Original Article

Non-Graphical Solutions for Cattell's Scree Test

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Abstract. Most of the strategies that have been proposed to determine the number of components that account for the most variation in a principal components analysis of a correlation matrix rely on the analysis of the eigenvalues and on numerical solutions. The Cattell's scree test is a graphical strategy with a nonnumerical solution to determine the number of components to retain. Like Kaiser's rule, this test is one of the most frequently used strategies for determining the number of components to retain. However, the graphical nature of the scree test does not definitively establish the number of components to retain. To circumvent this issue, some numerical solutions are proposed, one in the spirit of Cattell's work and dealing with the scree part of the eigenvalues plot, and one focusing on the elbow part of this plot. A simulation study compares the efficiency of these solutions to those of other previously proposed methods. Extensions to factor analysis are possible and may be particularly useful with many low-dimensional components.

Keywords: principal components analysis, number of components, dimensionality, eigenvalues, Cattell's scree plot

Several strategies have been proposed to determine the number of components that account for the most variation in a principal components analysis of a correlation matrix. Most of these rely on the analysis of the eigenvalues of the correlation matrix and on numerical solutions. For example, Kaiser's eigenvalue greater than one rule (Guttman, 1954; Kaiser, 1960), parallel analysis (Buja & Eyuboglu, 1992; Horn, 1965; Hoyle & Duvall, 2004), or hypothesis significance tests, like Bartlett's test (1950), make use of numerical criteria for comparison or statistical significance criteria. Independently of these numerical solutions, Cattell (1966) proposed the scree test, a graphical strategy to determine the number of components to retain. Along with the Kaiser's rule, the scree test is probably the most used strategy and it is included in almost all statistical software dealing with principal components analysis. Unfortunately, it is generally recognized that the graphical nature of the Cattell's scree test does not enable clear decision-making about the number of components to retain.

The previously proposed non-graphical solutions for Cattell's scree test have some limitations. First, the Cattell, Nelson, and Gorsuch's *Cng* test (1983; Gorsuch & Nelson, 1981) and the Zoski and Jurs' (1993, 1996) multiple regression approach can be used only when there are more than two components to retain. These solutions are not of interest when only one or two components are hypothesized. Second, Bartlett (1950), Anderson (1965), and Lawley (1956; Lawley & Maxwell, 1963) hypothesis of eigenvalues equality is appropriate for the asymptotic case and the normal probability distribution condition, but more work has to be done to study the properties of this approach in other cases. Third, the Nelson's (2005) coefficient of determination is not very clear about the threshold of the coefficient at which to decide to retain a given number of components. Finally, and most importantly, none of these solutions take into account the fact that the retained eigenvalues could be so small that the components could be rather meaningless. These solutions would also have to consider results from a parallel analysis or from a similar approach, and from results generated under Kaiser's rule.

Despite this fact, according to Velicer (1976; Velicer, Eaton, & Fava, 2000), the scree test is very efficient in recovering the number of components underlying a correlation matrix. They found that this test has a tendency for overestimation, but seldom leads to retaining more than two components over the true number. They also found that the average error in the number of retained components is superior to that observed with parallel analysis and minimum average partial correlations approaches, but generally inferior to other usual methods such as the Bartlett's test or the Kaiser's eigenvalue greater than one rule. However, given the ease of using the scree test, and given that it is well advised to examine components above and below the dimensionality estimation, and that it is usually better to apply more than one method of estimation, Cattell's scree test has practical utility in determining the number of components to retain. In order to assess the limits of the scree test in this context, some numerical solutions are proposed: A family of numerical solutions to deal with the acceleration of the plot of the eigenvalues,

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and, in the spirit of Cattell's work, to deal formally with the scree part of the plot.

Alternative Non-Graphical Solutions to Cattell's Scree Test

To address some of the limitations of previously put forward strategies, we propose a non-graphical solution to the Cattell's scree test, very much in line with the spirit of Cattell's approach, that deals formally with the scree part of the plot of the eigenvalues. We also propose a second approach focusing on the elbow of the plot. None of these are based on a statistical hypothesis test.

