13 Angles of a polygon

You should know about the sum of angles around a point, on a line and in a triangle.

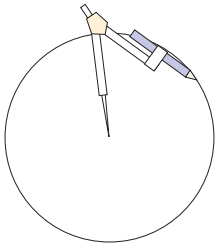
This work will help you use the interior and exterior angles of a regular or irregular polygon.

You need a pair of compasses and an angle measurer.

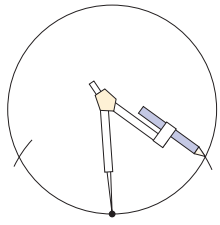
A Angles of a regular polygon

T

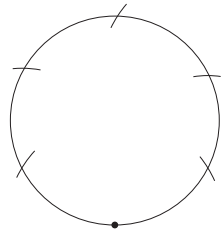
A **regular polygon** is one that has all its angles and all its sides equal.
Follow these instructions to draw a regular hexagon inside a circle.



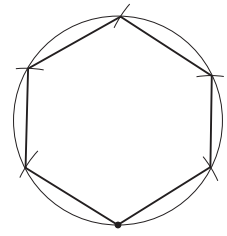
Draw a circle of radius 6 cm.



Mark a point on the circle and two spaced 6 cm each side.



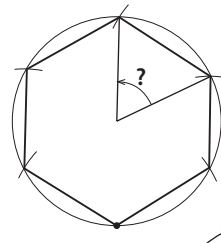
Repeat, using each mark as the centre for another mark.



Join the marks up to make a hexagon.

Join the vertices of the hexagon to the centre.

- How many of these angles are there at the centre?
- What must the sum of these angles be?
- So how big is each of the angles? Check by measuring.

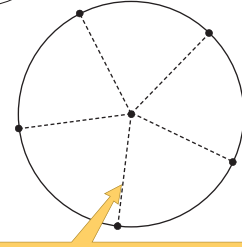


A1 For a regular pentagon you need five points spaced equally around a circle.

(a) When lines are drawn from the points to the centre of the circle, how many angles are there at the centre?

(b) What must these angles add up to?

(c) Work out the size of one angle at the centre.



Each of these lines is a **radius** of the circle (plural **radii**).

A2 Copy and complete this table.

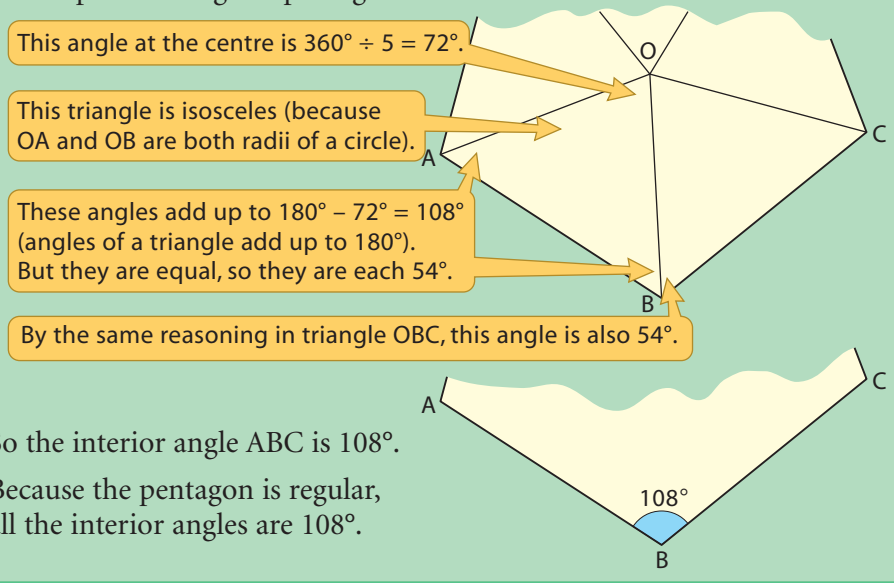
Regular polygon	Pentagon	Hexagon	Octagon	Decagon	Dodecagon
Number of sides	5	6	8	10	12
Angle at the centre					

A3 Use your table from A2 to draw each of the regular polygons in a 6 cm radius circle.
Use an angle measurer to draw the angles at the centre.

