MANNING Robert

Born in Normandy, France in 1816 - Dead in Ireland in 1897.



Figure 1 : Portrait of Robert Manning.

1. SHORT BIOGRAPHY

Robert Manning was born in Normandy, France in October 1816, as his Irish father was stationed there as adjutant to the 40th regiment, after the Battle of Waterloo. After the death of his father, his mother returned to Ireland in her family home in Waterford. From 1834 to 1845, Manning was employed by his uncle John Stephens on the management of his estates, and thought to become a lawyer. However, after two short term positions in 1846, he entered the Irish Office of Public Works in Dublin in October 1846, as an assistant engineer on the improvement of rivers, and worked there until June 1855 after having been promoted District Engineer in January 1848. Manning received his formation as an hydraulic engineer during the years of the Great Famine of 1845-1847. He initially learned hydraulics from the book published by the French hydraulician D'Aubuisson de Voisins, as he explained himself: Manning asked a colleague "a book on hydraulics which was not too difficult for a beginner, and thus was obtained the second edition of the Traité d'Hydraulique of M. d'Aubuisson de Voisins, published in Paris and Strasbourg in 1840. The author (i.e. Manning) "devoured" this book, as Du Buat said he did the Hydrodynamique of Bossut, and received the greatest benefit from its study." (Manning, 1895, p. 180). In the Irish Office of Public Works, Manning's tasks were related to the Arterial Drainage Act published in 1842, with a lot of activities in flood control and drainage for which he had to learn and to use hydraulics. In 1855, he started to work on the estates of the Marquis of Downshire, until the death of the Marquis in 1869. During this period, he worked in various engineering fields, including the construction of a harbour and of the water

supply of the city of Belfast. He then re-entered the Irish Department of Public Works as Second Engineer in October 1869, and was promoted Chief Engineer of the Board in April 1874. At the age of 75, he retired in December 1891 and died in 1897.

Robert Manning was elected as associate member of the Institution of Civil Engineers of Ireland in 1848, became a member of its Council in 1874 and served as president in 1877-1878. He was also elected member of the Institution of Civil Engineers of London in 1858.

According to Dooge (1992, p. 138), Manning "had a good command of French but not of German. In his writings he always makes a direct reference to authors writing in English of French, but cites English translations of works published in German". A detailed biography of Robert Manning, including scientific and engineering context, has been given by Dooge (1987, 1992), from which the above elements have been synthesised.

2. MANNING FORMULAS

After comparison with seven other formulas well known at his time (Chézy* and Eytelwein, Du Buat, Darcy and Bazin*, Weisbach, Saint-Venant*, Ganguillet* and Kutter*, and Neville, as given in the appendix of his 1891 paper) and by using experimental data issued from 20 data series from Revy, Humphreys and Abbott, Du Buat, Ganguillet and Kutter, Bazin, Darcy, Ftely and Stearns, and Smith, Manning established in 1885 the following formula which was presented at a meeting of the Institution of Civil Engineers of Ireland on 4th December 1889 and published in 1891 (Manning, 1891) (Figure 2):

$$U = C\sqrt{Sg} \left(R_h^{0.5} + \frac{0.22}{m^{0.5}} (R_h - 0.15m) \right)$$
 Eq. 1

with U mean velocity in seconds, for all measures of length

- *S* the sine of the angle of inclination of the water surface
- g the gravitational acceleration
- R_h the hydraulic radius
- *C* a coefficient which varies with the nature of the channel bed
- *m* the height of the barometric column.

In the same paper, Manning presented another formula, based on 170 experiments and written as (Figure 3):

$$U = CR_h^{2/3} S^{1/2}$$
 Eq. 2

Manning himself (Manning, 1891, p. 177) recognised that a similar relationship had been proposed earlier by the German hydraulician Hagen* in 1881 (Hagen, 1881), as he read in Cunningham (1882). However, another similar expression had been established as early as 1867 by the French engineer Gauckler* (Gauckler, 1867, 1868). But it appears that Manning, who very seriously cited and discussed the results and formulas

from many other hydraulicians, ignored the Gauckler formula while he carried out his own work (Dooge, 1992).

