Cowles Foundation Paper 465

Vol. 37 (1977), No. 3-4, pp. 337—354

Zeitschrift für
Nationalökonomie
Journal of Economics

The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money*

By

Martin Shubik** and Charles Wilson, New Haven, Conn., U. S. A.

(Received August 18, 1977)

1. Introduction

In several previous papers [1, 2, 3, 4, 5, 6, 7] models of a monetary economy have been solved as a noncooperative game. This problem of granting credit and the possibility of bankruptcy was avoided by the artifact of considering that all traders were supplied with "enough" of a commodity serving as a "money" or means of payment so that there was no need to borrow.

In this paper an outside bank, and borrowing are considered explicitly and the meaning of an optimal bankruptcy rule is considered. We stress that if credit or paper money are introduced into an economy described as a game of strategy, rules describing penalties to be levied against those who cannot pay back what they have borrowed, become a logical necessity in order to fully define all possible outcomes. Our approach is to specify such rules and study them parametrically; i. e. we carry out a sensitivity analysis to see what happens as the severity of the penalties is varied.

* This work relates to Department of the Navy Contract N00014-76-C0085 issued by the Office of Naval Research under Contract Authority NR 047-007. However, the content does not necessarily reflect the position or the policy of the Department of the Navy or the Government, and no official endorsement should be inferred.

The United States Government has at least a royalty-free, nonexclusive and irrevocable license throughout the world for Government purposes to publish, translate, reproduce, deliver, perform, dispose of, and to authorize others so to do, all or any portion of this work.

** The authors wish to thank Pradeep Dubey and Donald Brown for helpful conversations. This work builds not only on one of the author's previous work but on joint work with Shapley, Dubey, Whitt and Evers.

This paper deals primarily with problems in modelling and interpretation. Thus the argument is carried out in terms of a specific relatively simple example to illustrate the market, banking and bankruptcy mechanism. General proofs pertaining to a broad class of trading models are given in a separate paper [8].

2. The Model

The model is a varient of the model originally suggested by Shubik [1] and investigated by Shapley [3], Shapley and Shubik [9], Shubik [10, 11] and Dubey and Shubik [5, 6]. The paper here however is self contained inasmuch as a complete model is built, although the references noted provide detailed discussion of some aspects of the model and proofs which are not supplied here.

2.1. A Trading Economy Without Uncertainty

The procedure adopted here is to begin by taking a simple model of trade. This is formulated and solved for the standard competitive equilibrium solution. We then take the same economic background and model trade as a noncooperative game with a bank issuing loans to finance trade.

We solve the game for a type symmetric noncooperative equilibrium point (T. S. N. E.). This is an equilibrium point at which traders of the same type obtain equal treatment. Equal treatment is not necessarily a property of a noncooperative equilibrium. We study the conditions under which the T. S. N. E. coincides or fails to coincide with the C. E. in terms of market prices and distribution of resources.

Consider 2 n traders trading in two commodities, n have endowments of $(A, 0)$ and n have endowments of $(0, B)$. Traders of the first type have utility functions of the form

$$u^1 = \log x_1^{\alpha} y_1^{1-\alpha} \tag{1}$$

and the second type

$$u^2 = \log x_2^{\beta} y_2^{1-\beta}. \tag{2}$$

2.2. The Competitive Equilibrium and Pareto Optimal Surface

It is easy to solve for the unique competitive equilibrium and the Pareto optimal surface. We obtain:

$$p_1 = 1,\ p_2 = \left(\frac{1-\alpha}{\beta}\right)\left(\frac{A}{B}\right),\ x = (1-\alpha)\ A,\ y = \beta B, \tag{3}$$

$$\lambda_1 = 1/A \ \text{and}\ \lambda_2 = \beta/A\ (1-\alpha),$$

where $(A-x)$ and y are the consumptions of trader 1; λ_1 and λ_2 are Lagrangian multipliers and p_1 and p_2 are prices.

The Pareto optimal surface is given by

$$\left\{\frac{\alpha-\beta}{\beta(1-\alpha)}\right\} xy = AB - Bx - Ay. \tag{4}$$

2.3. The Money Game

2.3.1. The Trading Mechanism

Let the amount bid by a trader i of type 1 be b^i and by a trader j of type 2 be d^j. Bids are made in quantities of bank money, to be defined later. As traders of type 1 only have the first commodity we may assume that they can offer only it for sale. Let the amount of the first commodity offered for sale by a trader i of the first type be x^i. A trader j of the second type offers an amount y^j of the second commodity.

Let the symbol $b = \sum_{i=1}^{n} b^i$ and similarly for the others.