You should have found that in a regular polygon each angle at the centre is $360° \div n$, where n is the number of sides of the polygon.

You can use this fact to find the **interior angles** in a regular polygon.

This is part of a regular pentagon with centre O.



So the interior angle ABC is 108°.

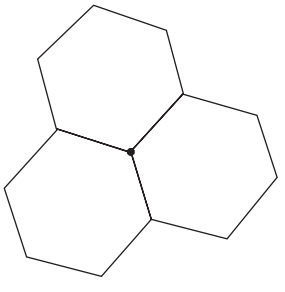
Because the pentagon is regular, all the interior angles are 108°.

A4 Copy this table and use the approach above to complete it.

Regular polygon	Pentagon	Hexagon	Octagon	Decagon	Dodecagon
Size of interior angle	108°				

Measure the interior angles of the polygons you drew in A3 and see how well they correspond to these answers.

A5 In this question you will use the table you completed for A4. So first make sure that it is correct.



(a) Three regular hexagons will fit exactly round a point. Explain why.

(b) Now decide whether each of these combinations of shapes will fit exactly round a point, giving a reason in each case.

(i) Three regular octagons

(ii) Six equilateral triangles

(iii) Two regular octagons and a square

(iv) A regular octagon, a regular hexagon and a regular pentagon

(v) Two regular hexagons and two equilateral triangles

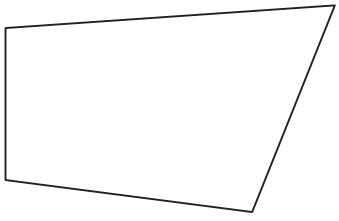
(vi) Two regular dodecagons and an equilateral triangle

(vii) Two regular decagons and a regular pentagon

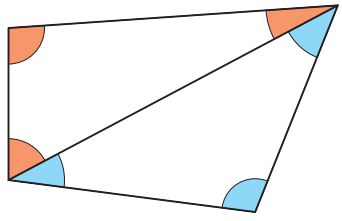
(viii) Two regular pentagons and a regular decagon

B Interior angles of a quadrilateral

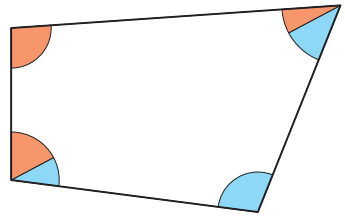
A **quadrilateral** is a shape with four straight sides.



A quadrilateral can be divided into two triangles. The sum of the angles in each triangle is 180°.

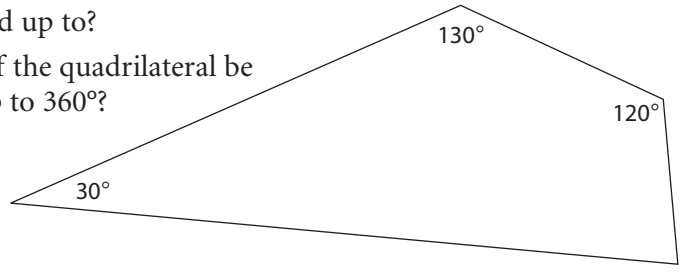


The angles of the two triangles make up the interior angles of the quadrilateral. So the interior angles of a quadrilateral add up to 360°.

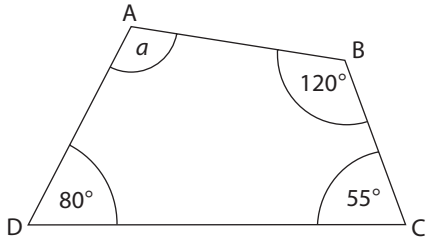


B1 **(a)** What do these three angles add up to?

