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# A SOURCE BOOK IN GREEK SCIENCE

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# SOURCE BOOKS IN THE HISTORY OF THE SCIENCES

# General Editor's Preface

The Source Books in this series are collections of classical papers that have shaped the structures of the various sciences. Some of these classics are not readily available and many of them have never been translated into English, thus being lost to the general reader and frequently to the scientist himself. The point of this series is to make these texts readily accessible and to provide good translations of the ones that either have not been translated at all or have been translated only poorly.

The series was planned originally to include volumes in all the major sciences from the Renaissance through the nineteenth century. It has been extended to include ancient and medieval Western science and the development of the sciences in the first half of the present century. Many of these books have been published already and several more are in various stages of preparation.

The Carnegie Corporation originally financed the series by a grant to the American Philosophical Association. The History of Science Society and the American Association for the Advancement of Science have approved the project and are represented on the Editorial Advisory Board. This Board at present consists of the following members:

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The series was begun and sustained by the devoted labors of Gregory D. Walcott and Everett W. Hall, the first two General Editors. I am in-

bottom, so that the flow of water may be facilitated. Call the sphere A, the tube B, and the vessel C.

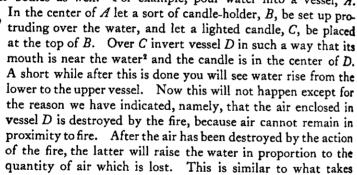
I say, then, that if you expose the sphere to the sun, part of the air enclosed in the tube will pass out when the sphere becomes hot. This will be evident because the air will descend from the tube into the water, agitating it and producing a succession of bubbles.

Now if the sphere is put back in the shade, that is, where the sun rays do not reach it, the water will rise and pass through the tube until it descends into the sphere. If you then put the sphere back in the sun the water will return to the vessel; but it will flow back to the sphere once more if you place the sphere in the shade. No matter how many times you repeat the operation the same thing will always happen.

In fact, if you heat the sphere with fire, or even if you pour hot water over it, the result will be the same. And if the sphere is then cooled, water passes from the vessel to the sphere.

# Combustion and Air

8.... Hence we shall prove that a place cannot be empty of air and of all other bodies as well. For example, pour water into a vessel, A.



place in the case of the tube described above.<sup>3</sup> Thus the air in this vessel (D) placed over the candle is destroyed because it is, so to speak, dissolved by the fire. For this reason the water is raised and entering fills the place left by the air, since that place was empty. The figure is appended.<sup>4</sup>

#### OPTICS

Various phases of optics were the object of speculation and investigation by the Greeks. Philosophers developed theories about the nature of light, color, and vision. Physiologists sought to explain the mechanism of seeing. Mathematicians and scientists studied perspective and mirrors with the help of the concept of visual rays; they arrived at the fundamental laws of reflection. Refraction was investigated empirically as well as mathematically, and its importance in connection with astronomical observation was not overlooked. There were practical applications of optical theory in the arts, e.g., in scene painting for the theater, and in the construction of devices employing mirrors. As for the literary remains, apart from the wealth of material in the writings of philosophers and physicians, there are treatises specially devoted to optics, extant either in the original Greek or in translation, by Euclid, Hero of Alexandria, Ptolemy (?), and Damianus or Heliodorus of Larissa. We have given selections representing the various branches. Note also Geminus's classification of these branches (p. 4, above). On theories of vision see pp. 543-546.

# Introduction to the Theory of Perspective

Euclid, Optics, Definitions and Propositions I-VIII, XLV, XLVIII (Heiberg) 1

# **Definitions**

Let it be assumed

- 1. That the rectilinear rays proceeding from the eye diverge indefinitely;<sup>2</sup>
- 2. That the figure contained by a set of visual rays is a cone of which the vertex is at the eye and the base at the surface of the objects seen;
- 3. That those things are seen upon which visual rays fall and those things are not seen upon which visual rays do not fall;

Tradition ascribed works on perspective to Democritus and Anaxagoras. See Vitruvius VII. Preface 11.

<sup>a</sup> The adoption of the theory of vision in which the visual rays proceed from the eye to the object, rather than from the object to the eye, does not affect the geometric development of the theory of perspective—the object of Euclid's work.

The precise meaning of the definition is doubtful and the version given conjectural, though according to a scholion the meaning would be substantially the same as that of Theon's revision, which reads: "Let it be assumed that the rays from the eye move in straight lines diverging from one another" (note the immediate application to Prop. I). Contrast the treatment in Ptolemy (?), Optics pp. 24f. (Govi).

<sup>&</sup>lt;sup>1</sup> For this purpose the end must always be immersed in water.

<sup>&</sup>lt;sup>2</sup> So both the Latin and the extant Arabic versions. But for the experiment to succeed the mouth must be under the water.

<sup>\*</sup> Pneumatics, ch. 7.

<sup>\*</sup>Though the idea of destruction of air in combustion is here expressed, there is, of course, no notion of oxidation.

Cf. Galen IV. 487-488 (Kühn): "For clearly we see these [flames], just as living things, swiftly extinguished when they are deprived of air. If a physician's cupping instrument or any narrow or concave vessel be put over the flames so as to cut off the access of air they are soon snuffed out. Now if we could discover why flames are in these cases extinguished, we should perhaps discover what advantage the heat in animals derives through respiration."

On Euclid, see p. 37. The Optics is extant in two versions, of which the earlier form is thought to be Euclid's own arrangement and the later that of Theon of Alexandria (latter part of the fourth century A.D.) The work consists of definitions (or rather assumptions) followed by 58 theorems geometrically demonstrated and constituting a treatise on perspective. It is the earlier version, as edited by Heiberg, from which the present translations have been made. On the Catoptrics attributed to Euclid, see p. 261. For a criticism of the Optics of Euclid from the point of view of modern optical theory see G. Ovio, L'Ottica di Euclide (Milan, 1918), and the introduction to Paul Ver Eecke's translation of Euclid's Optics and Catoptrics (Paris, 1938).

4. That things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal angles appear equal:

5. That things seen by higher visual rays appear higher, and things seen by lower visual rays appear lower:

6. That, similarly, things seen by rays further to the right appear further to the right, and things seen by rays further to the left appear further to the left:

7. That things seen under more angles are seen more clearly.2

# Proposition I

No visible object is seen completely at one time.



Let AD be a visible object, B the eye, BA, BG, BK. and BD visual rays from B to the object. Then, since the incident rays move at an interval from one another, they cannot fall continuously over AD.3 Hence there are intervals along AD upon which the rays will not fall. The whole of AD will, therefore, not be seen at one time. We think that we see the whole of AD at one time because the rays move along the object very quickly.

# Proposition II

Of equal magnitudes situated at a distance those that are nearer are seen more clearly.

Let B be the eye, and GD and KL the visible objects, which we are to consider as equal and parallel, GD being nearer the eye. Let BG, BD, BK, and BL be incident visual rays. The visual rays to KL will not pass through points G G and D. For if they did, in the resulting triangle, BDLKGB, KL would be larger than GD. But they were assumed to be equal. Therefore, GD will be seen by more visual rays than will KL. GD will, conse-



quently, be seen more clearly than will KL. For objects seen under a larger number of angles are seen more clearly.

<sup>1</sup> The angle referred to is that at the vertex of the cone (Definition 2).

\* The meaning is brought out in Prop. II and is essentially contained in Definition 4, the angles being those between each pair of successive rays.

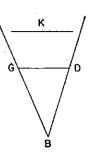
The assumption of discontinuity (see Definition 1 and note) seems to run counter to the geometrical continuity of the Elements. Euclid is here seeking to explain geometrically facts that are really due to limitations in the sensitivity of the retina. He does not, however, fully explain the meaning of the discontinuity he assumes.

4 The discontinuity of the rays is also assumed here and leads to an application of Definition 7.

# Proposition III

For every object there is a distance at which it is no longer seen.

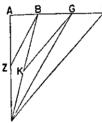
Let B be the eye and GD the visible object. I say that at a certain distance GD will no longer be seen. For suppose that GD is situated in an interval, K, between visual rays.1 Hence none of the visual rays from B will fall upon K. But an object upon which visual rays do not fall is not seen. Therefore, for each object there is a distance at which it is no longer seen.



# Proposition IV

Of equal intervals on the same straight line those seen from a greater

D distance appear smaller.



Let AB, BG, and GD be equal intervals on the same straight line. Draw AE perpendicular to this line; let the eye be at E. I say that AB will appear larger than BG, and BG larger than GD.

Let EB, EG, and ED be incident visual rays. Draw BZ through B parallel to GE. AZ = ZE, for since BZ was drawn parallel to one side, GE, of  $\triangle AEG$ , it follows that EZ:ZA = GB:BA.

Hence, as we have said. AZ = ZE. BZ > ZA. But BZ > ZE. Therefore  $\angle ZEB > \angle ZBE$ . and  $\angle ZBE = \angle BEG.$ But  $\angle ZEB > \angle BEG$ . Therefore

Consequently AB will appear larger than BG.

Similarly, if a parallel to DE be drawn through G, it may be shown that BG will appear larger than GD.

# Proposition V

Equal magnitudes situated at different distances from the eye appear unequal, and the nearer always appears larger.2 . . .

# Proposition VI

Parallel lines when seen from a distance appear to be an unequal distance

<sup>2</sup> The proof is similar to that of Prop. II.

<sup>1</sup> I.e., between proximate visual rays, discontinuity being assumed.

**OPTICS** 

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Let AB and GD be two parallels, and E be the eye. I hold that AB and GD seem to be an unequal distance apart, and that the interval be-

tween them at a point nearer the eye seems greater than at a point more remote from the eye.

Let EB, EZ, ET, ED, EH, and EK be visual rays. Draw BD, ZH, and TK.

Now since  $\angle BED > \angle ZEH$ , BD appears greater than ZH.

Again, since  $\angle ZEH > \angle TEK$ , ZH appears greater than TK.

That is, BD > ZH > TK in appearance.

The intervals, then, between parallels will not appear equal but unequal.1 . . .

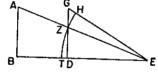
# Proposition VII

Equal but non-contiguous intercepts on the same straight line if unequally distant from the eye appear unequal. . . .

# Proposition VIII

Equal and parallel magnitudes unequally distant from the eye do not appear [inversely] proportional to their distances from the eye.

Let AB and GD be two such magnitudes unequally distant from the eye, E. I say that it is not the case that the apparent size of GD is to the apparent size of AB as BE is to ED, as might seem plausible.



Let AE and EG be visual rays. With E as center and EZ as radius describe arc HZT.

Since	$\triangle EZG > \text{sector } EZH$ ,
ind	$\triangle EZD < \text{sector } EZT$
t follows that	$\triangle EZG/\text{sector } EZH > 1$

it follows that  $\triangle EZG/\text{sector } EZH > \triangle EZD/\text{sector } EZT$ , and, by alternation,  $\triangle EZG/\triangle EZD > \text{sector } EZH/\text{sector } EZT$ .

Whence, by composition,  $\triangle EGD/\triangle EZD > \text{sector } EHT/\text{sector } EZT$ .

But  $GD/DZ = \Delta EDG/\Delta EZD$ .

and GD = AB.

Again, BE/ED = AB/DZ.

Therefore BE/ED > sector EHT/sector EZT.

But sector EHT/sector  $EZT = \angle HET$ / $\angle ZET$ .

Therefore  $BE/ED > \angle HET/\angle ZET$ .

Now GD is seen under ∠HET, and AB under ∠ZET.

The equal magnitudes do not, therefore, appear in [inverse] proportion to their distances from the eye.<sup>1</sup>

# Proposition XLV

There is a common point from which unequal magnitudes appear equal. Let BG be greater than GD. About BG describe a segment of a circle greater than a semicircle, and about GD describe a segment of a circle

similar to that about BG, i.e., a segment containing an angle equal to that contained in segment BZG. The segments, then, will intersect, let us say at Z. Draw ZB, ZG, and ZD.

B G D

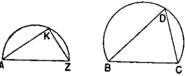
Since angles inscribed in similar segments are equal, the angles in segments BZG and

GZD are equal. But things seen under equal angles appear equal. Therefore, if the eye is placed at point Z, BG will appear equal to GD. But BG > GD. There is, then, a common point from which unequal magnitudes appear equal.

# Proposition XLVIII

To find points from which a given magnitude will appear half as large or a fourth as large, or, in general, in any fraction in which the angle may be divided.

Let magnitude AZ be equal to BC. Describe a semicircle about line AZ and inscribe right angle K therein.



Let line BC be equal to AZ and around BC describe a segment of a circle such that an angle inscribed therein will be half of angle K. Then angle K is double angle D, and AZ will, therefore,

appear twice as large as BC when the eye is on circumferences AKZ and BCD, respectively.<sup>2</sup>

# REFLECTION

#### THE THEORY OF MIRRORS

Hero, Catoptrics 1-6, 7, 10, 15, 18 (Schmidt)

1.... The science of vision is divided into three parts: optics, diop-

<sup>1</sup> In this proposition Euclid in effect proves  $\tan a/\tan b < a/b$  (where a and b are acute angles and a < b).

<sup>2</sup> The process may then be repeated indefinitely, the magnitude appearing  $\frac{1}{4}$ ,  $\frac{1}{8}$ , ... as large.

<sup>4</sup> The subject of mirrors, catoptrics, was a branch of the scientific study of optics in antiquity (see p. 4, above). Archimedes wrote a treatise on the subject, but it is not extant. The Catoptrics ascribed to Euclid is probably a compilation by Theon of Alexandria at the

<sup>&</sup>lt;sup>1</sup> The theorem of convergence is fundamental in the theory of perspective. There follows a proof of convergence for the case where the eye is not in the same plane as the parallels.

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trics.1 and catoptrics. Now optics has been adequately treated by our predecessors and particularly by Aristotle,2 and dioptrics we have ourselves treated elsewhere as fully as seemed necessary. But catoptrics, too, is clearly a science worthy of study and at the same time produces spectacles which excite wonder in the observer. For with the aid of this science mirrors are constructed which show the right side as the right side, and, similarly the left side as the left side, whereas ordinary mirrors by their nature have the contrary property and show the opposite sides. It is also possible with the aid of mirrors to see our own backs,3 and to see ourselves inverted. standing on our heads, with three eyes, and two noses, and features distorted, as if in intense grief. The study of catoptrics, however, is useful not merely in affording diverting spectacles but also for necessary purposes. For who will not deem it very useful that we should be able to observe, on occasion, while remaining inside our own house, how many people there are on the street and what they are doing?4 And will anyone not consider it remarkable to be able to tell the hour, night or day, with the aid of figures appearing in a mirror? For as many figures appear as there are hours of the day or of the night, and if a [given] part of the day has passed a end of the fourth century A.D. The Catoptrics of Hero of Alexandria is therefore our earliest extant work on the subject. The original Greek is lost but we have a Latin version thought to have been made by William of Moerbeke in the thirteenth century. This text, called De Speculis, was generally ascribed to Ptolemy until Ptolemy's (?) Optics, which contains in its third book a treatment of catoptrics (pp. 268, 271), became known in the Latin translation made in the twelfth century from an Arabic version. Since there is independent evidence for the ascription of the De Speculis to Hero, the identification of the De Speculis with Hero's Catoptrics seems plausible.

Hero bases his treatment on the proposition that if the speed of light is incomprehensibly great (that is probably the sense in which he uses the adjective "infinite"), it travels by a straight line and is so reflected that the path from eye to mirror to object is a minimum. From this the equality of the angles of incidence and reflection is easily deduced.

Quite apart from the method of proof, it is impossible to say when this fundamental principle of catoptrics was first formulated. That the formulation is pre-Aristotelian is shown by C. B. Boyer in *Isis* 36 (1946), 94-95. The reflection of sound at equal angles is referred to, e.g., in [Aristotle], *Problemata* XI. 23.

We may mention here an extant fragment of a work on burning mirrors by Anthemius (beginning of sixth century) in which ellipsoidal and paraboloidal reflectors as well as combinations of plane mirrors are discussed. There is a tradition (in Lucian, Galen, and others), probably without foundation, that Archimedes set fire to the fleet of Marcellus by using burning mirrors.

<sup>1</sup> The word "dioptrics" is probably not here used in its modern sense, in which it refers to the study of refraction. The reference in the following sentence seems to be to Hero's work On the Dioptra, an instrument for taking sightings (see pp. 139, 336).

<sup>2</sup> E.g., in the *De Anima* or in the *De Sensu*, where the theory of vision is treated. Ancient lists of Aristotle's works also mention one on optics.

3 The meaning might possibly be "to see those who are behind us," but see ch. 15.

<sup>4</sup> This interpretation (different from that of Schmidt) seems to be supported by ch. 16 of the work.

[given] figure will appear. Again, who will not be astonished when he sees, in a mirror, neither himself nor another, but whatever we desire that he see? Such, then, being the scope of the science, I think it necessary and proper to describe the views held by my predecessors, that my account may not be incomplete.

OPTICS

2. Practically all who have written of dioptrics and of optics have been in doubt as to why rays proceeding from our eyes are reflected by mirrors and why the reflections are at equal angles. Now the proposition that our sight is directed in straight lines proceeding from the organ of vision may be substantiated as follows. For whatever moves with unchanging velocity moves in a straight line.¹ The arrows we see shot from bows may serve as an example. For because of the impelling force the object in motion strives to move over the shortest possible distance, since it has not the time for slower motion, that is, for motion over a longer trajectory. The impelling force does not permit such retardation. And so, by reason of its speed, the object tends to move over the shortest path.² But the shortest of all lines having the same end points is the straight line.

That the rays proceeding from our eyes move with infinite velocity may be gathered from the following consideration. For when, after our eyes have been closed, we open them and look up at the sky, no interval of time is required for the visual rays to reach the sky. Indeed, we see the stars as soon as we look up, though the distance is, as we may say, infinite. Again, if this distance were greater the result would be the same, so that, clearly, the rays are emitted with infinite velocity. Therefore they will suffer neither interruption, nor curvature, nor breaking, but will move along the shortest path, a straight line.

3. That our vision is directed along a straight line has, then, been sufficiently indicated. We shall now show that rays incident on mirrors and also on water and on all plane surfaces are reflected. Now the essential characteristic of polished bodies is that their surfaces are compact. Thus, before they are polished, mirrors have some porosities upon which the rays fall and so cannot be reflected. But these mirrors are polished by rubbing until the porosities are filled by a fine substance; then the rays incident upon the compact surface are reflected. For just as a stone violently hurled against a compact body, such as a board or wall, rebounds, whereas

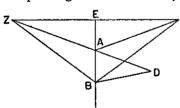
<sup>&</sup>lt;sup>1</sup>The meaning seems to be that as long as an object moves very swiftly it retains its rectilinear motion. A similar notion with respect to projectile motion prevailed until the time of Galileo.

<sup>&</sup>lt;sup>2</sup> The point seems to be that if visual rays were not propagated in straight lines they would not traverse a given distance as swiftly as possible.

<sup>&</sup>lt;sup>3</sup> The reference is not to reflection or refraction but to a change in the continuous rectilinear character of the motion.

a stone hurled against a soft body, such as wool or the like, does not (for the projecting force1 accompanies the stone and then, in the case of the hard obstacle, gives way, not being able to accompany the stone any further or move it forward, while in the case of the soft obstacle, the force merely slackens and is separated from the stone), so the rays that are emitted by us with great velocity, as we have shown, also rebound when they impinge on a body of compact surface. Now in the case of water and glass not all such rays are reflected since both these substances have irregularities, composed as they are of units having minute parts, and of solid particles.2 For in looking through glass and water we see our own reflection and also what lies beyond the surface of the glass or water. That is, in the case of standing water, we see what is at the bottom, and in the case of glass, what lies beyond its surface. For those rays which fall upon solid bodies are themselves turned back and reflected, while those which penetrate through porous bodies enable us to see that which lies beyond. Hence images reflected from such bodies are imperfectly seen because not all the visual rays are reflected to the objects, but some of them, as we have indicated, are lost through the pores.

4. That rays incident upon polished bodies are reflected has, then, in our opinion, been adequately proved. Now by the same reasoning, that is, by a consideration of the speed of the incidence and the reflection, we shall prove that these rays are reflected at equal angles in the case of plane and spherical mirrors. For our proof must again make use of minimum lines. I say, therefore, that of all incident rays [from a given point] reflected to a given point by plane and spherical mirrors the shortest are those that are reflected at equal angles; and if this is the case the reflection at equal angles is in conformity with reason.



Consider AB a plane mirror, G the eye, and D the object of vision. Let a ray GA be incident upon this mirror. Draw AD, and let  $\angle EAG = \angle BAD$ . Let another ray GB also be incident upon the mirror. Draw BD. I say that GA + AD < GB + BD.

