Electric Rocket Propulsion

A. Background

Electric Propulsion - Accelerators

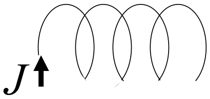
	Electrothermal	**Electromagnetic**	**Electrostatic**
Accel. Force	Pressure, ∇p Electrically heat propellant and use nozzle expansion	Lorentz, $\vec{F}_m$ Magnetic and elec. fields accelerate charged particles	Electrostatic, $\vec{F}_e$ Static electric field alone accelerates charged particles
I_{sp}(s)	300-1,500	1,000-10,000	2,000-100,000+
$\frac{\text{Thrust}}{\text{Weight}}$	$<10^{-3}$	$<10^{-4}$	$<10^{-4}–10^{-6}$

Electrical Heating

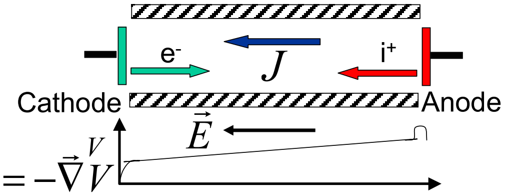
- Current passing through conductor heats it by amount proportional to its resistance

$$\dot{Q} = \overset{\text{Current (A=C/s)}}{J^2}\,\overset{\text{Resistance (Ohms)}}{R}$$

- Wire



- Gas (Plasma) Discharge
  - resistance due to collisions

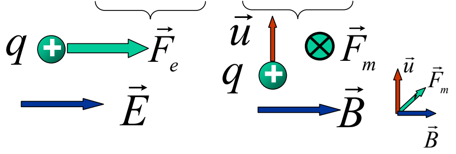


$$\vec{E} = -\vec{\nabla} V$$

Forces on a Charged Particle

- To examine how to use electrical energy to accelerate a propellant, consider acceleration of a particle with mass ***m*** and charge ***q***

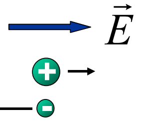
$$m\frac{d\vec{u}}{dt} = \underbrace{q\overbrace{\vec{E}}^{\text{Elec. Field}}}_{\text{Electrostatic Force}} + \underbrace{q\left(\vec{u}\times\overbrace{\vec{B}}^{\text{Mag. Field}}\right)}_{\text{Lorentz Force}} + \underbrace{\vec{p}}_{\text{Collisional Force (Momentum Transfer)}} \qquad (1)$$



Motion of Charged Particle in E&B Fields

- How does charged particle move in electric and magnetic fields
- **Electric field only**
  - electron lighter, higher accel.

$$\frac{d\vec{u}}{dt} = \frac{q}{m}\vec{E}$$

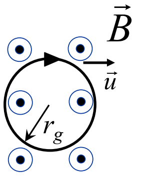


- **Magnetic field only**
  - particle gyrates (centripedal accel.)
  - radius of gyration
  - frequency of gyration
  - No work; B and F perpendicular

$$\frac{d\vec{u}}{dt} = \frac{q}{m}\left(\vec{u}\times\vec{B}\right)$$

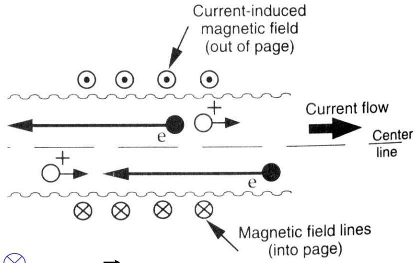
$$r_g = \frac{m\left|\vec{u}\times\vec{B}\right|}{qB^2}$$

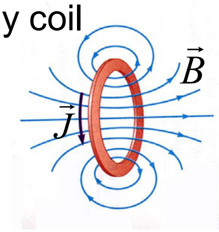
$$\omega_g = \frac{qB}{m}$$

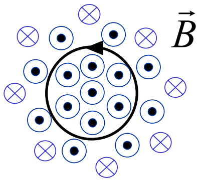


Induced Magnetic Fields

- In general, current flow induces a B field
  - B field induced by linear current
  - B field induced by coil



