

CURRICULUM OF MATHEMATICS

(Revised 2005)



**HIGHER EDUCATION COMMISSION
ISLAMABAD**

CURRICULUM DIVISION, HEC

Prof. Dr. Altaf Ali G. Shaikh	Adviser (Acad/R&D)
Malik Ghulam Abbas	Deputy Director
Miss Ghayyur Fatima	Deputy Director (Curri)
Mr. M. Tahir Ali Shah	Assistant Director
Mrs. Noshaba Awais	Assistant Director

Composed by Mr. Zulfiqar Ali, HEC Islamabad

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PREFACE

Curriculum of a subject is said to be the throbbing pulse of a nation. By looking at the curriculum one can judge the state of intellectual development and the state of progress of the nation. The world has turned into a global village; new ideas and information are pouring in like a stream. It is, therefore, imperative to update our curricula regularly by introducing the recent developments in the relevant fields of knowledge.

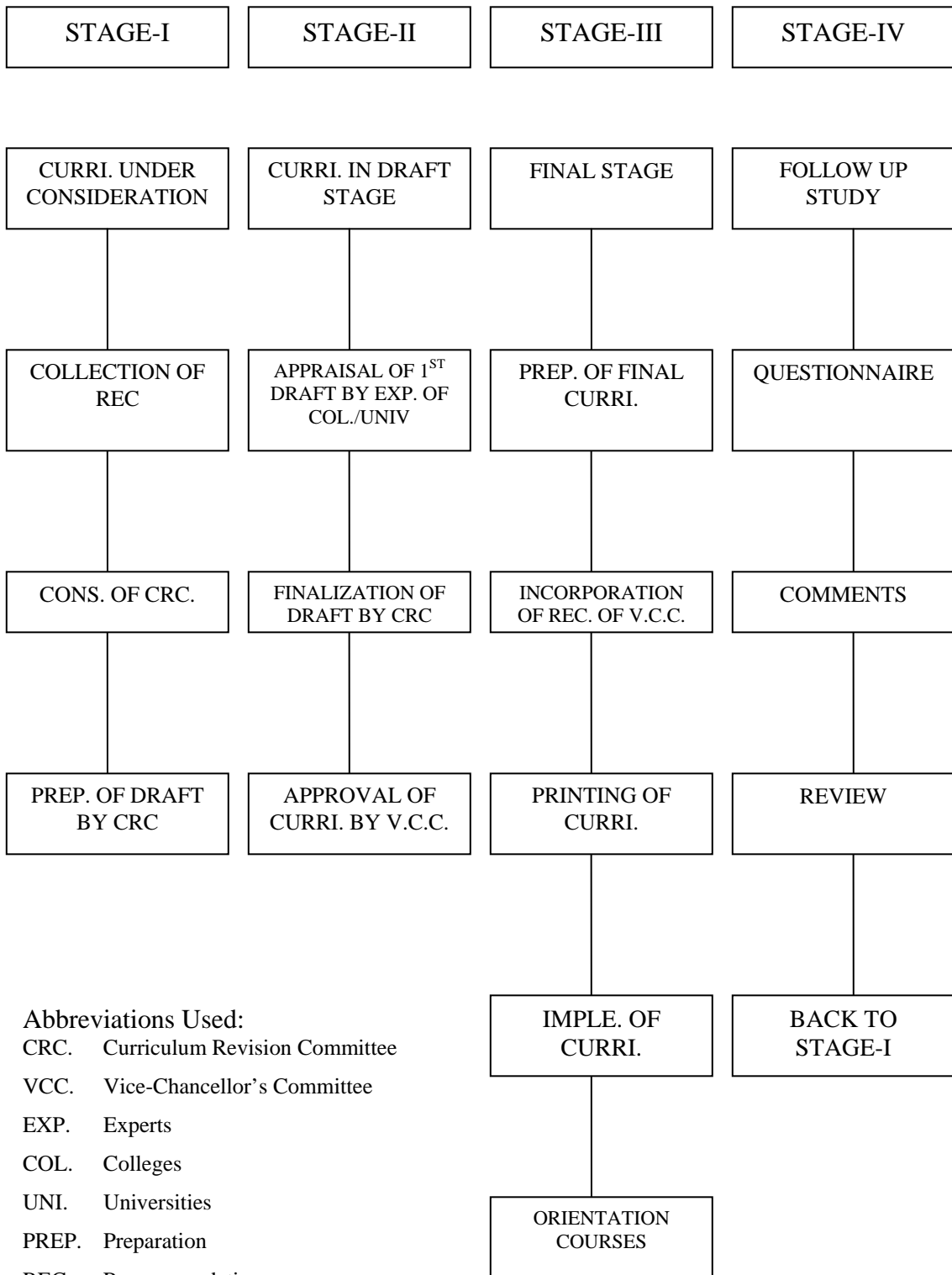
In exercise of the powers conferred by sub-section (1) of section 3 of the Federal Supervision of Curricula Textbooks and Maintenance of Standards of Education Act 1976, the Federal Government vide notification no. D773/76-JEA (Cur.), dated December 4, 1976, appointed University Grants Commission as the competent authority to look after the curriculum revision work beyond class XII at bachelor level and onwards to all degrees, certificates and diplomas awarded by degree colleges, universities and other institutions of higher education.

In pursuance of the above decisions and directives, the Higher Education Commission (HEC) is continually performing curriculum revision in collaboration with universities. According to the decision of the special meeting of Vice-Chancellors' Committee, curriculum of a subject must be reviewed after every 3 years. For the purpose, various committees are constituted at the national level comprising senior teachers nominated by universities. Teachers from local degree colleges and experts from user organizations, where required, are also included in these committees. The National Curriculum Revision Committee for Mathematics in its meeting held in June 9-11, 2005 at the HEC Regional Centre, Lahore revised the curriculum after due consideration of the comments and suggestions received from universities and colleges where the subject under consideration is taught. The final draft prepared by the National Curriculum Revision Committee duly approved by the Competent Authority is being circulated for implementation by architectural institutions.

(PROF. DR. ALTAF ALI G. SHAIKH)
Adviser (Acad/R&D)

August 2005

CURRICULUM DEVELOPMENT



Abbreviations Used:

- CRC. Curriculum Revision Committee
- VCC. Vice-Chancellor's Committee
- EXP. Experts
- COL. Colleges
- UNI. Universities
- PREP. Preparation
- REC. Recommendations

INTRODUCTION

The final meeting of National Curriculum Revision Committee in Mathematics was held at HEC Regional Center, Lahore, during June 9-11, 2005. Earlier, draft curricula for BS, MS and PhD were prepared in its preliminary meeting held at the same venue during January 31 – February 2, 2005. The following experts attended:

Convener

Dr Syed Arif KAMAL

Chairman

Department of Mathematics

University of Karachi

Karachi.

Members

1. **Muhammad ABBAS**
Associate Professor
Department of Mathematics
Gomal University
DI Khan, NWFP.
2. **Professor (Dr) Zafar ALI**
Professor
Department of Mathematics
University of Peshawar
Peshawar.
3. **Professor (Dr) Ahsanullah BALOCH**
Chairman
Department of Basic Sciences
and Related Studies
Mehran University of Engineering
and Technology
Jamshoro, Sindh.
4. **Mrs Tahira Nasreen BUTTER**
Associate Professor
Department of Mathematics,
Lahore College for Women
University, Lahore.
15. **Professor (Dr) Riaz Ahmed MEMON**
Professor of Mathematics
Institute of Mathematics and Computer
Science, University of Sindh, Jamshoro.
16. **Professor (Dr) Ghulam MURTAZA**
Director
National Center for Mathematics
GC University Campus
Lahore.
17. **Professor (Dr) Jawaid QUAMAR**
Dean
Faculty of Computer Science and
Informatics
Dadabhoy Institute of Higher Education
Karachi.
18. **Shuja Muhammad QURESHI**
Associate Professor
The Sir Syed University of Engineering
and Technology
Karachi.