Scree Test Optimal Coordinate (n_{oc})

In order to determine the location of the scree, in fact one has to inspect eigenvalue one by one by tracing lines from the coordinates of the last eigenvalue through each of the preceding coordinates. In this way it is possible to project each line to the preceding eigenvalue and verify if the observed eigenvalue is superior than or not equal to the estimated projected eigenvalue. The number of principal components to retain corresponds to the last observed eigenvalue that is greater than or equal to the estimated predicted eigenvalue.

This strategy can be automatized easily by computing $p-2$ two-point regression models, and verifying if the observed eigenvalue is, or is not, greater than or equal to the one estimated by these models. The last of these positive verifications, beginning at the second eigenvalue, and without interruption of the verification, is used to determine the number of principal components to retain. Of course, the value has to be superior than or equal to 1.00, according to Kaiser's rule (Equation 1):

$$n_{oc} = \sum_i \mathrm{I}\left[(\lambda_i \geq 1) \& \left(\lambda_i \geq \hat{\lambda}_i\right)\right] \tag{1}$$

or to the location statistics criteria, by a parallel analysis (Equation 2):

$$n_{\mathrm{oc}} = \sum_i \mathrm{I}\left[(\lambda_i \geq LS_i) \& \left(\lambda_i \geq \hat{\lambda}_i\right)\right]. \tag{2}$$

The location statistic is usually the mean eigenvalue or one of the 0.05, 0.50, or 0.95 quantiles.

In these equations, the predicted eigenvalue $\hat{\lambda}_i$, also referred to as the optimal coordinate, is obtained according to linear regression using only the last eigenvalue and the $(i+1)^{\text{th}}$ eigenvalue. Thus

$$\hat{\lambda}_i = a_{i+1} + b_{i+1}(i), \tag{3}$$

where

$$b_{i+1} = \frac{\lambda_p - \lambda_{i+1}}{(p-i-1)} \text{ and } a_{i+1} = \lambda_{i+1} - b_{i+1}(i+1). \tag{4}$$

Scree Test Acceleration Factor (n_{af})

The acceleration factor emphasizes the point on the coordinate where the slope of the curve changes abruptly. At each of the i eigenvalues (from 2 to $p-1$), the second derivative of Equation 3 evaluated at the i eigenvalue is approximated by the approach of Yakovitz and Szidarovszky (1986), as shown in (Equation 5):

$$f''(i) = \frac{f(i+h) - 2f(i) - f(i-h)}{h}. \tag{5}$$

But because h corresponds to a step in this function and is always equal to 1.00, the function simplifies to a second-order finite difference, like the one used by Hong, Mitchell, and Harshman (2006):

$$f''(i) = f(i+1) - 2f(i) - f(i-1). \tag{6}$$

According to the earlier description of visual inspection in the context of Cattell's scree test, the number of principal components to retain is determined by the eigenvalue preceding the coordinate where there is a maximum acceleration factor. At the same time, again, the value of the observed eigenvalue has to be greater than or equal to 1.00, according to Kaiser's rule (Equation 7):

$$n_{af} = \sum_i \mathrm{I}(\lambda_i \geq 1 \;\&\; i < k) \text{ with } k \equiv \arg\max_j (af_j) \tag{7}$$

or based on location statistics criteria, in a parallel analysis (Equation 8):

$$n_{af} = \sum_i \mathrm{I}(\lambda_i \geq LS_i \;\&\; i < k) \text{ with } k \equiv \arg\max_j \left(af_j\right). \tag{8}$$

While Hong (2003) used the acceleration factor in the context of a hypothesis test of the linearity and equality of eigenvalues, here only the maximum value of the acceleration factor is sought conditional on the Kaiser's rule or on a parallel analysis criterion.