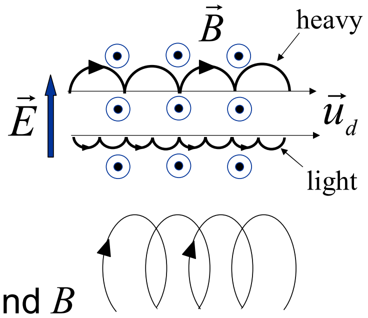




From *Space Propulsion Analysis and Design,* Humble, Henry and Larson, 1995

Motion of Charged Particle in E&B Fields

- **Crossed E and B fields**
  - E field accelerates particle upward
  - B field causes acceleration perpendicular to u
  - overall result is drift velocity normal to E and B



$$\vec{u}_d = \frac{\vec{E}\times\vec{B}}{B^2}$$

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Plasmas

- Gas composed of equal “amount” of negatively and positively charged particles
  - electrically neutral
  - negative particles usually e^-
  - positive particles are positive ions
  - typically most of gas molecules remain neutral
- **Momentum equation for plasma**

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}\right)=-\nabla p+\vec{j}\times\vec{B}$$

Current Density (A/m^2)

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Induced E Fields – Hall Effect

- **Electron acceleration**
  - lighter e^- accelerate more quickly and accommodate to flow field (most of j)
  - collisional coupling (momentum) of electrons and heavies (ions and neutrals) is weak

$$\Rightarrow \vec{E} = \eta\,\vec{j} - \underbrace{\vec{u}\times\vec{B}} + \underbrace{\vec{j}\times\frac{\vec{B}}{n_e e}} - \frac{\nabla p_e}{n_e e} \qquad (2)$$

plasma resistivity (Ωm)

$$\eta = m_e \nu_{ce} / n_e e^2$$

collision freq. electrons with heavies

Induced E field due to plasma motion

Hall Effect accelerates heavy particles in charge neutral plasma

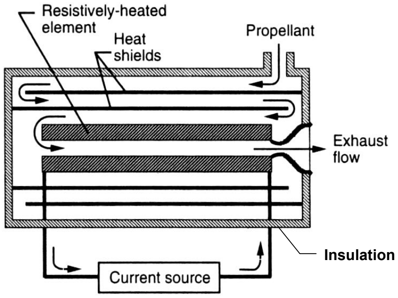
electron pressure term

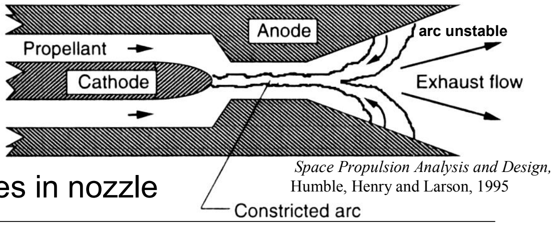
Electric Rocket Propulsion

B. Examples

Electrothermal Rockets

- **Resistojet (1W-10kW)**
  - walls hotter than gas, lower T_{max}
  - can combine with chemical heating, e.g., hydrazine monopropellant
- **Arcjet (1 kW- 10 MW)**
  - high T_{arc}>T_{wall} ($1\text{-}4\times10^4$ K)
  - must mix with rest of gas
  - very high T_o, frozen flow losses in nozzle





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Resistojets

- Heating methods
  - flow over coils of wire
  - flow through hollow tubes
  - flow over heated cylinder or plate
- Material limits temperature/performance
  - conducting materials < 2700 K rhenium, refractory metals/alloys (tungsten, tantalum, molybdenum), platinum, cermets
  - max I_{sp} ~ 300 sec
- Power supply
  - AC or DC
  - Power $=\dot{Q}=V^2/R=\dot{m}c_p\Delta T$

Arcjets

- Gas heated by electrical discharge (arc)
  - electrically conducting gas: plasma
- Joule heating

from (2)

$$\vec{E} = \eta\vec{j} - \vec{u}\times\vec{B} + \vec{j}\times\frac{\vec{B}}{n_e e} - \frac{\nabla p_e}{n_e e}$$