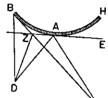
Draw GE from G perpendicular to

AB, and prolong GE and AD until they meet, say at Z. Draw ZB.

```
Now
                  \angle BAD = \angle EAG
                  \angle ZAE = \angle BAD (as vertical angles).
 and
                  \angle ZAE = \angle EAG.
   Therefore
   And since the angles at E are right angles.
                      ZA = AG
and
                      ZB = BG.
   But
                     ZD < ZB + BD
and
                          ZA = AG, ZB = BG.
   Therefore GA + AD < GB + BD.
                  \angle EAG = \angle BAD
   Now
                 \angle EBG < \angle EAG
and
                 \angle HBD > \angle BAD.
and
```

Therefore \(\angle HBD\) is, \(afortiori\), greater than \(\angle EBG.^1\) B

5. Let AB be the surface of a spherical mirror, G the eye, and D the object seen. Let GA and AD make equal angles with the mirror, while GB and BD make unequal angles. I say that GA + AD < GB + BD.



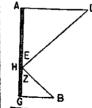
Draw EAZ tangent at A.

Then  $\angle HAE = \angle BAZ$ , and the remainder  $\angle EAG = \angle ZAD$ .

If ZD be drawn, GA + AD < GZ + ZD, as was proved above.

But GZ + ZD < GB + BD. Therefore GA + AD < GB + BD.

In general, then, in the case of mirrors [both plane and spherical], one must consider whether there is or is not a point from which incident rays may be reflected at equal angles in such a way that the ray incident from the organ of vision and the ray reflected to the object of vision, when added together, make a sum less than that of all other pairs of rays similarly incident and reflected.<sup>3</sup>



6. In the case of plane mirrors there is a place at the covering of which an image will no longer be seen.

Let AG be a plane mirror ..., B the eye, and D the visible object. Draw AD and BG perpendicular to the mirror, and divide AG at H in such a way that AD:BG = AH:HG. I say, then, that if H is covered the image of D is no longer seen.

\*I.e., the remainder of the angle between the visual ray and the mirror. The equality was proved in ch. 4, for the tangent plane may be considered a plane mirror.

Reading emittens at p. 322.21 (see p. 410 of the edition of Schmidt). Some have seen here an early instance of the anti-Aristotelian doctrine of vis impressa (p. 223, above).

<sup>&</sup>lt;sup>2</sup> A type of atomism, not very clearly defined, is invoked here to aid the explanation, as in the *Pneumatics* (see pp. 249 ff.)

<sup>\*</sup> I.e., the angle of incidence will equal the angle of reflection.

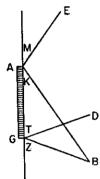
<sup>4</sup> I.e., the sum of the incident ray from eye to mirror and the reflected ray from mirror to object must be a minimum.

<sup>&</sup>lt;sup>1</sup> That is, not only is the path of the ray shortest when the angles of incidence and reflection are equal, but there is only one incident ray which can be reflected at equal angles.

But not reflected at an angle equal to the angle of incidence. There are probably lacunae in the text as we have it, but the reference seems to be to a test whether the image of a given object will be seen in a mirror (plane or spherical) from a given fixed point.

For if BH and HD are drawn, the triangles, because of the proportionality of their sides, will be similar. Hence  $\angle E = \angle Z$ , and D will be visible through point H. Therefore, if this point is covered with wax or some other material, D will no longer be visible.

If, however, the covering at H is removed from the mirror, the image will again appear in the mirror. For all rays incident upon a mirror will be reflected at equal angles.



7. In the case of plane mirrors the reflected rays neither will converge nor are parallel.

For let AG be a plane mirror, B the eye, [D and E the objects, GB and BA the incident rays,  $]^1$  and GD and AE the reflected rays.

$$\therefore ZZ = ZT$$

But 
$$\angle Z > \angle K$$
,

that is, 
$$\angle Z > \angle M$$
.

$$\therefore \angle T > \angle M$$
.

 $\therefore$  GD and AE are neither parallel nor will they meet in the direction of D and E.<sup>2</sup>

10. In the case of concave mirrors, when the eye is situated at the circumference the reflected rays converge.

Let BGA be a concave mirror, and let the eye be placed at B. Let

BG and BA be incident rays and GX and AN reflected rays. I say that GX and AN will meet on the side of X and N.

For since arc AB > arc GB,

$$\angle Z > \angle T$$
.

$$\therefore \angle E > \angle H$$
.

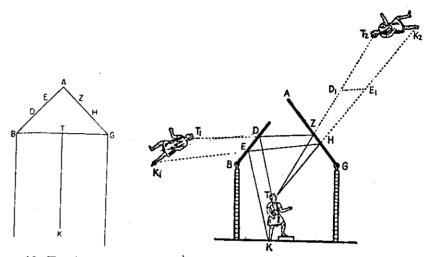
$$\therefore \angle L > \angle K$$
 (as remainders).

But 
$$\angle M > \angle L$$
.

 $\therefore \angle M > \angle K$ , and, consequently, GX and AN will meet on the side of N and X.

15. It is desired to secure the same effect<sup>3</sup> by another construction. Let ABG be a right triangle. Bisect BG at T. Let ZH and DE be plane mirrors on lines AG and AB, respectively. Consider TK as an observer with the eye at point T capable of looking into either mirror as desired. And so the problem will be solved.

If one mirror (ZH) into which the observer looks) is kept unmoved, while the other (DE), behind the observer) is moved up and down, the ray will reach a point where the image of the heel of the observer will appear in the mirror and he will think that he is flying.



18. To place a mirror so that one approaching it sees neither his own image nor that of another but only the image which we select.

Let AB [p. 268] be the wall where the mirror is to be put and let the mirror be inclined to it at a given angle. If this angle is one-third of a right angle the measurements will be suitable. Let BG be the surface of the mirror, and let BD be perpendicular to AB. D, the point on BD at which the eye is, is so situated that a perpendicular drawn from it to BG falls outside BG. Let this perpendicular be ED. Draw DG to the end G of the mirror and let  $\angle EGD = \angle BGH$ . If then, a visual ray from the eye D falls on G, the end of the mirror, it will be reflected to H. Now let HN be drawn from H at right angles to DB. Now let  $DT^2$  be another incident ray and draw HT.

 $\therefore \angle BTH > \angle ETD,^3$ d  $\angle BTK = \angle GTD$ .

 $\therefore$  TK intersects HN, as do all rays incident upon the mirror when reflected.

<sup>&</sup>lt;sup>1</sup> The material within [ ] translates Schmidt's restoration of a lacuna assumed here.

A similar proof is given for the case of convex mirrors (Prop. VIII).

A similar proof is given for the case of convex mirrors (riop. 411).

3 I.e., the effect of surprising the observer. The reference may, however, be to a combination of mirrors having the same purpose as that described in the previous chapter (14), the speculum theatrale.

<sup>1</sup> K2, in the figure at the right.

<sup>2</sup> T being any point on BG.

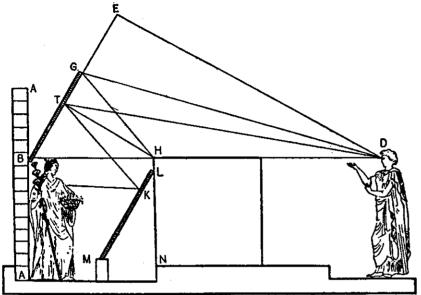
<sup>&</sup>lt;sup>3</sup> For  $\angle BTH > \angle BGH$ ,

and  $\angle ETD < \angle EGD$ .

But  $\angle BGH = \angle EGD$ .

 $<sup>\</sup>therefore \angle BTH > \angle ETD$ 

Now let a plane [mirror] LM be drawn parallel to mirror GB and intersected by a ray reflected from that mirror. Clearly, then, the eye will see only that which lies within HN, since all the reflected rays fall within HN.



Therefore, if we place whatever object we wish near plane LM, those approaching will see not their own image but merely that of the aforesaid object. It will consequently be necessary, as we have said, to place LM within HN so that the object in question may be between the parallel plane mirrors<sup>1</sup>...

# EXPERIMENTAL CONFIRMATION OF LAWS OF REFLECTION Prolemy (?), Optics III, pp. 60.23-64.28 (Govi) <sup>2</sup>

Now in seeking knowledge in any field we must start with certain general principles, and must make assumptions which are definite and selfevident either from the point of view of their practical effect or of their internal consistency. Only from such assumptions may the subsequent demonstrations be derived.

Now the basic principles required for the study of mirrors are three in number, and they are matters of primary knowledge, knowable in and of themselves. They are as follows: (1) objects seen in mirrors are seen in the direction of the visual ray which is reflected from the mirror to the object, depending on the position of the eye; (2) images in mirrors appear to be on the perpendicular drawn from the object to the surface of the mirror, and produced; (3) the position of the reflected ray, from the eye to the mirror and from the mirror to the object, is such that each of the two parts contains the point of reflection and makes equal angles with the perpendicular to the mirror at that point.<sup>1</sup>

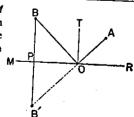
Now in the case of spherical mirrors what is meant by the perpendicular to the surface at a given point is the line perpendicular at the given point to the plane containing all the lines tangent to the sphere at that point. Hence all perpendiculars to the surface of a sphere must, when produced, pass through the center of the sphere.

The truth of the elementary principles which we have set forth is corroborated by the actual phenomena, as we shall now explain. For in the case of all mirrors we find that if we mark the points on the surface through which images are seen, and cover these points, the images will no longer be visible. When, however, we uncover the points successively and direct our vision toward these uncovered points, both the points and the images in question will be seen in the direction of the visual ray.

Again, if we place long, straight objects at right angles to the surfaces of mirrors and take a position some distance off, both the images of the objects and the objects themselves as actually seen outside the mirror will appear to form a single straight line.<sup>3</sup>

From both these circumstances it follows that the image of an object must appear in the mirror at the intersection of the visual ray and the perpendicular from the object to the mirror. Now these lines lie in the same plane, since they intersect. Again, this plane is perpendicular to the surface of the mirror, since one of the aforesaid lines is perpendicular to that surface. Finally, the visual ray, since it is reflected to the visible object, is in the aforesaid plane, and the perpendicular to the surface of the mirror at the point of reflection is the common boundary of all planes

<sup>&</sup>lt;sup>4</sup> Both, of course, produced. The visual ray referred to is that which, in the first instance, lies along the straight line from eye to image.



<sup>&</sup>lt;sup>1</sup> Reading, as Schmidt suggests, inter plana equidistantia specula. The rest of the paragraph, which is not given here, deals with the placing of the mirrors in a temple so that the apparition may be seen by one approaching. The figure shows the apparatus. Temple magic is also an important motive in Hero's Pneumatics (see pp. 327-329).

<sup>&</sup>lt;sup>2</sup> On this work see p. 271.

With reference to the figure where MR is the mirror, A the eye, B the object, B' the image, O the point at which the visual ray strikes the mirror, and TO perpendicular to the mirror, the assumptions are: (1) B' lies on AO, (2) B' lies on BP, perpendicular to MR, (3)  $\angle TOA = \angle TOB$ .

<sup>&</sup>lt;sup>2</sup> This is a confirmation of the first assumption.

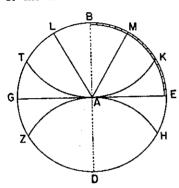
This is a confirmation of the second assumption.

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containing incident visual rays to the point in question and the corresponding reflected rays. . . . 1

But this will become clearer and still more obvious and its truth will be amply demonstrated by the following experiment.

Take a round copper disk of moderate size, such as the one illustrated, with center at A and both surfaces as even as possible. Have the edges of the circumference well rounded and smoothed. Draw a small circle



BGDE about center A on one surface of the disk, and draw two diameters, BD and GE, intersecting at right angles. Divide each quadrant of the circle into ninety equal parts. With B and D, respectively. as centers, and BA and DA as radii, draw ZAH and TAK, arcs of two circles. Now take three small, thin bars of iron, squared off and straight. Let one bar remain straight, and smooth one of its sides, making of it a polished mirror. Let the other two bars be curved so that a surface of one is

convex and of the other concave over an arc of a circle equal to circle BGDE. Polish these two surfaces of these bars so that they may act as two mirrors.

Now take arcs ZAH and TAK on each of the two curved bars, respectively, draw BA in white and AL in some other color, and set up a small dioptra2 upon AL, placing the disk in such a way that the line of sight of the dioptra coincides with AL passing through L. . . . Place the plane mirror on GAE, the convex mirror on ZAH, and the concave mirror on TAK. Place at the common midpoint of the upper edge of the three mirrors a knob protruding from the disk so as to mark its position over point A.

Now if we place one eye at the dioptra at L, on line AL, and look toward the point at which the axis meets the mirrors, and if we then pass some small colored object over the surface of the disk, moving it until it appears to us on the other side of point A, which is on the line of vision, then points L and A and the image of the object in the three mirrors<sup>3</sup> will appear to us on a single straight line. If, then, we mark the position of the object on the surface of the disk, that is, the point from which the image of the object is produced in the mirrors, say point M, and draw AM, we shall find that arc BM is always equal to arc BL. Angle LAB will, consequently, be equal to angle MAB, and BD will be perpendicular to all the mirrors.

Furthermore, AL is the path of the ray from the eye incident upon the surface of the mirror, and AM the path of the ray reflected from the mirror's surface to the object. Again, if we place an object of moderate length at R and place the eye on line AB, produced, the whole will appear upon a single straight line, AD.

The truth of the principles which we have assumed is, then, evident from our illustrations, and it may readily be seen that in these cases our reasoning accords with the evidence of our senses. Now it is the nature of a visual ray to proceed in a straight line from its source to all objects which are seen directly. A reflected ray, however, which proceeds from a mirror is not, in general, collinear with the visual ray. Our senses, therefore, must have recourse to an action which is natural and customary, and so we join the reflected ray to the first part of the visual ray, the part before reflection. Thus we have the impression that both parts constitute one straight ray, as if that were actually the case and nothing had happened to the ray. Hence the image of the object will be seen as if it were an object in the direct line of sight.

# REFRACTION

# AN INVESTIGATION OF REFRACTION

Ptolemy(?), Optics V, pp. 142.1-150.5 (Govi)1

Visual rays may be altered in two ways: (1) by reflection, i.e., the rebound from objects, called mirrors, which do not permit of penetration [by the visual ray], and (2) by bending [i.e., refraction] in the case of media

The original work seems to have been in five books. The first two dealt with the general theory of vision, the third and fourth with the theory of mirrors, and the fifth with the subject of refraction. Great interest attaches to this last book, for here is set forth the experimental procedure for the measurement of angles of refraction corresponding to given angles of incidence for a visual ray passing (a) from air to water, (b) from air to glass, and (c) from water to glass. (The incident ray, it is to be recalled, was treated in Greek geometrical optics as that which passes from the eye toward the reflecting or the refracting surface.) The figures obtained experimentally have evidently been corrected by the author to correspond to a set

<sup>1</sup> What may be meant is that the said perpendicular is the boundary between all planes containing the incident rays and all planes containing the reflected rays.

<sup>\*</sup> A simple sighting tube is meant.

<sup>\*</sup> Are we to understand that the plane mirror stood higher than the concave and the convex higher than the plane, to prevent interference, and that the object was sufficiently tall to appear in all three mirrors so arranged? Or was it intended that the mirrors be used consecutively and the place of the image noted in each case? The experiment is an empirical test of the equality of the angles of incidence and reflection.

<sup>1</sup> Claudius Ptolemy, to whose work reference has been made (p. 162), was the author of a treatise on optics which is no longer extant in its original form. There is, however, some evidence that an extant Latin translation of a lost Arabic version of a Greek work on optics is to be identified with the otherwise lost Optics of Ptolemy. A. Lejeune, L'Optique de Claude Ptolemée (1956) argues persuasively for this identification.

which permit of penetration and have a common designation ["transparent"] for the reason that the visual ray penetrates them.

Now in the preceding books we dealt with mirrors, and explained so far as was possible according to the aforementioned principles of the science of optics, the relation of images and objects, and how the object in each case comes to produce the particular image. It remains for us to consider here the illusions involved when the objects are seen in media through which sight penetrates.

Now it has been shown in what precedes: (1) that this type of bending of a visual ray does not take place in all liquids and rare media, but that a definite amount of bending takes place only in the case of those media that have some likeness to the medium from which the visual ray originates, so that penetration may take place, (2) that a visual ray proceeds along a straight line and may be naturally bent only at a surface which forms a boundary between two media of different densities, (3) that the bending takes place not only in the passage from rarer and finer media to denser (as is the case in reflection) but also in the passage from a denser medium to a rarer, and (4) that this type of bending does not take place at equal angles but that the angles, as measured from the perpendicular, have a definite quantitative relationship.<sup>2</sup>

We must now consider the effect of particular increments in the angles under discussion. Let us, however, first point out what refraction has in common with reflection, namely, that in either case the image appears to

formula (note the constancy of the "second differences" in the tables below).

The extant Latin version, made in the twelfth century by Eugenio of Sicily, lacks the first book and part of the fifth, as did the copy of the Arabic version on which it was based. Moreover the Latin is in many places obscure and unintelligible. This is due not only to the barbaric style of the translator but to the number of steps by which the translation is removed from the original. Our English version is consequently to be considered in many places as tentative and merely suggestive.

<sup>1</sup> This very obscure passage seems to mean that the process of refraction (as opposed to that of reflection) requires two different media, but not so different that one of them does and the other does not allow a visual ray to penetrate.

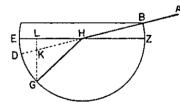
<sup>2</sup> Precisely what is meant is difficult to say because of the obscurity of the text. It is tempting, however, to interpret the passage as referring to a quantitative relation between angle of incidence and angle of refraction (the incident ray passing from the eye, and the angles being measured from the perpendicular upon the surface dividing the media). From the way in which the results of the observations seem to have been corrected, as the tables given on p. 278 indicate, the relation was of the form  $r = ai - bi^2$  (where i is the angle of incidence, r the angle of refraction, and a and b constants depending on the specific media). This does not accord with accurate modern observation as well as does the relation si / sin r = constant, but it is more accurate than the relation i/r = constant, which seems to have been widely adopted (see the tables, p. 278). Throughout the discussion it is to be remembered that the ray from the eye is considered the incident ray, in contrast with modern usage, in which the ray from the object is generally so named.

be on the prolongation of the straight line forming the initial portion of that ray which passes from the eye and is reflected or refracted to the object. That is, the image appears at the intersection of this ray (produced), which passes from the eye to the reflecting or refracting surface, and the perpendicular from the object to the same surface. It follows, therefore, in the case of refraction, just as in the case of reflection, that the plane passing through this altered ray is perpendicular to the surface at which the alteration takes place.<sup>2</sup>

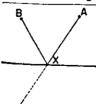
From this basis follow conclusions relevant to the nature of perception and involving a quantitative relation<sup>3</sup> as we have shown in the passage dealing with the principles of mirrors.

This is quite clear and obvious and we may understand it immediately with the help of a coin placed in a vessel called a baptistir. For suppose that the position of the eye is such that the visual ray emanating from it and just passing over the edge of the vessel reaches a point higher than the coin. Then, allowing the coin to remain in its position, pour water gently into the vessel until the ray which just passes over the edge is bent downward and falls upon the coin. The result is that objects not previously seen are then seen along the straight line passing from the eye to a point above the true position of the object. Now the observer will not suppose that the visual ray has been bent toward the objects but that the objects are themselves afloat and are raised toward the ray. The objects, therefore, will appear on the perpendicular drawn from them to the surface of the water, in accordance with the principles set forth above.

Thus [see fig.] if the eye be placed at point A... and the coin at point G in the lower part of the vessel, the coin will not be seen so long as the vessel is empty. The reason is that the portion of the vessel just below point B blocks the visual ray which could proceed directly to the coin.



But when enough water is poured into the vessel so that it stands at the



<sup>1</sup> That is, if A be the eye, B the object, B' the image, and X the point where the initial portion of the ray which is reflected to the object impinges on the reflecting or refracting surface, the image appears on a prolongation of AX.

<sup>2</sup> What seems to be meant is that the plane containing the visual and refracted rays is perpendicular to the refracting surface, just as, in the case of reflection, the plane containing the visual and reflected rays is perpendicular to the reflecting surface.

In the case of reflection, the equality of the angles of incidence

and reflection.

<sup>4</sup> The reference seems to be to the rule for the location of the image in the case, now, of refraction.

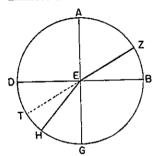
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level ZHE, the ray ABH is bent in the direction of GH, below the prolongation of AH. In that case, the position of the coin will appear to be on the perpendicular from G to EH, that is on LKG which meets AHD at K. The apparent position of the coin, then, will be on the straight line proceeding from the eye and produced so that it passes through K, which is above the line that the visual ray actually takes and nearer to the surface of the water. The coin will, therefore, be seen at point K.