Simulations

The performance of these new alternative non-graphical solutions for the Cattell's scree test is now evaluated with a simulation study. The estimated number of components retained from the optimal coordinates and acceleration factor indices is compared with the estimation from Kaiser's rule, parallel analysis procedure (100 replications based on the 5$^{\text{th}}$, median and 95$^{\text{th}}$ percentiles eigenvalues), Cattell, Nelson, and Gorsuch's *Cng*, Zoski and Jurs' b and standard error of the scree (*sescree*), Nelson's determination coefficient (R^2), and Bartlett, Anderson, and the Lawley's likelihood ratio tests. The *oc* and the *af* indices are computed according to a parallel analysis criterion.

Simulated correlation matrices are generated from correlation matrices with factor structures similar to the ones previously used in a study by Zwick and Velicer (1986). Consequently, comparison with previous results will be possible, here with Table 1 from Zwick and Velicer. An *R* package, *nFactors*, was specifically developed for the purpose of these simulations (Raîche & Magis, 2010). For the purpose of this study, it was judged more appropriate to use a unique software than to use different previous stand-alone ones (e.g., Ledesmar & Valero-Mora, 2007; Lorenzo-Seva & Ferrando, 2006) or relying on SAS and SPSS (O'Connor, 2000) that do not fully integrate all the solutions under investigation.

More specifically, various component structures are simulated so that 3 or 6 major components account for most of the variance from 36 or 72 variables (*var*) correlation matrices, each major component being constituted of 6 or 12 major loadings taking values of 0.50 or 0.80, the other being minor loadings taking values of 0.0 or 0.20. All these component structures are simulated fixing the sample size at 72, 144, 180, and 360. For each component structure, 100 replications are generated. Table 1 shows an example of a typical 36-variable component structure with 3 major components, each constituted of 6 major loadings fixed at 0.20, all other loadings on the last components are fixed at 0.00.

Assessment of the simulation results is pursued by tabulating the decision error according to the mean, median, minimum, maximum, and range of the number of components retained between each determination method. Also, the percentage of correct determination, underdetermination, and overdetermination of components is also computed.

Table 1. Example of a component structure use for the simulation study

	Component								
Variable	1	2	3	4		5	...	35	36
1	0.8	0.2	0.2	0.0		0.0		0.0	0.0
2	0.8	0.2	0.2	0.0		0.0		0.0	0.0
3	0.8	0.2	0.2	0.0	...	0.0		0.0	0.0
7	0.8	0.2	0.2	0.0		0.0		0.0	0.0
12	0.8	0.2	0.2	0.0		0.0	·	0.0	0.0
13	0.2	0.8	0.2	0.0		0.0	·	0.0	0.0
14	0.2	0.8	0.2	0.0	...	0.0	·	0.0	0.0
24	0.2	0.8	0.2	0.0		0.0		0.0	0.0
25	0.2	0.2	0.8	0.0		0.0		0.0	0.0
36	0.2	0.2	0.8	0.0	...	0.0		0.0	0.0

Results of Simulations

The first comparisons are based on the average decision error according to the mean between the number of components retained by each method from each component structure and the number of major components. So, a negative value corresponds to underdetermination of the number of components, while a positive value corresponds to overdetermination. From Table 2, the smallest error is obtained from the Cattell, Nelson, and Gorsuch's *Cng*, with an underdetermination never higher than one component and frequently without any error. The second best solution is from the Zoski and Jurs' *b* coefficient, while the third one is the parallel analysis followed by similar fourth best solutions from *oc* and *af* indices. In all cases, all these solutions never over- or underdetermine the number of components to retain by more than two components.

A closer look at the *Cng* results shows that the underdetermination is related specifically to loadings of 0.5 and a small number of variables (36), while overdetermination from the *b* index is related to loadings of 0.8 and a higher number of variables (72). This could mean that over or under bias correction could eventually be possible conditional on the values of the estimated loadings. Future work will further consider this possibility.