Hall Effect
normal to j

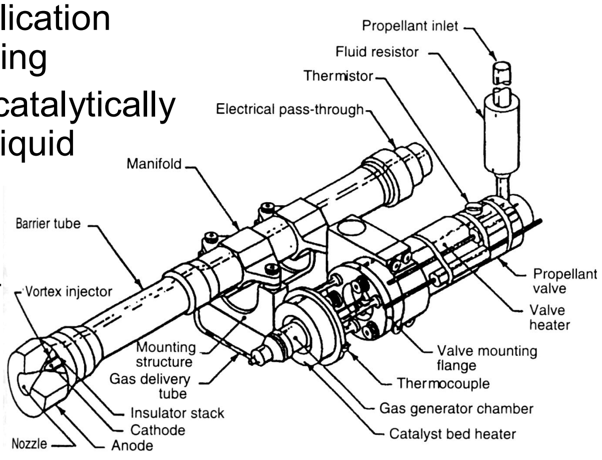
$$\dot{Q} = \vec{j}\cdot\vec{E} = \dot{m}c_p\Delta T$$
$$= \eta j^2 - \vec{j}\cdot\left(\vec{u}\times\vec{B}\right) - \vec{j}\cdot\nabla p_e / n_e e$$
$$= \underbrace{\eta j^2}_{\text{Resistive Heating}} + \underbrace{\vec{u}\cdot\left(\vec{j}\times\vec{B}\right)}_{\text{Work from EM force}} - \underbrace{\vec{j}\cdot\nabla p_e / n_e e}_{\text{Work from pressure grad.}}$$

no work

- Nozzle and electrode erosion
  - high temperature, current arc
- I_{sp} for H_2 ~1200-1500 sec

1.8 kW Hydrazine Arcjet

- Telstar IV application
  - stationkeeping
- Hot gas from catalytically decomposed liquid N_2H_4
- Heated valve prevents N_2H_4 from freezing
- I_{sp}~570 sec
- τ~230 mN (~40mg/s)



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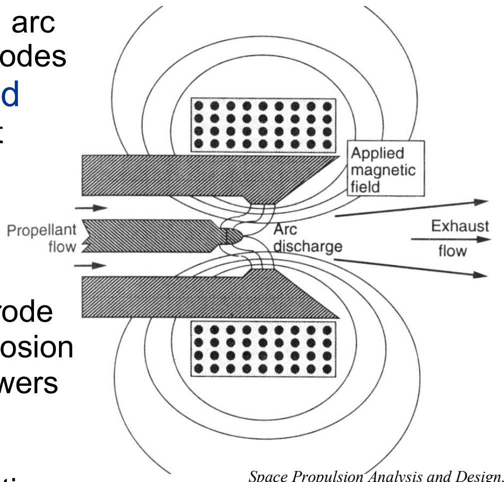
Arcjet with Applied Magnetic Field

- High current density in arc tends to destroy electrodes
- Add soleniod B Field
  - adds azimuthal drift velocity to arc
  - improves azimuthal symmetry of gas temperature
  - reduces local electrode heating, reduces erosion
  - needed for high powers (≥100 kW)
- Microwave discharge
  - alternate to arc heating



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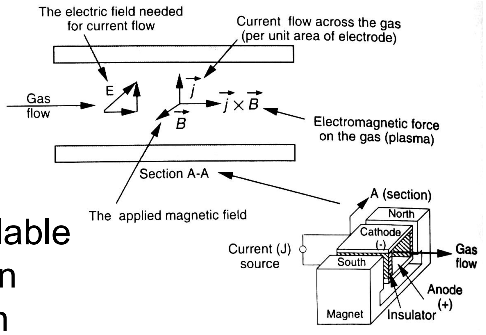
Electromagnetic Propulsion Systems

- Use applied or induced magnetic fields to produce acceleration of propellant
  - high currents/ powers required to produce significant induced fields
  - high power available only (normally) in pulsed operation



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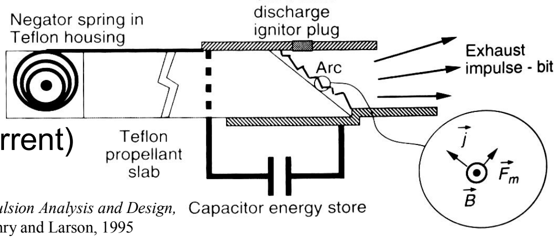
Pulsed Plasma Thruster