5. **Professor (Dr) Lal CHAND**
Chairman
Department of Mathematics
Shah Abdul Latif University
Khairpur, Sindh.
6. **Dr Mahmood-ul-HASSAN**
Associate Professor
Department of Mathematics
COMSATS Institute of
Information Technology
Islamabad.
7. **Professor (Dr) SM
HUSNAINE**
Chairman
Department of Mathematics
National University of
Computer & Emerging
Sciences FAST, B-Block
Faisal Town, Lahore.
8. **Muhammad Asif JAN**
Assistant Professor
Institute of Information
Technology
Kohat University of Science
and Technology
Banu Road, Off Jarma, Kohat.
9. **Dr Ataulлах KALIM**
Chairman
Department of Mathematics
COMSATS Institute of
Information Technology
Islamabad.
10. **Dr M Nazir KHAN**
Professor
CECOS University of I.T &
Emerging Sciences, P.O. Box
494, Phase VI, Hayatabad
Peshawar.
19. **Professor (Dr) Ghazala SADIQ**
Professor of Mathematics
Institute of Space Technology
Islamabad
Islamabad Highway, Near GT Road
Box 2750, Rawat, Islamabad 4400.
20. **Professor (Dr) Muhammad Bashir
SADIQ**
Dean
Superior College
Lahore.
21. **Professor (Dr) Nawazish Ali SHAH**
Chairman
Department of Mathematics
University of Engineering and
Technology
Lahore.
22. **Dr Farhana SHAHEEN**
Associate Professor
Shaheed Zulfiqar Ali Bhutto Institute
of Science and Technology, 90
Clifton
Karachi.
23. **Miss Raheela SHAHNAZ**
Department of Mathematics
Lahore College for Women
University, Lahore.
24. **Dr Shahid S SIDDIQI**
Associate Professor
Department of Mathematics,
University of the Punjab, Lahore.

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|---|--|
| <p>11 Dr Safeer H KHAN
<i>Assistant Professor</i>
GIK Institute of Engineering
Science and Technology
Topi, Distt. Swabi
NWFP.</p> | <p>25 Dr Nazra SULTANA
<i>Chairwoman</i>
Department of Mathematics
University of Sargodha, Sargodha.</p> |
| <p>12 Dr Tahir MAHMOOD
<i>Associate Professor</i>
Department of Mathematics
Islamia University
Bahawalpur.</p> | <p>26 Dr Shahadat Ali TAJ
<i>Associate Professor</i>
Department of Mathematics,
GC Univesity
Lahore.</p> |
| <p>13 Professor (Dr) Abdul MAJEED
<i>Former Head</i>
Department of Mathematics
University of the Punjab
Lahore.</p> | <p>27 Dr Muhammad YOUSAF
<i>Associate Professor</i>
Department of Mathematics
Hazara University
Mansehra, NWFP.</p> |
| <p>14 Ms Soofia MALIK
<i>Assistant Professor</i>
Jinnah University for Women
5-C Nazimabad
Karachi.</p> | <p>28 Muhammad YOUSUF
<i>Chairman</i>
Department of Mathematics
University of Malakand
Chakdara.</p> |

Secretary

Professor (Dr) Fazal RAHMAN

Chairman

Department of Mathematics
Bahria University
Islamabad.

On January 31, 2005, the preliminary meeting started with recitation of verses from the Holy Quran. Professor Dr. Altaf Ali G Shaikh welcomed the participants and briefed them about the curriculum-development exercise taken by HEC. He, also, requested the participants to develop 4-year BS program for the new scheme.

The Committee, unanimously, appointed Dr. Syed Arif Kamal as Convener and Dr. Fazal Rahman as Secretary. Dr. Kamal, in his opening remarks, thanked the participants for reposing confidence in him. It was decided to undertake the course formulation so as to accommodate 2-year BSc program offered by colleges. The participants thoroughly discussed each and every aspect of the curricula and the courses for BS, MS and PhD

programs. The curriculum as compiled by the Convener in the light of discussion of the committee was circulated among members on May 24, 2005.

On June 9, 2005, the final meeting started with recitation from the Holy Quran by Professor (Dr.) Zafar Ali of University of Peshawar. Mr. Riaz Cheema, Director, Regional Center, welcomed the participants on behalf of Chairman, HEC. The participants critically scrutinized and finalized the curricula for 4-year BS, 2-year MS and 2-year PhD Programs.

It was the feeling of the house that much more than mere curriculum development is needed to bring mathematics teaching at par with the developing countries, like Korea, Malaysia, Singapore, and others:

Teachers' Training

To handle such a demanding curriculum teachers need to be trained. Teachers should be prepared to teach all core courses. This can be achieved through refresher and special courses for college and university teachers. The teachers should maintain office hours to help students overcome their difficulties, guide and counsel students for further studies and career.

The fundamental responsibility of a mathematics teacher is to first enhance the teaching of basic theory, to execute closely such education of scientific methodology as induction, contrast, analogy and imitation.

Teaching Load

Teaching load should be balanced so that a teacher can find time to do some research. The class size should not exceed 25 students. Each senior instructor should be provided at least two junior instructors (teaching assistants), one to teach tutorial sessions and the other to grade problems. In courses having laboratory, an additional teaching assistant is needed to conduct the laboratory.

Textbooks and Teaching Aids

To teach effectively there is a need to make available textbooks at cheaper prices. These textbooks should be supplemented with solution manuals for instructors and study guides for students (these are not the “keys”, or the “guides” available in the market) as well as softwares, slides, films, charts, demonstrations, models and journals. Students should be encouraged to make drawings, models and softwares, which illustrate difficult concepts.

Getting the Best Students

In Pakistan, the brightest students compete for places in engineering and medical colleges as well as computer science and information technology programs. Those unable to find places in the professional institutions come to Mathematics Departments. In order to attract the very best to mathematics one needs to make effective contact with the prospective students by organizing open houses, summer programs for high-school seniors, lectures in schools and colleges, *etc.* Mathematics should be presented as fun through games and simulations, dramatic demonstrations, activities done outdoors and illustrations of mathematics in sports. Usefulness of mathematics can be demonstrated by daily-life applications and as a preparation of a scientific career. The goal is to enable the students to see the bridge leading them from mathematics to technology passing through physics and engineering.

Student Feedback

To promote effective learning and teaching there should be continuous feedback from students — much more than the confidential evaluation form filled by each student at the end of a semester.

Regulating Body

There is a need for a regulating body, which could maintain quality control over the choice of textbooks and journals published within the country. Such a body may be 'Pakistan Mathematics Society', which could publish one or a series of journals, *e. g.*, Pakistan Journal of Mathematics, instead of every university bringing out its own journal.

Schemes of Studies

The schemes of studies for BS, MS and PhD are given as following:

Bachelors Program in Mathematics [to replace the existing BSc (Pass/Honors) & MSc Programs]

- Degree Awarded:* **Bachelor of Science (BS)**
Entrance HSC (Pre-Engineering Group) or equivalent
Requirements:
Duration of the Program: 4 years (8 semesters); students NOT from the Pre-Engineering Group have to attend intensive courses in Mathematics, Physics and Chemistry during the semester breaks of first and second years depending on their deficiencies. Students cannot get the degree of BS before completing 4 years of study.
- Total Credit Hours:* 136 (maximum 18 hours per semester)
Compulsory Courses: 6 (11 credit hours)
Humanities Course: 1 (3 credit hours)
Core Courses: 24 (72 credit hours)
Minor Courses: 8 (24 credit hours)
Electives A: 4 (12 credit hours) [Electives in Specialization]
Electives B: 2 (6 credit hours); mathematics courses outside the field of specialization [Free Electives]
Class Size: 50 (maximum) for lectures; 25 (maximum) for tutorial and laboratory session
- Academic Standards:* In order to bring the standard of education at par with the developed countries, the notion that anyone who gets admitted to a university shall end up with a degree should be abolished.
- Those, who fail in more than two courses in a semester, are, automatically, dismissed.
 - Those, who fail in one or two courses, may retake the examination during the 40-day grace period.
 - Those, who have not cleared all courses after the grace period are dismissed.
- Attendance Requirement:* Students are required to maintain 75% attendance in order to sit in the final examination. Dean, under special circumstances, may condone attendance between 60-74%. Attendance registers, bearing the list of students, should be provided to the instructors on the first

day of classes. Instructors take the attendance at the start of every class. At the end of each class associate instructor keys in the hard-copy attendance record to a centralized database. On the first of every month list of students having attendance less than 75% is displayed on the notice board. Those having attendance less than 60% should be required to attend fresh classes.