The parallel analysis (median and 95th percentiles), *oc* index, *af* index, and Nelson's R^2 always underdetermine the number of components to retain. A simple correction for this constant directional bias would be to simply add a constant from the result. For example, adding a value of 1 from the result of *oc* would give an equivalent solution as the parallel analysis at the 5th percentile and even a better one than at the median or 95th percentile. However, parallel analysis computed according to the 5th percentile gives better results, that could be associated as well to underdetermination or overdetermination. In general, here, the use of the 5th percentile gives better results than median and 95th percentiles.

The worst solutions were obtained from Kaiser's eigenvalue greater than 1, Zoski and Jurs' standard error

Table 2. Average decision error according to the mean number of components retained

					Parallel analysis										
n	*var*	Load	*oc*	*af*	0.05	0.5	0.95	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
72	36	0.5	−2	−3	−1	−2	−2	7	−1	0	12	−3	−2	16	−1
180	36	0.5	−2	−2	−1	−2	−2	5	−1	0	4	−3	0	3	2
144	72	0.5	0	−2	1	−1	−2	20	0	0	37	−3	−1	49	1
360	72	0.5	0	−2	1	0	−1	18	0	1	22	−3	1	7	2
72	36	0.8	−1	−2	0	−1	−1	2	0	1	8	−1	3	19	4
180	36	0.8	−1	−2	0	0	0	2	0	1	5	−1	5	6	5
144	72	0.8	0	−2	1	0	0	13	0	1	34	−1	21	53	24
360	72	0.8	0	−2	1	0	0	13	0	1	32	0	31	33	32

of the scree (*sescree*), and all the likelihood ratio tests (Bartlett, Anderson, and Lawley). In each of these cases, an overdetermination is observed, moreover a very important one with the Kaiser's rule and the standard error of the scree. For Kaiser's solution, this was not a surprise because this problem has been described often. For the likelihood ratio tests, like all asymptotic hypothesis statistical tests, the decisions are affected by sample size and the results are probably not really related to the determination of the number of components to retain, but are more likely related to a judgment about the importance of eigenvalues. Note also the discrepancy of Anderson's test with Bartlett's and Lawley's tests. The way the Anderson's likelihood ratio is computed does not take into account the number of variables and eigenvalues considered, so its value, using the same number of degrees of freedom for the involved χ^2 test, leads more frequently to rejection of the null hypothesis.

These results are partly in accordance with Zwick and Velicer (1986), overdetermination from Bartlett's test and the underdetermination from parallel analysis being more important in our study. It is important to underline that Zwick and Velicer used only five replications of the simulation in each situation, while 100 replications are used in our study. Small sample size from this previous study can explain the difference in results.

The same conclusion comes from the analysis of the comparison of the decision error according to the median number of components retained between each determination method. As displayed in Table 3, only slight differences arise, the most important with the Anderson's likelihood ratio method showing a value of 2. But generally the difference is null or at worst not more than one component to retain.

The results of the analysis of the comparison of the decision error according to minimum, maximum, and range of the number of components retained between each determination methods are available in Tables 4, 5, 6. This information is useful to observe the variability of the decision about the number of components to retain according to each determination method. The lowest variability is shown by the use of the *b* index, the range being very small and equal to 1. The *Cng* index is the second candidate, the range never exceeding 2 and usually not more than 1. The *Cng* is the third candidate, followed by the parallel analysis computed according to the median and 95th percentiles. Parallel analysis computed according to the 5th percentile shows an important range with a value as high as 8. The *oc* and *af* indices also show important variability. Finally, the likelihood ratio approaches display generally huge variability, again questioning seriously their usefulness for the determination of the number of components to retain.