- Propellant produced by vaporizing solid material with discharge
- B field induced by discharge also acts to accelerate vaporized propellant
- Advantage of simplicity
- Acceleration force $\sim j^2$ (discharge current)



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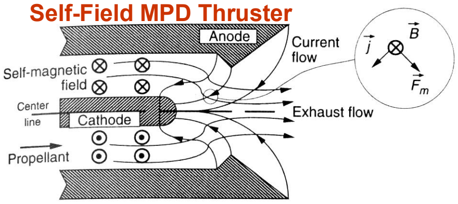
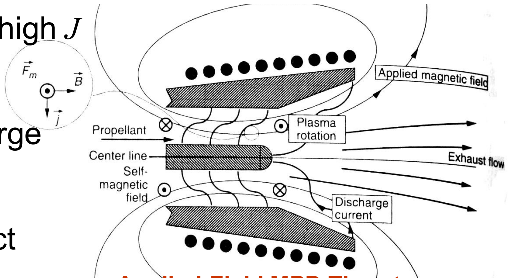
Magnetoplasmadynamic (MPD) Thrusters

- Resemble arcjets
- Lower flow densities to attain higher exhaust velocity
- Diffuse discharge, low erosion
- Self field requires high J
- Applied B field
  - allows higher V at lower discharge currents
  - increase accel.
  - larger Hall effect

Self-Field MPD Thruster





Applied-Field MPD Thruster

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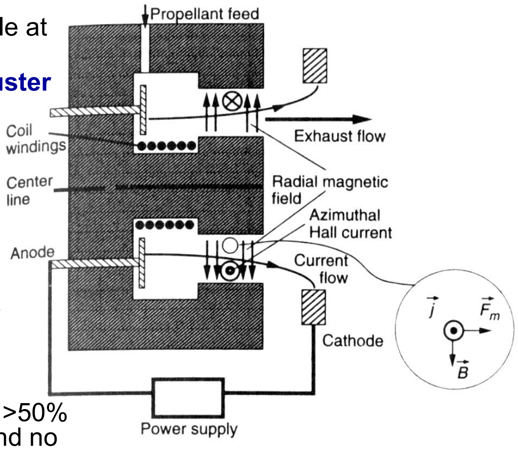
MPD Thrusters (con’t)

- Most efforts focused on applications with exhaust velocities (Isp) greater than arc jets
- Typically require higher powers than currently available on in-space vehicles
- Exhaust speed $u_e \propto j^2 / \dot{m}$
  - limited by erosion and oscillations at high j

Hall Thruster

- Hall effect most noticeable at low particle densities
- **Stationary Plasma Thruster** (SPT)
  - developed in Russia
  - 10’s kW
  - axial current flow across radial B generates azimuthal electron flow
  - induced (axial) Hall E accelerates ions
  - also called “gridless” (neutral) ion thruster
  - 1500-2000 s Isp with >50% efficiency using Xe and no oscillations like MPD



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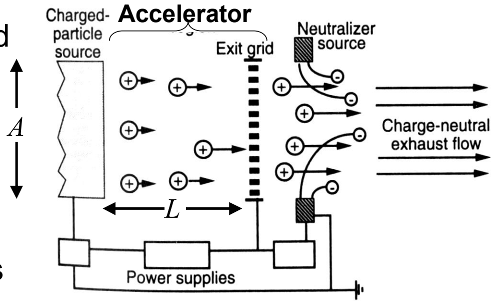
Electrostatic Thrusters

- Ion engine example
- Near vacuum required
- Ion source
  - usually electron bombardment plasma
  - also ion contact: propellant flowing through hot porous tungsten
  - field emission: charged spray droplets/particles
- Electrons added to neutralize exhaust
  - thermionic emitters



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Electrostatic Thruster Performance

- Maximum exhaust velocity (Isp) theoretically limited by voltage difference across accelerator

$$u_e = \sqrt{2\frac{q}{m}\left(V_{exit} - V_{inlet}\right)}$$

$$= 13{,}800\sqrt{\frac{\Delta V(\text{volts})}{MW}}\ \frac{m}{s}$$

for singly ionized

- High thrust requires high mass flowrates and therefore current (and number, n) densities

$$j = nqu_e$$

- Ion current limited by space-charge
  - field from lots of ions overcomes applied E field