Summer Activity: Students could be assigned independent study. Third- and fourth-year students may be placed in local industry and financial institutions for internships during semester breaks. During the fourth year the students should receive career counseling for suitable placement after completion of BS degree.

Project: 4 credit hours, with accompanying written report and presentation

Seminars: During each semester student must participate in Guest Seminar (presentations by eminent mathematicians on theoretical aspects and applications) and Students' Seminar (presentations by students); Guest Seminars and Students' Seminars are held fortnightly during the same time slot (8 credit hours) weeks — graded on the basis of presentation and write up by the student, a weekly quiz on the contents of Guest/Students' Seminar held during the previous week

Comprehensive Viva: Conducted by senior faculty members of the department at the end of eighth semester

Qualifications for Course Supervisors Faculty holding PhD, MS or MPhil degree is entitled to teach lecture session of a course

(Instructors):

Qualifications for Associate Instructors Associate Instructors (Teaching Assistants) must hold BS, MSc or MA and may handle tutorial, laboratory as well as assignment, quiz and problem grading

(Teaching Assistants):

Syllabus: Course Supervisor is required to distribute syllabus breakdown into 14 units, each unit completed within a week. There should be a discussion session every week on the unit covered during the last week (conducted by Associate Instructor/Teaching Assistant), a review session prior to each hourly (conducted by

Course Supervisor/ Instructor) and a comprehensive review prior to final examination (conducted by Course Supervisor/Instructor)

Pedagogical Techniques: Lecture sessions of each unit (normally, 2-3 lectures) are followed by a discussion session (reinforcing the concepts taught through examples, alternate derivations and proofs) as well as a problem-solving session (teaching skills of problem formulation, qualitative analysis and finding solutions), each of these sessions conducted, separately, by the Associate Instructor at the end of lecture session of every unit. In addition, a review session should be arranged prior to each monthly test (hourly) and a comprehensive review before the final examination, both sessions conducted by the Course Supervisor.

Continuous-Evaluation Report: Course Supervisor (Instructor) is required to prepare a progress report after each hourly mentioning tentative grade (cumulative grade based on all hourlies, quizzes, problem sets and assignments taken to date) and attendance record. It may, also, include qualitative description of student's weaknesses and areas needing special attention. Laboratory and theory portions are separate passing heads. This report is discussed with the student as well as student's parents, if unsatisfactory.

Withdrawal Policy: Student shall have the option to withdraw from a course during a period of 45 days from the start of course. This is possible, only, after the student submits the required forms in the Semester-Examination Section through the Chairman of Department. The first progress report (mentioning tentative grade of the student after the first hourly) is supposed to be available before the expiry of withdrawal period. A grade of 'W' is to be assigned in such a course, and it is not counted in the computation of GPA (Grade-Point Average).

Final Grades: Final grades are due within 48 hours of terminal examination in the Semester Examination Section. It is the responsibility of Semester-

Examination Section (SES) to provide attendance sheets (in duplicate) and award sheets (5 sets), both of them printed with the names and the seat numbers of students before the commencement of terminal examination (of 2-hours duration). One hour after the start of terminal examination One set of original attendance sheets must be sent duly sealed to the SES after an hour has passed. The other set of original attendance sheets has to be submitted with the award sheets (scripts must accompany award sheets).

Grading of Problem Sets and Assignments: An Associate Instructor (A Teaching Assistant) is available to mark problem sets (weekly), quizzes (weekly) and assignments (one or two during the semester).

Final Examinations: Final Examination (Part I) shall be set up and graded by the Course Supervisor. Final Examination (Part II) shall be set up and graded by a faculty member designated by Board of Studies. The student is supposed to attempt Part I and Part II on separate answer books.

Marks Breakdown: Passing grade is set at 50% (60% in core courses). Tables 1 and 2 list marks breakdowns for courses without laboratory (3 + 0) and courses with laboratory (2 + 1), respectively. Table 3 gives the equivalence of numerical and alphabetical grades.

Table 1: Marks Breakdown for Courses without Laboratory (3 + 0)

Item	Maximum Marks for courses without Laboratory (3 + 0)	Maximum Marks for courses with Laboratory (2 + 1)
Monthly Tests (2-3) ^a	30	15
Quizzes (10-14) ^b	10	10
Problem Sets (10-14) ^c	10	10
Assignments ^d	10	5
Laboratory	--	20
Final Examination	40	40
Total	100	100

- a 3 monthly tests (hourlies) for courses without laboratory, best 2 counted; 2 monthly tests (hourlies) for courses with laboratory, best of the 2 counted
- b All quizzes are surprise and conducted at the beginning of class/seminar; best 10 counted
- c One problem set given each week; best 10 counted
- d 2 for courses without laboratory; 1 for courses with laboratory

Table 3: Numerical and Alphabetical Grades

Numerical Grade	Alphabetical Grade	Grade Point
90-100	A ⁺	4.00
85-89	A	4.00
80-84	A ⁻	3.67
75-79	B ⁺	3.33
71-74	B	3.00
68-70	B ⁻	2.67
64-67	C ⁺	2.33
60-63	C	2.00
57-59	C ⁻	1.67
53-56	D ⁺	1.33
50-52	D	1.00
0-49	F	Zero

BS Scheme of Studies (Four-Year Program)

FIRST YEAR

First Semester			Second Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Calculus I	3 + 0	1	Calculus II	3 + 0
2	Discrete Structures	3 + 0	2	Guest/Students' Seminar II	1 + 0
3	English Structure	3 + 0	3	Minor-A II	2 + 1
4	Guest/Students' Seminar I	1 + 0	4	Minor-B II	2 + 1
5	Islamic Studies	2 + 0	5	Number Theory	3 + 0
6	Minor-A I	2 + 1	6	Pakistan Studies	2 + 0
7	Minor-B I	2 + 1	7	Social Science Course	3 + 0
Total 16 + 2			Total 16 + 2		

SECOND YEAR

Third Semester			Fourth Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Calculus III	3 + 0	1	Algebra I	3 + 0
2	Communication Skills	1 + 0	2	Guest/Students' Seminar IV	1 + 0
3	Computer Language	1 + 1	3	Minor-A IV	2 + 1
4	Guest/Students' Seminar III	1 + 0	4	Minor-B IV	2 + 1
5	Minor-A III	2 + 1	5	Numerical Computing	2 + 1
6	Minor-B III	2 + 1	6	Probability Theory	3 + 0
7	Vector and Tensor Analysis	4 + 0	7	Technical Writing	1 + 0
Total 14 + 3			Total 14 + 3		

THIRD YEAR

Fifth Semester			Sixth Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Algebra II	3 + 0	1	Algebra III	3 + 0
2	Elective-B I	3 + 0	2	Classical Mechanics	3 + 0
3	Guest/Students' Seminar V	1 + 0	3	Complex Analysis	3 + 0
4	Mathematical Statistics	3 + 0	4	Guest/Students' Seminar VI	1 + 0
			5	Partial-Differential	3 + 0

