet in order to satisfy the optimization criterion just described. We shall then prove that this set of conditions is both necessary and sufficient for that purpose.

To express this set of conditions, let k be the number of black dots in the basic 2^m by 2^m subsection defined by integers x and y such that $0 \leq x < 2^m$ and $0 \leq y < 2^m$. Then consider the sequence of dot patterns with k equal to 0, 1, ..., 2^{2m}. By the rules that we have adopted, each new pattern is obtained from the last by adding one black dot to an empty position. Now for each k, let 2^n be the largest power of 2 which divides k. That is, $k = j2^n$ where j is an odd integer. The conditions are most easily expressed in two parts:

(1) When n is even, we must have

$$f_{x+2^{m-\frac{n}{2}},y} = f_{x,y+2^{m-\frac{n}{2}}} = f_{xy}. \tag{7}$$

(2) When n is odd, we must have

$$f_{x+2^{m-(\frac{n+1}{2})},\ y+2^{m-(\frac{n+1}{2})}} = f_{x+2^{m-(\frac{n+1}{2})},\ y-2^{m-(\frac{n+1}{2})}} = f_{xy}. \tag{8}$$

The sequence will be optimum if and only if these conditions are satisfied for every value of k. Furthermore, the value of Λ is given by

$$\Lambda = 2^{m-\frac{n}{2}} \tag{9}$$

for each value of k, $0 < k < 2^{2m}$.

To prove that these conditions are both necessary and sufficient, we first apply the optimization criterion that we have chosen to the basic 2^m by 2^m subarray. From Eq. (5), the largest possible value of Λ is equal to 2^m. We wish to make as many patterns as possible satisfy $\Lambda < 2^m$. We shall show that to meet that criterion, we must have

$$f_{x+2^{m-1},\ y+2^{m-1}} = f_{x+2^{m-1},\ y-2^{m-1}} = f_{xy} \tag{10}$$

whenever k is even. This condition amounts to a rule that we must apply to each successive pair of dot positions.

We first show that we cannot have $\Lambda < 2^m$ for two successive values of k. From Eqs. (3) and (5) it is easy to show that we can have $\Lambda < 2^m$ if and only if $F_{10} = F_{01} = F_{-1,0} = F_{0,-1} = 0$. All of the remaining coefficients F_{uv} other than F_{00} are associated with a wavelength shorter than 2^m. Suppose that a Fourier coefficient is zero for some value of k. Adding one dot to an empty position (x_α, y_α) results in adding a nonzero term to Eq. (3). Thus a Fourier coefficient cannot equal zero for k+1 black dots if it equals zero for k black dots.

We can, however, have $\Lambda < 2^m$ for k+2 black dots. On adding a second black dot to an empty position (x_β, y_β), we have

$$\begin{cases} F_{10} = F_{-1,0}^{\ *} = e^{2\pi i(\frac{x_\alpha}{2^m})} + e^{2\pi i(\frac{x_\beta}{2^m})} \\ F_{01} = F_{0,-1}^{\ *} = e^{2\pi i(\frac{y_\alpha}{2^m})} + e^{2\pi i(\frac{y_\beta}{2^m})}, \end{cases} \tag{11}$$

where the asterisk indicates the complex conjugate. Equations (11) will equal zero if and only if

$$\begin{cases} |x_\beta - x_\alpha| = 2^{m-1} \\ |y_\beta - y_\alpha| = 2^{m-1} \end{cases} \tag{12}$$

for the pair of positions chosen.

Clearly, to maximize the number of patterns in the sequence for which $\Lambda < 2^m$, we can and must begin with k equal to zero and choose every successive pair of positions by Eqs. (12). Such a mandate is precisely equivalent to requiring that Eqs. (10) be satisfied whenever k is even. We can always follow this choice, because Eqs. (10) ensure that when the position (x_α, y_α) is empty, so is the position (x_β, y_β) given by Eqs. (12). The process leaves $\Lambda = 2^m$ whenever k is odd.

