# CHROMATIC DISPERSION (OPTICS)

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Introduction

In optics, dispersion is the phenomenon in which the phase velocity of a wave depends on its frequency. Media having such a property are termed dispersive media.

The most familiar example of dispersion is probably a rainbow, in which dispersion causes the spatial separation of a white light into components of different wavelengths (different colors). However, dispersion also has an effect in many other circumstances: for example, it causes pulses to spread in optical fibers, degrading signals over long distances; also, a cancellation between dispersion
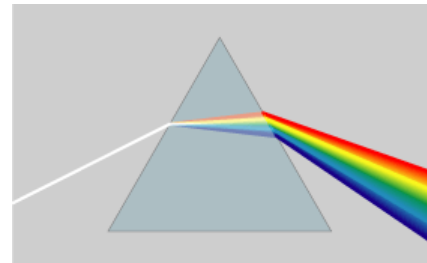
and nonlinear effects leads to soliton waves. Dispersion is most often described for light waves, but it may occur for any kind of wave that interacts with a medium or passes through an inhomogeneous geometry (e.g. a waveguide), such as sound waves. Dispersion is sometimes called chromatic dispersion to emphasize its wavelength-dependent nature.

There are generally two sources of dispersion: material dispersion and waveguide dispersion. Material dispersion comes from a frequency-dependent response of a material to waves. For example, material dispersion leads to undesired chromatic aberration in a lens or the separation of colors in a prism. Waveguide dispersion occurs when the speed of a wave in a waveguide (such as an optical fiber) depends on its frequency for geometric reasons, independent of any frequency dependence of the materials from which it is constructed. More generally, "waveguide" dispersion can occur for waves propagating through any inhomogeneous structure (e.g. a photonic crystal), whether or not the waves are confined to some region. In general, both types of dispersion may be present, although they are not strictly additive. Their combination leads to signal degradation in optical fibers for telecommunications, because the varying delay in arrival time between different components of a signal "smears out" the signal in time.

Material dispersion in optics

Material dispersion can be a desirable or undesirable effect in optical applications. The dispersion of light by glass prisms is used to construct spectrometers and spectroradiometers. Holographic gratings are also used, as they allow more accurate discrimination of wavelengths. However, in lenses, dispersion causes chromatic aberration, an undesired effect that may degrade images in microscopes, telescopes and photographic objectives.
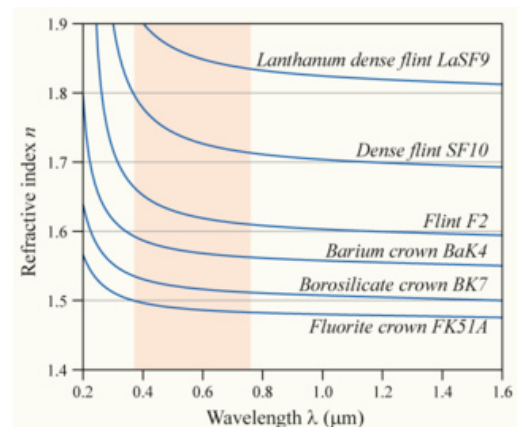
The phase velocity, v, of a wave in a given uniform medium is given by

$$v = \frac{c}{n}$$

The variation of refractive index vs. wavelength for various glasses. The wavelengths of visible light are shaded in red

where c is the speed of light in a vacuum and n is the refractive index of the medium.

In general, the refractive index is some function of the frequency f of the light, thus n = n(f), or alternately, with respect to the wave's wavelength n = n(λ). The wavelength dependency of a material's refractive index is usually quantified by an empirical formula, the Cauchy or Sellmeier equations.

The most commonly seen consequence of dispersion in optics is the separation of white light into a color spectrum by a prism. From Snell's law it can be seen that the angle of refraction of light in a prism depends on the refractive index of the prism material. Since that refractive index varies with wavelength, it follows that the angle that the light is refracted will also vary with wavelength, causing an angular separation of the colors known as angular dispersion.

1

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For visible light, most transparent materials (e.g. glasses) have:

$$1 < n(\lambda_{\text{red}}) < n(\lambda_{\text{yellow}}) < n(\lambda_{\text{blue}}) \,,$$

or alternatively:

$$\frac{dn}{d\lambda} < 0,$$

that is, refractive index n decreases with increasing wavelength λ. In this case, the medium is said to have normal dispersion. Whereas, if the index increases with increasing wavelength the medium has anomalous dispersion.