5	Numerical Analysis	3 + 0	Eqns.		
6	Ordinary-Differential Eqns.	3 + 0	6	Real Analysis II	2 + 0
7	Real Analysis I	2 + 0	7	Mathematical Spaces	3 + 0
Total			Total		
18 + 0			18 + 0		

FOURTH YEAR

Seventh Semester			Eighth Semester		
#	Course Title	Cr. Hrs.	#	Course Title	Cr. Hrs.
1	Elective-A I	3 + 0	1	Elective-A III	3 + 0
2	Elective-A II	3 + 0	2	Elective-A IV	3 + 0
3	Elective-B II	3 + 0	3	Functional Analysis	3 + 0
4	Guest/Students' Seminar VII	1 + 0	4	Guest/Students' Seminar VIII	1 + 0
5	Mathematical Modeling	3 + 0	5	Optimization Theory	3 + 0
6	Project I	2 + 0	6	Project II	2 + 0
Total			Total		
15 + 0			15 + 0		

BS Course Contents (Four-Year Program)

Course contents for Communication Skills, English Structure, Islamic Studies, Minor A and Minor B (could be chosen from the list of subjects offered in the institution, e. g., chemistry, economics, geology, physics, statistics), Pakistan Studies, Social Sciences Course as well as Technical Writing, are to be drafted by the respective National-Curriculum-Revision Committees.

Course contents for the mathematics courses are, alphabetically, given below:

Algebra I

Prerequisite(s): None

Credit Hours: 3 + 0

Recommended Books:

Anton H, *Linear Algebra with Applications* (8th edition), John Wiley, New York

Herstein IN, *Topics in Algebra* (2nd edition), John Wiley, New York

Hill RO, *Elementary Linear Algebra with Application* (3rd edition), 1995, Brooks/Cole

Leon SJ, *Linear Algebra with Applications* (6th edition), 2002, Prentice Hall, Englewood Cliffs, NJ, USA

Nicholson WK, *Elementary Linear Algebra with Applications* (2nd edition), 1994, PWS Publishing Co.

Background and Goals: This is the first course in groups, matrices and linear algebra, which provides basic background needed for all mathematics majors, a prerequisite for many courses. Many concepts presented in the course are based on the familiar setting of plane and real three-space, and are developed with an awareness of how linear algebra is applied.

Contents:

Group Theory: basic axioms of a group with examples, subgroups, order of a group, subgroups generated by subset of a group, system of generators, cyclic groups, cosets, Lagrange's theorem, introduction to permutations, even and odd permutations, cycles, lengths of cycles, transpositions, symmetric group, alternating groups, rings, fields (definitions and examples), vector spaces, subspaces, linear dependence and independence, linear span of a subset of a vector space, bases and dimensions of a vector space

Linear Algebra: Algebra of matrices, determinants, matrix of a linear transformation. row and column operations, rank, inverse of matrices, solution of homogeneous and non-homogeneous equations, orthogonal transformation, eigenvalue problem with physical significance

Algebra II

Prerequisite(s): Algebra I

Credit Hours: 3 + 0

Recommended Books:

Allenby RBJT, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, 1983, Edward Arnold

Farleigh JB, *A First Course in Abstract Algebra* (7th edition), Addison-Wesley, Reading, Ma., USA

Macdonald ID, *The Theory of Groups*, 1975, Oxford Clarendon Press, Ma., USA

Background and Goals: This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I.

Contents:

Group Theory: Normalizers and centralizers of a subset of a group, Centre of a group, Normal subgroup, quotient groups, Conjugacy relation between elements and subgroups, homomorphism and isomorphism between groups, Homomorphism and isomorphism theorems, finite p-groups, internal and external direct products, group action on sets, isotropy subgroups, orbits, 1st, 2nd and 3rd Sylow theorems.

Ring Theory: Types of rings, matrix rings, rings of endomorphisms, polynomial rings, integral domain, characteristic of a ring, ideal, types of

ideals, quotient rings, homomorphism of rings, fundamental theorem of homomorphism of rings.

Algebra III

Prerequisite(s): Algebra II

Credit Hours: 3 + 0

Recommended Books:

Axle SJ, *Linear Algebra Done Right*, Undergraduate Texts in Mathematics, 1996, Springer, New York

Birkhoff G, Maclane S, *A Survey of Modern Algebra* (4th edition), AKP Classics

Perry WL, *Elementary Linear Algebra*, 1988, McGraw-Hill, New York

Background and Goals: This is a course in abstract linear algebra. The majority of follow up courses in both pure and applied mathematics assume the material covered in this course.

Contents: Vector spaces; sums and direct sums of subspaces of a finite dimensional vector space, Dimension theorem, linear transformation, null space, image space of linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of the domain of a linear transformation, matrix of linear transformation, change of basis, inner product spaces, projection of a vector along another vector, norm of a vector, Cauchy Schwartz inequality, Orthogonal and orthonormal basis, similar matrices and diagonalization of a matrix, $\text{Home}(V,W)$, dimension and basis of $\text{Home}(V,W)$, dual space and dual basis, annihilators.

Calculus I

Prerequisite(s): None

Credit Hour s: 3 + 0

Recommended Books:

Anton H, *Calculus: A New Horizon* (6th edition), 1999, John Wiley, New York

Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)

Thomas GB, Finney AR, *Calculus* (10th edition), 2002, Addison-Wesley, Reading, Ma, USA

Objectives and Goals: This is the first course of the basic sequence, Calculus I-III, serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. Calculus I & II focus on the study of functions of a single variable.

Contents: Limits and continuity; derivative of a function and its applications; optimization problems; mean value theorem (Taylor's theorem and the infinite Taylor series with applications) and curve sketching; anti-derivative and integral; definite integral and applications; the fundamental theorem of calculus; inverse functions (Chapters 1-6 of the text)

Calculus II

Prerequisite(s): Calculus I

Credit Hours: 3 + 0

Recommended Books:

Anton H, *Calculus: A New Horizon* (6th edition), 1999, John Wiley, New York

Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)

Thomas GB, Finney AR, *Calculus* (10th edition), 2002, Addison-Wesley, Reading, Ma, USA

Background and Goals: This is the second course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. As continuation of Calculus I, it focuses on the study of functions of a single variable.

Contents: Continuation of Calculus I: Techniques of integration; further applications of integration; parametric equations and polar coordinates; sequences and series; power series representation of functions (Chapters 7-10 of the text)

Calculus III

Prerequisite(s): Calculus II

Credit Hours: 3 + 0

Recommended Books:

Anton H, *Calculus: A New Horizon* (6th edition), 1999, John Wiley, New York

Stewart J, *Calculus* (3rd edition), 1995, Brooks/Cole (suggested text)

Thomas GB, Finney AR, *Calculus* (10th edition), 2002, Addison-Wesley, Reading, Ma, USA

Background and Goals: This is the third course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics.

Contents: This course covers vectors and analytic geometry of 2 and 3 dimensional spaces; vector-valued functions and space curves; functions of several variables; limits and continuity; partial derivatives; the chain rule; double and triple integrals with applications; line integrals; the Green theorem; surface area and surface integrals; the Green, the divergence and the Stokes theorems with applications (Chapters 11-14 of the text)

Classical Mechanics

Prerequisite(s): Vector and Tensor Analysis

Credit Hours: 3 + 0

Recommended Books:

Bedford A, Fowler W, *Dynamics: Engineering Mechanics*, Addison-Wesley, Reading, Ma, USA

Chow TL, *Classical Mechanics*, 1995, John Wiley, New York

Goldstein H, *Classical Mechanics* (2nd edition), 1980, Addison-Wesley, Reading, Ma, USA

Marion JB, *Classical Dynamics of Particles and Fields* (2nd edition), 1970, Academic Press, New York

(suggested text)

Synge JL, Griffith BA, *Principles of Mechanics*, McGraw Hill, New York

Background and Goals: This course builds grounding in principles of classical mechanics, which are to be used while studying quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics and continuum mechanics.