The amount of refraction which takes place in water and which may be observed is determined by an experiment like that which we performed, with the aid of a copper disk, in examining the laws of mirrors.<sup>1</sup>

On this disk draw a circle [see fig.] ABGD with center at E and two diameters AEG and BED intersecting at right angles. Divide each quad-



rant into ninety equal parts and place over the center a very small colored marker. Then set the disk upright in a small basin and pour into the basin clear water in moderate amount so that the view is not obstructed. Let the surface of the disk, standing perpendicular to the surface of the water, be bisected by the latter, half the circle, and only half, that is, BGD, being entirely below the water. Let diameter AEG be perpendicular to the surface of the water.

Now take a measured arc, say AZ, from point A, in one of the two quadrants of the disk which are above the water level. Place over Z a small colored marker. With one eye take sightings until the markers at Z and at E both appear on a straight line proceeding from the eye. At the same time move a small, thin rod along the arc, GD, of the opposite quadrant, which is under the water, until the extremity of the rod appears at that point of the arc which is on a prolongation of the line joining the points Z and E.

Now if we measure the arc between point G and the point H, at which the rod appears on the aforesaid line, we shall find that this arc, GH, will always be smaller than arc AZ. Furthermore, when we draw ZE and EH, angle AEZ will always be greater than angle GEH. But this is possible only if there is a bending, that is, if ray ZE is bent toward H, according to the amount by which one of the opposite angles exceeds the other.

If, now, we place the eye along the perpendicular AE the visual ray will not be bent but will fall upon G, opposite A and in the same straight line as AE.

In all other positions, however, as arc AZ is increased, arc GH is also

increased, but the amount of the bending of the ray will also be progressively greater.1

When AZ is 10°, GH will be about 8°

" " 20°, " " " 15½°

" " 30°, " " " 22½°

" " 40°, " " " 29°

" " 450°, " " " 35°

" " 460°, " " " 40½°

" " 70°, " " " 45½°

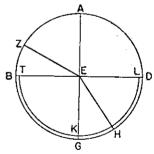
" " 80°, " " " 50°...²

This is the method by which we have discovered the amount of refraction in the case of water. We have not found any perceptible difference in this respect between waters of different densities.

Now if we make our observation from the relatively dense natural water to the rarer medium, there will be considerable difference in the amount of refraction, corresponding to various increments in the angle of incidence, in the passage of the ray from the denser medium, water, to the rarer. But since it is impossible for us to determine, by an experiment such as that just now described, the amount of refraction which takes place when a visual ray proceeds from a denser to a rarer medium, we have applied the following method of measuring the angles.

Construct a semicylinder of pure glass similar to half the circular disk.

Let the base of this semicylinder take the position TKL [see fig.] and let its diameter be smaller than that of the aforesaid metal disk. Fit the base of the semicylinder to the disk so that the whole base is fastened to the disk, the common center is at E, the diameter TL lies along diameter BD, and AE is perpendicular to the plane side of the glass surface. Hence all lines drawn from E to arcs BGD and TKL will be perpendicular thereto.



Now we arrange this experiment as we did the preceding experiment, placing a small marker on the glass just over E, the midpoint of the straight edge of the base of the semicylinder. We then look, with one eye, along line AE toward the edge of the glass, and keep moving an object along that part of the circumference opposite that from which we are observing, until it appears before our eye. Now this object will be found to be over point G, for AEG is perpendicular both to TEL and TKL.

<sup>&</sup>lt;sup>1</sup> See p. 270, above.

<sup>&</sup>lt;sup>2</sup> The amount of bending depends on the law connecting  $\angle$  AEZ and  $\angle$  GEH, angles of incidence and refraction, respectively.

<sup>&</sup>lt;sup>1</sup> That is, GH increases but at a progressively slower rate than AZ,

<sup>&</sup>lt;sup>2</sup> See the comparisons in Table A, From Air to Water, below. Note that r, the angle of refraction, is given by the equation  $r = ai - bi^2$ , where a = 0.825, b = 0.0025, and i is the angle of incidence (the incident ray being that which passes from the eye to the refracting surface: modern usage generally calls this the angle of refraction).

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Again, if we shift our viewpoint to a point opposite A and look in the direction of GE, an object moved over the opposite circumference of the disk will come into view on a prolongation of GE, that is, above EA. For the same reason as before, there will be no bending of the visual ray in its passage from glass to air.

But now take a certain arc measured from A, say arc AZ, draw ZE, coloring it black, and direct the vision along this line until an object moved back and forth behind the glass is seen in the direction of that line. If we place a marker at the point, H, reached when EH appears collinear with the black line ZE, we shall again in this case find that angle AEZ is larger than angle GEH. Moreover we shall find the excess of the one angle over the other greater than in the case of water, for given angles of incidence.

And again, when the eye is at point H on the other side of E and looks from H in the direction of HE, both points will appear to be on precisely the same line as in the preceding case. But since there was a bending of the ray at point E, it follows that whether the ray proceeded from the air to the glass, as did ZE, and was bent along EH, or proceeded from the glass to the air, as did HE, and was bent along EZ, in either case there was a bending in the direction of T. And since the perpendiculars which are drawn from E to are TKL are all similar, they are not bent, whether the rays which they represent are considered as beginning or as ending at E.<sup>3</sup>

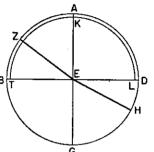
Now if in this case, too, we wish to find the amount of the refraction in each position, we place the eye successively in each of the positions taken in the former experiment [where the visual ray passed from air to water] and thus we vary the angle made on the disk<sup>4</sup> between the perpendicular AE and the visual ray EZ. The results are as follows:

When ∠AEZ is 10°, ∠GEH is approximately 7°
" "20°, " "13½°
" "30°, " "19½°
" "40°, " "25°
" "50°, " "30°
" "60°, " "34½°

" " 60°, " " 34½° " " 70°, " " 38½° " " 80°. " " 42° But the amount of refraction will be less when the glass is placed next to water, since the difference between angles of incidence and refraction in the passage of a visual ray from one of these bodies to the other is not large. For the difference in density between water and glass is less than that between air and water or between air and glass. But we are able again in this case to determine the amount of refraction, as we shall now explain.

Attach a semicylinder of glass [see fig.] to the bronze disk and adjust it so that the center of the straight edge is the same as that of the

disk. Again color point E, and set up the bronze disk in a basin so that the disk is at right angles to the surface of the water and half under the water. Place the curved side of the glass, TKL, above, and pour into the basin an amount of water so that edge TEL of the cylinder will be just above the surface of the water.



Now take arc GH in the less dense medium, that is, in the water, containing, say, 10°.

Mark H with a small colored marker, and sight it with one eye until an object Z, which is being moved over arc AB, is seen along the line joining H with the marked point E. When this has been done, draw the two lines EH and EZ.

If, then, we wish to measure on arc AB the angle subtended in the denser medium, that is, in the glass, as the angle in the water measured from the perpendicular, that is, angle GEH, varies, we shall find the following results:

When	∠GEH	is	10°,	<i>LAEZ</i>	is	approximately 91/2°
66	**	"	20°,	46	**	18½°
44	£¢.	"	30°,	66	"	27°
44			40°,	"	"	35°
66	44	"	50°,	**	"	421⁄3°
**	**	"	60°,	46	"	491/5°
**	"	"	70°,	**	"	56°
**	**	"	80°,	**	"	62° 1

<sup>&</sup>lt;sup>1</sup> That is,  $r = ai - bi^2$ , where a = 0.975 and b = 0.0025 (Table C, below).

<sup>&</sup>lt;sup>1</sup> I.e., there is no refraction when the visual ray is perpendicular to the boundary between the media. Thus there is no refraction in the light from a star at the zenith (see p. 282).

<sup>&</sup>lt;sup>2</sup> I.e., the excess of the angle of incidence over the angle of refraction is greater in the passage of the visual ray from air to glass than for the passage from air to water.

The point seems to be that so long as the ray is within the same medium, glass, there is no bending.

<sup>&#</sup>x27;Possibly, "in the air."

<sup>&</sup>lt;sup>5</sup> See the comparisons in Table B, From Air to Glass. Note that the angle of refraction is given by the equation  $r = ai - bi^2$ , where a = 0.725 and b = 0.0025.

The following tables will serve to compare the theory of refraction as given in our text with that in which the ratio of the angles of incidence and refraction is a constant, and with the modern theory, in which the ratio of the sines of the angles is a constant. See P. Brunet and A. Mieli, *Histoire des sciences: antiquité*, pp. 826-827.

It may be noted that the correction of experimental data to correspond to a set formula is discussed by Kepler in connection with Witelo's (thirteenth century) tables of refraction, which are much the same as those in the present text. Kepler contented himself with the formula r = ki for relatively small angles of incidence. The relation between refraction and

#### SUMMARY OF THE LAWS OF REFRACTION

Ptolemy(?), Optics V, pp. 154.1-156.6 (Govi)

Now it is possible, on the basis of our investigations, to draw general conclusions about this type of refraction. Thus, near the point at which the velocity of light in different media (which leads to the sine law) was not explicitly stated until the seventeenth century.

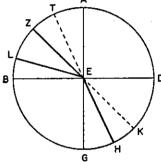
Angles of	Angles of refraction (r),	First	Second	i ir	sin i:sin r
incidence (i)	according to text	differences	differences	(on the basis of the angles in the first two columns)	
	·····	A. From Ai	r to Water		
0°	0°	80			
10°	8°	7°30′	30′	1.25	1.248
20°	15°30′	7°30	30'	1.29	1.270
30°	22°30′	•	30′	1.33	1.308
40°	29°	6°30′ 6°	30'	1.38	1.369
50°	35°	*	30′	1.43	1.336
60°	40°30′	5°30′	30′	1.48	1.333
70°	45°30′	5°	30'	1.55	1.329
80°	50°	4°30′	<b>!</b> .	1.60	1.286
		B. From A	ir to Glass		
0.	O°	7°			
10°	7°	6°30′	30,	1.43	1.425
20⁰	13°30′		30′	1.48	1.465
30°	19°30′	6°	30′	1.54	1.498
40°	25°	5°30′ 5°	30′	1.60	1.521
50°	30°	4°30′	30′	1.67	1.531
60°	34°30′		30′	1.74	1.529
70°	38°30′	40	30′	1.82	1.509
80°	42°	3°30′		1.91	1.472
		C. From Wa	ter to Glass		
0°	0°	9°30′			
10°	9°30′	90	30′	1.07	1.052
20°	18°30′	8°30′	30′	1.09	1.078
30°	27°	, 8°30	30′	1.11	1.101
40°	35*	7°30′	30'	1.14	1.121
50°	42°30′	7-30	30′	1.18	1.134
60°	49°30′	. /- 6°30'	30′	1.22	1.139
70°	56°	6°	30'	1.25	1.133
80°	62°	0-		1.28	1.115

the visual ray bends and at which a perpendicular drawn from any external point reaches the surface that bounds the two aforesaid dissimilar bodies, objects in the denser medium appear larger than they do in the rarer medium (the same position in each medium being preserved). The visual ray passes in this case from the rarer to the denser medium. The opposite will be the case when the passage of the visual ray is from the denser to the rarer medium.<sup>2</sup>

Our proposition is that the *amount* of the refraction is the same in each of the two types of passage but that the two refractions differ in type. For in its passage from a rarer to a denser medium the ray inclines toward the perpendicular, whereas in its passage from a denser to a rarer medium it inclines away from the perpendicular.

For consider a plate such as we have previously described, with diameter BD [see fig.] lying on the surface which divides two dissimilar media.

Draw the perpendicular AEG. Let a ray inclined to the perpendicular, for example EH, make  $\angle GEH$  with the perpendicular. Now the position of the refracted ray remains exactly the same when the visual ray passes through point  $E^4$  and the position of B the eye is at point Z. For the line beyond the point of refraction, that is EK, inclines, in its course, toward the perpendicular, whereas the visible object appears to be along the straight line (EK). Again, if the



eye is at point H, and EZ is in the rarer medium (BAD), line ET (beyond the point of refraction) will incline away from perpendicular AE, an action exactly the opposite of that in the former case. That is, the ray is further from the perpendicular than if it were to proceed in a straight line.

Again when the media differ considerably (in density), so do the angles (of incidence and refraction), and the difference in the angles becomes greater as the density of (the denser) one of the media is increased. For if we assume that semicircle BAD is in a rarer and BGD in a denser medium, and take angle AEZ as it is [see fig.], then, if the medium of section

<sup>&</sup>lt;sup>1</sup> The boundary between the air and the aether had been referred to, but the sequel considers any two media of different densities.

<sup>&</sup>lt;sup>2</sup> The point seems to be that if the eye and the object exchange positions the object (now in the rarer medium) appears smaller than it did in the denser medium.

<sup>\*</sup>Here we have the general statement of the reversibility of refraction. The experimental evidence is set forth in the following paragraphs.

I.e., originates further back than E.

 $<sup>^{5}</sup>$  Obviously what is meant is that the visual ray, which if unbent would lie along EK, is bent toward the perpendicular.

<sup>&</sup>lt;sup>6</sup> I.e., the ray, which if unbent would lie along ET, is bent away from the perpendicular-

BGD is made denser, the excess of  $\angle AEZ$  over  $\angle GEH$  will vary with the excess of the density of the new medium over the old. For example, when  $\angle AEZ$  in air is a third of a right angle,  $\angle GEH$  will, in water, be about a fourth part of a right angle, and, in glass a fifth part of a right angle plus a sixtieth thereof, approximately. In this latter case the amount of the refraction and the excess of the angle of incidence over the angle of refraction will, as the former approaches 90°, be greater, since glass is a denser substance than water.

In the same way, if we take the path of one of the refracted visual rays, say LEK, other than that of perpendicular AE,

and, alternando, AL:AZ > GK:HGAl:GK > AZ:GH. Again, separando, LZ:AZ > KH:HGand LZ:KH > AZ:GH.

Now it is possible for us to understand these various points from a quantitative study of the refractions we have investigated, if we assume certain numbers,<sup>3</sup> and, with their help take up the several changes indicated on the basis of such initial assumptions, as we did in the case of the two arcs AZ and AL.

But some one in opposition to this may ask why it is that in the first principles set forth—i.e., about the perpendiculars, and the appearance of the image in the direction of the visual ray—there is a similarity between the type of bending just discussed [i.e., refraction] and reflection, as it takes place in mirrors, but there is no such similarity in the measure of

$$\begin{array}{c} i_2 \colon i_1 > r_2 \colon r_1, \\ i_2 \colon r_2 > i_1 \colon r_1, \\ (i_2 - i_1) \colon i_1 > (r_2 - r_1) \colon r_1, \\ (i_2 - i_1) \colon (r_2 - r_1) > i_1 \colon r_1. \end{array}$$

and

But  $i_2$ :  $i_1 > r_2$ :  $r_1$  (from which the other results follow immediately) is valid only in the case where the visual ray from the eye passes from the less dense to the denser medium (the angle of incidence being the angle made by the line of vision with the perpendicular), as in all the cases given by the author. Since we, however, consider the light as passing from the object to the eye, and take the angle of incidence as that angle which the ray from the object makes with the perpendicular, it follows that, for us,  $i_2$ :  $i_1 > r_2$ :  $r_1$  only in the passage of light from the less refrangent to the more refrangent medium: otherwise,  $i_2$ :  $i_1 < r_2$ :  $r_1$ .

The point seems to be that the theoretical results set forth above may be better grasped if we assign specific values to  $i_2$  and  $i_1$ , and note the corresponding values of  $r_2$  and  $r_1$  (from tables such as those on p. 278).

the angles.<sup>1</sup> The answer, as well as the necessity that things be so, will be found in what we are to set forth. And from this will be seen something even more remarkable, namely, the operation of nature in conserving the activity of force.<sup>2</sup>

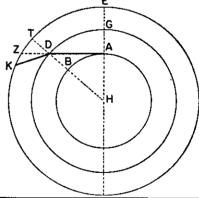
# ATMOSPHERIC REFRACTION

# Ptolemy(?), Optics V, pp. 151.1-153.20 (Govi)

Again, it is possible for us to see from the phenomena which I am about to discuss that at the boundary between air and aether there is a bending of the visual ray because of a difference between these two bodies. We find that stars which rise and set seem to incline more toward the north when they are near the horizon and are measured by the instrument used for such measurement.<sup>8</sup> For the circles, parallel to the equator, described upon these stars when they are rising or setting are nearer the north than the circles described upon them when they are in the middle of the heaven.<sup>4</sup> As they draw nearer the horizon they have a greater inclination toward the north. In the case of stars which do not rise and set, their distance from the north pole will be smaller when their position on the meridian is nearer the horizon. For when their position on the meridian is nearer the zenith, the circle parallel to the equator will, at that point, be larger, whereas it is smaller in the other position.<sup>5</sup> This is due to the bending of

the visual ray at the surface which divides the air from the aether, a spherical surface, of necessity, whose center is the common center of all the elements and of the earth.

Consider, then, in the first place point E [see fig.] as the zenith and the great circles of the various spheres which we have mentioned, circle AB on the surface of the earth, circle GD on the surface dividing air and aether, and EZ which passes through certain stars. Let the center of all the spheres



<sup>&</sup>lt;sup>1</sup> The angles of incidence and reflection are equal, but the angles of incidence and refraction are unequal.

Neither the method of measuring relative densities nor the precise way in which the angular differences vary with the differences in densities can be gathered from the example that follows, viz., when  $\angle AEZ = 30^{\circ}$ ,  $\angle GEH = 22\frac{1}{2}^{\circ}$  in water and  $19\frac{1}{2}^{\circ}$  in glass (see p. 278, above).

<sup>&</sup>lt;sup>2</sup> If  $i_1$  and  $i_2$  are angles of incidence (the incident visual ray being that which passes from the eye to the refracting surface) with  $i_2 > i_1$ , and  $r_1$  and  $r_2$  the corresponding angles of refraction, the preceding paragraph gave the result  $(i_2 - r_2) > (i_1 - r_1)$ . The present paragraph gives the results:

<sup>&</sup>lt;sup>2</sup> This statement of a principle of least action or conservation of energy is to be noted, though its application to the problem of refraction is not clearly made by the author in the portion of Book V that is extant.

<sup>&</sup>lt;sup>1</sup> The dioptra or the astrolabe (see p. 139).

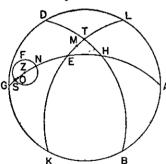
<sup>&</sup>lt;sup>4</sup> I.e., the small circle (parallel to the celestial equator) that represents the apparent path of a star in its diurnal rotation appears to have a greater declination when taken at the rising or setting of the star in question than, say, midway between the rising and setting.

<sup>&</sup>lt;sup>8</sup> I.e., the apparent distance from a circumpolar star to the pole is greater at its upper than at its lower transit across the meridian.

be H. Draw EAH, take point A as the position of the eye, and line ADZ to meet the boundary common to the horizon and to circle GD. Again, let DT be perpendicular to the circle, and consider ADK as a visual ray bent from point D along KD. Suppose there is a star at point K. Since the visual ray is bent at the surface, toward a position more remote from point E, the angle E0 (which is in the rarer medium) formed with the line E1 perpendicular to the bounding surface from which a reflection would be at equal angles, is greater E1. The stars will therefore be seen from point E2 along line E3 and the distance of the star from the zenith will appear less than the true distance. For its distance will appear to be arc E2 instead of arc E3.

The higher, then, the position of the star in the heavens, the smaller will be the difference between its apparent and its real position. If the star is at E there is no bending, since the visual ray from point A to point E is not subject to such bending (refraction), for it is perpendicular to the bounding surface at which any refraction would take place.

These, then, are the preliminary propositions. Let us now consider ABG [see fig.] as the circle of the horizon, and AEZG as that half of the circle of the meridian which is above the earth, E being the zenith and E the visible pole of the celestial sphere. Let E be the portion above



earth of the circle parallel to the celestial equator and passing through certain stars. Let there be a star at point T on this circle near the horizon, and let KETL be the visible half of the circle which passes through the zenith and through the star at T. Since, then, when a star is near the horizon it appears nearer the zenith than it really is, and its apparent position differs from its true position on the great circle [passing through the zenith and the

star in question and] intersecting the horizon, it follows that the apparent position of the star, which is above point T, will be between E and T, let us say at point M. Then the circle parallel to the celestial equator and passing through M will lie further to the north than the circle (parallel to the celestial equator) passing through point T, which, in our part of the inhabited world, lies toward the north. When the star has risen to point H it is in a position where the bending of the visual ray is insufficient to cause a perceptible difference between the apparent and the true position.