coefficients. Such a formula the author now submits to the judgment of the members of the Institution. Formula. I. $V=C\sqrt{S g}$. $\left[R^{\frac{1}{2}}+\frac{0.22}{m^{\frac{1}{2}}}(R-0.15 m)\right]$ V represents the mean velocity in seconds for all measures of length; C, a coefficient which varies with the nature of the bed; S, the sine of the angle of inclination of the surface; R, "the mean radius" or "mean hydraulic depth" (which is found by dividing the area of the channel by the length of the perimeter in contact with the fluid); g, the velocity acquired by a falling body in a second of time; and m, the height of the column of mercury which balances the atmospheric pressure. For units of English feet and seconds this formula becomes for channels in earth in good order:— II. $V=62 S^{\frac{1}{2}} (R^{\frac{1}{2}} + \frac{R}{7} - 0.05)$,

and, for metres-seconds, it takes the simple form :---

III.
$$V = 34 S^{\frac{1}{2}} (R^{\frac{1}{2}} + \frac{R}{4} - 0.03).$$

In both cases a mean height of barometer is taken=30 English inches.

Figure 2 : Manning formula nr. I (Manning, 1891, p. 162).

The method adopted in these calculations was to take the first observation of each series as unity, and to reduce all the others to it, so that the exponent of R might be easily found; of course a similar operation should be performed successively on each of the other experiments, and a mean of all the results taken; this was not done, it being considered sufficiently accurate to take the value of the exponent at 0.666' or $\frac{2}{5}$, and so the formula—

 $V. \quad V = C S^{i} R^{i}$

was established, and was applied to 170 experiments, with the following results :---

Figure 3 : Manning formula nr. V (Manning, 1891, p. 175).

It is particularly interesting to note that Manning himself preferred Eq. 1 instead of Eq. 2 which is used nowadays, while Eq. 1 has been completely abandoned. His choice was based on both mathematical and theoretical considerations: "Although the author's monomial formula (V) is practically accurate within the wide limits of the experiments discussed, it has the practical disadvantage of requiring the extraction of a cube root, which is tedious by the arithmetical process, and moreover some theoretical objections may be taken as to its form." (Manning, 1891, p. 183). Additionally, Manning was not satisfied with the lack of homogeneity in the variables in Eq. 2, especially for coefficient *C* (in m^{1/3}s⁻¹) which was not dimensionless and could not therefore be related to any theory of hydraulics. Regarding Eq. 1, he claimed " for this formula that the equation is homogeneous, and therefore each of its sides has the same dimensions. That is consistent with such natural laws as we are acquainted with, and that it gives very closely the experimental velocities in Table I, the range of which is very extended" (Manning, 1891, p. 191).

In his second paper dated 1895, Manning compared his formula I (i.e. Eq. 1) with the three formulas from Humphreys and Abbott, Bazin, Ganguillet and Kutter (Manning, 1895). The comparison was based on 643 experiments from several hydraulicians. Manning concluded that the two best formulas were the Ganguillet and Kutter formula and his own one. But as the Ganguillet and Kutter formula was more complex to use for practical calculations, he recommended to use his own formula and, in the appendix F of his paper, gave the results he obtained with his formula for the 643 experiments (Figure 4).

In textbooks using English units, as 1 m = 3.2808 ft, one gets (3.2808)1/3 = 1.4858, and Eq. 2 is rewritten as follows:

$$U = \frac{1.4858}{n} R_h^{2/3} S^{1/2}$$
 Eq. 3

with *n* roughness coefficient as initially proposed by Ganguillet and Kutter.

Manning himself observed this equivalence between both coefficients: "It is quite true that Ganguillet and Kutter's formula gives a near approximation to the observed velocities within wide limits, and it is worthy of remark that the value of the reciprocal of C (the coefficient of formula V) corresponds closely with that of n, as determined by them; both C and n being constant for the same channel." (Manning, 1891, p. 204-205). However, Manning himself did not suggest to use Kutter's n instead of his own coefficient C. This re-writting of his formula appeared later and has been likely initiated by Bovey (1901) as reported by Williams (1970). Chow (1955) also suggested that all digits in 1.4858 are not necessary for practical purposes and that the numerical coefficient could be replaced by 1.49.