Contents:

Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), newtonian mechanics, the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations, calculus of variations, Hamilton's principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, Noether's theorem, central-force motion, two-body problem, orbit theory, Kepler's laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, geostationary and polar satellites, kinematics of two-particle collisions, special theory of relativity, motion in non-inertial reference frame, rigid-body dynamics (3-D-rigid bodies and mechanical equivalence, center of mass and gravity, motion of a rigid body, inverted pendulum and stability, gyroscope), coupled oscillations, vibrating strings, wave equation in one dimension

Complex Analysis

Prerequisite(s): Real Analysis I

Credit Hours: 3 + 0

Recommended Text: Churchill RV, Brown JW: *Complex Variables and Applications* (5th edition), 1989, McGraw Hill, New York

Background and Goals: This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an

emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

Contents: The algebra and the geometry of complex numbers, Cauchy-Riemann equations, harmonic functions, elementary functions, branches of the logarithm, complex exponents. Contours and contour integrals, the Cauchy-Goursat Theorem, Cauchy integral formulas, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra. Convergence of sequences and series, the Taylor series, the Laurent series, uniqueness of representation, zeros of analytic functions. Residues and poles and the residue theorem, evaluation of improper integrals involving trigonometric functions, integrals around a branch point. Linear functions, the inversion function, transformations as mappings, preservation of angles, analytical continuation, the argument principle, the Roche theorem.

Computer Language

Prerequisite(s): Calculus II

Credit Hours: 1 + 1

Recommended Books:

Aho, AV, Ulman JD, *Foundation of Computer Science*, 1995, Computer Science Press, WH Freeman, New York

Hein JL, *Theory of Computation: An Introduction* (1st edition), Jones & Bartlett, Boston

Background and Goals: The purpose of this course is to introduce students to operating systems and environments

Contents: Introduction to operating systems, C language, building blocks, variables, input/output, loops (FOR, WHILE, DO), decisions (IF, IF ELSE, ELSE IF) construct switch statement, conditional statement, function hat returns a value using argument to pass data to another function, external variable, arrays and strings, pointers, structure, files and introduction to C++

Discrete Structures

Prerequisite(s): None

Credit Hours: 3 + 0

Recommended Text: Rosen KH, *Discrete Mathematics and its Applications* (12th edition), 1999, McGraw Hill, New York

Background and Goals: This course shall assume background in number theory. It lays a strong emphasis on understanding and utilizing various strategies for composing mathematical proofs.

Contents:

Set and Relations: Basic notions, set operations, Venn diagrams, extended-

set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction.

Elementary Logic: Logics of order zero and one, Propositions and connectives, truth tables, conditionals and biconditionals, quantifiers, methods of proof, proofs involving quantifiers.

Functional Analysis

Prerequisite(s): Complex Analysis

Credit Hours: 3 + 0

Recommended Books:

Curtain RF, Pritchard AJ, *Functional Analysis in Modern Applied Mathematics*, Academic Press, New York

Friedman A, *Foundations of Modern Analysis*, 1982, Dover

Kreyszig E, *Introductory Functional Analysis with Applications*, John Wiley, New York

Rudin W, *Functional Analysis*, 1973, McGraw Hill, New York

Background and Goals: This course extends methods of linear algebra and analysis to spaces of functions, in which the interaction between algebra and analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.

Contents:

Metric Spaces: A quick review, completeness and convergence, completion.

Normed Spaces: Linear spaces, Normed spaces, Difference between a metric and a normed space, Banach spaces, Bounded and continuous linear operators and functionals, Dual spaces, Finite dimensional spaces, F. Riesz Lemma, The Hahn-Banach Theorem, The HB theorem for complex spaces, The HB theorem for normed spaces, The open mapping theorem, The closed graph theorem, Uniform boundedness principle and its applications

Banach-Fixed-Point Theorem: Applications in Differential and Integral equations

Inner-Product Spaces: Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, representation of functionals, Reiz-representation theorem, weak and weak* Convergence.

Mathematical Modeling

Prerequisite(s): Partial-Differential Equations

Credit Hours: 3 + 0

Recommended Books:

Giordano FR, Weir MD, *Differential Equations: A Modeling Approach*, 1994, Addison- Wesley, Reading, Ma, USA (suggested text)

Jerri AJ, *Introduction to Integral Equations with Applications*, 1985, Marcel Dekker, New York

Myint UT, Debnath L, *Partial Differential Equations for Scientists and Engineers* (3rd edition), 1987, North Holland, Amsterdam

Background and Goals: Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

Contents:

Concepts of model, modeling and simulation

Functions, linear equations, linear-differential equations, nonlinear-differential equations and integral equations as models, introduction to simulation techniques

Ordinary-Differential Equations: Modeling with first order differential equations: Newton's law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; application to biological systems; modeling with periodic or impulse forcing functions. Modeling with systems of first order differential equations; competitive hunter model; predator prey model.

Partial-Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding. Modeling wave phenomena (wave equation); shallow water waves, uniform transmission line, traffic flow, RC circuits. Modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc. Modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems. Equation of continuity.

Mathematical Spaces

Prerequisite(s): Discrete Structures, Real Analysis I

Credit Hours: 3 + 0

Recommended Books:

Friedmann A, *Foundations of Modern Analysis*, 1982, Dover

Holmos PR, *Measure Theory*, van Nostrand, New York

Munkres JR, *Topology: A First Course*, Prentice Hall, Englewood Cliffs, NJ, USA

Simmon GF, *Introduction to Topology and Modern Analysis*, 1963, McGraw Hill, New York

Background and Goals: This course is designed primarily to develop pure mathematical skills of students. Students will need some background in writing proofs. They will learn notions of spaces, metric, measure and topology

Contents:

- a) *Notion of Spaces:* Example of set, group, field, ring, affine space, Banach space, normed space, Hilbert space [Simmon GF, *Introduction to Topology and Modern Analysis*, 1963, McGraw Hill, New York]
- b) *Notion of Topology:* Calculus on manifolds, continuity of functions on spaces, neighborhoods, topological spaces, finer and weaker topologies, homomorphism, homomorphic spaces, compactness, connectedness, normal spaces, Urysohn's lemma [Munkres JR, *Topology: A First Course*, Prentice Hall, Englewood Cliffs, NJ, USA]
- c) *Notion of Metric:* Metric space, complete metric space, Baire category theorem, metrization of spaces [Friedmann A, *Foundations of Modern Analysis*, 1982, Dover]
- d) *Notion of Measure:* Spaces with measure, measurable function, idea of σ -fields [Holmos PR, *Measure Theory*, van Nostrand, New York]

Mathematical Statistics

Prerequisite(s): Probability Theory

Credit Hours: 3 + 0

Recommended Books:

DeGroot MH, Schervish MJ, *Probability and Statistics* (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text)

Johnson RA, *Probability and Statistics for Engineers*, 1994, Prentice-Hall, Englewood Cliffs, NJ, USA

Papoulis A, *Probability, Random Variables, and Stochastic Processes*, (3rd edition), 1991, McGraw Hill, New York

Sincich T, *Statistics by Examples*, 1990, Dellen Publication Company

Background and Goals: In the course “Probability Theory” the students learnt how to set up mathematical models of processes and systems that are affected by *chance*. In the present course the students would learn how to check these models against reality, to determine whether they are reliable/accurate enough for practical purposes or otherwise. This helps in making predictions and decisions

Contents: Sampling theory: sampling distributions; sampling procedures; estimation of parameters: estimation of mean, variance; confidence intervals; decision theory: hypothesis testing and decision making; types of errors in tests; quality control; control charts for mean, standard deviation, variance, range; goodness of fit, chi-square test. Regression analysis; method of least squares; correlation analysis

Number Theory

Prerequisite(s): Calculus I, Discrete Structures

Credit Hours: 3 + 0

Recommended Text: Rosen KH, *Elementary Number theory and its Applications* (4th edition), 2000, Addison-Wesley, Reading, Ma, USA

Background and Goals: This course shall assume no experience or background in number theory or theoretical mathematics. The course introduces various strategies for composing mathematical proofs.