Similarly, take Z as the north celestial pole, and let NSF be the circle parallel to the celestial equator, described by one of the stars which are always above the horizon. When the star is at point S of this circle it will appear nearer point E, the zenith, and will seem to be at about point O. But when the star is at point N there is no difference, or only an imperceptible difference, between the apparent and the real position. And therefore when such a star, in its revolution, is nearer the horizon, its distance from the north pole of the celestial sphere will seem less than the distance from the aforesaid pole when the star in its revolution is nearer the zenith. For arc ZN will be greater than arc OZ. We have therefore seen why stellar phenomena must, because of the bending of visual rays, appear as they do. I.

# Cf. Ptolemy, Almagest, p. 13.3-9 (Heiberg)

Now the fact that heavenly bodies appear larger when they are near the horizon is due not to their smaller distance from the horizon but to the vaporous moisture surrounding the earth between our eye and these heavenly bodies.<sup>2</sup> It is the same as when objects immersed in water appear larger and in fact the more deeply immersed the larger.

# Cf. Cleomedes, On the Circular Motion of the Heavenly Bodies II. 1, pp. 122.15-124.8 (Ziegler)

The sun appears larger to us as it rises and sets, but smaller when it is in mid-heaven, for the reason that in viewing it at the horizon we see it through a thicker layer of air and also one that is more humid, for such is the air next to the earth. But when we view it in mid-heaven, we see it through clearer air. Thus in the latter case the ray issuing from the eye toward the sun is not refracted, but in the former case, where the ray is directed toward the horizon when the sun rises or sets, the ray is necessarily refracted when it encounters the thicker and moister air. Thus the sun appears larger to us, just as the appearance of objects in water is altered because they are not seen in a straight line. Such phenomena are in every case to be considered as due to disturbances of our vision and not connected with the visible objects themselves. It is also said that where it is possible to view the sun from deep wells, its appearance is much larger since it is seen through the humid air of the well. It certainly cannot be said that the sun grows larger when viewed from the bottom of a well and smaller when viewed from the top; but evidently the dimness and dampness of the air in the well cause the sun to appear larger to the observer.

<sup>1</sup> l.e., the surface marking the boundary between aether and air.

<sup>&</sup>lt;sup>2</sup> The text reads E, which can hardly be right.

<sup>&</sup>lt;sup>3</sup> The text reads "nearer the zenith than to its true position," which is obviously wrong.

<sup>&</sup>lt;sup>4</sup> At H the star is on the meridian, and its angular distance from the zenith is least.

<sup>&</sup>lt;sup>1</sup> The author goes on to indicate the impossibility of measuring astronomical refraction because of the absence of data as to the relative extent of the atmosphere and the aether.

<sup>&</sup>lt;sup>2</sup> The apparent increase of size of sun and moon at the horizon is largely due to an optical illusion, not to refraction (cf. also p. 123).

But by a refracted ray.

# A "PARADOXICAL" ECLIPSE EXPLAINED BY REFRACTION

Cleomedes, On the Circular Motion of the Heavenly Bodies II. 6.

Translation of T. L. Heath, Greek Astronomy<sup>1</sup>

These facts having been proved with regard to the moon, the argument establishing that the moon suffers eclipse through falling into the earth's shadow would seem to be contradicted by the stories told about a class of eclipses seemingly paradoxical. For some say that an eclipse [sometimes] occurs, even when both the luminaries are seen above the horizon.<sup>2</sup> This should make it clear that [in that case] the moon does not suffer eclipse through falling into the earth's shadow, but in some other way, since, if an eclipse occurs when both sun and moon appear above the horizon, the moon cannot suffer eclipse through falling into the earth's shadow. For the place where the moon is, when both bodies appear above the horizon, is still being lit up by the sun, and the shadow cannot yet be at the place where the moon gives the impression of being eclipsed. Accordingly, if this be the case, we shall be obliged to declare that the cause of the eclipse of the moon is a different one.<sup>3</sup>...

Nevertheless, having regard to the many and infinitely various conditions which naturally arise in the air, it would not be impossible that, when the sun has just set, and is under the horizon, we should receive the impression of its not yet having set, if there were cloud of considerable density at the place of setting and the cloud were illuminated by the sun's rays and transmitted to us an image of the sun, or if there were "anthelium." Such images are indeed often seen in the air, especially in the neighbourhood of Pontus. The ray, therefore, proceeding from the eye and meeting the air in a moist and damp condition might be bent, and so might catch the sun although just hidden by the horizon. Even in ordinary life we have observed something similar. For, if a gold ring be thrown into a drinking

cup or other vessel, then, when the vessel is empty, the object is not visible at a certain suitable distance, since the visual current goes right on in a straight line as it touches the brim of the vessel. But, when the vessel has been filled with water up to the level of the brim, the ring placed in the vessel is now, at the same distance, visible, since the visual current no longer passes straight on past the brim as before, but, as it touches, at the brim, the water which fills the vessel up to the brim, it is thereby bent, and so, passing to the bottom of the vessel, finds the ring there. Something similar, then, might possibly happen in a moist and thoroughly wet condition of the air, namely, that the visual ray should, by being bent, take a direction below the horizon, and there catch the sun just after its setting, and so receive the impression of the sun's being above the horizon. Perhaps, also, some other cause akin to this might sometimes give us the impression of the two bodies being above the horizon, though the sun had already set. But the observed phenomena make it as clear as day that the moon is not eclipsed otherwise than by falling within the earth's shadow.

# THE NATURE AND VELOCITY OF LIGHT

Aristotle, On the Soul II. 7.1 Translation of R. D. Hicks (Cambridge, 1907)

And so we shall have first to explain what light is.

There is, then, we assume, something transparent; and by this I mean that which, though visible, is not, properly speaking, visible in itself, but by reason of extrinsic colour. Air, water, and many solid bodies2 answer to this description. For they are not transparent quâ air or quâ water, but because there is a certain natural attribute present in both of them which is present also in the eternal body on high. Light is the actuality of this transparent qua transparent. But where the transparent is only potentially present, there darkness is actually. Light is a sort of colour in the transparent when made transparent in actuality by the agency of fire or something resembling the celestial body: for this body also has an attribute which is one and the same with that of fire. What the transparent is, and what light is, has now been stated; namely, that it is neither fire nor body generally nor an effluence from any body4 (for even then it would still be a sort of body), but the presence of fire or something fiery in the transparent. For it is impossible for two bodies to occupy the same space at the same time.

<sup>&</sup>lt;sup>1</sup> It is not known when Cleomedes lived. Some place him in the first century B.C., others in the first or second A.D. He was a compiler rather than an original scientist, and his work is important for the wealth of historical information that it contains. His treatise On the Circular Motion of the Heavenly Bodies, extant in two books, deals with various astronomical questions and is often controversial, reflecting the Stoic viewpoint against the Epicurean. A passage containing the account of the methods used by Eratosthenes and Posidonius in estimating the size of the earth has been quoted above (p. 149). The present passage discusses possible explanations of a "paradoxical" eclipse of the moon when both sun and moon appear above the horizon. [Edd.]

<sup>&</sup>lt;sup>2</sup> Pliny refers to an instance of such an eclipse (*Natural History* II. 57) as follows: "And [it was discovered by Hipparchus] why it is that, though after sunrise the eclipsing shadow must be below the earth, it has once happened that the moon was eclipsed in the west while both sun and moon were visible above the horizon."

Such an eclipse was visible in the vicinity of New York City on Nov. 7, 1938. [Edd.]

<sup>&</sup>lt;sup>3</sup> Cleomedes is skeptical about the reported observation, but considers various explanations before turning to the possibility of refraction as an explanation. [Edd.]

<sup>&</sup>lt;sup>1</sup> In connection with Aristotelian optical theory note also the discussion of the rainbow (Meteorologica III. 4-5) and the comments of T. E. Lones, Aristotle's Researches in Natural Science (London, 1912), pp. 36-42, and A. M. Sayili in Isis 30 (1939) 65-83. [Edd.]

E.g., glass. [Edd.]

I.e., not as air or as water, but as transparent. [Edd.]

<sup>&</sup>lt;sup>4</sup> Aristotle is here refuting the view of Empedocles and Plato that light is fire or like fire, and the view of Democritus that light is a corporeal emission from the surface of a body.

Light is held to be contrary to darkness. But darkness is absence from the transparent of the quality above described: so that plainly light is the presence of it. Thus Empedocles and others who propounded the same view are wrong when they represent light as moving in space and arriving at a given point of time between the earth and that which surrounds it without our perceiving its motion. For this contradicts not only the clear evidence of reason, but also the facts of observation: since, though a movement of light might elude observation within a short distance, that it should do so all the way from east to west is too much to assume.

Aristotle, De Sensu 6, 446a26-b3. Translation of J. I. Beare (Oxford, 1908)

Empedocles, for example, says that Light from the Sun arrives first in the intervening space before it comes to the eye, or reaches the Earth. This might plausibly seem to be the case. For whatever is moved [in space], is moved from one place to another; hence there must be a corresponding interval of time also in which it is moved from the one place to the other. But any given time is divisible into parts; so that we should assume a time when the sun's ray was not as yet seen, but was still travelling in the middle space.

#### Cf. Lucretius VI. 195-204

But we hear the thunder with our ears after our eyes perceive the flash of lightning, because things always reach our ears more slowly than they affect our vision. This may also be seen from the fact that if you perceive someone in the distance cutting down a tall tree with a double edged axe, you see him strike the blow before the sound of it comes to the ears. So also we see the lightning before we hear the thunder clap which arises at the same time and from the same cause as the lightning, being born of the same collision.

#### Cf. also Pliny, Natural History II. 142

It is certain that a lightning flash is seen before the thunder is heard, though they both take place at the same time. And this is not strange for light is swifter than sound.

#### ACOUSTICS AND MUSICAL THEORY

THE NATURE OF SOUND
Archytas, Frag. 1 (Diels) 2

"Now they [the mathematicians] observed in the first place that there can not be sound without the striking of bodies against one another....

Now as we are constituted many sounds cannot be heard by us, some because of the feebleness of the blow, others because of the distance from us. still others because of the intensity of the sound. For just as when one seeks to pour a great deal of water into a narrow-necked vessel and none of the water enters, so the very intense sounds fail to penetrate to our hearing. Of the sounds that we do perceive, those that reach us swiftly and violently from the blow seem high pitched, those that reach us slowly and weakly seem to be low pitched.1 For if one takes a stick and moves it slowly and feebly, the blows produce a low sound, whereas if one strikes swiftly and intensely, the blows produce a high sound. This we may observe not only in the circumstances just described but also when we wish to produce a loud and sharp sound in speaking or singing. In that case we make the sound by increasing the intensity of our breath. Now it is the same as with projectiles. Those that are hurled with greater force are carried far. those that are hurled weakly move only a short distance. For the air gives way more readily before those projectiles that are hurled with great force, but less readily before those hurled weakly. So it is with sounds. Those that are projected by an intense breath are loud and sharp, while those projected with weak breath are soft and low.2 And we can see this clearly with the help of a most compelling example. For when the same person makes a loud sound we can hear him from a considerable distance. but when he makes a soft sound we cannot hear him even at close range. And in the case of pipes, air blown from the mouth into holes near the mouth produces a sharper note because of the strong pressure. But if the air passes through the holes further from the mouth, the note is lower. Clearly, then, swift motion produces a high-pitched sound and slow motion a low-pitched sound.<sup>3</sup> Now the same thing happens in the case of the bull-roarers that are used in the celebration of the mysteries. Those that are moved slowly give out a low sound, those moved intensely give out a sharp sound. So it is also with the reed. If one blows into it, closing off the bottom, it gives us a low-pitched sound, but if one blows into it using

following passage from a no longer extant work of Archytas (see p. 35). The passage is interesting for its connection of sound with motion and its connection of pitch with speed of motion. That there is no clear idea, however, of wave propagation of sound and frequency of vibrations is seen from the statement that high-pitched sounds travel more swiftly than those of low pitch. On the relative velocity of light and sound see the preceding passages.

<sup>1</sup> Cf. Aristotle, De Sensu 448a19: "Some of those who treat of concords say that the sounds do not reach us at the same time, but only appear to do so."

Theophrastus in Porphyrius's Commentary on Ptolemy's Harmonics, p. 64.20 (Düring): "The higher note does not differ [from the lower] in speed, for if it did it would reach the hearing sooner, and there would be no concord. If there is a concord, both notes must have the same speed."

<sup>2</sup>There seems to be some confusion, in Archytas's account, between intensity and pitch.

<sup>3</sup>The inverse relation between the length of the vibrating column of air and the rate of

vibration is not very clearly put. Cf. the case of the reed pipe mentioned below.

<sup>&</sup>lt;sup>1</sup> Cf. p. 214, above. Having denied both the corporeality and the (spatial) motion of light, Aristotle rejects the notion of a finite velocity. Empedocles and the atomists had held, as we do, that light from the sun reached the intervening spaces between sun and earth before it reached the earth and our eyes. [Edd.]

<sup>2</sup> Porphyrius (third century A.D.) in his commentary on Ptolemy's Harmonics quotes the

# PIGMENTS AND DYES

Vitruvius, On Architecture VII. 10-12. Translation of M. H. Morgan

# Artificial Colours. Black

- 1. I shall now pass to those substances which by artificial treatment are made to change their composition, and to take on the properties of colours; and first I shall treat of black, the use of which is indispensable in many works, in order that the fixed technical methods for the preparation of that compound may be known.
- 2. A place is built like a Laconicum, and nicely finished in marble. smoothly polished. In front of it, a small furnace is constructed with vents into the Laconicum, and with a stokehole that can be very carefully closed to prevent the flames from escaping and being wasted. Resin is placed in the furnace. The force of the fire in burning it compels it to give out soot into the Laconicum through the vents, and the soot sticks to the walls and the curved vaulting. It is gathered from them, and some of it is mixed and worked with gum for use as writing ink, while the rest is mixed with size, and used on walls by fresco painters.1
- 3. But if these facilities are not at hand, we must meet the exigency as follows, so that the work may not be hindered by tedious delay. Burn shavings and splinters of pitch pine, and when they turn to charcoal, put them out, and pound them in a mortar with size. This will make a pretty black for fresco painting.
- 4. Again, if the lees of wine are dried and roasted in an oven, and then ground up with size and applied to a wall, the result will be a colour even more delightful than ordinary black; and the better the wine of which it is made, the better imitation it will give, not only of the colour of ordinary black, but even of that of India ink.

#### Blue. Burnt Ochre

1. Methods of making blue were first discovered in Alexandria, and afterwards Vestorius set up the making of it at Puzzuoli. The method of obtaining it from the substances of which it has been found to consist is strange enough. Sand and the flowers of natron are brayed together so finely that the product is like meal, and copper is grated by means of coarse files over the mixture, like sawdust, to form a conglomerate. Then it is made into balls by rolling it in the hands and thus bound together for drying. The dry balls are put in an earthen jar, and the jars in an oven.

As soon as the copper and the sand grow hot and unite under the intensity of the fire, they mutually receive each other's sweat, relinquishing their peculiar qualities, and having lost their properties through the intensity of the fire, they are reduced to a blue colour.

2. Burnt ochre, which is very serviceable in stucco work, is made as follows. A clod of good yellow ochre1 is heated to a glow on a fire. It is then quenched in vinegar, and the result is a purple colour.

# White Lead, Verdigris, and Artificial Sandarach

- 1. It is now in place to describe the preparation of white lead and of verdigris, which with us is called "aeruca." In Rhodes they put shavings in jars, pour vinegar over them, and lay pieces of lead on the shavings; then they cover the jars with lids to prevent evaporation. After a definite time they open them, and find that the pieces of lead have become white lead.2 In the same way they put in plates of copper and make verdigris,3 which is called "aeruca"
- 2. White lead on being heated in an oven changes its colour on the fire, and becomes sandarach.4 This was discovered as the result of an accidental fire. It is much more serviceable than the natural sandarach dug up in mines.

# Dioscorides, De Materia Medica V. 98 (Wellmann)

Blue vitriol (chalcanthum) is generically one and the same, for it is a liquid which has been solidified. But it appears in three different states. It appears as a concretion of liquids that filter drop by drop through the roofs of mines. It is for this reason called "stalacton" by those who work in the mines of Cyprus. Another type trickles down abundantly in caves, and is then led off into trenches where it solidifies. This is specifically called "solid" chalcanthum. The third type is called "boiled." This is the most colorless and the weakest kind. It is made in Spain and the method of preparing it is as follows. It is dissolved in water and cooked, then poured off into tanks and left to stand. After a definite number of days it solidifies, dividing into many cube-shaped particles, adhering to one another in clusters.5

The type that is heavy, blue, dense, clear, and transparent is considered

<sup>&</sup>lt;sup>1</sup> Any highly carbonaceous material burned in the type of large enclosed furnace described by Vitruvius would produce the required soot. [Edd.]

<sup>&</sup>lt;sup>1</sup>Probably hydrated ferric oxide. [Edd.]

<sup>&</sup>lt;sup>3</sup> May the wood shavings be the source of the requisite carbon dioxide, corresponding to the tanbark used in the so-called "Dutch process"? See p. 360, n. 1. [Edd.]

<sup>&</sup>lt;sup>1</sup> I.e., basic copper acetate. [Edd.]

<sup>4</sup> Sandarach is usually identified with realgar, but here the meaning seems to be red lead, which is of similar appearance. [Edd.]

This observation of crystallization is noteworthy.

the best. Of this type is "stalacton," called by others "lonchoton." Next best is the "solid" type. The "boiled" type seems to be more useful Next best is the "solid" type. The "boiled" type seems to be more useful than the others in the preparation of dyes, especially black.<sup>2</sup> Experience shows it to be quite ineffective in medicine.

Stockholm Papyrus, selections.<sup>3</sup> Translation of E. R. Caley, Journal of Chemical Education 4 (1927) 979-1002

17. Preparation of emerald.

# 17. Preparation of emerald.

Take and put so-called topaz4 stone in liquid alum and leave it there 3 days. Then remove it from this and put it in a small copper vessel in which you have placed pure unadulterated verdigris along with sharp vinegar. Put the cover upon the vessel, close up the cover, and gently keep a fire under the vessel with olive wood for 6 hours, otherwise the longer you maintain the fire, the better and deeper will the stone beonly, as I say, with a gentle fire. Cool and lift the stone out. Its condition will show whether it has become emerald. That is to say, you will observe that a green film has formed upon it. Let it become slowly cooled, however; if not, it soon breaks. Put oil in a small box-tree vessel a sufficient number of days beforehand so that the oil is purified and the product from it can be taken off. Put in the stone and leave it under cover 7 days. On taking out you will have an emerald which resembles the natural ones.5

18. Manufacture of a pearl.

Take and grind an easily pulverized stone such as window mica. Take

1 I.e., lance-shaped.

<sup>2</sup> Cf. Pliny, Natural History XXXIV. 123-127, on which K. C. Bailey writes:

"There can be no question that the compound whose properties were the origin of the name 'shoemakers' black' was green vitriol or ferrous sulphate (Fe SO<sub>4.7</sub>H<sub>2</sub>O).... On the other hand, the connection with copper is, as Pliny notes, indicated by the name chalcanthos. . . . Both Pliny and Dioscorides say that the best variety was blue, although there was a kind whose colour was lighter. The blue variety, in its pure form, was blue vitriol or cupric sulphate (CuSO4.5H2O), and no doubt many intermediate kinds were prepared, for ferrous and cupric sulphates will crystallize together from solution. The confusion has left its trace on chemical nomenclature, for ferrous sulphate is known to this day as 'copperas.' "

\* See p. 360, n. 3. [Edd.]

4 The interpretation is doubtful. E. O. von Lippmann has suggested that the reference is to a certain stone found in India (Chemiker-Zeitung 27 [1913] 963). [Edd.]

<sup>5</sup> The translator notes that in the imitation of precious stones, as practised in ancient Egypt, the base was so treated as to roughen it and make the surface porous. For this purpose oil, wax, alum, native soda, common salt, vinegar, calcium sulfide, or mixtures of these were used. After the stone was thus corroded a dye was applied. This general method was frequently used in the recipes of the Stockholm papyrus.