Despite the fact that, as suggested by Williams (1970), Eq. 2 should be renamed the Gauckler-Manning formula, it is frequently referred to as the Manning-Strickler formula where *C* is replaced by *K*. This is due to the fact that Strickler* (1923), on the basis of 17 sets of Swiss data, suggested to calculate the roughness parameter *K* proportionally to the sixth-root of the median diameter d_{50} of the bed material (with *n* the Ganguillet and Kutter's coefficient):

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| | | APPENDIX | F. | | | |
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| | | OPEN CHANNE | ELS. | <u>.</u> | n in the Marine State | |
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| | COFFFICIENTS OF | Mean Velocity in | MANN | wa's T | TOPMIT | 27 C |
| | | | | • . | | |
| | $V = C_1 \sqrt{Sg} \left[R^{\frac{1}{2}} + \frac{C_1}{2} \right]$ | $\frac{122}{m^{\frac{1}{2}}}(R-0.15m)$]. | $V = C_2 S$ | $\frac{1}{2}(R^{\frac{1}{2}}+$ | $\frac{R}{7}$ - 05 |). |
| | • • L | - | | N S | | |
| | | $V = C_s S^{\frac{1}{2}} \left(R^{\frac{1}{2}} + \frac{R}{4} - \frac{R}{4} \right)$ | - 03). | · · · · · | | |
| Series | Channel | Surface of Perimeter | No. of Obser- vations | Coefficients | | |
| | | | | C ₁ | Cg | C3 |
| 43 | Marseilles Canal | Very smooth | 1 | 21.31 | 120.93 | 66.98 |
| 43 | Same | Quite smooth | - 1 | | 124.67 | 69.06 |
| 49 | Sudbury Conduit | Hard brick, fairly | 8 | 20.70 | 117.78 | 65.07 |
| | | clean and smooth | 1.1 | | | |
| 50 | Same | Same | 13 | 20.53 | 116.50 | 64.38 |
| 8 | Chazilly Canal | Earth, stony | 4 | 18.33 | 104.02 | 57.61 |
| i co Angles | Bayon la Fourche | - | 4 | 14.38 | 81.60 | 45.20 |
| 25 | Neva | | 1 | 14.17 | 80.41 | 44.51 |
| 24 9 | Great Nevka Burgundy Canal | Ashlan allalat | 1 | 13.93 | 79.05 | 43.69 |
| 3 | Lungunuy Canal | Ashlar, slightly damaged | 8 | 13.13 | 74.51 | 41.27 |
| 22 | Ohio | uamagea | 1 | 12.84 | 72.85 | 40.36 |
| 51 | Missouri | Sand | 8 | 12.62 | 71.61 | 39.67 |
| 43 | Marseilles Canal | Hammered stone, | 4 | 12.36 | 70-13 | 38.86 |
| | | rather rough | | | | |
| 56 | Mississippi | Very fine sand | 16 | 12.32 | 69.91 | 38.73 |
| 11 | Grünbachschale | Dry rubble | 6 | 12.12 | 68:78 | 38.10 |
| | Linth Canal | | 10 | 12.10 | 68-85 | 38.04 |
| 14 | Rbine at Bâle | | 1 | 12.05 | 68·38 | 37.89 |
| 14 84 | Germersheim | - | 1 | 11.92 | 67.64 | 37.47 |
| 14 34 33 | | alay sa n a kasar | 11 | 11.78 | 66.85 | 37.03 |
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Figure 4 : First page of Appendix F (Manning, 1895, p. 201).

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3. DISSEMINATION OF THE MANNING FORMULA

As Eq. 2 (or Eq. 3) is widely used nowadays as the Manning formula, many authors have tried to trace its dissemination in the end of the 19^{th} century and in the beginning of the 20^{th} century (see e.g. Powell, 1960, 1962, 1968; Williams, 1970; Dooge, 1992; Fischenich, 2000).

Many similar formulas have been proposed around the end of the 19th century. Most of them had the following expression:

$$U = CR_h^x S^y Eq. 5$$

where C, x and y were experimentally determined for various types of pipe and channel material. For example, Tutton (1899) suggested the following formula for uniform flows in open channels (Figure 5):

$$U = \frac{1.54}{n} R_h^{2/3} S^{1/2}$$
 Eq. 6

where n is the Kutter's coefficient. In his paper, Tutton quoted Gauckler, Hagen, and several other authors but not Manning.