(b) What must the fourth angle of the quadrilateral be to make all four angles add up to 360°?



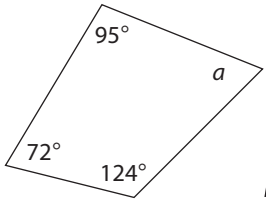
B2 ABCD is a quadrilateral.
Work out the value of *a*.

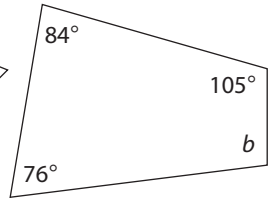


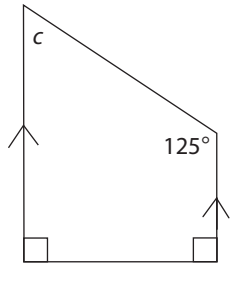
Not drawn accurately

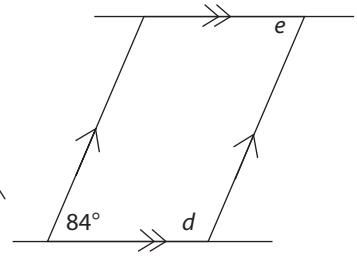
AQA

B3 Find the angles labelled by letters in these quadrilaterals.





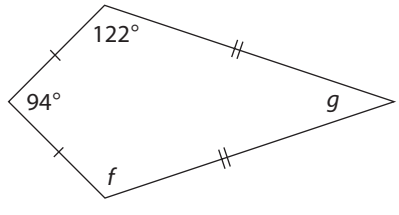




B4 This diagram shows a kite.

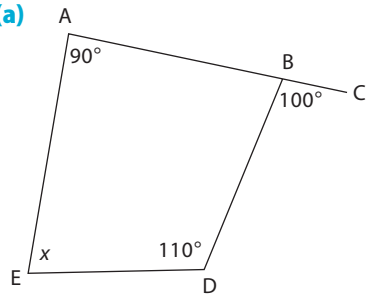
(a) What is angle *f*?
Explain how you know.

(b) Calculate angle *g*.

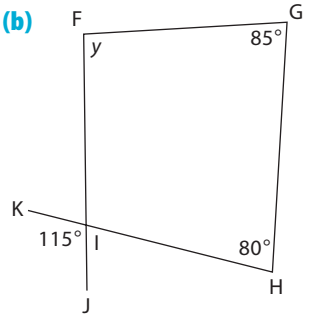


B5 Find the angles marked x, y and z.
Explain how you worked out each angle.

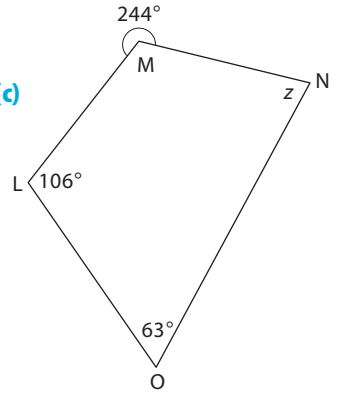
(a)



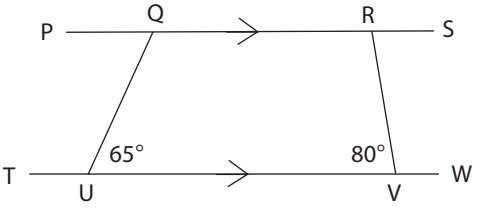
(b)



(c)



B6 In this diagram lines PS and TW are parallel.



(a) What kind of quadrilateral is QRVU?