Influences of selected glass component additions on the mean dispersion of a specific base glass (nF valid for **λ** = 486 nm (blue), nC valid for **λ** = 656 nm (red))

that is, refractive index n decreases with increasing wavelength **λ**. In this case, the medium is said to have normal dispersion. Whereas, if the index increases with increasing wavelength the medium has anomalous dispersion.

At the interface of such a material with air or vacuum (index of ~1), Snell's law predicts that light incident at an angle θ to the normal will be refracted at an angle arcsin(sin (θ) / n) . Thus, blue light, with a higher refractive index, will be bent more strongly than red light, resulting in the well-known rainbow pattern.

Group and phase velocity

Another consequence of dispersion manifests itself as a temporal effect. The formula above, v = c / n calculates the phase velocity of a wave; this is the velocity at which the phase of any one frequency component of the wave will propagate. This is not the same as the group velocity of the wave, which is the rate that changes in amplitude (known as the envelope of the wave) will propagate. The group velocity vg is related to the phase velocity by, for a homogeneous medium (here **λ** is the wavelength in vacuum, not in the medium):
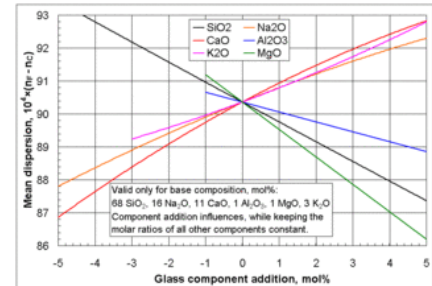
$$v_g = c \left(n - \lambda \frac{dn}{d\lambda} \right)^{-1}.$$

The group velocity vg is often thought of as the velocity at which energy or information is conveyed along the wave. In most cases this is true, and the group velocity can be thought of as the signal velocity of the waveform. In some unusual circumstances, where the wavelength of the light is close to an absorption resonance of the medium, it is possible for the group velocity to exceed the speed of light (vg > c), leading to the conclusion that superluminal (faster than light) communication is possible. In practice, in such situations the distortion and absorption of the wave is such that the value of the group velocity essentially becomes meaningless, and does not represent the true signal velocity of the wave, which stays less than c.

The group velocity itself is usually a function of the wave's frequency. This results in group velocity dispersion (GVD), which causes a short pulse of light to spread in time as a result of different frequency components of the pulse travelling at different velocities. GVD is often quantified as the group delay dispersion parameter (again, this formula is for a uniform medium only):

$$D = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2}.$$

If D is less than zero, the medium is said to have positive dispersion. If D is greater than zero, the medium has negative dispersion.If a light pulse is propagated through a normally dispersive medium, the result is the higher frequency components travel slower than the lower frequency components. The pulse therefore becomes positively chirped, or up-chirped, increasing in frequency with time. Conversely, if a pulse travels through an anomalously dispersive medium, high frequency components travel faster than the lower ones, and the pulse becomes negatively chirped, or down-chirped, decreasing in frequency with time.

Chromatic Dispersion

2

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The result of GVD, whether negative or positive, is ultimately temporal spreading of the pulse. This makes dispersion management extremely important in optical communications systems based on optical fiber, since if dispersion is too high, a group of pulses representing a bit-stream will spread in time and merge together, rendering the bit-stream unintelligible. This limits the length of fiber that a signal can be sent down without regeneration. One possible answer to this problem is to send signals down the optical fibre at a wavelength where the GVD is zero (e.g. around ~1.3-1.5 μm in silica fibres), so pulses at this wavelength suffer minimal spreading from dispersion—in practice, however, this approach causes more problems than it solves because zero GVD unacceptably amplifies other nonlinear effects (such as four wave mixing). Another possible option is to use soliton pulses in the regime of anomalous dispersion, a form of optical pulse which uses a nonlinear optical effect to self-maintain its shape—solitons have the practical problem, however, that they require a certain power level to be maintained in the pulse for the nonlinear effect to be of the correct strength. Instead, the solution that is currently used in practice is to perform dispersion compensation, typically by matching the fiber with another fiber of opposite-sign dispersion so that the dispersion effects cancel; such compensation is ultimately limited by nonlinear effects such as self-phase modulation, which interact with dispersion to make it very difficult to undo.