Here the stone is etched and green cupric acetate is absorbed into the pores. The green film that forms is quite impermanent, and though the treatment with oil makes the stone appear smoother, the resemblance to the natural gem cannot have been very striking. The method of annealing to prevent fracture of the stone is noteworthy. It is still in use. [Edd.] gum tragacanth and let it soften for ten days in cow's milk. When it has become soft, dissolve it until it becomes as thick as glue. Melt Tyrian wax: add to this the white of egg and mercury. The mercury should amount to 2 parts and the stone 3 parts, but all remaining substances 1 part each. Mix [the ground mica and the molten wax] and knead the mixture with mercury. Soften the paste in the gum solution and the contents of the hen's egg. Mix all of the liquids in this way with the paste. Then make the pearl that you intend to, according to a pattern. The paste very shortly turns to stone. Make deep round impressions and bore through it while it is moist. Let the pearl thus solidify and polish it highly. If managed properly it will excel the natural.1

# 19. Production of ruby.

The treating of crystal so that it appears like ruby. Take smoky crystal and make the ordinary stone from it. Take and heat it gradually in the dark; and indeed until it appears to you to have the heat within it. Heat it once more in gold-founder's waste. Take and dip the stone in cedar oil mixed with natural sulphur and leave it in the dye, for the purpose of absorption, until morning.2

# 24. Corroding of stones.

A corrosive for any stone. Equal amounts of alum and natron are boiled in an equal amount of water. The small stones are then etched. Previously warm them slightly near the fire and dip them in the corrosive. Do this for a while once to three times while the corrosive boils: dip and leave again three times but no more, so that the small stones do not break.

# 31. Boiling of stones.

If you wish to make ruby from crystal, which has been worked to any desired end, take and put it in the pan and stir up turpentine balsam and a little pulverized alkanet there until the dye liquid rises; and then take care of the stone.8

# 74. Preparation of verdigris for emerald.

Clean a well-made sheet of Cyprian copper by means of pumice stone and water, dry, and smear it very lightly with a very little oil. Roll it thin and tie a cord around it. Then hang it in a cask with sharp vinegar so that it does not touch the vinegar, and carefully close the cask so that no evaporation takes place. Now if you put it in in the morning, then

A white cement will form, but this should be quite distinguishable from a real pearl. besides being impermanent and easily disintegrated. [Edd.]

<sup>3</sup> A colloidal gold suspension reddish in color would probably form on the surface. but. unlike the real ruby, would be impermanent, opaque, and easily abraded. [Edd.]

The dye is dissolved in the balsam, and the balsam coats the crystal. This is essentially lacquering. [Edd.]

scrape off the verdigris carefully in the evening, but if you put it in in the evening, then scrape if off in the morning, and suspend it again until the sheet becomes used up. However, as often as you scrape it off again smear the sheet with oil as explained previously. The vinegar is [thus rendered] unfit for use.<sup>1</sup>

# 84. A dye liquor for 3 colors.

A dye liquor from which three dye solutions can come. Bruise and mix with water  $\frac{2}{3}$  of a part of krimnos and 1 part of dyer's alum. Put the wool in and it becomes scarlet red. If it is to be leek-green add ground sulphur with water. If, however, it is to be quince-yellow then add unadulterated natron along with water.

# 86. For purple.

Boil asphodel and natron, put the wool in it 8 drachmas at a time, and rinse it out. Then take and bruise 1 mina of grape skins, mix these with vinegar and let stand 6 hours. Then boil the mixture and put the wool in.

# 87. Mordanting.

Boil chalcanthum and scorpiurus and employ for any desired color. These substances, however, also mordant all kinds of stones and skins.<sup>2</sup>

# 88. Dissolving of alkanet.

Alkanet is dissolved by oil, water, and nuts. The best of all dissolving mediums is, however, camel's urine. For this makes the alkanet dye not only fast, but also durable.

# 89. Another [recipe].

Bruise alkanet and mix natron with it until it becomes the color of blood. The boiling is done with water. Then dye what you desire. Or else bruise alkanet in the same way with safflower; afterwards put it in and let the blood color be absorbed. And if you bruise alkanet with telis<sup>3</sup> then proceed likewise. Alkanet in company with chalcanthum, however, dyes linen as well as cambric. For with chalcanthum, alkanet red changes into purple.

90. Making purple brilliant.

To make purple brilliant cook alkanet with purging weed and this will dissolve it; or with wild cucumber, purgative cucumber, or hellebore.

93. Mordanting for Sardian purple.

For a mina of wool put in 4 minas of dross of iron and 1 choenix of sour pomegranate; but if not this latter, then use 1 chus of vinegar and 8 chus of water heated over the fire until half of the water has disappeared. Then take the fire away from under it, put the cleaned wool in and leave it there until the water becomes cold. Then take it out, rinse it and it will be mordanted.<sup>1</sup>

94. Mordanting for Sicilian purple.

Put in the kettle 8 chus of water, a half a mina of alum, 1 mina of flowers of copper<sup>2</sup> and 1 mina of gall-nuts. When it boils put in 1 mina of washed wool. When it has boiled two or three times take the wool out. For if you leave it in a longer time, the purple becomes red. Take the wool out, however, rinse it out, and you will have it mordanted.<sup>3</sup>

95. Mordanting and dyeing of genuine purple.

For a stater of wool put in a vessel 5 oboli of alum and 2 kotylae of water. Boil and let it become lukewarm. Leave it until early morning, then take it off and cool it. Then prepare a secondary mordant, putting 8 drachmas of pomegranate blossoms and two kotylae of water in a vessel. Let it boil and put the wool in. However, after you have dipped the wool in several times, lift it out. Add to the pomegranate blossom water about a ball of alumed archil and dye the wool by judging with the eye. If you wish, however, that the purple be dark, add a little chalcanthum and let the wool remain long in it. In another passage it is put in the following way: But if you wish that the purple be dark, then sprinkle natron and a little chalcanthum in the dye bath.

101. Cold dyeing of purple which is done in the true way.

Keep this as a secret because the purple has an extremely beautiful luster. Take scum of woad from the dyer, and a sufficient portion of foreign alkanet of about the same weight as the scum—the scum is very light—and triturate it in the mortar. Thus dissolve the alkanet by grinding in the scum and it will give off its essence. Then take the brilliant color prepared by the dyer—if from kermes it is better, or else from krimnos—heat, and put this liquor into half of the scum in the mortar. Then put

<sup>&</sup>lt;sup>1</sup> Here we have a clever and effective method for producing basic copper acetate (verdigris). The oil prevents atmospheric oxidation without hindering the reaction of the copper with the acetic acid (see p. 367). [Edd.]

<sup>&</sup>lt;sup>2</sup> The term "chalcanthum" was used to denote various products of the weathering of iron and copper pyrites and hence was either copper or iron sulfate or mixtures of these salts. The Greek word "scorpiurus" was, according to some, a name given to a sapindaceous plant. [A fairly good mordant is obtained. The iron sulfate will, on boiling, deposit ferric hydroxide in the fibers of the fabrics. This will mordant the dye. Edd.]

<sup>&</sup>lt;sup>3</sup> Probably our fenugreek. [Edd.]

<sup>&</sup>lt;sup>1</sup> An excellent recipe for mordanting. Though wool does not, in general, require a mordant, the method would be effective with other fabrics. [Edd.]

<sup>&</sup>lt;sup>2</sup> Impure cuprous oxide. [Edd.]

This and the following are good recipes for mordanting. [Edd.]

the wool in and color it unmordanted and you will find it beyond all description.1

105. Dyeing in dark blue.

Put about a talent of woad in a tube, which stands in the sun and contains not less than 15 metretes, and pack it in well. Then pour urine in until the liquid rises over the woad and let it be warmed by the sun; but on the following day get the woad ready in such a way that you can tread around in it in the sun until it becomes well moistened. One must do this, however, for 3 days together.<sup>2</sup>

110. Dyeing in bright red purple.

To dye in genuine bright red purple grind archil and take 5 cyathi of the juice for a mina of wool. If you wish a bright tint, mix in ground natron; if you desire a still brighter one, chalcanthum.<sup>3</sup>

111. From the book of Africanus: Preparation of bright red purple.

Take and put the mordanted wool into 1 choenix of krimnos and 4 choenices of archil. Boil these materials, put the wool in, and leave it there until later. Take it out and rinse it with salt water, then with fresh water.

133. Preparing genuine purples.

Iron rust, roasted misy, and pomegranate blossom adapt themselves to mordanting in water and make it possible to give the wool a good deep purple color in 4 hours.<sup>5</sup>

139. Dyeing of colors.

Celandine is a plant root. It dyes a gold color by cold dyeing. Celandine is costly, however. You should accordingly use the root of the pomegranate tree and it will act the same. And if wolf's-milk is boiled and dried it produces yellow. If, however, a little verdigris is mixed with it, it produces green; and safflower blossoms likewise.

153. Making of madder-purple.

After bluing, sprinkle the wool with ashes and trample it down with them in a convenient manner. Then press the liquid out of potter's clay and wash off the blued wool therein. Rinse it in salt water and mordant

1 Dyeing will probably take place in this case. [Edd.]

This is simply a case of direct dyeing. The dye does not appear to be fast, according to our standards. [Edd.]

Direct dyeing, as in the preceding recipe, with the use of a mordant. [Edd.]

it. You will know if it is sufficiently mordanted when it sinks down in the kettle and the fluid becomes clear. Then heat rain water so that you cannot put your hand in it. Mix roasted, pulverized and sifted madder root, i.e., madder, with white vinegar, a half a mina of madder to a mina of wool, and mix a quarter of a choenix of bean meal with the madder root. Then put these in a kettle and stir up. Then put the wool in; in doing so, stir incessantly and make it uniform. Take it out and rinse it in salt water. If you wish the color to take on a beautiful gloss and not to fade, then brighten it with alum. Rinse the wool out again in salt water; let it dry in the shade and in doing so protect it from smoke.

It appears that the solid woad is simply macerated in the urine mechanically, so that it is finely dispersed and consequently suitable for use in dyeing. [Edd.]

From rust will not dye wool. The misy may accomplish this result, and pomegranate blossoms probably will. [Edd.]

Another case of direct dyeing. [Edd.]

most insects may be banished with burnt hart's horn, or better still by the burning of the gum styrax. The cuttle-fish, the octopus, and the crawfish may be caught by bait. The octopus, in fact, clings so tightly to the rocks that it cannot be pulled off, but remains attached even when the knife is employed to sever it; and yet, if you apply fleabane to the creature, it drops off at the very smell of it.1 The facts are similar in regard to taste. For the food that insects go in quest of is of diverse kinds, and they do not all delight in the same flavours: for instance, the bee never settles on a withered or wilted flower, but on fresh and sweet ones; and the conops or gnat settles only on acrid substances and not on sweet. The sense of touch, by the way, as has been remarked, is common to all animals. Testaceans have the senses of smell and taste. With regard to their possession of the sense of smell, that is proved by the use of baits, e.g., in the case of the purple-fish: for this creature is enticed by baits of rancid meat, which it perceives and is attracted to from a great distance. The proof that it possesses a sense of taste hangs by the proof of its sense of smell; for whenever an animal is attracted to a thing by perceiving its smell, it is sure to like the taste of it. Further, all animals furnished with a mouth derive pleasure or pain from the touch of sapid juices.

With regard to sight and hearing, we cannot make statements with thorough confidence or on irrefutable evidence. However, the solen or razor-fish, if you make a noise, appears to burrow in the sand, and to hide himself deeper when he hears the approach of the iron rod<sup>2</sup> (for the animal, be it observed, juts a little out of its hole, while the greater part of the body remains within), and scallops if you present your finger near their open valves, close them tight again as though they could see what you were doing. Furthermore, when fishermen are laying bait for neritae, they always get to leeward of them, and never speak a word while so engaged, under the firm impression that the animal can smell and hear; and they assure us that, if any one speaks aloud, the creatures make efforts to escape. With regard to testaceans, of the walking or creeping species the urchin appears to have the least developed sense of smell; and, of the stationary species, the ascidian and the barnacle.

So much for the organs of sense in the general run of animals.

Aristotle, History of Animals I. 15 (494b11-18). Translation of D. W. Thompson

As for the senses and for the organs of sensation, the eyes, the nostrils, and the tongue, all alike are situated frontwards; the sense of hearing, and

the organ of hearing, the ear, is situated sideways, on the same horizontal plane with the eyes. The eyes in man are, in proportion to his size, nearer to one another than in any other animal.

Of the senses man has the sense of touch more refined than any animal, and so also, but in less degree, the sense of taste; in the development of the other senses he is surpassed by a great number of animals.

Aristotle, History of Animals I. 3 (489a17-19); I. 4 (489a23-26). Translation of D. W. Thompson

One sense, and one alone, is common to all animals—the sense of touch. Consequently, there is no special name for the organ in which it has its seat; for in some groups of animals the organ is identical, in others it is only analogous.

Touch has its seat in a part uniform and homogeneous, as in the flesh or something of the kind, and generally, with animals supplied with blood, in the parts charged with blood. In other animals it has its seat in parts analogous to the parts charged with blood; but in all cases it is seated in parts that in their texture are homogeneous.

# Cf. Aristotle, On the Soul III. 13 (435b4-7)

Clearly, then, animals deprived of this sense alone [the sense of touch] must die. For it is impossible that that which is not an animal should have this sense, and, again, it is unnecessary that an animal possess any other sense but this.

# HUMAN PSYCHOLOGY

# SENSATION AND PERCEPTION

#### GENERAL CONSIDERATIONS

Theophrastus, On the Senses 1-2, 19.1 Translation of G. M. Stratton (London, 1917)

1. The various opinions concerning sense perception, when regarded broadly, fall into two groups. By some investigators it is ascribed to similarity, while by others it is ascribed to contrast: Parmenides, Empedocles, and Plato attribute it to similarity; Anaxagoras and Heraclitus attribute it to contrast.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>The translator notes that this procedure is still common in Greece, either fleabane or tobacco being used. [Edd.]

<sup>&</sup>lt;sup>2</sup> The translator notes that an iron rod with a conical knob at the head is still used in the Adriatic for the capture of razor fish. The rod is let down into the burrow between the creature's valves. These close upon the iron and the fish is thus drawn up. [Edd.]

<sup>&</sup>lt;sup>1</sup>Theophrastus' work On the Senses is of the greatest value for its account and criticism of Greek psychology before Aristotle and is concerned not merely with perception but with such psychological subjects as pleasure and pain, temperaments and emotion, and the relation of bodily to psychic states.

Material within [ ] is added by the translator for the sake of clearness. [Edd.]

<sup>&</sup>lt;sup>2</sup> Besides those philosophers mentioned there are others, e.g., Alcmaeon, Diogenes of Apollonia, and Democritus, whom Theophrastus finds it more difficult to fit into this classification. The justice of citing Plato as an adherent of the "likeness" theory may be questioned. [Edd.]

The one party is persuaded by the thought that other things are, for the most part, best interpreted in the light of what is like them; that it is a native endowment of all creatures to know their kin; and furthermore, that sense perception takes place by means of an effluence, and like is borne toward like

- 2. The rival party assumes that perception comes to pass by an alteration; that the like is unaffected by the like, whereas opposites are affected by each other. So they give their verdict for this [idea of opposition]. And to their mind further evidence is given by what occurs in connection with touch, since a degree of heat or cold the same as that of our flesh arouses no sensation.
- 19.... Although it is a fairly difficult task to explain the facts of vision, yet how could we by *likeness* discern the objects with which the other senses deal? For the word "likeness" is quite vague. [We do] not [discern] sound by sound, nor smell by smell, nor other objects by what is kindred to them; but rather, we may say, by their opposites. To these objects it is necessary to offer the sense organ in a passive state. If we have a ringing in the ears, or a taste in the tongue, or a smell in the nostrils, these organs all become blunted; and the more so, the fuller they are of what is like them, unless there be a further distinction of these terms.

# Aristotle, On the Soul II. 6, 12. Translation of R. D. Hicks

6. In considering each separate sense we must first treat of their objects. By the sensible object may be meant any one of three things, two of which we say are perceived in themselves or directly, while the third is perceived per accidens or indirectly. Of the first two the one is the special object of a particular sense, the other an object common to all the senses. By a special object of a particular sense I mean that which cannot be perceived by any other sense and in respect to which deception is impossible, for example, sight is of colour, hearing of sound and taste of flavour, while touch no doubt has for its object several varieties. But at any rate each single sense judges of its proper objects and is not deceived as to the fact that there is a colour or a sound; though as to what or where the coloured object is or what or where the object is which produces the sound, mistake is possible. Such, then, are the special objects of the several senses. By common sensibles are meant motion, rest, number, figure, size: for such qualities are not the special objects of any single sense, but are common to all.

For example, a particular motion can be perceived by touch as well as by sight. What is meant by the indirect object of sense may be illustrated if we suppose that the white thing before you is Diares' son. You perceive Diares' son, but indirectly, for that which you perceive is accessory to the whiteness.¹ Hence you are not affected by the indirect sensible as such. Of the two classes of sensibles directly perceived it is the objects special to the different senses which are properly perceptible: and it is to these that the essential character of each sense is naturally adapted.

12. In regard to all sense generally we must understand that sense is that which is receptive of sensible forms apart from their matter, as wax receives the imprint of the signet-ring apart from the iron or gold of which it is made: it takes the imprint which is of gold or bronze, but not qua gold or bronze. And similarly sense as relative to each sensible is acted upon by that which possesses colour, flavour or sound, not in so far as each of those sensibles is called a particular thing, but in so far as it possesses a particular quality and in respect of its character or form. The primary sense-organ is that in which such a power resides, the power to receive sensible forms. Thus the organ is one and the same with the power, but logically distinct from it. For that which perceives must be an extended magnitude. Sensitivity, however, is not an extended magnitude, nor is the sense: they are rather a certain character or power of the organ. From this it is evident why excesses in the sensible objects destroy the sense-organs. For if the motion is too violent for the sense-organ, the character or form (and this, as we saw, constitutes the sense) is annulled, just as the harmony and the pitch of the lyre suffer by too violent jangling of the strings. It is evident, again, why plants nave no sensation, although they have one part of soul and are in some degree affected by the things themselves which are tangible: for example, they become cold and hot. The reason is that they have in them no mean, no principle capable of receiving the forms of sensible objects without their matter, but on the contrary, when they are acted upon, the matter acts upon them as well.

#### VISION

# Alcmaeon and Anaxagoras

Theophrastus, On the Senses 26, 36. Translation of G. M. Stratton

26. [Alcmaeon<sup>2</sup> states that] eyes see through the water round about. And the eye obviously has fire within, for when one is struck [this fire]

¹ In addition to the five senses Aristotle recognizes a synthetic faculty. The perception of the "common sensibles" here mentioned is one of the functions of this synthetic faculty. With the latter is also connected the power of discriminating between and comparing the data of the special senses, as well as the faculties of consciousness, imagination, memory, and reminiscence; sleeping and dreaming are affections of this same synthetic faculty. [Edd.]

<sup>&</sup>lt;sup>1</sup> For the special object of vision is color. [Edd.]

<sup>&</sup>lt;sup>2</sup> Alcmacon of Croton seems to have been influenced by Pythagorean views, though the tradition of his having been a pupil of Pythagoras is doubtful. According to Chalcidius (Commentary on the Timaeus 246), "he was the first who ventured to dissect the eye." His theory of health and disease may be a foreshadowing of the humoral view (see p. 490). [Edd.]

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flashes out.¹ Vision is due to the gleaming—that is to say, the transparent—character of that which [in the eye] reflects to the object; and sight is the more perfect, the greater the purity of this substance. All the senses are connected in some way with the brain;² consequently they are incapable of action if [the brain] is disturbed or shifts its position, for [this organ] stops up the passages through which the senses act.

36. Anaxagoras' doctrine of the visual image is one somewhat commonly held; for nearly everyone assumes that seeing is occasioned by the reflection in the eyes.<sup>3</sup> They took no account of the fact, however, that the size of objects seen is incommensurate with the size of their reflection; and that it is impossible to have many contrasting objects reflected at the same time; and farther, that motion, distance, and size are visual objects and yet produce no image. And with some animals nothing whatever is reflected—for example, with those that have horny eyes, or that live in the water. Moreover, according to this theory many lifeless things would possess the power of sight; for there is a reflection certainly in water, in bronze, and in many other things.

# Criticism of Democritus by Theophrastus

# Theophrastus, On the Senses 50-53. Translation of G. M. Stratton

Vision he [Democritus] explains by the reflection [in the eye], of which he gives a unique account. For the reflection does not arise immediately in the pupil. On the contrary, the air between the eye and the object of sight is compressed by the object and the visual organ, and thus becomes imprinted; since there is always an effluence of some kind arising from everything. Thereupon this imprinted air, because it is solid and is of a hue contrasting [with the pupil], is reflected in the eyes, which are moist. A dense substance does not receive [this reflection], but what is moist gives it admission. Moist eyes accordingly have a better power of vision than have hard eyes; provided their outer tunic be exceedingly fine and close-knit, and the inner [tissues] be to the last degree spongy and free from dense and stubborn flesh, and free too, from thick oily moisture; and provided, also, the ducts connected with the eyes be straight and dry that they may

"perfectly conform" to the entering imprints. For each knows best its kindred.

