Brahmagupta

Brahmagupta (598-670) was the most prominent Indian mathematician and astronomer of the 7th century. He may have been born at Bhillamala in Gujarat, Western India. Little is known about his life, except that he lived and worked in the great astronomical center of Uljain, located in the Gwalior state of Central India. He spent his early days as court astronomer to King Vyaghramukha. He is best known for his seminal text, the *Bahmasphutasiddhanata* (628, *The Opening of the Universe*) or simply the *Siddhanta*. A corrected and updated version of the old astronomical text the *Brahma Siddhanta (The system of Brahma in astronomy*) is a comprehensive treatment of the astronomical knowledge of the time; two chapters, the *Ganitād'hāya* (*Lectures on Arithmetic*) and the *Kutakhādyaka* (*Lectures on Indeterminate Equations*) are devoted to mathematics.

Brahmagupta begins the *Ganitād'hāya* by identifying a *ganaca,* that is, a calculator who is competent enough to study astronomy, as one "… who distinctly and severally knows addition and the rest of the twenty logistics and the eight determinations, including measurement by shadow…." The chapter covers arithmetic progressions, the rule of three, simple interest, shadow reckoning, the mensuration of plane figures and finding volumes. In the *Kutakhādyka*, Brahmagupta defined zero as the result of subtracting a number from itself, and he used dots underneath numbers to indicate a zero. The English word "zero" comes from the Arabic *sifur*, which was a translation of the Sanskrit *shûnya*, meaning, "void" or "emptiness." *Sifur* also passed into the English language, as an alternative to "zero," as the word "cipher." "Zero" is a contraction of the Italian word *zepiro*. Brahmagupta gave rules for operating with zero and the "rules of signs," stated in the language of the marketplace, using "dhana" (fortunes) to represent positive numbers and "rina" (debts) for negative numbers.

The following rules for algebra given by Brahmagupta should be familiar except for terminology, although he incorrectly claimed that zero divided by zero is zero. 1. A debt minus zero is a debt. 2. A fortune minus zero is a fortune. 3. Zero minus zero is zero. 4. A debt subtracted from zero is a fortune. 5. A fortune subtracted from zero is a debt. 6. The product of zero multiplied by a debt or a fortune is zero. 7. The product of zero multiplied by zero is zero. 8. The product or quotient of two fortunes is a fortune. 9. The product or quotient of two debts is a fortune. 10. The product or quotient of a debt and a fortune is a debt. 11. The product or quotient of a fortune and a debt is a debt.

In addition, Brahmagupta gave a method of solving indeterminate equations of the second degree, and rules for solving simple quadratic equations of various types. He made an impressive start in solving the quadratic Diophantine equation $y^2 = Ax^2 + 1$, mistakenly called Pell's equation. Brahmagupta developed a rule for the formation of Pythagorean triads expressed in the form m, $\frac{1}{2}(m^2/n - n)$, and $\frac{1}{2}(m^2/n + n)$. He seems to have been the first to treat arithmetic and algebra as different subjects. Brahmagupta is also known for expressing the identity, which states that the product of two numbers, each of which is the sum of two squares, is itself a sum of two squares. Specifically, in modern notation, his identity is,

$$(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2 .$$

Another of his achievements was in expanding the method, usually known as Heron's formula for finding the area of a triangle, to cyclic quadrilaterals, that is quadrilaterals that have all four of their vertices on the circumference of a circle. In Figure 9.8, a quadrilateral with sides *a*, *b*, *c* and *d* is inscribed in a circle. Heron used the semiperimeter of the triangle, s = (a + b + c)/2 in his area formula, Area = $\sqrt{[s(s-a)(s-b)(s-c)]}$, where a, b, and c are the sides of a triangle. Brahmagupta used $s = (a + b + c + d)/2$ in his formula for the area of cyclic quadrilateral.

$$\text{Area} = \sqrt{[s(s-a)(s-b)(s-c)(s-d)]}.$$

If one side of the quadrilateral is diminished to form a triangle, Heron's formula is a special case.

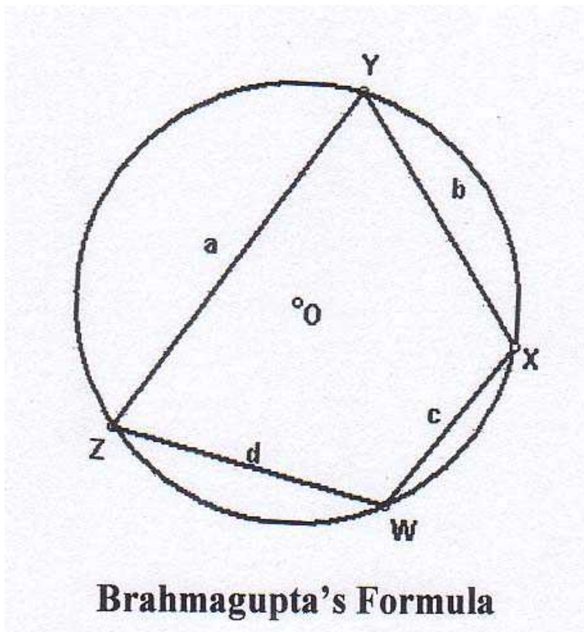

Figure 9.8

When Brahmagupta was 67, he wrote the *Khandakhadyaka*, literally meaning "sweetmeat." With it, he became the first to use algebra to solve astronomical problems. He anticipated the gravitational theory, writing: "Bodies fall towards the earth as it is the nature of earth to attract bodies, just as it is the nature of water to flow." He gave the sidereal periods for many heavenly bodies, including the sun, which he claimed, made 30 circuits of the ecliptic in 10,960 days; that is, the sun moves 3/1096 of a sidereal year every day. He then imagined the ecliptic divided into 10,960 congruent arcs, with the sun at the beginning of the first arc if the first day of a 10,960-day cycle. He offered no proofs of his results, but this shouldn't be taken as a sign that he was unaware of the nature of proof or the need to demonstrate the validity of the rules. His works were intended as algorithms for solving a variety of problems, mostly relating to astronomy and astrology, and he likely didn't feel that those who would use them needed proofs. With the spread of Buddhism around 500 CE, Chinese scholars became acquainted with the *Siddhanta*, and made translations. Of greater significance in terms of his influence, were the Arabic translations.

Quotation of the Day: "As the sun eclipses the stars by his brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them." – Brahmagupta