Dispersion control is also important in lasers that produce short pulses. The overall dispersion of the optical resonator is a major factor in determining the duration of the pulses emitted by the laser. A pair of prisms can be arranged to produce net negative dispersion, which can be used to balance the usually positive dispersion of the laser medium. Diffraction gratings can also be used to produce dispersive effects; these are often used in high-power laser amplifier systems. Recently, an alternative to prisms and gratings has been developed: chirped mirrors. These dielectric mirrors are coated so that different wavelengths have different penetration lengths, and therefore different group delays. The coating layers can be tailored to achieve a net negative dispersion.

Dispersion in waveguides

Optical fibers, which are used in telecommunications, are among the most abundant types of waveguides. Dispersion in these fibers is one of the limiting factors that determine how much data can be transported on a single fiber.

The transverse modes for waves confined laterally within a waveguide generally have different speeds (and field patterns) depending upon their frequency (that is, on the relative size of the wave, the wavelength) compared to the size of the waveguide.

In general, for a waveguide mode with an angular frequency $\omega(\beta)$ at a propagation constant β (so that the electromagnetic fields in the propagation direction z oscillate proportional to ei(βz – ωt)), the group-velocity dispersion parameter D is defined as:

$$D = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} = \frac{2\pi c}{v_g^2 \lambda^2}\frac{dv_g}{d\omega}$$

where $\lambda = 2\pi c / \omega$ is the vacuum wavelength and vg = dω / dβ is the group velocity. This formula generalizes the one in the previous section for homogeneous media, and includes both waveguide dispersion and material dispersion. The reason for defining the dispersion in this way is that |D| is the (asymptotic) temporal pulse spreading t per unit bandwidth Δλ per unit distance travelled, commonly reported in ps / nm km for optical fibers.

A similar effect due to a somewhat different phenomenon is modal dispersion, caused by a waveguide having multiple modes at a given frequency, each with a different speed. A special case of this is polarization mode dispersion (PMD), which comes from a superposition of two modes that travel at different speeds due to random imperfections that break the symmetry of the waveguide.

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Dispersion in gemology

In the technical terminology of gemology, dispersion is the difference in the refractive index of a material at the B and G Fraunhofer wavelengths of 686.7 nm and 430.8 nm and is meant to express the degree to which a prism cut from the gemstone shows "fire", or color. Dispersion is a material property. Fire depends on the dispersion, the cut angles, the lighting environment, the refractive index, and the viewer

Dispersion in imaging

In photographic and microscopic lenses, dispersion causes chromatic aberration, distorting the image, and various techniques have been developed to counteract it

Dispersion in pulsar timing

Pulsars are spinning neutron stars that emit pulses at very regular intervals ranging from milliseconds to seconds. It is believed that the pulses are emitted simultaneously over a wide range of frequencies. However, as observed on Earth, the components of each pulse emitted at higher radio frequencies arrive before those emitted at lower frequencies. This dispersion occurs because of the ionised component of the interstellar medium, which makes the group velocity frequency dependent. The extra delay added at frequency ν is

$$D = 4.15ms\left(\frac{\nu}{GHz}\right)^{-2} \times \left(\frac{DM}{cm^{-3}pc}\right)$$

where the dispersion measure DM is

$$DM = \int_0^d n_e\, dl$$

is the integrated free electron column density ne out to the pulsar at a distance d[4].

Of course, this delay cannot be measured directly, since the emission time is unknown. What can be measured is the difference in arrival times at two different frequencies. The delay ΔT between a high frequency νhi and a low frequency νlo component of a pulse will be

$$\Delta T = 4.15ms\left[\left(\frac{\nu_{lo}}{GHz}\right)^{-2} - \left(\frac{\nu_{hi}}{GHz}\right)^{-2}\right] \times \left(\frac{DM}{cm^{-3}pc}\right)$$

and so DM is normally computed from measurements at two different frequencies. This allows computation of the absolute delay at any frequency, which is used when combining many different pulsar observations into an integrated timing solution.