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as sewers and water conduits, but, as it has already reached a sufficient length, that will be reserved for a future communication. It may, however be briefly stated regarding open channels that the formula $v = \frac{1.54}{n} R^{\frac{2}{3}} S^{\frac{1}{3}}$ will apply as long as we can consider the flow uniform and surface parallel to bottom inclination, but it will not correctly apply to rapidly rising or falling rivers or to those discharging against tidal action, on which conditions Kutter's formula is, unfortunately, principally based.

Figure 5 : Formula for open channel flow proposed by Tutton (1899, p. 165)

According to Dooge (1992, p. 170-171) who had access to the private papers and correspondence of Manning, the first dissemination of the Manning formula was made by the French hydraulician Alfred Aimé Flamant who exchanged letters with Manning after he received from him an off-print of his 1891 paper. In his book *Mécanique appliquée – Hydraulique* published in Paris in 1891, Flamant quoted the Manning formula as:

$$U = CR_{h}^{2/3}S^{1/2}$$

Eq. 7

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with the coefficient C defined as the reciprocal of the Kutter's coefficient n in metric units. The other Manning equation was only mentioned in a footnote (Dooge, 1992, p. 173), with a reference to Manning surprisingly dated 1890 by Flamant (Figure 6).

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écarts que l'on a constatés paraissent être fonction de la pente et sont surtout appréciables lorsque celle-ci est très petite.

On peut d'ailleurs objecter à cette formule qu'il est bien extraordinaire que, lorsque le rayon moyen a la valeur $\mathbf{R} = \mathbf{1}$ m., le coefficient C devient indépendant de la pente, tandis qu'il varie avec elle lorsque le rayon moyen a une valeur ou plus grande ou plus petite. D'après cette formule, le coefficient C croît avec la pente pour les valeurs de R inférieures à 1 et il décroît au contraire quand la pente augmente lorsque R est supérieur à l'unité. On ne voit pas les causes de cette différence qui semble difficile à expliquer.

M. Robert Manning, professeur au Collège Royal de Dublin, a proposé la formule ¹:

$$U = C_{1} \sqrt[3]{R^{2}} \sqrt{I} = C_{1} R^{\frac{2}{3}} I^{\frac{1}{2}};$$

dans laquelle le coefficient C_1 , variable avec la nature de la paroi, a la valeur $\frac{4}{n}$ de l'inverse du coefficient de rugosité n de la formule de Kutter. On devrait donc prendre, pour les parois en terre, $C_1 = 40$, et de même pour les autres. Cette formule revient à :

$$\mathbf{U} = \mathbf{C}_{\mathbf{i}} \sqrt[6]{\mathbf{R}} \cdot \sqrt{\mathbf{R}} \mathbf{I},$$

1. Cette formule n'est qu'une forme simplifiée d'une formule plus générale donnée par M. Rob. Manning sous la forme

$$\mathbf{U} = \mathbf{C} \sqrt{g\mathbf{I}} \left[\sqrt{\mathbf{R}} + \frac{0.22}{\sqrt{m}} \left(\mathbf{R} - 0.45 \, m \right) \right],$$

dans laquelle *m* représente la hauteur de la colonne de mercure qui équilibre la pression atmosphérique. Les valeurs des coefficients numériques y restent les mêmes quelle que soit l'unité adoptée pour les longueurs. Pour la valeur $m=0^m76$, en prenant le mètre pour unité de longueur, elle devient simplement

$$\mathbf{U} = \mathbf{C}_{1}\sqrt{\mathbf{I}}\left(\sqrt{\mathbf{R}} + \frac{\mathbf{R}}{4} - \mathbf{0}^{\mathrm{m}}, \mathbf{03}\right).$$

C'est la parenthèse que remplace le facteur $\sqrt[3]{R^2}$ de la forme plus simplifiée donnée ci-dessus. Voir la brochure On the flow of water in open channels and pipes, par Robert Manning. Dublin, 1890.