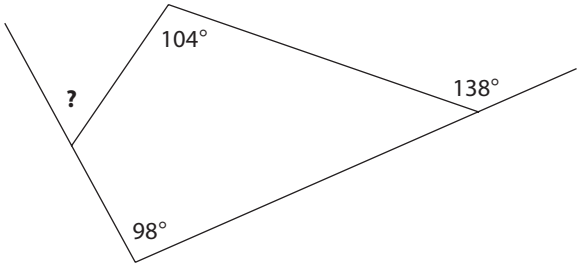
(b) What is the size of angle PQU?

(c) What is the size of angle RQU?

(d) What is the size of angle QRV?

(e) Check that the sum of the angles of quadrilateral QRVU is what you expect.

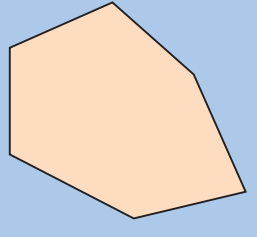
B7 Find the missing angle.



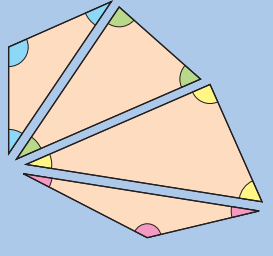
C Interior angles of any polygon

T

This is an irregular hexagon.



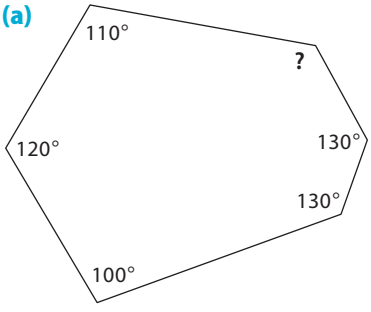
Here it has been split into triangles by cutting from one vertex to the other vertices.



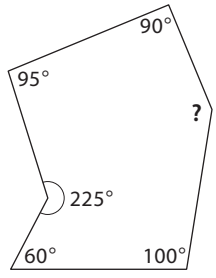
- How many triangles has the hexagon been split into?
- What is the sum of all the interior angles of these triangles?
- What is the sum of the interior angles of the hexagon?

C1 Find the missing angle in each of these hexagons.

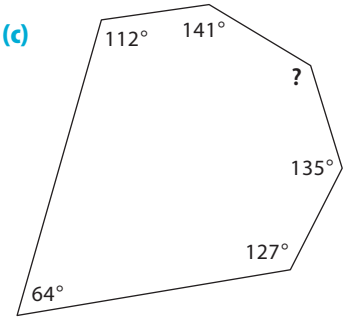
(a)



(b)



(c)



C2 In question A4 you should have found that each interior angle of a regular hexagon is 120°.
Use this to find the sum of the interior angles of a regular hexagon.

Check that this answer agrees with sum of the angles of a hexagon that you used in question C1.

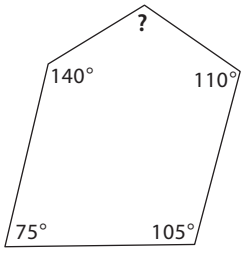
C3 **(a)** Draw a pentagon with a ruler: it does not have to be regular.
Draw lines from a single vertex to all the other vertices.

(b) How many triangles are there inside your pentagon?

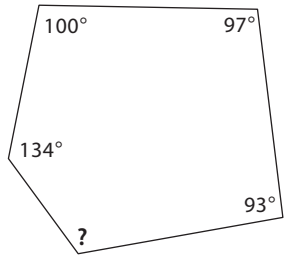
(c) What is the sum of all the interior angles of a pentagon?

C4 Find the missing angle in each of these pentagons.

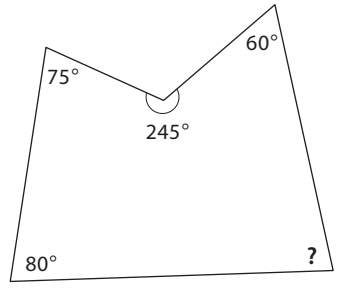
(a)



(b)



(c)



C5 On page 100 you saw that each interior angle of a **regular** pentagon is 108°.
Use this to find the sum of the interior angles of a regular pentagon.