We may assume in this simple market that traders of type 1 bid only for the second good and traders of type 2 bid only for the first good. It has been shown elsewhere [6] that there is no loss of generality in making this assumption in large markets[1].

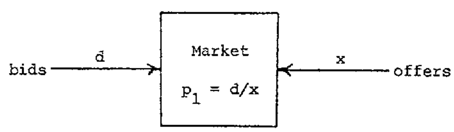

Fig. 1

The markets for the goods are extremely simple. Fig. 1 illustrates the market for the first good. All bids are aggregated (d) and all offers are aggregated (x) and the price is fixed by dividing the amount of good offered.

There are several other price formation mechanisms which could have been selected. These are discussed elsewhere [11]. This was selected because of its basic simplicity.

---

[1] When numbers are few, "wash sales" may be of importance, but we leave this problem aside at this time. A "wash sale" occurs when an individual simultaneously sells and buys back the same item in order to increase the thickness or activity in a market. (See Dubey and Shubik [6] for an example with wash sales.)

When the price has been formed, individual j who has bid d^j for the first good will obtain

$$z^j = \frac{d_j}{p_1} = \frac{d_j}{d} x \tag{5}$$

where

$$p_1 = \frac{d}{x} \tag{6}$$

similarly trader i of the first type obtains

$$w^i = \frac{b^i}{p_2} = \frac{b^i}{b} y \tag{7}$$

where

$$p_2 = \frac{b}{y} \tag{8}$$

2.3.2. *Banking: Credit and Promissory Notes*

In 2.3.1 we did not specify the currency in which bids are to be made. We now assume that all bids are made using a fiat or bank money which must be obtained from an "outside bank" which is modelled as a mechanism or "dummy" in the sense that it is given a fixed strategy. In particular the bank fixes an amount of money it will issue. In an economy with 2 n traders let this amount be nK.

The loan mechanism to lend out the nK units will be a simple "money market". Each trader i of type 1 is permitted to create a financial instrument of his own, to wit, a nonnegotiable promissory note of size or denomination u^i. Similarly a trader j of type 2 bids v^j.

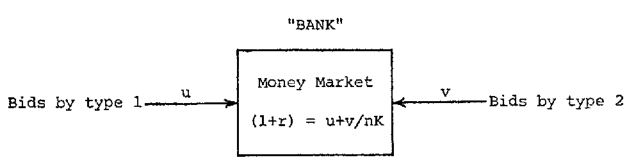

Fig. 2

The traders use the promissory notes to bid for the money supply. The meaning of a promissory note is that it is a contract between an individual i and the bank, which promises to return to the bank an amount of bank money u^i at the end of the period in return for an amount obtained at the start of the period. This amount is determined by the supply of loanable funds nK and the offers of promissory notes, i. e. if s^i and t^j are the loans obtained by i and j they are respectively:

$$s^i = \frac{u^i}{u+v}(nK) \tag{9}$$

and

$$t^j = \frac{v^j}{u+v}(nK). \tag{10}$$

The model is naively simple. Banks in general would be suspicious of borrowers wishing to promise to pay back enormous sums for loans whose size they do not know in advance. However the mechanism is well defined and in a mass market where individuals have estimates of the aggregate money supply and demand this can be interpreted more reasonably, as a single individual will not expect the price of money to move very much in response to his actions.

As can be seen from Fig. 2 the price of money is given by:

$$1 + r = \frac{u+v}{nK} \tag{11}$$

where r can be interpreted as a money rate of interest[2].

2.3.3. Settlement and Bankruptcy

After the market is over individuals obtain their final allocations of goods, obtain their money incomes and must make their final settlements at the bank.

If an individual ends up with a positive amount of money after having paid the bank, this has no positive value to him. If on the other hand he is unable to honor his debts in full, a penalty is leveled against him. This penalty does not necessarily have to be the same for all individuals. In this model it is proposed to study it parametrically. Two parameters μ_1 and μ_2 are introduced associated with a linear term[3] which measures the "punishment" leveled against anyone who in unable to repay his debts. The specific structure of the payoff functions is shown in 2.3.5 below.

We may consider the bankruptcy act as economic or not directly so. An example of the former might involve a sale of assets of the

---

[2] In a one period model the concept of an interest rate seems somewhat strange. A different way of phrasing the role of r is that it is a loss reserve payment protecting the bank against default.

[3] This term does not need to be linear, as is discussed elsewhere [8]. It is much simpler to make it linear for the example.

debtor, an example of the latter would be a prison sentence or death. In a one period model we may imagine that assets which are confiscated are sold off in a subsequent period.