Now in the first place this imprint upon the air is an absurdity. For the substance receiving such an imprint must have a certain consistence and not be "fragile"; even as Democritus himself, in illustrating the character of the "impression," says that "it is as if one were to take a mould in wax." In the second place, an object could make a better imprint upon water [than upon air], since water is denser. While the theory would require us to see more distinctly [an object in water], we actually see it less so. In general, why should Democritus assume this *imprint*, when in his discussion of forms he has supposed an *emanation* that conveys the object's form? For these images [due to emanation] would be reflected.

But if such an imprint actually occurs and the air is moulded like wax that is squeezed and pressed, how does the reflection [in the eye] come into existence, and what is its character? For the imprint here as in other cases will evidently face the object seen. But since this is so, it is impossible for a reflection facing us to arise unless this imprint is turned around. What would cause this reversal, and what the manner of its operation, ought, however, to be shown; for in no other way could vision come to pass. Moreover, when several objects are seen in one and the same place, how can so many imprints be made upon the self-same air? And again, how could we possibly see each other? For the imprints would inevitably clash, since each of them would be facing [the person] from whom it sprung. All of which gives us pause.

Furthermore, why does not each person see himself? For the imprints [from ourselves] would be reflected in our own eyes quite as they are in the eyes of our companions, especially if these imprints directly face us and if the effect here is the same as with an echo—since Democritus says that [in the case of the echo] the vocal sound is reflected back to him who utters it. Indeed the whole idea of imprints made on the air is extravagant.<sup>1</sup>

#### Aristotle

# Aristotle, On the Soul II. 7.2 Translation of R. D. Hicks

The object seen in light is colour, and this is why it is not seen without light. For the very quiddity of colour is, as we saw, just this, that it is capable of exciting change in the operantly transparent medium: and the

<sup>&</sup>lt;sup>1</sup> In the absence of knowledge of the mechanism of the retina and optic nerve the flash "seen" when the eyeball is struck was attributed to an inner fire. This played an important role in ancient optical theory. [Edd.]

Aristotle criticizes this view and holds the region of the heart to be the sensory center (c.g., On the Parts of Animals II. 10). [Edd.]

<sup>\*</sup>Observation of the image seen in the pupils of the eyes by one who looks closely into a mirror may have given rise to conjectures of this kind. The structure of the eye and optic nerve were quite accurately demonstrated in the early Alexandrian period, but the physiology of vision was never understood in antiquity. [Edd.]

<sup>&</sup>lt;sup>1</sup> The atomistic theory of effluences or idols is considered at length in Epicurus's Letter to Herodotus and in Lucretius IV. [Edd.]

<sup>&</sup>lt;sup>2</sup> In the explanation of sight Aristotle proceeds from the cardinal facts that by this sense we distinguish objects (1) at a distance, (2) as coloured. Hence he assumes a medium upon which colour can act. The medium, in itself neutral, has two determinations, a positive state when it is illuminated and we say there is light, a negative state when we say there is darkness....

activity of the transparent is light. There is clear evidence of this. If you lay the coloured object upon your eye, you will not see it. On the contrary, what the colour excites is the transparent medium, say, the air, and by this, which is continuous, the sense-organ is stimulated. For it was a mistake in Democritus to suppose that if the intervening space became a void, even an ant would be distinctly seen, supposing there were one in the sky. That is impossible. For sight takes place through an affection of the sensitive faculty. Now it cannot be affected by that which is seen, the colour itself: therefore it can only be by the intervening medium: hence the existence of some medium is necessary. But, if the intermediate space became a void, so far from being seen distinctly, an object would not be visible at all.<sup>2</sup>

Cf. Aristotle, De Sensu 438b2-16. Translation of J. I. Beare (Oxford, 1908)

That without light vision is impossible has been stated elsewhere; but, whether the medium between the eye and its objects is air or light, vision is caused by a process through this medium.

Accordingly, that the inner part of the eye consists of water is easily intelligible, water being translucent.

Now, as vision outwardly is impossible without [extra-organic] light, so also it is impossible inwardly [without light within the organ]. There must, therefore, be some translucent medium within the eye, and, as this is not air, it must be water. The soul or its perceptive part is not situated at the external surface of the eye, but obviously somewhere within: whence the necessity of the interior of the eye being translucent, i.e., capable of admitting light. And that it is so is plain from actual occurrences. It is matter of experience that soldiers wounded in battle by a sword slash on the temple, so inflicted as to sever the passages of [i.e., inward from] the eye, feel a sudden onset of darkness, as if a lamp had gone out; because what is called the pupil, i.e., the translucent, which is a sort of inner lamp, is then cut off Ifrom its connexion with the soul].

#### HEARING<sup>3</sup>

Theophrastus, On the Senses 25, 9. Translation of G. M. Stratton

25.... Hearing is by means of the ears, he [Alcmaeon] says, because within them is an empty space, and this empty space resounds. A kind of

noise is produced by the cavity, and the internal air re-echoes this sound.

9. He [Empedocles] says that hearing results from sounds within [the head], whenever the air, set in motion by a voice, resounds within. For the organ of hearing, which he calls a "fleshy off-shoot," acts as the "bell" of a trumpet, ringing with sounds like [those it receives]. When set in motion [this organ] drives the air against the solid parts and produces there a sound.

#### Plato, Timaeus 67 B. Translation of R. G. Bury

The third organ of perception within us which we have to describe in our survey is that of hearing, and the causes whereby its affections are produced. In general, then, let us lay it down that sound is a stroke transmitted through the ears, by the action of the air upon the brain and the blood, and reaching to the soul; and that the motion caused thereby, which begins in the head and ends about the seat of the liver, is "hearing"; and that every rapid motion produces a "shrill" sound, and every slower motion a more "deep" sound; and that uniform motion produces an "even" and smooth sound and the opposite kind of motion a "harsh" sound; and that large motion produces "loud" sound, and motion of the opposite kind "soft" sound.

# Aristotle, On the Soul II. 8 (420a4-5)

Now there is air naturally connected with the organ of hearing. And because this organ is in air, motion of the external air produces motion of the air within the organ.

#### SMELL AND TASTE

# Empedocles

Theophrastus, On the Senses 9. 21-22. Translation of G. M. Stratton

9.... Smell, according to Empedocles, is due to the act of breathing. As a consequence, those have keenest smell in whom the movement of the breath is most vigorous. The intensest odour emanates from bodies that are subtile and light. Of taste and touch severally he offers no precise account, telling us neither the manner nor the means of their operation, save the [assertion he makes with regard to all the senses in] common, that perception arises because emanations fit into the passages of sense.¹ Pleasure is excited by things that are similar [to our organs], both in their constituent parts and in the manner of their composition; pain, by things opposed.

¹ The films that fly from objects and cause vision, according to Democritus, are prevented by the intervening air from reaching our eyes. Instead they impress their form on the air; but since this process entails progressive distortion the clearness of perception decreases as the distance increases. If, however, the intervening air were removed, such distortion would not take place and the image would reach the eye unaltered. [Edd.]

On Aristotle's theory of light, see p. 285. [Edd.]

The Greeks attacked the problem of the nature of sound more successfully than that of

the nature of hearing (see pp. 286ff.). Here we have merely suggested a few viewpoints of the latter problem. [Edd.]

<sup>&</sup>lt;sup>1</sup> The reference is to the pores, which played a part in all sense perceptions in the theory of Empedocles. [Edd.]

21.... It is silly to assert [as does Empedocles] that those have the keenest sense of smell who inhale most; for if the organ is not in health or is, for any cause, not unobstructed, mere breathing is of no avail. It often happens that a man has suffered injury [to the organ] and has no sensation at all. Furthermore, persons "short of breath" or at hard labour or asleep—since they inhale most air—should be most sensitive to odours. Yet the reverse is the fact. For in all likelihood respiration is not of itself the cause of smell, but is connected with it incidentally.

#### Democritus

Theophrastus, On the Causes of Plants VI. 1.6. Translation of J. I. Beare, Greek Theories of Elementary Cognition from Alemacon to Aristotle, p. 164 (Oxford, 1906)

Democritus investing each taste with its characteristic figure makes the sweet that which is round and large in its atoms; the astringently sour that which is large in its atoms, but rough, angular, and not spherical; the acid, as its name imports, that which is sharp in its bodily shape, angular, and curving, thin, and not spherical; the pungent that which is spherical, thin, angular, and curving; the saline, that of which the atoms are angular, and large, and crooked and isosceles; the bitter, that which is spherical, smooth, scalene, and small. The succulent is that which is thin, spherical, and small.<sup>1</sup>

#### Aristotle

# Aristotle, On the Soul II. 9-10. Translation of R. D. Hicks

9.... As with flavours, so with odours: some are sweet, some bitter. (But in some objects smell and flavour correspond; for example, they have sweet odour and sweet flavour: in other things the opposite is the case.) Similarly, too, an odour may be pungent, irritant, acid or oily. But because, as we said above, odours are not as clearly defined as the corresponding flavours, it is from these latter that the odours have taken their names, in virtue of the resemblance in the things. Thus the odour of saffron and honey is sweet, while the odour of thyme and the like is pungent; and so in all the other cases. Again, smell corresponds to hearing and to each of the other senses in that, as hearing is of the audible and inaudible, and sight of the visible and invisible, so smell is of the odorous and inodorous. By inodorous may be meant either that which is wholly incapable of having odour or that which has a slight or faint odour. The term tasteless involves a similar ambiguity.

Further, smell also operates through a medium, namely, air or water. For water animals too, whether they are, or are not, possessed of blood, seem to perceive odour as much as the creatures in the air: since some of them also come from a great distance to seek their food, guided by the scent. . . .

The inability to perceive what is placed immediately on the sense organ man shares with all animals: what is peculiar to him is that he cannot smell without inhaling. . . . <sup>1</sup>

10. The object of taste is a species of tangible. And this is the reason why it is not perceived through a foreign body as medium: for touch employs no such medium either. The body, too, in which the flavour resides, the proper object of taste, has the moist, which is something tangible, for its matter or vehicle. Hence, even if we lived in water, we should still perceive anything sweet thrown into the water, but our perception would not have come through the medium, but by the admixture of sweetness with the fluid, as is the case with what we drink. But it is not in this way, namely, by admixture, that colour is perceived, nor yet by emanations. Nothing, then, corresponds to the medium; but to colour, which is the object of sight, corresponds the flavour, which is the object of taste. But nothing produces perception of flavour in the absence of moisture, but either actually or potentially the producing cause must have liquid in it: salt, for instance, for that is easily dissolved and acts as a dissolvent upon the tongue. . . .

The organ of taste, then, which needs to be moistened, must have the capacity of absorbing moisture without being dissolved, while at the same time it must not be actually moist. A proof of this is the fact that the tongue has no perception either when very dry or very moist. In the latter case the contact is with the moisture originally in the tongue, just as when a man first makes trial of a strong flavour and then tastes some other flavour; or as with the sick, to whom all things appear bitter because they perceive them with their tongue full of bitter moisture.

As with the colours, so with the species of flavour, there are, first, simple flavours, which are opposites, the sweet and the bitter; next to these on one side the succulent, on the other the salt; and, thirdly, intermediate between these, the pungent, the rough, the astringent, and the acid. These seem to be practically all the varieties of flavour. Consequently, while the faculty of taste has potentially the qualities just described, the object of taste converts the potentiality into actuality.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup> The dependence of sensations on atomic sizes, shapes, arrangements, and motions is cardinal in the philosophy of the Greek atomists. Thus Democritus formulated the distinction between what came to be called primary and secondary qualities when he said, "By convention we speak of color, and of sweet and bitter, but in reality there are atoms and void." (Frag. 125, Diels.) [Edd.]

<sup>&</sup>lt;sup>1</sup> Taken literally, this is not true. Other air-breathing animals also smell while inhaling breath....

<sup>&</sup>lt;sup>2</sup> Cf. the treatment in Plato, *Timaeus* 65C-66C. Theophrastus in his discussion of plant saps (On the Causes of Plants VI) has much to say about flavors. [Edd.]

# [Aristotle], Problemata XIII. 2. Translation of E. S. Forster

Why is it that things of unpleasant odour do not seem to have an odour to those who have eaten them? Is it because, owing to the fact that the sense penetrates to the mouth through the palate, the sense of smell soon becomes satiated and so it no longer perceives the odour inside the mouth to the same extent—for at first every one perceives the odour, but, when they are in actual contact with it, they no longer do so, as though it had become part of themselves—and the similar odour from without is overpowered by the odour within?

# Theophrastus

# Theophrastus, On Odors 1, 5. Translation of Arthur Hort

1. Odours in general, like tastes, are due to mixture: for anything which is uncompounded has no smell, just as it has no taste: wherefore simple substances have no smell, such as water, air, and fire: on the other hand earth is the only elementary substance which has a smell, or at least it has one to a greater extent than the others, because it is of a more composite character than they.

Of odours some are, as it were, indistinct and insipid, as is the case with tastes, while some have a distinct character. And these characters appear to correspond to those of tastes, yet they have not in all cases the same names, as we said in a former treatise; nor in general are they marked off from one another by such specific differences as are tastes: rather the differences are, one may say, in generic character, some things having a good, some an evil odour. But the various kinds of good or evil odour, although they exhibit considerable differences, have not received further distinguishing names, marking off one particular kind of sweetness or of bitterness from another: we speak of an odour as pungent, powerful, faint, sweet, or heavy, though some of these descriptions apply to evil-smelling things as well as to those which have a good odour.

5. Now the odour of some things which have a good odour resides in things which are used for food, for instance that of stone-fruits, pears, and apples, the smell of which is sweet even if one does not eat them; indeed it may be said to be sweeter in that case. However, to make a general distinction, some odours exist independently, while others are incidental; those of juices and things used for food are incidental, those of flowers exist independently. And, as was said above, things which have a good odour are generally of unpleasant, astringent, or somewhat bitter taste. Again some things which have a good taste have also an evil odour, such as the carob, which is sweet (this is true of some regions, if not of all). Again

the Phoenician cedar, though it is sweet to the taste, when chewed produces a sort of evil odour, though it makes the water fragrant.

#### TOUCH

# Aristotle, On the Soul II. 11.1 Translation of R. D. Hicks

If touch is not a single sense but includes more senses than one, there must be a plurality of tangible objects also. It is a question whether touch is several senses or only one. What, moreover, is the sense-organ for the faculty of touch? Is it the flesh or what is analogous to this in creatures that have not flesh? Or is flesh, on the contrary, the medium, while the primary sense-organ is something different, something internal? We may argue thus; every sense seems to deal with a single pair of opposites, sight with white and black, hearing with high and low pitch, taste with bitter and sweet; but under the tangible are included several pairs of opposites, hot and cold, dry and moist, hard and soft and the like. A partial solution of this difficulty lies in the consideration that the other senses also apprehend more than one pair of opposites. Thus in vocal sound there is not only high and low pitch, but also loudness and faintness, smoothness and roughness, and so on. In regard to colour also there are other similar varieties. But what the one thing is which is subordinated to touch as sound is to hearing is not clear.

But is the organ of sense internal or is the flesh the immediate organ? No inference can be drawn, seemingly, from the fact that the sensation occurs simultaneously with contact. For even under present conditions, if a sort of membrane were constructed and stretched over the flesh, this would immediately on contact transmit the sensation as before. And yet it is clear that the organ of sense is not in this membrane; although, if by growth it became united to the flesh, the sensation would be transmitted even more quickly. Hence it appears that the part of the body in question, that is, the flesh, is related to us as the air would be if it were united to us all round by natural growth. We should then have thought we were perceiving sound, colour and smell by one and the same instrument: in fact, sight, hearing and smell would have seemed to us in a manner to constitute a single sense. But as it is, owing to the media, by which the various motions are transmitted, being separated from us, the difference of the organs of these three senses is manifest. But in regard to touch this point is at present obscure. . . .

It is, then, the distinctive qualities of body as body which are the objects of touch: I mean those qualities which determine the elements, hot or cold, dry or moist, of which we have previously given an account in our discussion of the elements. And their sense-organ, the tactile organ, that

<sup>1</sup> I.c., the smell is a kind of "accident" or by-product of the taste.

<sup>&</sup>lt;sup>1</sup> Cf. p. 541, above. [Edd.]

is, in which the sense called touch primarily resides, is the part which has potentially the qualities of the tangible object. For perceiving is a sort of suffering or being acted upon: so that when the object makes the organ in actuality like itself it does so because that organ is potentially like it. Hence it is that we do not perceive what is just as hot or cold, hard or soft. as we are, but only the excesses of these qualities: which implies that the sense is a kind of mean between the opposite extremes in the sensibles. This is why it passes judgment on the things of sense. For the mean is capable of judging, becoming to each extreme in turn its opposite. And, as that which is to perceive white and black must not be actually either, though potentially both, and similarly for the other senses also, so in the case of touch the organ must be neither hot nor cold. Further, sight is in a manner, as we say, of the invisible as well as the visible, and in the same way the remaining senses deal with opposites. So, too, touch is of the tangible and the intangible: where by intangible is meant, first, that which has the distinguishing quality of things tangible in quite a faint degree, as is the case with the air; and, secondly, tangibles which are in excess, such as those which are positively destructive.

# Lucretius, On the Nature of Things III. 374-395

Not only are the atoms of the soul much smaller than those of which our body and flesh are composed, but they are also fewer in number and are scattered only here and there over our body. Thus one may say that the intervals between the atoms of the soul are equal to the size of the smallest bodies that produce sensation by impact upon our body.<sup>1</sup>

For sometimes we do not feel the adhesion of dust to our body, or the settling of powdered chalk on our limbs, or a mist at night, or a spider's fine threads when we become entangled in them, or its fine-spun web falling on our head, or the feathers of birds, or the seeds of thistle-down as they are wafted to us and fall so slowly because they are so light, or the passage of every crawling creature, or every footstep that gnats and their like place on our body. So true is it that many particles must be moved in us before the atoms of the soul, interspersed through all the parts of our bodies, can perceive the impact and move in these intervals to buffet others, and meeting them leap apart in turn.

# THE AFTERIMAGE

Aristotle, On Dreams 2 (459a24-b23). Translation of J. I. Beare (Oxford, 1908)

The objects of sense-perception corresponding to each sensory organ produce sense-perception in us, and the affection due to their operation is present in the organs of sense not only when the perceptions are actualized, but even when they have departed.

What happens in these cases may be compared with what happens in the case of projectiles moving in space. For in the case of these the movement continues even when that which set up the movement is no longer in contact [with the things that are moved]. For that which set them in motion moves a certain portion of air, and this, in turn, being moved excites motion in another portion; and so, accordingly, it is in this way that [the bodies], whether in air or in liquids, continue moving, until they come to a standstill.<sup>1</sup>

This we must likewise assume to happen in the case of qualitative change; for that part which [for example] has been heated by something hot, heats [in turn] the part next to it, and this propagates the affection continuously onwards until the process has come round to its point of origination. This must also happen in the organ wherein the exercise of senseperception takes place, since sense-perception, as realized in actual perceiving, is a mode of qualitative change. This explains why the affection continues in the sensory organs, both in their deeper and in their more superficial parts, not merely while they are actually engaged in perceiving, but even after they have ceased to do so. That they do this, indeed, is obvious in cases where we continue for some time engaged in a particular form of perception, for then, when we shift the scene of our perceptive activity, the previous affection remains; for instance, when we have turned our gaze from sunlight into darkness. For the result of this is that one sees nothing, owing to the motion excited by the light still subsisting in our eyes. Also, when we have looked steadily for a long while at one colour, e.g., at white or green, that to which we next transfer our gaze appears to be of the same colour. Again if, after having looked at the sun or some other brilliant object, we close the eyes, then, if we watch carefully, it appears in a right line with the direction of vision (whatever this may be), at first in its own colour; then it changes to crimson, next to purple, until it becomes black and disappears. And also when persons turn away from looking at objects in motion, e.g., rivers, and especially those which flow very rapidly, they find that the visual stimulations still present themselves, for the things really at rest are then seen moving: persons become very deaf after hearing loud noises, and after smelling very strong odours their power of smelling is impaired; and similarly in other cases. These phenomena manifestly take place in the way above described.

#### Association of Ideas

Aristotle, On Memory and Recollection 2. Translation of J. I. Beare (Oxford, 1908)

Acts of recollection, as they occur in experience, are due to the fact that one movement has by nature another that succeeds it in regular order....

<sup>1</sup> Le., a smaller body may fail to touch any atoms of the soul.

<sup>&</sup>lt;sup>1</sup> A reference to the doctrine of antiperistasis. See p. 221. [Edd.]