Check that this answer agrees with sum of the angles of a pentagon that you found in question C3.

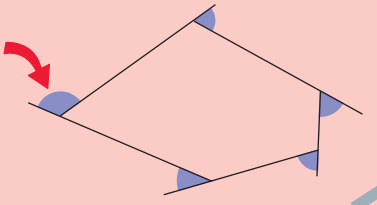
***C6** **(a)** If a polygon is split into triangles by lines drawn from a single vertex to all the other vertices, what rule connects the number of sides of the polygon with the number of triangles produced? (Draw sketches and experiment.)

(b) What rule connects the number of sides of a polygon with the sum of its interior angles?

D Exterior angles of any polygon

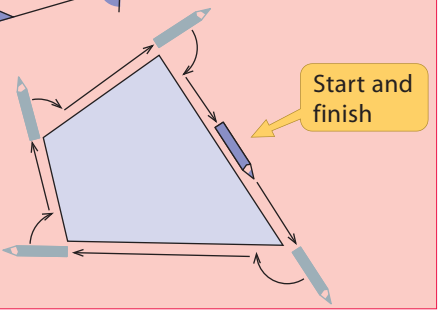
T

If you extend a side of a polygon, the angle made is called an **exterior angle**.

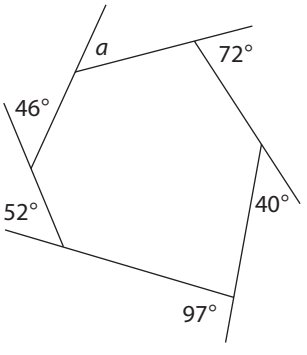


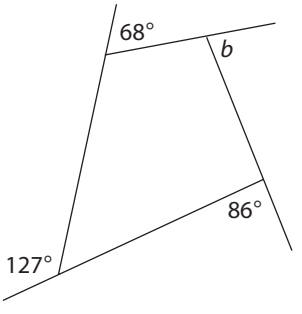
If a pencil is moved around the sides of a polygon, at each vertex it turns through the exterior angle.

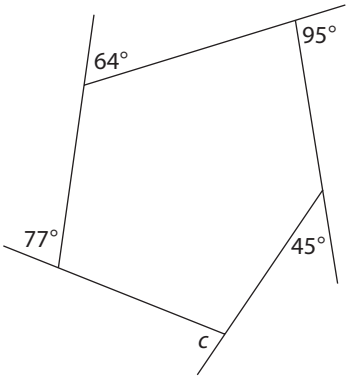
- When the pencil gets back to where it has started from, it will point in the same direction as before. What angle has it turned through?
- What is the sum of the exterior angles of a polygon?



D1 Find the angles labelled by letters.



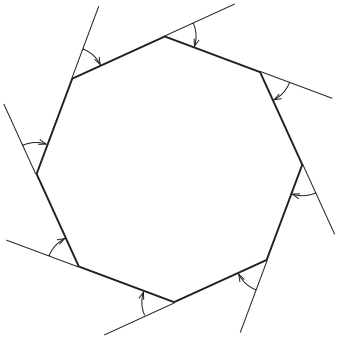




D2 This diagram shows a regular octagon.

(a) How many exterior angles are shown here?

(b) What must be the size of one exterior angle?

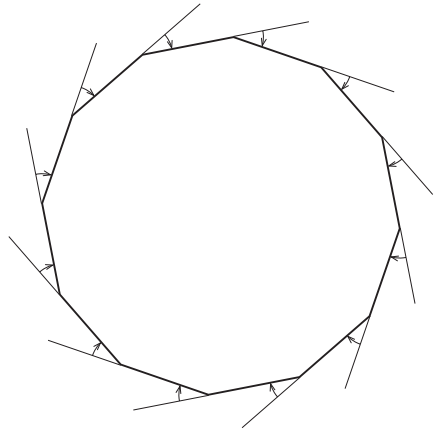


D3 In section A you should have found that each **interior** angle of a regular octagon is 135°.
With the help of a diagram, explain how this value relates to the size of an exterior angle that you have just found.