Regardless of how we regard the punishment, its presence is needed if we wish to influence individuals against making exorbitant repayment commitments to increase the current loans, knowing that they will be unable to meet their obligations.

2.3.4. *The Payoff Functions*

We have seen that the utility functions of the traders [shown in (1) and (2)] involve the only two consumer commodities. We define the payoff functions in terms of strategies and outcomes including the final disposition of money. We may write the payoff to a trader i of type 1 as:

$$\Pi_1{}^i = \log\,(A - x^i)^\alpha \left\{\frac{b^i y}{d}\right\}^{1-\alpha} + \mu_1 \min\left[0, \left\{\frac{d}{x}\,x^i - b^i + \frac{u^i n K}{u+v} - u^i\right\}\right] \tag{12}$$

and the payoff to a trader j of type 2 is:

$$\Pi_2{}^j = \log\left\{\frac{d^j x}{d}\right\}^\beta (B - y^j)^{1-\beta} + \mu_2 \min\left[0, \left\{\frac{b}{y}\,y^j - d^j + \frac{v^j n K}{u+v} - v^j\right\}\right] \tag{13}$$

where the x^i and y^j are considered as functions[4] of the u^i and v^j, and where $0 \le b^i \le u^i nK/u+v$ and $0 \le d^j \le v^j nK/u+v$.

$$p_1 = d/x \quad \text{and} \quad p_2 = b/y. \tag{14}$$

We note that the bankruptcy conditions enter as linear terms, or as zero.

We assume $0 < \alpha$, $\beta < 1$ to ensure a solution with some trade. The preferences of the individuals are represented by concave utility functions. The introduction of the bankruptcy condition can be considered as though the utility function for an individual had been defined to include money holdings where nothing is added for positive holdings but debt (which may be interpreted as negative hold-

---

4 If traders have information about prior moves in a multistage market, the selection of subsequent moves may be regarded as a general function of the information state of the trader. If a trader does not know the amount of money he obtained in stage 1 he may still bid, but the bid may be interpreted as a fraction of his wealth.

ings) has a negative worth. Limiting our illustration to one real commodity and money and debt Fig. 3 shows the modified utility function extending into a negative money or debt zone.

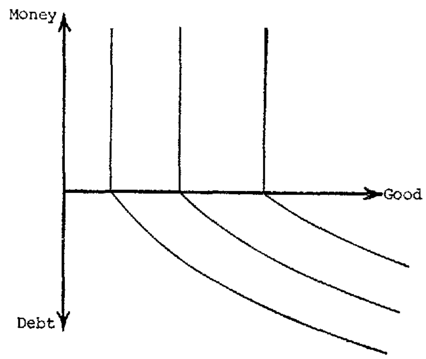

Fig. 3

2.4. The Noncooperative Equilibrium With No Information

Equilibria may be extremely sensitive to information conditions. In multistage games such as this, the less information the traders have the easier it may be to examine the equilibrium point of the games[5].

The simplest situation is where all traders are required to announce simultaneously not only u^i and v^j but[6] also b^i, d^j and x^i, y^j [7].

We may consider that a trader i of the first type attempts to maximize:

$$G_1^i(u^i, b^i, x^i) = \alpha \log (A - x^i) + (1-\alpha) \log (y b^i / b) + \lambda_1^i \{ d x^i / x - b^i + (M-1) u^i \} + \gamma_1^i (M u^i - b^i). \tag{15}$$

---

[5] In the analysis which follows, only the first order conditions for utility maximization are used to describe the equilibrium. The general proof given elsewhere [8] shows that they are noncooperative equilibria.

[6] As an individual may not be sure of his money supply before his bid b^i (d^j) is fixed, we require a feasibility or acceptability condition in the game. For example a nonfeasible bid is replaced by a zero bid or some convention is given which transforms the bid into a feasible act. The simplest convention is to interpret b^i (d^j) as percentages of money available.

[7] It is straightforward to check that degenerate equilibria always exist — that is, equilibria with x^i, $y^i = 0$ and no bankruptcy. In what follows, we will be restricting our attention to non-degenerate equilibria.