Whenever, therefore, we are recollecting, we are experiencing certain of the antecedent movements until finally we experience the one after which customarily comes that which we seek. This explains why we hunt up the series having started in thought either from a present intuition or some other, and from something either similar, or contrary, to what we seek, or else from that which is contiguous with it. Such is the empirical ground of the process of recollection; for the mnemonic movements involved in these starting-points are in some cases identical, in others, again, simultaneous, with those of the idea we seek, while in others they comprise a portion of them, so that the remnant which one experienced after that portion [and which still requires to be excited in memory] is comparatively small.

Thus, then, it is that persons seek to recollect, and thus, too, it is that they recollect even without the effort of seeking to do so, viz., when the movement implied in recollection has supervened on some other which is its condition. For, as a rule, it is when antecedent movements of the classes here described have first been excited, that the particular movement implied in recollection follows. . . . Accordingly, therefore, when one wishes to recollect, this is what he will do: he will try to obtain a beginning of movement whose sequel shall be the movement which he desires to reawaken. This explains why attempts at recollection succeed soonest and best when they start from a beginning [of some objective series]. For, in order of succession, the mnemonic movements are to one another as the objective facts [from which they are derived]. Accordingly, things arranged in a fixed order, like the successive demonstrations in geometry, are easy to remember [or recollect], while badly arranged subjects are remembered with difficulty. . . .

Hence it is that [from the same starting-point] the mind receives an impulse to move sometimes in the required direction, and at other times otherwise, [doing the latter] particularly when something else somehow deflects the mind from the right direction and attracts it to itself. This last consideration explains too how it happens that, when we want to remember a name, we remember one somewhat like it, indeed, but blunder in reference to the one we intended.

Thus, then, recollection takes place.

THE INTERRELATION OF BODILY AND MENTAL STATES
TEMPERATURE AND EMOTIONS

Aristotle, On the Motion of Animals, 7-8. Translation of A. S. L. Farquharson

Sensations are obviously a form of change of quality, and imagination and conception have the same effect as the objects so imagined and conceived. For in a measure the form conceived be it of hot or cold or pleasant or fearful is like what the actual objects would be, and so we shudder and are frightened at a mere idea. Now all these affections involve changes of quality, and with those changes some parts of the body enlarge, others grow smaller. And it is not hard to see that a small change occurring at the centre makes great and numerous changes at the circumference, just as by shifting the rudder a hair's breadth you get a wide deviation at the prow. And further, when by reason of heat or cold or some kindred affection a change is set up in the region of the heart, or even in an imperceptibly small part of the heart, it produces a vast difference in the periphery of the body—blushing, let us say, or turning white, goose-skin and shivers and their opposites.

But to return, the object we pursue or avoid in the field of action is, as has been explained, the original of movement, and upon the conception and imagination of this there necessarily follows a change in the temperature of the body. For what is painful we avoid, what is pleasing we pursue. We are, however, unconscious of what happens in the minute parts; still anything painful or pleasing is generally speaking accompanied by a definite change of temperature in the body. One may see this by considering the affections. Blind courage and panic fears, erotic motions, and the rest of the corporeal affections, pleasant and painful, are all accompanied by a change of temperature, some in a particular member, others in the body

images of these pleasures and pains, are now more and now less causes of the same changes of temperature.

ABERRATIONS OF THE SENSES

Aristotle, On Dreams 2. Translation of J. I. Beare

generally. So, memories and anticipations, using as it were the reflected

We are easily deceived respecting the operations of sense-perception when we are excited by emotions, and different persons according to their different emotions; for example, the coward when excited by fear, the amorous person by amorous desire; so that, with but little resemblance to go upon, the former thinks he sees his foes approaching, the latter, that he sees the object of his desire; and the more deeply one is under the influence of the emotion, the less similarity is required to give rise to these illusory impressions. Thus too, both in fits of anger, and also in all states of appetite, all men become easily deceived, and more so the more their emotions are excited. This is the reason too why persons in the delirium of fever sometimes think they see animals on their chamber walls, an illusion arising from the faint resemblance to animals of the markings thereon when put together in patterns; and this sometimes corresponds with the emotional states of the sufferers, in such a way that, if the latter be not very ill, they know well enough that it is an illusion; but if the illness is more severe they actually move according to the appearances. The cause of these occurrences

HUMAN PSYCHOLOGY

is that the faculty in virtue of which the controlling sense judges is not identical with that in virtue of which presentations come before the mind. A proof of this is that the sun presents itself as only a foot in diameter, though often something else gainsays the presentation. Again, when the fingers are crossed, the one object [placed between them] is felt [by the touch] as two; but yet we deny that it is two; for sight is more authoritative than touch. Yet, if touch stood alone, we should actually have pronounced the one object to be two. The ground of such false judgments is that any appearances whatever present themselves, not only when its object stimulates a sense, but also when the sense by itself alone is stimulated, provided only it be stimulated in the same manner as it is by the object.

#### PSYCHOPATHOLOGY3

#### Plato, Timaeus 86 B-87 A. Translation of R. G. Bury

Such is the manner in which diseases of the body come about; and those of the soul which are due to the condition of the body arise in the following way. We must agree that folly is a disease of the soul; and of folly there are two kinds, the one of which is madness, the other ignorance. Whatever affection a man suffers from, if it involves either of these conditions it must be termed "disease"; and we must maintain that pleasures and pains in excess are the greatest of the soul's diseases. For when a man is overjoyed or contrariwise suffering excessively from pain, being in haste to seize on the one and avoid the other beyond measure, he is unable either to see or to hear anything correctly, and he is at such a time distraught and wholly incapable of exercising reason. . . . And again, in respect of pains likewise the soul acquires much evil because of the body.

For whenever the humours which arise from acid and saline phlegms, and all humours that are bitter and bilious wander through the body and find no external vent but are confined within, and mingle their vapour with the movement of the soul and are blended therewith, they implant diseases of the soul of all kinds, varying in intensity and in extent; and as these humours penetrate to the three regions of the soul, according to the region which they severally attack, they give rise to all varieties of bad temper and

bad spirits, and they give rise to all manner of rashness and cowardice, and of forgetfulness also, as well as of stupidity.<sup>1</sup>

Aretaeus, On the Causes and Symptoms of Chronic Diseases 1.5. Translation of Francis Adams

# Melancholy

But if it [black bile] be determined upwards to the stomach and diaphragm, it forms melancholy;<sup>2</sup> for it produced flatulence and eructations of a fetid and fishy nature, and it sends rumbling wind downwards, and disturbs the understanding. On this account, in former days, they were called melancholics and flatulent persons. And yet, in certain of these cases there is neither flatulence nor black bile, but mere anger and grief, and sad dejection of mind. . . .

It is a lowness of spirits from a single phantasy, without fever; and it appears to me that melancholy is the commencement and a part of mania. For in those who are mad, the understanding is turned sometimes to anger and sometimes to joy, but in the melancholics to sorrow and despondency only. But they who are mad are so for the greater part of life, becoming silly, and doing dreadful and disgraceful things; but those affected with melancholy are not every one of them affected according to one particular form; but they are either suspicious of poisoning, or flee to the desert from misanthropy, or turn superstitious, or contract a hatred of life. Or if at any time a relaxation takes place, in most cases hilarity supervenes, but these persons go mad. . . .

But if it also affects the head from sympathy, and the abnormal irritability of temper change to laughter and joy for the greater part of their life, these become mad rather from the increase of the disease than from change of the affection.

Dryness is the cause of both. Adult men, therefore, are subject to mania and melancholy, or persons of less age than adults. Women are worse affected with mania than men. As to age, towards manhood, and those actually in the prime of life. The seasons of summer and of autumn engender, and spring brings it to a crisis.

The characteristic appearances, then, are not obscure; for the patients are dull or stern, dejected or unreasonably torpid, without any manifest cause: such is the commencement of melancholy. And they also become peevish, dispirited, sleepless, and start up from a disturbed sleep.

Unreasonable fear also seizes them, if the disease tend to increase, when

<sup>&</sup>lt;sup>1</sup> The Epicureans, it will be recalled, maintained that the sun and moon were actually about a foot in diameter. [Edd.]

<sup>&</sup>lt;sup>2</sup> This experiment of crossing the fingers is often referred to by Aristotle. [Edd.]

Numerous passages in biological, medical, philosophical, and purely literary works illustrate this topic, and in this connection we may note references to the effect of age on character (e.g., Aristotle, *Rhetoric II.* 12-14), the effect of drugs on the mind (e.g., Theophrastus, *History of Plants IX.* 19), and the effect of music (Plato *Republic* 398-400, Aristotle, *Politics* VIII. 7, Theophrastus, *Fragments* 87, 88). [Edd.]

I.e., the brain, the heart, and the liver, which are the centers, respectively, of the rational, "spirited," and appetitive aspects of soul. [Edd.]

<sup>&</sup>lt;sup>1</sup> The connection here alluded to between the humors and psychological states has persisted in language and in folklore, as when we speak of sanguine, phlegmatic, melancholic, and choleric types. There is a classic discussion of the melancholic type in [Aristotle], *Problemata* XXX. 1. [Edd.]

<sup>&</sup>lt;sup>2</sup> Cf. pp. 488-490. [Edd.]

their dreams are true, terrifying, and clear: for whatever, when awake. they have an aversion to, as being an evil, rushes upon their visions in sleep. They are prone to change their mind readily; to become base, mean-spirited. illiberal, and in a little time, perhaps, simple, extravagant, munificent, not from any virtue of the soul, but from the changeableness of the disease. But if the illness become more urgent, hatred, avoidance of the haunts of men, vain lamentations: they complain of life, and desire to die. In many. the understanding so leads to insensibility and fatuousness, that they become ignorant of all things, or forgetful of themselves, and live the life of the inferior animals. The habit of the body also becomes perverted. . . . Therefore the bowels are dried up, and discharge nothing; or, if they do, the dejections are dried, round, with a black and bilious fluid, in which they float; urine scanty, acrid, tinged with bile. They are flatulent about the hypochondriac region; the eructations fetid, virulent, like brine from salt: and sometimes an acrid fluid, mixed with bile, floats in the stomach. Pulse for the most part small, torpid, feeble, dense, like that from cold.

A story is told, that a certain person, incurably affected, fell in love with a girl; and when the physicians could bring him no relief, love cured him. But I think that he was originally in love, and that he was dejected and spiritless from being unsuccessful with the girl, and appeared to the common people to be melancholic. He then did not know that it was love; but when he imparted the love to the girl, he ceased from his dejection, and dispelled his passion and sorrow; and with joy he awoke from his lowness of spirits, and he became restored to understanding, love being his physician.

# SOME IMPORTANT BOOKS ON GREEK SCIENCE

The books here cited do not constitute an exhaustive bibliography of the subject; for such an undertaking a separate volume would be necessary. There are, however, certain outstanding contributions that should be made known to the student. We also wish here to indicate the range of primary sources, for which more precise bibliographical data may be found in the body of this volume.

Much of the literature on Greek science is dispersed among philological books, periodicals, and encyclopedias. We mention first the two great encyclopedias devoted to classical antiquity:

Paulys Real-Encyclopädie der classischen Altertumswissenschaft. Ed. by G. Wissowa, W. Kroll, and K. Mittelhaus. Stuttgart, publication begun in 1893; not yet complete. Hereafter abbreviated RE.

Daremberg, C., and E. Saglio. Dictionnaire des antiquités grecques et romaines. 5 volumes, Paris. 1873-1917.

Older than these, but occasionally very helpful:

A Dictionary of Greek and Roman Biography and Mythology. Ed. by William Smith. 3 volumes, London, 1876.

A Dictionary of Greek and Roman Antiquities. Ed. by W. Smith, W. Wayte, and G. E. Marindin. 3d ed., 2 volumes, London, 1890-1891.

#### Note also:

A Companion to Latin Studies. Ed. by J. E. Sandys, Cambridge, 1910.

A Companion to Greek Studies. Ed. by L. Whibley, Cambridge, 1905.

In many types of investigation the student of Greek science will find it profitable to employ the bibliographical resources of the classicist, e.g.:

Marouzeau, J. Dix Années de bibliographie classique (covering the period 1914-1924). Paris, 1927-1928.

L'Année philologique (Paris).

Bibliotheca philologica classica (Leipzig).

Klassieke bibliographie (Utrecht).

Bursian's Jahresbericht über die Fortschritte der klassischen Altertumswissenschaft. Note the classified list of articles (1873-1923) in McFayden, D., "Fifty Years of Bursian's Jahresbericht." Washington University Studies. Humanistic Series 12 (1924) 111.

Of treatises dealing wholly or in large measure with Greek science note the following:

Sarton, George. Introduction to the History of Science. Vol. I, From Homer to Omar Khayyam. Baltimore, 1927. This book includes rich bibliographies which are supplemented in the issues of the periodical Isis.

Brunet, P., and A. Mieli. Histoire des sciences: Antiquité. Paris, 1935 (a history and an anthology, with extensive bibliographies).

Mieli, A. Manuale di storia della scienza. Storia, antologia, bibliografia. Rome, 1925. Enriques, F., and G. De Santillana. Storia del pensiero scientifico. Vol. I, Il Mondo antico. Bologna, 1932.

Lovesickness is found to be the cause of insomnia in a case described by Galen, XIV. 631 (Kühn). The quickening of the pulse at the mention of the name of the beloved gives the clue. [Edd.]

Heiberg, J. L. Geschichte der Mathematik und Naturwissenschaften im Altertum. Munich, 1925.

Mathematics and Physical Science in Classical Antiquity. London, 1922. A translation by D. C. Macgregor of Naturwissenschaften Mathematik und Medizin im klassischen Altertum, Leipzig, 1912.

Rehm, A., and K. Vogel. Exakte Wissenschaften. Leipzig, 1933. (Einleitung in die Altertumswissenschaft II. 2, ed. by A. Gercke and E. Norden.)

Rey, Abel. La Science orientale avant les Grecs. Paris, 1930.

- La Jeunesse de la science grecque. Paris, 1933.

La Maturité de la pensée scientifique en Grèce. Paris, 1939.

—— L'Apogée de la science technique grecque. Paris, 1946. The emphasis in these works is on the philosophy of science.

Milhaud, G. Leçons sur les origines de la science grecque. Paris, 1894.

Etudes sur la pensée scientifique chez les Grecs et chez les modernes. Paris, 1906.

Les Philosophes géomètres de la Grèce. Paris, 1906.

--- Nouvelles Etudes sur l'histoire de la pensée scientifique. Paris, 1912.

Reymond, A. Histoire des sciences exactes et naturelles dans l'antiquité gréco-romaine. Paris, 1924. English translation by R. G. de Bray, New York [1927].

Heidel, W. A. The Heroic Age of Science: the Conception, Ideals, and Methods of Science among the Ancient Greeks. Baltimore, 1933.

Farrington, B. Science in Antiquity. London, 1936.

Science and Politics in the Ancient World. London, 1939.

Thorndike, Lynn. A History of Magic and Experimental Science during the First Thirteen Centuries of Our Era. Vol. I (New York, 1923) begins with the Roman Empire.

Thompson, D. W. Science and the Classics. Oxford, 1940.

Tannery, Paul. Mémoires scientifiques. 14 volumes, 1912-1937. Volumes I-III are especially devoted to ancient science, but there is extensive material on antiquity in the other volumes.

The intimate relation of philosophy and science in the period covered by our volume makes it advisable to mention here, for its bibliographical richness:

Friedrich Ueberwegs Grundriss der Geschichte der Philosophie. Erster Teil. 12th ed., ed. by K. Praechter. Berlin, 1926.

#### Note also:

Zeller, E. Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung. English translation of various parts by Alleyne, Alleyne and Goodwin, Costelloe and Muirhead, Reichel.

- Grundriss der Geschichte der griechischen Philosophie. English translation by L. R. Palmer. New York, 1931.

Gomperz, T. Griechische Denker. 3 volumes, Leipzig, 1896-1909. English translation, 4 volumes, London, 1911-1912.

Burnet, John. Greek Philosophy. Part I, Thales to Plato. London, 1914.

--- Early Greek Philosophy. 3d ed., London, 1920.

Tannery, P. Pour l'Histoire de la science hellène. 2d ed., Paris, 1930.

Robin, Léon. La Pensée grecque et les origines de l'esprit scientifique. Paris, 1923. English translation by M. R. Dobie, New York, 1928.

Mieli, Aldo. La Scienza greca prearistotelica. Florence, 1916.

Frank, E. Platon und die sogenannten Pythagoreer. Halle, 1923.

Jaeger, W. Aristoteles. English translation by R. Robinson. Oxford, 1934.

Paideia. English translation by G. Highet. 3 volumes, Oxford, 1939-1944.

In addition to the extant philosophical works, such as those of Plato, Aristotle, and Lucretius, the collections of fragments furnish many of our sources. Note, e.g.:

Diels, H. Die Fragmente der Vorsokratiker. 5th ed., ed. by W. Kranz, 3 volumes, Berlin, 1934-1937.

Diels, H. Doxographi graeci. Berlin, 1879.

Usener, H. Epicurea. Leipzig, 1887.

Arnim, H. von. Stoicorum veterum fragmenta. Leipzig, 1903-1924.

Useful English translations are contained in:

Fairbanks, A. The First Philosophers of Greece. London, 1898.

Selections from Early Greek Philosophy. Ed. by Milton C. Nahm. New York, 1935. Selections from Hellenistic Philosophy. Ed. by G. H. Clark. New York, 1940.

#### Periodicals

Among those periodicals dealing generally with the history of science, including the ancient period:

Archeion: archivio di storia della scienza. Founded in 1919 by A. Mieli,

Isis: International Review Devoted to the History of Science and Civilization. Founded in 1913 by George Sarton. Includes critical bibliographies of current literature in the history of science and thus forms a continuing supplement to Sarton's Introduction to the History of Science.

Osiris. Founded in 1936 by George Sarton for the publication of longer papers and monographs.

Mitteilungen zur Geschichte der Medizin, der Naturwissenschaften und der Technik. Founded in 1902. Especially important for its bibliographies.

Archiv für Geschichte der Naturwissenschaften und der Technik. Issued 1908-1922, 1927-1931.

Thalès. Recueil annuel des travaux de l'institut d'histoire des sciences et des techniques de l'université de Paris. Founded in 1934.

Lychnos. Organ of the Swedish History of Science Society. Founded in 1936.

The series of translations, Klassiker der exacten Naturwissenschaften, founded by Wilhelm Ostwald, contains translations of various works of Greek mathematics, astronomy, and physics.

Many historical articles are published in periodicals dealing with special sciences (some will be noted below) and in such general scientific periodicals as Science (New York), Nature (London), and Scientia (Bologna).

#### MATHEMATICS

An extensive bibliography of the history of mathematics (including the ancient period) is contained in George Sarton, The Study of the History of Mathematics, Cambridge, Mass., 1936, and in Gino Loria, Guida allo studio della storia delle matematiche, 2d. ed., Milan, 1946.

The standard histories of mathematics take up the Greek period quite fully, e.g.:

Cantor, Moritz. Vorlesungen über Geschichte der Mathematik. 4 volumes. Volume I (4th ed., Leipzig, 1922) includes the ancient period.

Smith, D. E. History of Mathematics. 2 volumes, Boston, 1923-1925.

Loria, Gino. Storia delle matematiche. 3 volumes, Turin, 1929-1933.

The following are devoted to Greek mathematics in particular.

Heath, T. L. A History of Greek Mathematics. 2 volumes, Oxford, 1921.

- A Manual of Greek Mathematics. Oxford, 1931.

Thomas, Ivor. Selections Illustrating the History of Greek Mathematics. With English translation. Loeb Classical Library, 2 volumes, Cambridge, Mass., and London, 1939, 1941. An excellent anthology, which closely follows Heath's treatment of the subject.

Loria, Gino. Le Scienze esatte nell' antica Grecia. 5 volumes, Modena, 1893-1902.

Histoire des sciences mathématiques dans l'antiquité hellénique. Paris, 1929.

# Among the older books note:

Gow, James. A Short History of Greek Mathematics. Cambridge, 1884.

Tannery, P. La Géométrie grecque. Paris, 1887.

Zeuthen, H. G. Die Lehre von den Kegelschnitten im Altertum. Copenhagen, 1886.

# On pre-Greek mathematics see:

Neugebauer, O. Vorgriechische Mathematik. Berlin, 1934.

Chase, A. B., H. P. Manning, and R. C. Archibald. The Rhind Mathematical Papyrus. 2 volumes, Oberlin, 1927-1929.

Vogel, Kurt. Die Grundlagen der ägyptischen Arithmetik. Munich, 1929.