Contents: Number systems: natural numbers, integers, rational numbers, real numbers, complex numbers, the equivalence and the difference of cardinality between them, de Morvie’s theorem with applications, hyperbolic and logarithmic functions, introduction to number theory including divisibility, the Euclidean algorithm, GCD and LCM of 2 integers, fundamental theorem of arithmetic (UFT), properties of prime numbers, congruences with applications, arithmetic functions, quadratic residues

Numerical Analysis

Prerequisite(s): Numerical Computing

Credit Hours: 3 + 0

Recommended Books:

Atkinson KE, *An Introduction to Numerical Analysis* (2nd edition), 1989, John Wiley, New York (suggested text)

Burden RL, Faires JD, *Numerical Analysis* (5th edition), 1993, PWS Publishing Company

Chapra SC, Canale RP, *Numerical Methods for Engineers*, 1988, McGraw Hill, New York

Background and Goals: This course is designed to teach the students about numerical methods and their theoretical bases. The students are expected to know computer programming to be able to write program for

each numerical method. Knowledge of calculus and linear algebra would help in learning these methods

Contents: Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regula falsi method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation, Newton's divided difference, forward-difference and backward-difference formulae, Hermite interpolation. Numerical integration and error estimates: rectangular rule, trapezoidal rule, Simpson's one-three and three-eight rules. Numerical solution of systems of algebraic linear equations: Gauss-elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle's, Crout's, Cholesky's methods; Gauss-Seidel and Jacobi methods; matrix norms; method of least squares; eigenvalues and eigenvectors: inclusion methods, power method

Numerical Computing

Prerequisite(s): Computer Language

Credit Hours: 2 + 1

Recommended Books:

Etter DM, Kuncicky D, Hull D, *Introduction to MATLAB 6*, 2001, Prentice Hall, Englewood Cliffs, NJ, USA

Garvan F, *The Maple Book*, 2002, Chapman & Hall/CRC

Kaufmann S, *Mathematica As a Tool: An Introduction with Practical Examples*, 1994, Springer, New York

Background and Goals: The purpose of this course is to teach students the use of mathematical software like MATLAB, MAPLE, MATHEMATICA for solving computationally difficult problems in mathematics. The student will become well versed in using mathematical software and will learn a number of techniques that are useful in calculus as well as in other areas of mathematics.

Contents: The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above.

Optimization Theory

Prerequisite(s): Algebra I, Real Analysis I

Credit Hours: 3 + 0

Recommended Books:

Elsgolts L, *Differential Equations and the Calculus of Variations*, 1970, Mir Publishers, Moscow

Gotfried BS, Weisman J, *Introduction to Optimization Theory*, 1973, Prentice Hall, Englewood Cliffs, NJ, USA

Luenberger DG, *Introduction to Linear and Non-Linear Programming*, 1973, Addison-Wesley, Reading, Ma, USA

Background and Goals: The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

Contents: Linear programming: simplex method, duality theory, dual and primal-dual simplex methods. Unconstrained optimization: optimality conditions, one-dimensional problems, multi-dimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem. The calculus of variations, the Euler-Lagrange equations, functionals depending on several variables, variational problems in parametric form, transportation models and networks

Ordinary-Differential Equations

Prerequisite(s): Calculus III, Numerical Computing

Credit Hours: 3 + 0

Recommended Text: Zill DG, Cullen MR, *Differential Equations with Boundary-Value Problems*, (3rd Edition), 1997, PWS Publishing Co.

Background and Goals: This course provides the foundation of all advanced subjects in Mathematics. Strong foundation and applications of Ordinary Differential Equations is the goal of the course.

Contents: Introduction; formation, solution and applications of first-order-differential equations; formation and solution of higher-order-linear-differential equations; differential equations with variable coefficients; Sturm-Liouville (S-L) system and boundary-value problems; series solution and its limitations; the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations; properties of the Bessel, the Legendre and the Hermite functions

Partial-Differential Equations

Prerequisite(s): Real Analysis I, Ordinary-Differential Equations

Credit Hours: 3 + 0

Recommended Text: Myint UT, *Partial Differential Equations for Scientists and Engineers* (3rd edition), 1987, North Holland

Background and Goals: The course provides a foundation to solve Partial Differential Equations with special emphasis on wave, heat and Laplace equations. Formulation and some theory of these equations are also intended.

Contents: First-order-partial-differential equations; classification of second-order p. d. e; canonical form for second-order equations; wave, heat and the Laplace equation in Cartesian, cylindrical and spherical-polar coördinates; solution of partial differential equation by the methods of: separation of variables; the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations

Probability Theory

Prerequisite(s): Calculus III

Credit Hours: 3 + 0

Recommended Books:

DeGroot MH, Schervish MJ, *Probability and Statistics* (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text)

Papoulis A, *Probability, Random Variables, and Stochastic Processes*, (3rd edition), 1991, McGraw Hill, New York

Sincich T, *Statistics by Examples*, 1990, Dellen Publishing Company

Background and Goals: This course is designed to teach the students how to handle data numerically and graphically. If data are influenced by *chance* effect, the concepts and rules of probability theory may be employed, being the theoretical counterpart of the observable reality, whenever *chance* is at work.

Contents: Statistical measures, statistical description and graphical representation of data. Sets; introduction to probability theory; permutations and combinations; random variables; probability distributions; mean, standard deviation, variance and expectation. Binomial, Poisson, hypergeometric and normal distributions; normal approximation to binomial distribution; distributions of several random variables.

Real Analysis I

Prerequisite(s): Calculus III

Credit Hours: 3 + 0

Recommended Books:

Brabenec RL, *Introduction to Real Analysis*, 1997, PWS Publishing Company

Gaughan ED, *Introduction to Analysis* (5th edition), 1997, Brooks/Cole

Bartle RG, Sherbert DR, *Introduction to Real Analysis* (3rd edition), 1999, John Wiley, New York

Background and Goals: This is the first rigorous course in analysis and has a theoretical emphasis. It rigorously develops the fundamental ideas of calculus and is aimed to develop the students' ability to deal with abstract mathematics and mathematical proofs.

Contents: Supremum and infimum, completeness properties of the real numbers, limits of numerical sequences; limits and continuity, properties of continuous functions on closed bounded intervals; derivatives in one variable; the mean value theorem; Sequences of functions, power series, point-wise and uniform convergence. Functions of several variables: open and closed sets and convergence of sequences in \mathbb{R}^n ; limits and continuity in several variables, properties of continuous functions on compact sets; differentiation in n-space; the Taylor series in \mathbb{R}^n with applications; the inverse and implicit function theorems.

Real Analysis II

Prerequisite(s): Real Analysis I

Credit Hours: 3 + 0

Recommended Text: Fulks W, *Advanced Calculus*, John Wiley, New York

Background and Goals: A continuation of Real Analysis I, this course rigorously develops integration theory. Like Real Analysis I, Real Analysis II emphasizes proofs.

Contents: Series of numbers and their convergence. Series of functions and their convergence. Darboux upper and lower sums and integrals; Darboux integrability. Riemann sums and the Riemann integral. Riemann integration in \mathbb{R}^2 , change of order of variables of integration. Riemann integration in \mathbb{R}^3 , and \mathbb{R}^n . Riemann-Steinitz integration. Functions of bounded variation. The length of a curve in \mathbb{R}^n . Lebesgue integration.