There will be a similar expression for a trader j of the second type. We define:

$$M = \frac{nK}{u+v} \geq 0, \quad u = \Sigma\, u^i \quad \text{(and similarly for the others)} \tag{16}$$

where $\lambda_1{}^i$ and $\gamma_1{}^i$ can be regarded as Lagrangian multipliers with the following conditions:

$$\text{if} \quad dx^i/x - b^i + (M-1)\, u^i \begin{cases} >0 & \text{then} \quad \lambda_1{}^i = 0 \\ =0 & \text{then} \quad 0 \leq \lambda_1{}^i \leq \mu_1 \\ <0 & \text{then} \quad \lambda_1{}^i = \mu_1 \end{cases} \tag{17}$$

$$\text{if} \quad (Mu^i - b^i) \begin{cases} >0 & \text{then} \quad \gamma_1{}^i = 0 \\ =0 & \text{then} \quad \gamma_1{}^i \geq 0. \end{cases} \tag{18}$$

First order maximization conditions give:

$$\frac{\partial G_1{}^i}{\partial x^i} = \frac{-\alpha}{A - x^i} + \lambda_1{}^i\, d \left\{ \frac{1}{x} - \frac{x^i}{x^2} \right\} = 0 \tag{19}$$

and from symmetry:

$$\frac{\alpha}{A - x^i} = \lambda_1{}^i \left(\frac{n-1}{n} \right) \frac{d}{x} \tag{20}$$

Similarly from $\partial G_1{}^i / \partial b^i = 0$ and $\partial G_1{}^i / \partial u^i = 0$ we obtain

$$(1-\alpha) \left(\frac{n-1}{n} \right) = b^i\, (\lambda_1{}^i + \gamma_1{}^i) \tag{21}$$

and

$$\lambda_1{}^i = M\, (\lambda_1{}^i + \gamma_1{}^i) \left\{ \frac{v + \left(\frac{n-1}{n} \right) u}{u+v} \right\} \tag{22}$$

From (20) and $x = nx^i$

$$x\alpha = \lambda_1{}^i \left(\frac{n-1}{n} \right) d\, (A - x^i)$$

gives

$$x^i = \frac{\lambda_1{}^i\, dA \left(\frac{n-1}{n} \right)}{n\alpha + \lambda_1{}^i\, d \left(\frac{n-1}{n} \right)} \tag{23}$$

If $\lambda_1{}^i = 0$, the only solution consistent with Eq. (23) is $x^i = 0$. But this means that each type i trader receives 0 rather than d^i units of money from type j traders. As a consequence, it is easy to verify that no solution exists for $\lambda_1^i = 0$. Therefore, (17) implies

$$d - b + (M-1)\, u \leq 0 \quad \text{and} \quad b - d + (M-1)\, v \leq 0 \tag{24}$$

or

$$(M-1)\,(u+v) \leq 0.$$

Hence if $u+v>0$ then

$$M \leq 1. \tag{25}$$

From (22) and (25) and symmetry[8] then $\gamma_1 > 0$, and similarly $\gamma_2 > 0$; hence

$$Mu = b \quad \text{and} \quad Mv = d. \tag{26}$$

From (21), (22) and (26) and symmetry:

$$u^i = \frac{1-\alpha}{\lambda_1} \left(\frac{n-1}{n}\right) \left\{ \frac{v + \left(\frac{n-1}{n}\right) u}{u+v} \right\}$$
$$v^i = \frac{\beta}{\lambda_2} \left(\frac{n-1}{n}\right) \left\{ \frac{u + \left(\frac{n-1}{n}\right) v}{u+v} \right\} \tag{27}$$

From (17) and (26):

$$\lambda_1 < \mu_1 \Rightarrow Mv = u \quad \text{and} \quad \lambda_2 < \mu_2 \Rightarrow Mu = v \tag{28}$$

and

$$Mv < u \Rightarrow \lambda_1 = \mu_1, \; Mu < v \Rightarrow \lambda_2 = \mu_2. \tag{29}$$

We now confine our attention to the limiting behavior in markets as $n \to \infty$. From (27) we obtain:

$$u^i \to \frac{1-\alpha}{\lambda_1} \quad \text{and} \quad v^j \to \frac{\beta}{\lambda_2} \tag{30}$$

We wish to consider all λ_1 and λ_2 consistent with (28) or (29).

From (16) and (30)

$$M = \frac{\lambda_1 \lambda_2 K}{\lambda_2 (1-\alpha) + \lambda_1 \beta}$$

hence

$$Mu^i = \frac{K \frac{1-\alpha}{\lambda_1}}{\frac{1-\alpha}{\lambda_1} + \frac{\beta}{\lambda_2}} \quad \text{and} \quad Mv^j = \frac{K \frac{\beta}{\lambda_2}}{\frac{1-\alpha}{\lambda_1} + \frac{\beta}{\lambda_2}} \tag{31}$$

[8] Assuming the existence of a symmetric solution we may drop superscripts from the λ and γ.