D4 This is a regular 12-sided polygon (a dodecagon).

(a) How many exterior angles are shown here?

(b) What must be the size of one exterior angle?



D5 Copy this table. Calculate the missing exterior angles and complete the table.

Regular polygon	Pentagon	Hexagon	Octagon	Decagon	Dodecagon
Size of exterior angle			45°		30°

D6 Each exterior angle of a certain regular polygon is 40°.
How many sides must this polygon have?

D7 How many sides does a regular polygon have if each exterior angle is

(a) 9° **(b)** 24° **(c)** 10° **(d)** 18°

E Mixed questions

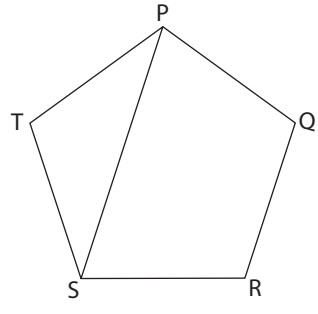
E1 PQRST is a regular pentagon.

(a) Calculate angle STP, giving your reasons.

(b) What kind of triangle is triangle STP?

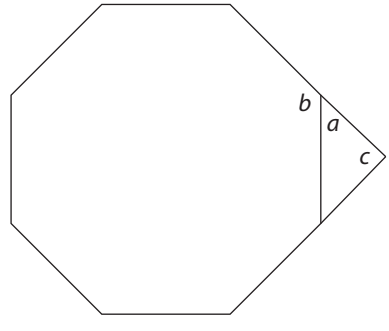
(c) Calculate angle TSP, giving your reasons.

(d) Calculate angle PSR, giving reasons.

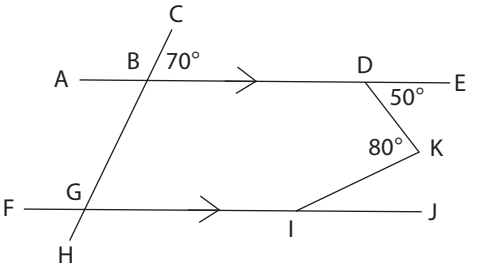


E2 Two sides of this regular octagon have been extended to make a triangle on one of the sides.

Find angles *a*, *b* and *c*.

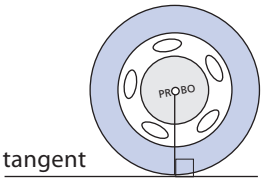


E3 Calculate the size of angle GIK, giving full reasons for your answer.



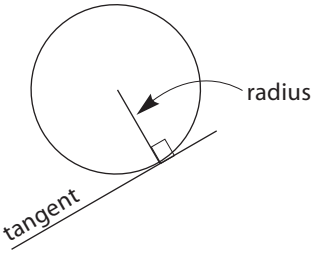
A **tangent** is a line that touches a circle at one point only.

Think of a wheel touching horizontal ground.
The centre of the wheel is above the point of contact, so the radius is at right angles to the horizontal tangent.



If the entire diagram is rotated it will make no difference to the fact that **the angle between the radius and tangent is a right angle**.

You will need this fact in the following question.



E4 The sides of the triangle ABC are tangents that touch a circle at points P, Q and R as shown.
O is the centre of the circle.

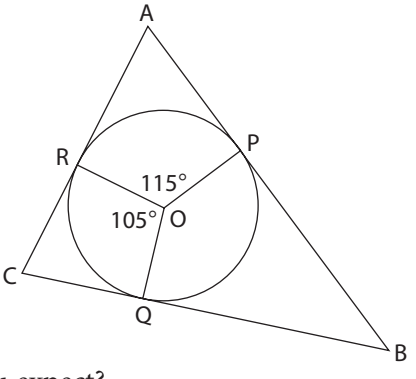
(a) Give the sizes of angles ARO and APO, explaining your reasons.