Vector and Tensor Analysis

Prerequisite(s): Calculus I & II

Credit Hours: 4 + 0

Recommended Books:

Bourne DE, Kendall PC, *Vector Analysis and Cartesian Tensors* (2nd edition), Thomas Nelson

Shah NA, *Vector and Tensor Analysis*, 2005, A-One Publishers, Lahore

Smith GD, *Vector Analysis*, Oxford University Press, Oxford

Spiegel MR, *Vector Analysis*, 1974, McGraw Hill, New York

Background and Goals: This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics

Contents:

3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with ϵ_{ijk} notation, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coördinate systems and their bases, the spherical-polar- and the cylindrical-coördinate meshes, alternating symbol, relation between alternating symbol and kronecker delta, tensors of first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, isotropic tensors, differentiation of tensors, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagonalization of inertia tensor as aligning coördinate frame with natural symmetries of the system

BS Electives (Four-Year Program)

Abstract Algebra I	Abstract Algebra II
Advanced Calculus I (Numerical Solution of Partial Differential Equations)	Advanced Calculus II (Integral Equations)
Advanced Numerical Analysis I	Advanced Numerical Analysis II
Astronomy I	Astronomy II
Differential Geometry I	Differential Geometry II
Electromagnetism I	Electromagnetism II
Fluid Dynamics I	Fluid Dynamics II
Group Theory I	Group Theory II (Study of Symmetries)
History and Philosophy of Mathematics I	History and Philosophy of Mathematics II
Measure Theory I	Measure Theory II
Modern Algebra I (Galois Theory & Applications)	Modern Algebra II (Commutative Rings & Fields)
Nonlinear Systems I	Nonlinear Systems II
Operations Research I	Operations Research II
Projective Geometry I	Projective Geometry II
Quantum Mechanics I	Quantum Mechanics II
Relativity I	Relativity II
Software Engineering I (Design & Development)	Software Engineering II (Analysis)
Theory of Processes I (Stochastic Processes)	Theory of Processes II (Renewal Processes & Theory of Ques)
Theory of Splines I	Theory of Splines II
Topology I (Topological-Dimension Theory)	Topology II (Differential Topology)

Masters Program in Mathematics

- Degree Awarded:* **Master of Science (MS)**
- Entrance Requirements:* BS (Mathematics) or equivalent; GRE (General); cutoff scores 50th percentile (each section); written test; personal interview
- Duration of the Program:* 2 years (4 semesters), students must submit thesis at the end of 2 years of study
- Total Credit Hours:* 34
- Course Requirements:* 24 credit hours [Research Methodology + 15-credit-hour elective courses in the area of specialization (Elective A) and 6-credit-hour free electives (Elective B) + Mathematics Teaching Techniques]; more courses could be prescribed by the supervisor depending on the results of masters examinations; students coming from allied disciplines have to complete additional courses; courses to be taken in consultation with the supervisor
- Academic Standards:* Same as in BS Scheme of Studies
- Attendance Requirements:* Same as in BS Scheme of Studies
- Masters Examinations:* Masters' Examinations (written + oral) must be passed before starting thesis research; any student, who fails to qualify these examinations in 2 attempts, or by the end of first year of study, is asked to withdraw from the program. Masters' examinations are to be offered by the department 4 times during an academic year. Written examination may consist of selected questions from PhD Qualifying Examination, testing the student in core areas. Oral examination, testing the student in the area of specialization, is scheduled only after the student passes the written examination. The student is, also, required to describe the proposed research work. Board of Advanced Studies and Research to conduct these examinations form committees.
- Thesis Requirements:* Student must write and publicly defend a thesis bearing original work, which should have enough substance to result in at least one publishable paper in a journal of international repute.
- Mathematics Undergraduate:* Student acts as Associate Instructor (Teaching Assistant) in a BS course. This is an opportunity for

- Teaching:* a student to gain teaching experience at the university level.
- Summer Activity:* Students, who have passed Masters Examination, may start independent study to find thesis problem during the first year. During the second year the students shall be involved in thesis research.
- Qualifications for Course Supervisors (Instructors):* Faculty holding PhD degree is entitled to teach lecture session; the electives offered are taken by both MS and PhD students
- Qualifications for Associate Instructors (Teaching Assistants):* Associate Instructors (Teaching Assistants) must hold MS (Two-Year Program), or MPhil and may handle laboratory, discussions as well as assignment, quiz and problem grading
- Syllabus:* Course Supervisor is required to distribute syllabus break-down into 14 units, each unit completed within a week.
- Seminars:* Same as in BS Scheme of Studies
- Pedagogical Techniques:* Lecture sessions of each unit (normally, 2-3 lectures) are followed by a discussion session (reinforcing the concepts taught through examples, alternate derivations and proofs, innovative solutions to the problems), conducted by the Associate Instructor. In addition, a review session should be arranged prior to each monthly test and a comprehensive review before the final examination, both sessions conducted by the Course Supervisor.
- Continuous-Evaluation Report:* Same as in BS Scheme of Studies with the difference that the report is discussed ONLY with the student and NOT with the student's parents.
- Withdrawal Policy:* Same as in BS Scheme of Studies
- Final Grades:* Same as in BS Scheme of Studies
- Grading of Problem Sets:* Same as in BS Scheme of Studies
- Final Examinations:* Same as in BS Scheme of Studies
- Marks Breakdown:* Passing grade is set at 60%. Marks breakdown and GPA assignment same as in BS Scheme of Studies, with the difference that the numerical grade 0-59 corresponds to alphabetical grade F, with grade point zero. There is no D, D⁺ or C⁻ for graduate courses.

MS Scheme of Studies (Two-Year Program)

FIRST YEAR

First Semester			Second Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Elective-A I	3 + 0	1	Elective-A II	3 + 0
2	Guest /Students' Seminar I	1 + 0	2	Elective-A III	3 + 0
3	Masters-Examination Preparation I	Non- Credit	3	Guest /Students' Seminar II	1 + 0
4	Mathematics Teaching Techniques	1 + 0	4	Masters-Examination Preparation II	Non- Credit
5	Mathematics Undergraduate Teaching I	Non- Credit	5	Mathematics Undergraduate Teaching II	Non- Credit
Total			Total		
5 + 0			8 + 1		

SECOND YEAR

Third Semester			Fourth Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Elective-A IV	3 + 0	1	Elective-A V	3 + 0
2	Elective-B I	3 + 0	2	Elective-B II	3 + 0
3	Mathematics Undergraduate Teaching III	Non- Credit	3	Mathematics Undergraduate Teaching IV	Non- Credit
5	Guest /Students' Seminar III	1 + 0	5	Guest/Students' Seminar IV	1 + 0
6	Thesis Research I	3 + 0	6	Thesis Research I	3 + 0
Total			Total		
10 + 0			10 + 0		

Note: The students who have passed Masters' Examinations before the start of Second Semester shall not be attending "Masters-Examination Preparation II"

Research Methodology

Prerequisite(s): Mathematical Statistics

Credit Hours: 1 + 1

Recommended Books:

Day RA, *How to Write and Publish a Scientific Paper*, 1979, ISI Press, Philadelphia, Pa., USA

Diamond WJ, *Practical Experiment Designs for Scientists and Engineers* (2nd Edition), 1989, John Wiley, New York

Harré R, *Great Scientific Experiments: Twenty Experiments that Changed the World*, 2002, Dover, New York

Background and Goals: This course is supposed to introduce the student with research tools and methods and particularly geared for mathematics research, both pure and applied. Student is supposed to choose a research problem, prepare and present a research proposal, and write a review/research paper (8-10 pages) in format HEC-approved research journal in mathematics.