There are four cases arising from (28) and (29). They are as follows:

(A) $u^i = Mv^j$ and $v^j = Mu^i$

(B) $u^i = Mv^j$ and $v^j < Mu^i$

(C) $u^i > Mv^j$ and $v^j = Mu^i$ and

(D) $u^i > Mv^j$ and $v^j > Mu^i$.

Fig. 4 shows the four cases as μ_1 and μ_2 are varied. The calculations of the values are tedious but straightforward and are given elsewhere [12].

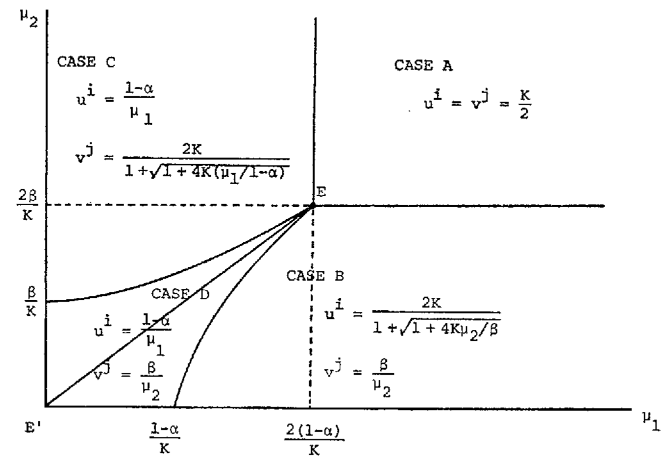

Fig. 4

In zones A and B traders of type 1 stay solvent; in zones A and C traders of type 2 stay solvent.

We observe that at the point E, $\mu_1 = 2\,(1-\alpha)/K$ and $\mu_2 = 2\,\beta/K$ where $K = 2A\,(1-\alpha)$. Hence $\mu_1 = 1/A = \lambda_1$ and $\mu_2 = \beta/A\,(1-\alpha) = \lambda_2$ which are the same as the Lagrangian multipliers obtained from solving the competitive equilibrium. Thus for these bankruptcy penalties the limit noncooperative equilibrium coincides with the competitive equilibrium.

For other bankruptcy penalties we see that if the μ_1 and/or the μ_2 are set to be less than λ_1 or λ_2 that one or both types will elect bankruptcy. If both go bankrupt and the penalties are selected such that $\mu_1 = g\lambda_1$ and $\mu_2 = g\lambda_2$ where $0 < g < 1$ then trade and prices are the same at the competitive equilibrium along the line EE' in Fig. 4

In Case A there is no bankruptcy and competitive equilibrium trade takes place.

When bankruptcy occurs off the ray EE', the final distribution of trade is not the same as at the competitive equilibrium. It lies on the Pareto optimal surface of the original trading model [see Eq. (4)]. This can be proved generally [8].

2.4.1. *A Finite, Infinite Game Solution Distinction*

We can see from (22) and (25) that for a finite n hoarding cannot take place. This is not so if there were a continuum of nonatomic traders. If each trader felt that his influence on the aggregates were negligible a new solution appears.

Suppose each trader takes x, y, b, d, and M as given, and chooses his optimal x^i, b^i and u^i accordingly. Then the first order conditions for a maximum [given by Eqs. (19), (21), and (22)] simplify to:

$$x^i = \frac{A\lambda_1{}^i d - \alpha x}{\lambda_1{}^i d} \tag{32}$$

$$b^i = \frac{1-\alpha}{\lambda_1{}^i + \gamma_1{}^i} \tag{33}$$

$$\gamma_1{}^i = \frac{1-M}{M}\lambda_1{}^i \tag{34}$$

Using (34), (33) becomes:

$$b^i = M\frac{1-\alpha}{\lambda_1{}^i} \tag{35}$$

Adding (32) n times and simplifying yields:

$$x^i = \frac{A\lambda_1{}^i d}{n\alpha + \lambda_1{}^i d}$$

From Eq. (15), $b^i > 0$ implies $\lambda_1{}^i > 0$, and therefore (34) requires $M \leqq 1$ to keep $\gamma_1{}^i \geqq 0$. Note that $\gamma_1{}^i = 0$ if and only if $M = 1$.