Finally, the percentage of correct determination, underdetermination, and overdetermination is available in Table 7. These statistics are less discriminative than the previous from Tables 4 to 6 and convey mostly the same conclusion. There are exceptions, like *Cng* and *b*, that now seem not so

Table 3. Average decision error according to the median number of components retained

					Parallel analysis										
n	*var*	Load	*oc*	*af*	0.05	0.5	0.95	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
72	36	0.5	−2	−3	−1	−2	−2	7	−1	0	12	−4	−1	18	−1
180	36	0.5	−2	−2	−1	−2	−2	5	−1	0	4	−3	1	4	2
144	72	0.5	0	−2	1	−1	−2	20	0	0	37	−3	0	50	1
360	72	0.5	0	−2	1	0	−1	18	0	1	21	−3	2	8	3
72	36	0.8	0	−2	0	−1	−1	2	0	1	8	−1	4	20	4
180	36	0.8	0	−2	0	0	0	2	0	1	5	−1	5	6	6
144	72	0.8	0	−2	0	0	0	13	0	1	34	−1	21	54	24
360	72	0.8	0	−2	0	0	0	12	0	1	32	0	32	34	32

Table 4. Average decision error according to the minimum number of components retained

					Parallel analysis										
n	*var*	Load	*oc*	*af*	0.05	0.5	0.95	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
72	36	0.5	−4	−4	−2	−2	−3	4	−2	0	6	−4	−3	0	−2
180	36	0.5	−4	−3	−2	−2	−2	3	−1	0	0	−3	−2	−1	−1
144	72	0.5	−4	−4	−1	−2	−2	18	−1	0	29	−4	−2	24	−2
360	72	0.5	−4	−3	−1	−2	−2	16	−1	0	14	−3	0	2	0
72	36	0.8	−4	−3	−1	−1	−1	1	−1	0	5	−1	1	6	3
180	36	0.8	−4	−2	−1	−1	−1	1	0	0	4	−1	4	5	5
144	72	0.8	−3	−3	−1	−1	−1	12	0	0	28	−2	14	40	17
360	72	0.8	−4	−2	0	0	0	11	0	0	26	−1	30	32	31

Table 5. Average decision error according to the maximum number of components retained

					Parallel analysis										
n	*var*	Load	*oc*	*af*	0.05	0.5	0.95	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
72	36	0.5	1	0	2	0	−1	9	1	1	18	−3	3	31	12
180	36	0.5	0	−2	0	0	−1	7	0	1	12	−2	6	12	9
144	72	0.5	4	−1	7	2	0	22	1	1	46	−3	5	66	9
360	72	0.5	3	−2	4	2	0	20	0	1	38	−2	6	16	8
72	36	0.8	0	−2	0	0	0	3	0	1	12	0	9	32	17
180	36	0.8	0	−2	0	0	0	2	0	1	6	0	8	16	12
144	72	0.8	2	−2	2	1	0	15	0	1	39	0	30	67	38
360	72	0.8	2	−2	2	1	1	14	0	1	34	0	34	43	38

Table 6. Average decision error according to the range of the number of components retained

					Parallel analysis										
n	*var*	Load	*oc*	*af*	0.05	0.5	0.95	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
72	36	0.5	5	4	4	2	4	5	3	1	12	1	6	31	14
180	36	0.5	4	5	2	2	3	4	1	1	12	1	8	13	10
144	72	0.5	8	5	8	4	2	4	2	1	16	1	7	42	11
360	72	0.5	7	5	5	4	2	4	1	1	24	1	6	14	8
72	36	0.8	4	5	1	1	1	2	1	1	7	1	8	10	14
180	36	0.8	4	4	1	1	1	1	0	1	2	1	4	62	7
144	72	0.8	5	5	3	2	1	3	0	1	11	2	16	27	21
360	72	0.8	6	4	2	1	1	3	0	1	8	1	4	9	7