Figure 6 : First citation of the Manning formulas by the French hydraulician Alfred Aimé Flamant (1891, p. 191).

The first reference to the Manning formula published in English is attributed to Willcocks and Holt in their book *Elementary Hydraulics* published in Cairo, Egypt in 1899, who recommended it as the "best formula of the day" (Willcocks and Holt, 1899, p. 10 cited in Powell, 1960, p. 1310). Willcocks and Holt cite Flamant (1891) but it is also likely that they received directly from Manning of copy of his paper (Dooge, 1992, p. 173-174). Powell (1968) and Williams (1970) indicate some other early references to the Manning formula in Church (1900), Bovey (1991), Buckley (1911), Parker (1913), Dougherty (1916) and King (1918) (Figure 8).

More information about the Manning formula, its context and its evolution may be found in the book edited by Yen (1992) from the material presented during the International Conference for the Centennial of Manning formula held at the University of Virginia, USA in May 1989. More information about previous, similar and following equations for open channel flow may be found e.g. in Rouse and Ince (1957), Williams (1970), Garbrecht (1987) and Levi (1995).

Additionally, it may be interesting to cite a similar formula proposed by Crimp and Bruges (1894) for the design of sewers. The authors (who did not cite any previous authors like Gauckler, Hagen or Manning) were looking for an improvement of the Chézy* formula:

$$U = C\sqrt{R_h S}$$
 Eq. 8

because they had observed that "recent research has shown that the resistances are not proportional to that [i.e. 1/2] power of R_h , but to other powers which depend upon the roughness of the wetted surface and upon other factors" (Crimp and Bruges, 1894, p. 198). But they were also looking for a simple formula to avoid the mathematical difficulties of Darcy and Ganguillet and Kutter formulas. Using data from Darcy and Bazin measured for semi-circular channels, they suggested the following formula (Figure 7):

$$U = 124 R_h^{3/2} S^{1/2}$$
 Eq. 9

Then they tested their formula by doing field measurements in the King Scholar's Pond sewer in London which had just been redesigned and rebuilt to avoid dry weather deposition observed in the previous pipe. In the new pipe made with high quality smooth bricks and mortar, they measured the flow velocity with both floating papers (to measure the surface velocity) and cream of lime as a dye tracer (to estimate the mean cross section velocity). In order to obtain the best fit with mean velocities U estimated as equal to 0.83 of the measured centre surface velocities, they had to replace 124 by 143 in Eq. 9. They concluded that this highest value was due to the exceptionally smooth walls of the new pipe, and that for safe practical use the initial value of 124 was more appropriate and equivalent to Kutter's coefficient n between 0.012 and 0.013 (Crimp and Bruges, 1894, p. 201).

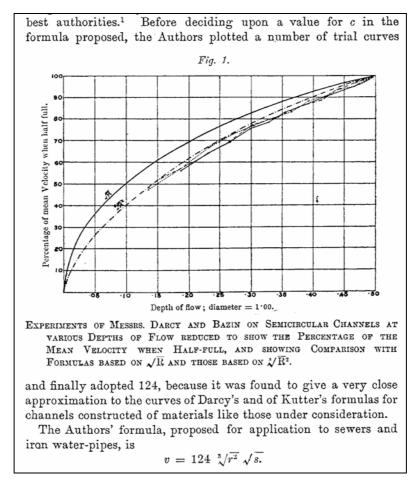


Figure 7 : Formula for in-sewer flow proposed by Crimp and Bruges (1894, p. 199).

In this book the older and commonly accepted formulas are given preference except where a gain in accuracy or simplicity or both will result from the adoption of new formulas or methods. The author departs from standard American practice in advocating the use of the Manning formula in place of the Kutter formula. He has not done this, however, until he has been able to prove that the two formulas give practically identical results by using the same coefficient. New weir formulas are also submitted which are shown to be simpler and to conform to existing experimental data more consistently than other formulas. Exponential formulas are advocated for pipes but a simplified method of using them is given in detail.

Figure 8 : Reference to Manning formula in the preface to the 1st edition the *Hanbook of Hydraulics* by King (1918) as given in the 2nd edition dated 1929 (King, 1929, p. viii).



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