The knowledge of pre-Greek mathematics has developed rapidly in recent years and may best be followed in the books and papers of O. Neugebauer, F. Thureau-Dangin, K. Vogel, S. Gandz, and others. See *Isis* 31 (1940) 399ff.

#### Periodicals

Many recent papers of great importance on Greek and pre-Greek mathematics are to be found in Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik. Begun in 1930.

Scripta Mathematica. Ed. by J. Ginsburg. Begun in 1932.

Of the earlier periodicals and series containing important material for the student of Greek mathematics note especially:

Bibliotheca Mathematica. Zeitschrift für Geschichte der mathematischen Wissenschaften. Ed. by Gustaf Eneström. 1884-1886; 1887-1899; 1900-1914.

Abhandlungen zur Geschichte der mathematischen Wissenschaften. Founded by Moritz Cantor. 1877-1913.

Bullettino di bibliografia e storia delle scienze matematiche e fisiche. Ed. by B. Boncompagni. 1868-1887.

Bollettino di bibliografia e storia delle scienze matematiche. Ed. by G. Loria, 1898-1917. (Later an appendix to Bollettino di matematica.)

#### Sources

See p. 1. Among the more important Greek writers on mathematics (and their modern editors) we may mention:

Euclid (Heiberg and Menge); Archimedes (Heiberg); Apollonius (Heiberg: this edition does not contain Conics V-VII, extant only in Arabic); Diophantus (Tannery); Pappus (Hultsch); Autolycus (Hultsch); Hero of Alexandria (Schöne and Heiberg); Theodosius, Sphaerica (Heiberg); Menelaus, Sphaerica (Arabic text ed. by Krause); Theon of Smyrna (Hiller); Nicomachus (Hoche); Serenus (Heiberg); Iamblichus, Introduction to Arithmetic (Pistelli); and Proclus, Commentary on Euclid's Elements I (Friedlein). Many of these editions are accompanied by Latin translations.

The English reader is fortunate in having T. L. Heath's series of translations, paraphrases, and commentaries:

The Thirteen Books of Euclid's Elements. 3 volumes, Cambridge, 1908.

The Works of Archimedes. Cambridge, 1897.

The "Method" of Archimedes. Cambridge, 1912.

Apollonius of Perga. Treatise on Conic Sections. Cambridge, 1896. Diophantus of Alexandria. 2d ed., Cambridge, 1910.

Note also Nicomachus of Gerasa, Introduction to Arithmetic. English translation by M. L. D'Ooge with studies in Greek arithmetic by F. E. Robbins and L. C. Karpinski. New York, 1926.

Paul Ver Eecke has in recent years published French translations of a series of Greek mathematical authors, including works of Archimedes, Apollonius, Pappus, Diophantus, Theodosius (Sphaerica), Euclid (Optics and Catoptrics), and Serenus.

Important mathematical passages sometimes occur in non-mathematical works, e.g., passages from Eudemus preserved in Simplicius' commentaries on Aristotle's *Physics* and *De Caelo*, passages in Plato, etc.

#### ASTRONOMY AND MATHEMATICAL GEOGRAPHY

Many of the books noted under Mathematics contain material on Greek astronomy and mathematical geography. Apart from general histories of astronomy, the following contain important treatments of Greek astronomy.

Heath, T. L. Aristarchus of Samos. Oxford, 1913. The text and translation of Aristarchus's treatise On the Sizes and Distances of the Sun and Moon are preceded by a study of early Greek astronomy.

Dreyer, J. L. E. History of the Planetary Systems from Thales to Kepler. Cambridge, 1906. Delambre, J. B. J. Histoire de l'astronomie ancienne. 2 volumes, Paris, 1817.

Duhem, P. Le Système du monde. Histoire des doctrines cosmologiques de Platon à Copernic. 5 volumes, Paris, 1913-1917.

Tannery, P. Recherches sur l'histoire de l'astronomie ancienne. Paris, 1893.

Schiaparelli, G. Scritti sulla storia dell' astronomia antica. 3 volumes, Bologna, 1925-1927.

The papers of J. K. Fotheringham and O. Neugebauer (some of them in Quellen und Studien zur Geschichte der Mathematik, Astronomie, und Physik) include important recent contributions to Babylonian and Greek astronomy. For a general summary see O. Neugebauer, "The History of Ancient Astronomy: Problems and Methods." Journal of Near Eastern Studies 4 (1945) 1-38: reprinted, with some amplification, in Publication of the Astronomical Society of the Pacific 58 (1946) 17-43, 104-142.

The books dealing with mathematical geography are substantially the same as those on astronomy. In addition to the latter, note Wilhelm Kubitschek, articles "Karten" (RE, vol. X) and "Erdmessung" (RE Supplement VI), and F. Gisinger, article "Geographie" (RE, vol. IV).

Bunbury, E. H. A History of Ancient Geography among the Greeks and Romans from the Earliest Ages till the Fall of the Roman Empire. 2d ed., 2 volumes, London, 1883.

Wright, J. K. The Geographical Lore of the Time of the Crusades. New York, 1925. Has extensive bibliographies and numerous references to antiquity.

Berger, Hugo. Geschichte der wissenschaftlichen Erdkunde der Griechen. 2d ed., Leipzig, 1903.

Heidel, W. A. The Frame of the Ancient Greek Maps. New York, 1937.

Warmington, E. H. Greek Geography. London and New York, 1934. An anthology with a considerable section on mathematical geography.

#### Sources

See p. 89. The basic texts include Euclid, *Phaenomena* (Menge); Autolycus (Hultsch: German translation by Czwalina); Aristarchus (Heath, with English translation); Geminus,

Elements of Astronomy (Manitius); Hipparchus, Commentaries on the "Phaenomena" of Aratus and Eudoxus (Manitius, with German translation); Ptolemy, Syntaxis Mathematica (=Almagest) and Opera Astronomica Minora (Heiberg: German translation of Syntaxis by Manitius, French translation by Halma), Catalogue of Stars [from Syntaxis VII and VIII] (Peters and Knobel), commentaries on Syntaxis by Theon of Alexandria and Pappus (A. Rome); Cleomedes (Ziegler: German translation by Czwalina); Theodosius, De Diebus et Noctibus (Heiberg), De Habitationibus (Fecht); Proclus, Hypotyposis (Manitius, with German translation); Ptolemy, Geography (Müller, incomplete, Nobbe: German translation and commentary on Book I—the part relating to mathematical geography—by H. v. Mžik); Strabo (Meinecke: English translation by H. L. Jones); Philoponus, On the Astrolabe (Hase: French translation by P. Tannery, English translation by H. W. Greene in R. T. Gunther, The Astrolabes of the World, vol. I).

In addition there is extensive source material on astronomy and mathematical geography in the philosophical and doxographical literature and in such Latin writers as Pliny the Elder, Macrobius, Martianus Capella, and Censorinus.

#### PHYSICS

Of the general histories of physics that of Ernst Gerland, Geschichte aer Physik (Munich, 1913), has a good treatment of the ancient period.

#### Mechanics

Duhem, P. L'Evolution de la mécanique. Paris, 1903.

Les Origines de la statique. 2 volumes, Paris, 1905-1906.

---- Σώζειν τὰ φαινόμενα. Essai sur la notion de théorie physique de Platon à Galilée. Paris, 1908.

- Etudes sur Léonard de Vinci. 3 volumes, Paris, 1906-1913.

La Théorie physique: son objet, sa structure. 2d ed., Paris, 1914.

Vailati, Giovanni. Scritti. Leipzig, 1911.

Mach, Ernst. Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt. 7th ed., Leipzig, 1912. English translation (The Science of Mechanics) by T. J. McCormack, 4th ed., Chicago, 1919.

#### Atomism

Mabilleau, L. Histoire de la philosophie atomistique. Paris, 1895.

Lasswitz, K. Geschichte der Atomistik im Mittelalter bis Newton. 2 volumes, Hamburg, 1890.

Bailey, C. The Greek Atomists and Epicurus. Oxford, 1928.

#### TECHNOLOGY

Enciclopedia delle scienze e delle loro applicazioni. 2 volumes, Milan, 1941-1943. Contains much material pertaining to antiquity.

Feldhaus, F. M. Die Technik der Antike und des Mittelalters. Potsdam, 1931.

Diels, Hermann. Antike Technik. 3d ed., Leipzig, 1924.

#### Sources

Mechanics: Archimedes, various works, especially On the Equilibrium of Planes and the Method (ed. by Heiberg: English translation by Heath); the Aristotelian Mechanics (text and English translation by W. S. Hett, English translation by E. S. Forster); Hero of Alexandria, Mechanics (ed. by L. Nix, with German translation of the Arabic; B. Carra de Vaux, with French translation); Pappus, Mathematical Collection, Book VIII (Hultsch: see under Mathe-

matics). For basic questions of dynamics, the philosophers and commentators (e.g., Simplicius and Philoponus) must be consulted.

Hydrostatics: Archimedes, On Floating Bodies (ed. by Heiberg: English translation by Heath).

Optics and catoptrics: Euclid (Heiberg: English translation by H. E. Burton, French translation by Ver Eecke).

Ptolemy (?) (Govi), Hero of Alexandria (W. Schmidt), Damianus (Schöne).

Acoustics and musical theory: Euclid (Menge); Ptolemy, Harmonics (Düring, also German translation); Porphyrius, Commentary on Ptolemy's Harmonics (Düring); Aristoxenus (Macran, with English translation); the acoustical and musical problems in the Aristotelian Problemata Books 11 and 19 (English translations by W. S. Hett [Loeb Classical Library] and E. S. Forster [Oxford] ); Theon of Smyrna (Hiller); Plutarch, On Music (Volkmann: French translation by Weil and T. Reinach); Boethius, On Music (Friedlein). Works of Nicomachus, Bacchius, Gaudentius, and Alypius in addition to some of the others mentioned above are included in K. von Jan, Musici Scriptores Graeci, Leipzig, 1895. In this branch, as in so many others of ancient science, the philosophers (notably in this case Plato and Aristotle) and their commentators (e.g., Alexander of Aphrodisias, Themistius, Proclus, Porphyrius, Simplicius, Philoponus, etc.) must be consulted.

Pneumatics: Hero of Alexandria, *Pneumatics* (Schmidt, with German translation; English translation by J. G. Greenwood); Philo of Byzantium, *Pneumatics* (Arabic ed. by Carra de Vaux, with French translation; Latin ed. by Schmidt; French translation by A. de Rochas).

For applied mechanics, see p. 314. Among the literary sources for our knowledge of ancient machinery—agricultural, industrial, military, theatrical, etc.—are passages in the Aristotelian *Problemata* and *Mechanica*, Hero of Alexandria, Philo of Byzantium, Biton, Vitruvius, Pappus, *Mathematical Collection* VIII, Cato, Varro, and the minor writers on geoponics.

# CHEMISTRY AND CHEMICAL TECHNOLOGY

Kopp, Hermann. Geschichte der Chemie. 4 volumes. Braunschweig, 1843-1847.

Beiträge zur Geschichte der Chemie. Part I. Braunschweig, 1869.

Meyer, Ernst von. Geschichte der Chemie von den ältesten Zeiten bis zur Gegenwart. 4th ed., Leipzig, 1914. English translation of 3d ed. by G. McGowan, New York, 1906.

Stillman, J. M. The Story of Early Chemistry. New York, 1924.

Berthelot, M. Les Origines de l'alchimie. Paris, 1885.

Introduction à l'étude de la chimie des anciens et du moyen âge. Paris, 1889.

Lippmann, E. O. von. Abhandlungen und Vorträge zur Geschichte der Naturwissenschaften. Leipzig, 1906.

- Entstehung und Ausbreitung der Alchemie. Berlin, 1919.

#### Periodicals

Ambix, Journal of the Society for the Study of Alchemy and Early Chemistry, was published in 1937-1938 under the editorship of F. S. Taylor. Publication resumed in 1946.

Chymia: Studies in the History of Chemistry. University of Pennsylvania Press. An annual begun in 1948.

#### Sources

See p. 352. Important source material is to be found in:

Berthelot, M., and C. E. Ruelle. Collection des alchimistes grees. 3 volumes, Paris, 1885-1888.

Lagercrantz, O. Papyrus graecus Holmiensis. Uppsala, 1913.

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1885 (containing Leyden Papyrus X).

Mieli, Aldo. Pagine di storia della chimica. Rome, 1922.

Bailey, K. C. The Elder Pliny's Chapters on Chemical Subjects. 2 volumes, London, 1929, 1932.

Medical and pharmacological authors contribute important evidence and, of course, on the basic questions of matter and its transformations the philosophical literature (e.g., Aristotle (?), *Meteorologica* Bk. IV) must be consulted.

#### GEOLOGY, PHYSIOGRAPHY, AND METEOROLOGY

Adams, F. D. Birth and Development of the Geological Sciences. Baltimore, 1938.

Gilbert, Otto. Die meteorologischen Theorien des griechischen Altertums. Leipzig, 1907.

Capelle, Wilhelm. Article "Meteorologie" in RE, Supplement VI.

Lones, T. E. (see under Biology).

Lenz, H. O. Mineralogie der alten Griechen und Römer. Gotha, 1861. An anthology (German translation of sources).

#### Sources

See p. 374.

#### BIOLOGY

Singer, Charles. A Short History of Biology. Oxford, 1931.

Nordenskiöld, E. History of Biology. English translation, New York, 1928.

Locy, W. A. The Growth of Biology. New York, 1925.

Singer, Charles. "Greek Biology and Its Relations to the Rise of Modern Biology." Studies in the History and Method of Science, ed. by C. Singer, vol. 2, Oxford, 1921.

Senn, Gustav. Die Entwicklung der biologischen Forschungsmethode in der Antike. Aarau,

Thompson, D'Arcy W. On Aristotle as a Biologist. Oxford, 1916.

Lones, T. E. Aristotle's Researches in Natural Science. London, 1913. The larger part is concerned with Aristotelian biology, though the non-biological sciences are also treated.

#### BOTANY

Meyer, Ernst H. F. Geschichte der Botanik. 4 volumes. Königsberg, 1854-1857.

Reed, Howard Sprague. A Short History of the Plant Sciences. Waltham, Mass., 1942. Greene. Edward L. Landmarks of Botanical History. Washington, 1909.

Strömberg, Reinhold. Theophrastea. Studien zur botanischen Begriffsbildung. Göteborg, 1937.

Bretzl. Hugo. Botanische Forschungen des Alexanderzuges. Leipzig, 1903.

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See pp. 394, 400, 438,

Sources

Botany. Theophrastus, History of Plants and Causes of Plants (ed. by F. Wimmer: English translation of the History by A. Hort and of Book I of the Causes by R. E. Dengler). Treatise On Plants of the Aristotelian Corpus (ed. by E. H. F. Meyer, English translations by E. S. Forster and W. S. Hett); Dioscorides (ed. by M. Wellmann, English translation by J. Goodyer, German translation by J. Berendes); Pliny, Natural History, especially Books 12-27. Note in addition Vergil (Georgics), Varro, Columella, Ps.-Democritus, the minor geoponic authors, and the medical authors who discuss pharmacology.

Zoology. Aristotle's History of Animals, Parts of Animals, Motion of Animals, Progression of Animals, and Generation of Animals; Pliny's Natural History, Books 8-11, Aelian's treatise On Animals, the poem Halieutica attributed to Ovid, and the anonymous Physiologus are the chief extant works. The veterinary literature may also be noted in this connection. (See Corpus Hippiatricorum Graecorum, ed. by E. Oder and C. Hoppe, 2 volumes, Leipzig, 1924, 1927; Mulomedicina Chironis, ed. by E. Oder, Leipzig, 1901; and Vegetius, Digesta Artis Mulomedicinae, ed. by E. Lommatzsch, Leipzig, 1903.)

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Singer, Charles. Greek Biology and Greek Medicine. Oxford, 1922.

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#### Sources

See p. 467 and bibliography of M. Drabkin. A series of definitive texts is in course of being published in the Corpus Medicorum Graecorum and Corpus Medicorum Latinorum.

Among the more important Greek and Roman writers on medicine (and their modern editors) we may mention Aëtius of Amida (Olivieri, still incomplete, separate books by Hirschberg, Costomiris, Zervos, others); Aretaeus (Adams, Hude); Caelius Aurelianus (Amman); Cassius Felix (Rose); Celsus (Marx); Diocles (Wellmann); Galen (Diels, Marquardt, Helmreich, Kalbsleisch, Müller, Simon, others; the edition by C. G. Kühn is not completely superseded); Hippocratic Collection (Littré, Kuehlewein, Heiberg, Jones, Withington, others); Oribasius (Bussemaker and Daremberg, Raeder); Paul of Aegina (Heiberg); Rufus of Ephesus (Daremberg and Ruelle); Soranus (Ilberg); Theodore Priscian (Rose); Fragments of the Empiric School (Deichgräber); of the Pneumatic School (Wellmann).

Note the following important English translations.

The Genuine Works of Hippocrates. Trans. by F. Adams. London, 1848. (Reprinted New York, 1929.)

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---- On Ancient Medicine. Ed. and trans. by W. H. S. Jones, in Philosophy and Medicine in Ancient Greece. Bulletin of the History of Medicine, Suppl. 8, 1946.

The Medical Writings of Anonymus Londinensis. Ed. and trans. by W. H. S. Jones, Cambridge, 1947.

Galen. On the Natural Faculties. Ed. and trans. by A. J. Brock. London, 1916. (Loeb Classical Library.)

Galen. On Medical Experience. Ed. and trans. by R. Walzer. Oxford, 1944.

Celsus. On Medicine. Ed. and trans. by W. G. Spencer. 3 volumes, London, 1935-1938. (Loeb Classical Library.)

The Extant Works of Aretaeus. Ed. and trans. by F. Adams. London, 1856.

The Seven Books of Paulus Aegineta. Trans. by F. Adams. 3 volumes, London, 1844-1846.

#### PHYSIOLOGICAL PSYCHOLOGY

See p. 530, where the chief primary sources are indicated. J. I. Beare, Greek Theories of Elementary Cognition from Alemaeon to Aristotle (Oxford, 1906) and G.M. Stratton, Theophrastus and the Greek Physiological Psychology before Aristotle (London, 1917) are important works in this field.

# ADDENDA

P. 80 (last line before footnotes). I.e., Euclid's Elements. We adopt Halley's emendation of the Greek text.

P. 80, n. 2. Although Guldin's Theorem is now sometimes called Pappus's Theorem, the latter term is usually employed to designate a quite different theorem of Pappus (Mathematical Collection VII. 139): If A, B, C are any three distinct points on a line l, and A', B', C' any three distinct points on a line l' intersecting l, the points of intersection of BC' and B' C, of CA' and C' A, and of AB' and A' B are collinear. This theorem and outgrowths of it have played an important part in projective geometry and in modern discussions of the foundations of geometry.

P. 82, n. 1. In modern times isoperimetric problems and their generalizations have had a central role in the development of the Calculus of Variations.

P. 128. Although Ptolemy in his mathematical exposition is generally concerned with the geometry and trigonometry of circles, his system, like that of his predecessors from the time of Eudoxus (and his successors until Kepler), is fundamentally a system of spheres. We have alluded (p. 102) to the question whether the spheres have physical reality, a question about which there was considerable discussion, particularly in medieval times. It was not until after Brahe's observations of planetary orbits (which would require that certain of the spheres intersect) and of the paths of comets (which would penetrate planetary spheres) that the idea of the physical reality of the spheres was seriously shaken; Kepler's work definitely ended the controversy.

The order of the heavenly bodies in Ptolemy's system, proceeding outward from the earth, was: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, fixed stars. But because he did not observe any planetary parallax, Ptolemy was unable to determine the distances of the planets from the earth; his system merely gave a ratio between the perigee and the apogee distances of any particular planet. But from first beginnings in the fifth century or earlier, the idea developed, primarily among Arab astronomers, that the apogee distance of any planet was just exceeded by the perigee distance of the next outer planet. Since the distance from the earth to the first heavenly body, the moon, could be found with some accuracy, the distance of every other heavenly body could also, on this theory, be found.

The history of this theory, which is linked with the philosophic concept of a full universe and with the controversy about the physical reality of the spheres, is traced by Edward Rosen in Scientific Monthly 63 (1946) 213-217.

P. 130 (top). Actually a planetary orbit is not an absolutely perfect ellipse; this is due to the presence of bodies other than the planet and the sun. Now planetary motion could theoretically be described, to any required degree of approximation, by sufficiently complex combinations of circular motions. But the great advantages of Kepler's system over Ptolemy's lay in the relative simplicity of its structure and in its making possible the unification of celestial and terrestrial mechanics under the same dynamical principles.

P. 130 (Star Catalogue). In classifying stars into six magnitudes, from the brightest stars (first magnitude) to those barely visible to the unaided eye (sixth magnitude), Ptolemy