Table 7. Percentage of correct determination (Ok), underdetermination (↓), and overdetermination (↑) according to the number of components retained

	n	*var*	Load	*oc*	*af*	PA (0.50)	Kaiser	*Cng*	*b*	*sescree*	R^2	Bartlett	Anderson	Lawley
Ok	72	36	0.5	23	17	24	2	35	1	0	3	20	0	19
↓				67	81	64	1	44	46	0	97	69	1	45
↑				10	2	12	97	21	52	100	0	11	99	36
Ok	180	36	0.5	31	34	35	7	37	0	8	12	35	11	20
↓				62	66	55	15	38	43	3	88	30	14	19
↑				8	0	10	78	35	57	89	0	35	74	60
Ok	144	72	0.5	20	30	21	0	34	1	0	0	25	0	18
↓				54	68	50	0	33	42	0	100	38	0	16
↑				25	3	30	100	34	57	100	0	37	100	66
Ok	360	72	0.5	30	47	32	1	26	0	0	15	22	1	9
↓				47	53	41	0	25	31	0	85	5	0	2
↑				23	0	27	99	49	69	100	0	74	99	89
Ok	72	36	0.8	62	42	66	53	30	0	8	67	24	0	8
↓				34	58	29	14	29	34	0	33	13	0	11
↑				4	0	4	32	41	66	92	0	62	99	81
Ok	180	36	0.8	70	43	82	57	29	0	46	71	26	6	9
↓				27	57	15	14	29	30	14	29	14	10	12
↑				2	0	3	29	43	70	40	0	60	85	79
Ok	144	72	0.8	71	49	73	23	25	0	0	50	6	0	2
↓				14	14	51	0	25	32	0	50	0	0	0
↑				15	0	16	77	50	68	100	0	94	100	98
Ok	360	72	0.8	82	50	88	25	25	0	12	59	11	0	3
↓				7	50	0	0	25	25	0	30	0	0	0
↑				11	0	12	75	50	75	88	11	89	100	97

powerful. The reason is that emphasis is now on percentages of the correct determination rather than on preceding more informative statistics.

Discussion

In order to redress the subjective weakness of the Cattell's scree test, two families of non-graphical solutions to this test were presented. These solutions are easy to apply and, like the parallel analysis, seem to give parsimonious results compared to Kaiser's rule. In particular, the *oc* solution could eventually be more efficient if a bias correction is applied by simply subtracting 1 from the number of components to retain. But surprisingly, this study shows that the better candidate solution to the determination of the number of components to retain is the Zoski and Jurs' *b* coefficient of regression index. The bias is very small, at worst leading to an overdetermination of one component, and the variability of the decision is the smallest one. Also important, the *Cng* is the second best candidate solution. This is not surprising if we consider that the *b* solution is a numerical improvement of *Cng*. However, one has to note that the *b* coefficient of regression and the *Cng* are only valid when the number of components to retain is at least equal to 3. Finally, the parallel analysis (5^{th}, median, and 95^{th} percentiles) shows good results, but it is not the best approach in the context of this simulation study.

More work has to be done to formally assess the strengths and weaknesses associated with *oc* compared with previous ones. Therefore, future research will formally study in which conditions each of the studied solutions is best suited. More variability in the dependent variables will be considered. Also, the estimated values of the major and minor loadings will also be used as independent variables. In this way, it may be possible to prescriptively make decisions about the solution to apply according to the estimated major and minor loadings.

The relevance of this approach is very clear, namely, that subjective bases for the number of components to extract are greatly reduced by the newly proposed solutions, but other previously proposed numerical ones are better candidates in general. The graphical approaches become very important, however, both in terms of replication of component structures and in consideration of many low-dimensional correlational structures in which the determination of the number of components to extract can be extremely difficult without a finer-grained solution. The new strategies will transfer readily to similar domains such as factor analysis and other areas of latent variable analysis. It has to be noted that these results are limited to correlation computed from continuous variables and orthogonal components. Much work is actually done with discrete dichotomous and polytomous variables, mostly related to parallel analysis (Cho, Li, & Bandalos, 2009; Crawford et al., 2010; Tran & Forman, 2009; Weng & Cheng, 2005; Yu, Popp, DiGangi, & Jannash-Pennell, 2007). Works with oblique components are also to be done (Beauducel, 2001), so that the decision about the number of components to retain can not only rely on these indices, the interpretability of the components must also be considered. For example, a sufficient number of variables with substantial loadings on each component is necessary to draw insightful conclusions.

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