Suppose $M = 1$, then $u^i + v^j = K$. From (35) and an equivalent expression for type 2 traders, $b^i \leqq Mu^i$ and $\lambda_1{}^i \leqq \mu_1$ then imply

$$\frac{1-\alpha}{\mu_1} + \frac{\beta}{\mu_2} = K \tag{36}$$

Furthermore, (24) implies that $b^i = d^j$. Therefore,

$$\frac{1-\alpha}{\lambda_1} = \frac{\beta}{\lambda_2} \tag{37}$$

which does not violate (36) only if:

$$\frac{1-\alpha}{\mu_1}, \frac{\beta}{\mu_2} \leq \frac{K}{2} \tag{38}$$

One can verify that $M=1$ if and only if (38) holds. If (38) does not hold, then the limiting results of the previous section hold. However if (38) is satisfied, not only do both types stay solvent, but the possibility of hoarding occurs. In this case the following values x^i, y^j, b^i, d^j, u^i, v^j satisfy the conditions for a noncooperative equilibrium.

$$x^i = A\ (1-\alpha);\ \ y^j = \beta B \tag{39}$$

$$b^i = d^j \geqq \max\left(\frac{1-\alpha}{\mu_1}, \frac{\beta}{\mu_2}\right) \tag{40}$$

$$u^i + v^j = K;\ \ u^i, v^j \geqq b^i = d^j. \tag{41}$$

Not only may there be an indeterminate solution for u and v, but even when u and v is specified any values of b and d satisfying (40) and (41) will generate a noncooperative equilibrium. The resultant relative prices and distribution coincide with the competitive equilibrium, but the price level may be low enough that not all of M is used.

3. Two Stage Noncooperative Equilibria

3.1. The Extensive Form, Information and Strategies

When we assume that individuals are informed about some aspects of what happened at the first stage of the game prior to selecting their moves in the second stage we may set up many different games which differ only in the shading of information. Two of these are illustrated.

Case 1: The game is played as follows: First, all individuals simultaneously bid for loans. They obtain no information beyond the size of the loan they have secured. After they have obtained their loans they then all bid simultaneously in the markets for goods.

If the individuals know the size of their own loans, as they know their own bids they can calculate the price of money. And if they all know the size of the total money supply they can calculate the aggregate amount of loans obtained by their competitors.

Let $\bar{u}^i$ signify $u+v-u^i$ and $\bar{v}^j$ signify $u+v-v^j$. Then a strategy for a trader i of type 1 is a number u^i and two functions $x^i = \phi_1{}^i(u^i, \bar{u}^i)$, $b^i = \phi_2{}^i\ (u^i, \bar{u}^i)$ and similarly for a trader j of tape 2.

Case 2: If we wished we might assume that trader i is given complete information as to who has borrowed how much. In this case a strategy would be a number u^i and two functions

$$x^i = \phi_1{}^i\,(u^1, u^2, \ldots, u^n;\ v^1, v^2, \ldots, v^n)$$

and

$$b^i = \phi_2{}^i\,(u^1, u^2, \ldots, u^n;\ v^1, v^2, \ldots, v^n).$$

In the first instance the trader bases his plan on *macroeconomic statistics,* in the second, on *microeconomic* detail. The refinement of information in a general noncooperative game could easily create many new equilibrium points associated with a noncooperative solution.

Consider Case 2, do we have guidance as to how to select strategies? Fortunately there is a simple way to pick these functions. Assume that u^i and v^j, $i = 1, \ldots, n$, $j = 1, \ldots, n$ are given. Any individual i will select the pair (x^i, b^i) by maximizing his payoffs in the one stage game. Any individual j will select the pair (y^j, d^j) in the same manner. We may solve for x^i, y^j, b^i, d^j as functions of the u^i and v^j then solve the bidding-for-loans stage of the market by maximizing with respect to (*wrt*) u^i and v^j.

3.2. Threats and Two Stage Equilibria

In 3.1 we have noted that a strategy in a two stage game can consist of a number in the first period and a function depending upon the information concerning the moves of all others in the first period. The extreme generality of this function enables traders to convey highly implausible threats which may nevertheless give rise to new (and improbable) equilibrium points. Thus in a two stage game, in general, the problem is not with existence of noncooperative equilibria but with a surfeit of them.

Is it possible to distinguish "plausible" or nice equilibria from the others? As yet there does not seem to be a completely general satisfactory way to do so[9]. However, we could use the type of backward solution used in dynamic programming. Unfortunately there are two basic difficulties in doing so. One concerns the information conditions. We need perfect information between the stages so that subgames can be well defined. It is likely that by making use of the special structure of these economic games where it is possible to aggregate moves we could weaken the information requirements, however this is not explored further here. The second difficulty concerns uniqueness. If we wish to replace the second

[9] Although the work of Harsanyi [13] has progressed on this problem.

stage games by the values of their noncooperative equilibria we need them either to be unique, or at least we require some sort of indexing scheme which enables us to associate the equilibrium points we select by some natural property such as continuity. Fortunately in the simple example considered here by appealing to continuity we can choose a unique value of the payoff function so that a backward solution is well defined.