Child-Langmuir Law

$$j_{\max} = \frac{4\varepsilon_o}{9}\sqrt{\frac{2q}{m}}\frac{\Delta V^{3/2}}{L^2}$$

accelerator electrode spacing

permittivity of free space

$$\varepsilon_o = 8.854\times10^{-12}\ \frac{Farad}{meter}$$

$$j_{\max} = 5.44\times10^{-8}\ \frac{\Delta V(\text{volts})^{3/2}}{\sqrt{MW}\ L^2(\text{m})}\ \frac{Amps}{m^2}$$

for singly ionized

Electrostatic Thruster Thrust

$$\tau = \dot{m}u_e = jA(m/q)u_e$$

A=cross-sectional area flow

- Maximum thrust

$$\tau_{\max} = j_{\max} A \frac{m}{q} u_e = \frac{4\varepsilon_o}{9}\sqrt{\frac{2q}{m}}\frac{\Delta V^{3/2}}{L^2} A \frac{m}{q}\sqrt{\frac{2q}{m}\Delta V}$$

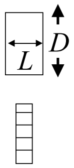
$$\tau_{\max} = (8\varepsilon_o/9)A\Delta V^2/L^2$$

for circular cross-section of diameter, D

$$\tau_{\max} = (2\pi/9)\varepsilon_o(D/L)^2\Delta V^2$$

$$= 6.18\times10^{-12}(D/L)^2\Delta V^2 \text{ in } Newtons$$

- High τ requires high ΔV and aspect ratio
  - space charge⇒D/L ~1
  - use many small ion beams to get more thrust



Electrostatic Thruster Thrust

- For fixed specific impulse $I_{sp} = u_e$

$$\tau_{\max} = \frac{4}{9\varepsilon_o}\sqrt{\frac{2q}{m}}\frac{\Delta V^{3/2}}{L_{accel}^2} A \frac{m}{q} I_{sp}$$

$$\frac{\tau_{\max}}{A} \propto \sqrt{m/q}$$

  - best propellant for high thrust
    - heavy molecules (particles)
    - xenon (Xe) good choice (MW=131.3)

Electrostatic Thruster Power

$$Power = I\,\Delta V = jA\,\Delta V$$

- Electrical energy converted to kinetic energy with some efficiency

$$\eta_{conv} I\,\Delta V = \frac{1}{2}\dot{m}u_e^2$$

- Must also account for energy used to create ions – **ionization losses**
  - given by ionization potential for atom times current

$$P_{ion\,loss} = \varepsilon_I I = \varepsilon_I \frac{\dot{m}}{m} q$$

Electron Bombardment Xe$^+$ Engine Example

- Operating conditions
  - ΔV=700 V, L=2.5 mm, 2200 holes (grids) each with D=2.0 mm
  - MW(Xe)=131.3, ε_I(Xe)=12.08 eV
- Determine
  - τ,
  - u_e, Isp
  - $\dot{m}$
  - power required including ionization loss

Solution

- $$\tau_{\max} = 6.18\times10^{-12}\left(D/L\right)^2 \Delta V^2$$
  $$= 6.18\times10^{-12}\left(2/2.5\right)^2 700^2 = 1.94\times10^{-6}\, N/grid$$
- $$\tau_{\max,total} = 2200\, grids\left(1.94\times10^{-6}\, N/grid\right) = 4.3mN$$
- $$u_e = 13{,}800\sqrt{\Delta V/MW}\; m/s = 13{,}800\sqrt{700/131.3}\; m/s$$
  $$u_e = 31{,}860\, m/s \quad \Rightarrow I_{sp} = 3248\, s$$
- $$\dot{m} = \tau/u_e = 1.34\times10^{-4}\, g/s$$
- $$P = P_{exhaust} + P_{ion\ loss} = \dot{m}/2u_e^2 + \varepsilon_I q\,\dot{m}/m$$
  $$= 67.9W + 1.18W$$
  $$P = 69.1W$$

$$m = 1.67\times10^{-27} MW\ kg$$
$$= 2.19\times10^{-25} kg$$
$$q = 1.602\times10^{-19} C$$

for singly ionized molec.

maximum effic. =67.9/69.1
= 98.3%