(b) By considering the angles of the quadrilateral APOR, calculate angle RAP.

(c) Find angle RCQ.

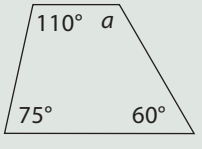
(d) Find angle POQ and hence find angle PBQ.

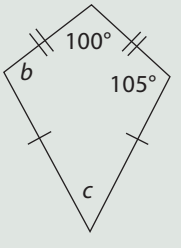
(e) Do the three angles of the triangle have the total you expect?



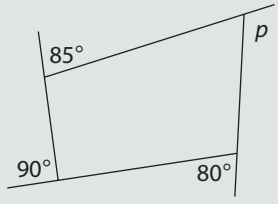
Test yourself

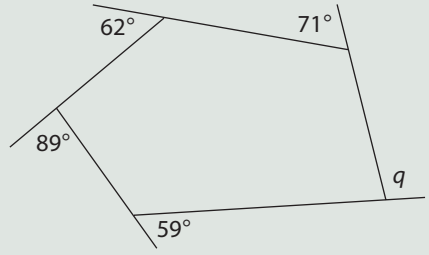
T1 Find the angles labelled by letters in these quadrilaterals.



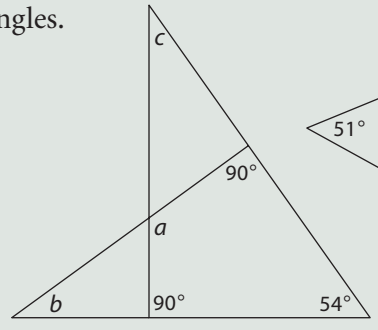


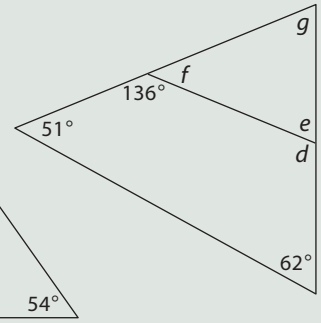
T2 Find the angles labelled by letters.





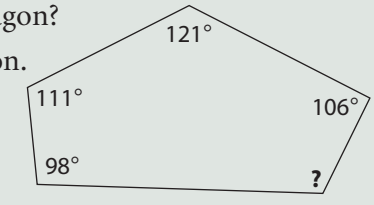
T3 Calculate the lettered angles.





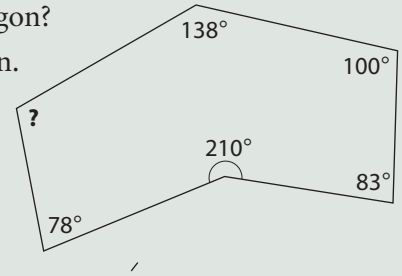
T4 **(a)** What is the sum of the interior angles of any pentagon?

(b) Calculate the missing interior angle in this pentagon.



T5 **(a)** What is the sum of the interior angles of any hexagon?

(b) Calculate the missing interior angle in this hexagon.

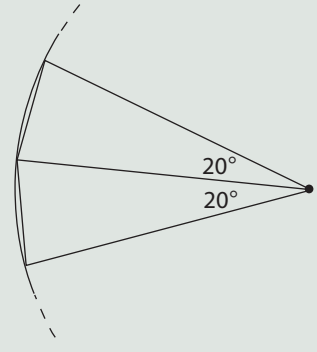


T6 Katya has started to draw a regular polygon by marking points round a circle.

(a) How many sides will her polygon have?

(b) What size will each exterior angle of her polygon be?

(c) What size will each interior angle be?



T7 Each exterior angle of certain regular polygon is 8°.

(a) How many sides does this regular polygon have?

(b) What is the size of an interior angle of this polygon?