3.3. Symmetric Perfect Equilibria

We have calculated the perfect equilibria for this example. The details of the second stage calculations of the backward solution together with a sketch of the first stage are given elsewhere [12].

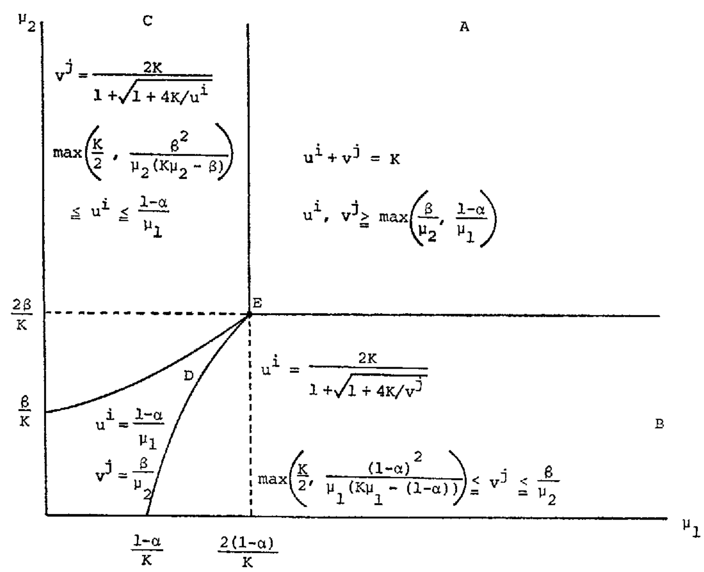

Fig. 5

The results are highly related to the previous analysis, but somewhat different as is shown in Fig. 5 which should be compared with Fig. 4. The qualitative differences of note are that even for the finite model hoarding may take place in regions A, B and C.

4. So What?

Recently there has been considerable interest in the construction of a microeconomic theory with money and with the reconciliation of micro- and macro-economics. There are currently several different approaches as evinced by the work of Clower [14], Hahn [15], Starr [16], Grandmont and Laroque [17] and many others. These approaches differ from each other and from the approach adopted here in both the basic questions asked and models constructed.

The approach adopted here calls for extremely detailed modelling up to the point that the test of completeness and consistency is: is the model sufficiently well specified that it could be played as a game? — i. e. are there rules to describe all moves and cover all contingencies?

In this paper we have labored through a large amount of tedious calculation on a special example in order to at least begin to disclose the fine structure of a general class of economic problems that we believe are of importance.

In particular even though our model may appear to be special certain phenomena have been encountered and problems solved.

(1) Because we wished to construct a model of trade with simultaneous independent bids and offers with a price formation mechanism defined for all positions of equilibrium and disequilibrium we needed to specify bids and offers and market clearance in equilibrium or disequilibrium. When numbers of competitors are large, many mechanisms lead to the same noncooperative outcome [1, 6, 10].

(2) In a previous model [1, 9] it was shown that a mechanism would work if one commodity were used to bid for the others. For the outcome of trade in a mass market to yield results comparable to the competitive equilibrium enough commodity money, appropriately distributed is required.

(3) If there is not enough commodity money present the outcomes will be nonoptimal unless credit is introduced. There are many ways of introducing credit. One is to imagine that each trader has an unbounded open credit line at a bank [9, 18, 19]; another one, adopted here, is to imagine an outside bank which auctions off a fixed amount of its "money" or universally accepted I. O. U. notes in return for individual traders' I. O. U. notes.

(4) No matter which method is adopted for the issue of credit, a rule must be adopted to prevent individuals from issuing unbounded quantities of I. O. U. notes. This rule is embodied here in the bankruptcy penalty.

(5) As there does not appear to be an *a priori* reason for the selection of a specific penalty we have studied the penalty parametrically and have observed that for certain penalty values we

obtain results from the noncooperative game which are comparable with the competitive equilibrium. For other penalties different results are obtained.

(6) We may interpret the ratio of the amount of personal I. O. U.'s offered for bank money as specifying a money interest rate. When the noncooperative and competitive trades coincide without any bankruptcies taking place the money rate of interest is zero, i. e. the credit supplied to finance trade has been provided at no cost beyond the requirement that settlement after trade is on a one for one basis.

(7) The zero rate of interest appears to be most reasonable when we observe that there is no time discount and no production. The only purpose of the outside money is to finance *the float* created by the requirement that individuals bid and offer simultaneously. The creation of the float enables each individual to view the process sequentially, i. e. he borrows, pays for purchases and then obtains income and settles debt. Price formation however appears to take place simultaneously in the "clearing houses" matching bids and offers.