A pattern which obeys Eqs. (10) can be specified by a basic 2^m by 2^{m-1} subsection given by positions x and y such that $0 \leq x < 2^m$ and $0 \leq y < 2^{m-1}$. The requirement that Eqs. (10) be met for every even value of in no way restricts the order in which p[illegible] tions are chosen from that 2^m by 2^{m-1} s[illegible] tion. We have therefore reduced the or[illegible] problem to one of smaller dimensions. [illegible] must now find the optimum order of filling a 2^m by 2^{m-1} sublattice, repeated by Eqs. (10).

To continue the proof, we apply the optimization criterion that we have chosen to the 2^m by 2^{m-1} subsection. Let k' be the number of black dots in that subsection. We are concerned with that sequence of patterns for which k' is equal to 0, 1, ..., 2^{2m-1}. From Eqs. (3), (5), and (10), the largest possible value of Λ is equal to $2^{m-(1/2)}$. By arguments similar to those used before, we can show that we cannot have $\Lambda < 2^{m-(1/2)}$ for two successive values of k'. To maximize the number of members of the sequence for which $\Lambda < 2^{m-(1/2)}$, we can and must choose every successive pair of positions from within the 2^m by 2^{m-1} subsection so that

$$f_{x+2^{m-1},\ y} = f_{x,\ y+2^{m-1}} = f_{xy} \tag{13}$$

whenever k' is even. This process leaves $\Lambda = 2^{m-(1/2)}$ whenever k' is odd.

The rest of the proof is now clear. We have already reduced the problem of filling a 2^m by 2^m sublattice, repeated by Eqs. (1), to that of filling a 2^{m-1} by 2^{m-1} sublattice, repeated by Eqs. (13). The latter problem is just like the former with m replaced by m-1. If we repeatedly apply the same arguments as before, we shall verify that the set of conditions set out in Eqs. (7) and (8) is both necessary and sufficient for an optimum sequence of dot positions. Furthermore, we shall show that Λ is always given by Eq. (9).

To construct an actual sequence, we need to follow the rules implied by Eqs. (7) and (8). The position of the first dot is arbitrary. The next position must satisfy Eqs. (10). The next pair must satisfy Eqs. (10) and (13). The next four must satisfy three relations, and so on. The procedure has a symmetry that is easy to discover. The 4 by 4 dither patterns given by Limb and by Lippel and Kurland are solutions.

The conditions that we have established are equally applicable to the construction of threshold patterns having a 2^m by 2^{m-1} subsection, repeated by Eqs. (10). Figure 5, for example, is an optimum arrangement for an 8 by 4 sublattice. That arrangement was used in Fig. 4.

The number of different threshold values is a compromise. If that number is 2^{2m}, then as gray level is increased, every other dot pattern will have a component of wavelength 2^m. We wish to keep wavelengths small. Too few gray levels, however, will lend the picture a paint-by-number appearance.

Picture Detail

In deriving a rule for an optimum threshold pattern, we have considered only unwanted texture. We can also show, however, that use of the rule ensures nearly ideal rendition of picture detail.

To define ideal rendition, consider a light or dark spot in the original picture. That spot will not be visible in the final image unless it contains at least one more or one less dot than a like area of surround. We can do no better than to provide within every area of q lattice points q different threshold absorptance values, spaced as uniformly as possible between zero and unity.

Now consider a 2^m by 2^m threshold pattern, and let p be any integer between zero and m, inclusive. If the rule that we have developed is used, then of the 2^{2m} different threshold values employed, every 2^p by 2^p sublattice, regardless of its position, will receive one and only one of each consecutive set of $2^{2(m-p)}$ threshold values. A similar statement can be made for each 2^p by 2^{p-1} sublattice. Therefore, we have approximately met the ideal.

Final Remarks and Conclusions

The use of a lattice of fixed-size dots to print pictures will undoubtedly receive continued attention in association with advanced digital methods. High resolution and fast digital logic naturally suggest such an approach. We have verified in this paper that optimizing picture quality may be approached mathematically in terms of visual response. When the dots are laid down as in Fig. 4, the picture will in theory exhibit a minimum of artificial texture and render picture detail well.

Acknowledgment

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