(8) As this is a sequential process the meaning of strategy, the feasibility of moves and the dependence of the process on information conditions becomes crucial. For this reason we have included a brief discussion of the two stage equilibria.

To most of us money and credit are more naturally associated with uncertainty, multiperiod trade and production [20]. Even in this one period model of trade, the float, bankruptcy penalties and the possibility of hoarding appear. A more satisfactory model with many periods and production is being considered — but new phenomena and difficulties appear. In particular (setting aside extra problems due to uncertainty) the model presented here can be immediately generalized for k time periods [8] if we are satisfied with a solution in which intertemporal prices are adjusted by hoarding outside money issued at period 1. This, though logically correct does not appear to be satisfactory. A way of avoiding this is to introduce an inside bank as well as an outside bank. After the traders bid for the outside money supply M using their I. O. U. notes, they then bid for shares in an inside bank using outside money. This model will be presented in a subsequent paper[10].

[10] There are extra difficulties encountered in defining short term profits, bad loans and roll over conditions on loans, as well as defining the strategies, payoffs and bank failure rules for the inside bank.

References

[1] M. Shubik: Commodity Model, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model, Western Economic Journal *10* (1972), pp. 24—38.

[2] M. Shubik: Fiat Money and Noncooperative Equilibrium in a Closed Economy, International Journal of Game Theory *1* (1971/72), pp. 243—268.

[3] L. S. Shapley: Noncooperative General Exchange, in: S. A. Y. Lin (ed.): Theory and Measurements of Economic Externalities, New York 1976, pp. 155—175.

[4] M. Shubik: Mathematical Models for a Theory of Money and Financial Institutions, in: R. H. Day and Th. Groves (eds.): Adaptive Economic Models, New York 1975, pp. 513—574.

[5] P. Dubey and M. Shubik: Trade and Prices in a Closed Economy with Exogenous Uncertainties, Different Levels of Information, Money and Compound Future Markets, forthcoming, Econometrica *45* (1977).

[6] P. Dubey and M. Shubik: The Noncooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies, CFDP 422, February 1976, forthcoming, Journal of Economic Theory (1978).

[7] L. S. Shapley and M. Shubik: Models of Noncooperative Exchange, in process, partially available from the authors.

[8] P. Dubey and M. Shubik: Trade Using a Borrowed Means of Payment, with Bankruptcy Conditions, CFDP 448, February 1977.

[9] L. S. Shapley and M. Shubik: Trade Using One Commodity as a Means of Payment, Journal of Political Economy *85* (1977), pp. 937—968.

[10] M. Shubik: A Trading Model to Avoid Tatonnement Metaphysics, in: Y. Amshud (ed.), Studies in Game Theory and Mathematical Economics, New York 1976, pp. 129—142.

[11] M. Shubik: On the Number of Types of Markets with Trade in Money, CFDP 416, January 1976.

[12] M. Shubik and C. Wilson: The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money, CFDP 424(R), June 7, 1976.

[13] J. Harsanyi: The Tracing Procedure: A Bayesian Approach to Defining a Solution for *n*-Person Noncooperative Games, International Journal of Game Theory *4* (1975), pp. 61—94.

[14] R. W. Clower: A Reconsideration of the Microfoundations of Monetary Theory, Western Economic Journal *6* (1967), pp. 1—9.

[15] F. H. Hahn: On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy, in F. H. Hahn and F. Brechling (eds.): The Theory of Interest Rates, New York 1965.

[16] R. M. Starr: The Price of Money in a Pure Exchange Monetary Economy with Taxation, Econometrica *42* (1974), pp. 45—54.

[17] J. M. Grandmont and G. Laroque: On Money and Banking, Review of Economic Studies *42* (1975), pp. 207—236.

[18] P. Dubey and L. S. Shapley: Noncooperative Exchange with a Continuum of Traders, CFDP 447, February 1977.

[19] A. Postlethwaite and D. Schmeidler: Approximate Efficiency of Non-Walrasian Nash Equilibria, mimeographed, University of Illinois, September-October 1975.

[20] P. Dubey and M. Shubik: The Money Rate of Interest (A Multi-period Nonatomic Trading and Production Economy with Outside Money, Inside Money and Optimal Bankruptcy Rules), April 1977.

Address of authors: Prof. Dr. Martin Shubik and Dr. Charles Wilson, Department of Economics, Cowles Foundation for Research in Economics, Box 2125, Yale Station; Yale University, New Haven, CT 06520, U. S. A.