Contents:

Scientific Method: Scientific statement, hypothesis, model, theory and law, steps in problem solving, limitation of science, calibration (mathematical definition: conversion of output, which is not desired, to desired output through a linear/nonlinear model), sensitivity, least count, reproducibility (stability and objectivity), difference between accuracy and precision, role of mathematician in industrial, business, financial and health-care research, challenges of research in pure mathematics

Research Proposals: Identifying problem, literature search, defining problem, feasibility study, pilot projects/field trials, formal research proposal, budgeting and funding, progress report, final technical and fiscal report

Experiment Design: Purpose of an experiment, good, bad and inefficient experiments, null and alternative hypothesis, α and β errors and their relationship to sensitivity and specificity, designing efficient experiments, calculation of sample size, identifying variables of interest and their interactions, operating-characteristic curves, power curves, surveys and field trials

Publishing Original Paper: Submission, role of the editor, peer-review process, importance of citations, impact factor, plagiarism, protection of your work from misuse

MS Electives Alphabetical Listing(Two-Year Program)

First Semester	Second Semester
3-D Optical Imaging & Image Processing I	3-D Optical Imaging & Image Processing II
Advanced Astronomy I (Solar-System Astrophysics)	Advanced Astronomy II (Cosmology)
Advanced Differential Equations I	Advanced Differential Equations II
Advanced Fluid Dynamics I	Advanced Fluid Dynamics II
Advanced Functional Analysis I	Advanced Functional Analysis II
Advanced Group Theory I (Abelian Groups)	Advanced Group Theory II (Soluble & Nilpotent Groups)
Advanced Operational Research I	Advanced Operational Research II
Advanced Quantum Mechanics I	Advanced Quantum Mechanics II
Advanced Relativity I	Advanced Relativity II
Advanced Topology I (Topological-Dimension Theory)	Advanced Topology II (Differential Topology)
Almost Periodic Functions I	Almost Periodic Functions II
Astrodynamics I	Astrodynamics II
Atmospheric Study I	Atmospheric Study II
Biomathematics & Bioinformatics I	Biomathematics & Bioinformatics II
Classical Electrodynamics I	Classical Electrodynamics II
Classical field Theory I	Classical field Theory II
Combinatorics & Measure Theory I	Combinatorics & Measure Theory II
Continuum Mechanics I	Continuum Mechanics II
Control Theory I (Basic Principles)	Control Theory II (Design of Control System)
Differential Geometric Techniques I	Differential Geometric Techniques II
Experiment Designs I	Experiment Designs II
Fourier Series I	Fourier Series II
Gauge Theory of Gravitation I	Gauge Theory of Gravitation II
General Topology I	General Topology II
Homological Algebra I	Homological Algebra II
Lie Algebra I	Lie Algebra II
Near Rings I	Near Rings II (Special Classes)
Plasma Dynamics I	Plasma Dynamics II
Quantum Field Theory I	Quantum Field Theory II
Reliability Analysis I	Reliability Analysis II
Representation Theory I	Representation Theory II
Special Classes of Rings I	Special Classes of Rings II
Summability Theory I	Summability Theory II
System Analysis I	System Analysis II

Theory of Rings I	Theory of Rings II
Topics in Astronomy I (Stellar & Galactic Astronomy)	Topics in Astronomy II (Extra-Galactic Astronomy)
Topics in Numerical Analysis I	Topics in Numerical Analysis II
Topological Groups I	Topological Groups II

Doctoral Program in Mathematics

Degree Awarded: **Doctor of Philosophy (PhD)**

Entrance Requirements: MS in Mathematics or equivalent; GRE (General) and GRE (Mathematics); cutoff scores 50th percentile (each section for the former); personal interview

Duration of the Program: 2 years (4 semesters); students cannot submit dissertation before completing 2 years of study; admission shall be valid till 7 years of initial enrolment

Total Credit Hours: 48 (12 credit hours must be completed during each semester)

Course Requirements: 18 credit hours [12-credit-hour elective courses in the area of specialization (Elective A) and 6-credit-hour free electives (Elective B)]; students coming from allied disciplines or possessing MPhil without course work/less than 8 courses have to complete additional courses

Class Size: Same as in MS Schemes of Studies

Academic Standards: Same as in MS Scheme of Studies

Attendance Requirements: Same as in MS Scheme of Studies

Qualifying Examinations:

Qualifying Examinations (written + oral) must be passed before starting dissertation research; any student who fails to qualify these examinations in 2 attempts, or by the end of first year of study is asked to withdraw from the program. Qualifying examinations are to be offered by the department 4 times during an academic year. Written examination tests the student in core areas. Oral examination, testing the student in the area of specialization, is scheduled only after the student passes the written examination. The student is, also, required to describe the proposed research work. Committees formed by Board of Advanced Studies and Research conduct these

	examinations.
<i>Dissertation Requirements:</i>	Student must write and publicly defend a dissertation bearing original work, which should result in at least 2 publishable papers in journals of international repute (acceptance of papers required for award of degree)
<i>Mathematics Graduate Teaching:</i>	Student acts as Associate Instructor (Teaching Assistant) in an MS/a PhD course, which the student has, already, passed. This is an opportunity for a student to gain teaching experience at the university level.
<i>Independent Study:</i>	Supervised by a faculty member, which may help student find dissertation problem
<i>Seminars:</i>	Same as in MS Scheme of Studies
<i>Summer Activity:</i>	Students, who have passed Qualifying Examination, may start dissertation research during the first year. They will be engaged in full-time research during the subsequent summers to finish their degrees in time.
<i>Qualifications for Course Supervisors (Instructors):</i>	Same as in MS Scheme of Studies
<i>Qualifications for Associate Instructors (Teaching Assistants):</i>	Same as in MS Scheme of Studies
<i>Syllabus:</i>	Same as in MS Scheme of Studies
<i>Pedagogical Techniques:</i>	Same as in MS Scheme of Studies
<i>Continuous-Evaluation Report:</i>	Same as in MS Scheme of Studies
<i>Withdrawal Policy:</i>	Same as in MS Scheme of Studies
<i>Final Grades:</i>	Same as in MS Scheme of Studies
<i>Grading of Problem Sets:</i>	Same as in MS Scheme of Studies
<i>Final Examinations:</i>	Same as in MS Scheme of Studies
<i>Marks Breakdown:</i>	Same as in MS Scheme of Studies

PhD Scheme of Studies (Two-Year Program)

FIRST YEAR

First Semester			Second Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Elective-A I	3 + 0	1	Elective-A II	3 + 0
2	Elective-B I	3 + 0	2	Elective-B II	3 + 0
3	Guest/Students' Seminar I	1 + 0	3	Guest/Students' Seminar II	1 + 0
4	Independent Study I	5 + 0	4	Independent Study II	5 + 0
5	Mathematics Graduate Teaching I	Non- Credit	5	Mathematics Graduate Teaching II	Non- Credit
6	Qualifying- Examination Preparation I	Non- Credit	6	Qualifying- Examination Preparation II	Non- Credit
Total		12 + 0	Total		12 + 0

SECOND YEAR

Third Semester			Fourth Semester		
#	<i>Course Title</i>	<i>Cr. Hrs.</i>	#	<i>Course Title</i>	<i>Cr. Hrs.</i>
1	Dissertation Research I	8 + 0	1	Dissertation Research II	8 + 0
2	Elective-A III	3 + 0	2	Elective-A IV	3 + 0
3	Guest/Students' Seminar III	1 + 0	3	Guest/Students' Seminar IV	1 + 0
4	Mathematics Graduate Teaching III	Non- Credit	4	Mathematics Graduate Teaching IV	Non- Credit
Total		12 + 0	Total		12 + 0

Notes:

- i) List of PhD electives is included with the MS Scheme of Studies.
- ii) The students who have passed Qualifying Examinations before the start of Second Semester shall enroll in "Dissertation Research I" (followed by "Dissertation Research II" and "Dissertation Research III" during the Third and the Fourth Semesters, respectively), instead of "Independent Study II". They shall not be attending "Qualifying-Examination Preparation II"

RECOMMENDATIONS

(to make the curriculum-development effort effective)

National-Curriculum-Revision Committee in Mathematics, unanimously, made the following recommendations:

- i) Every public/private sector university should have a mathematics department employing full-time faculty.
- ii) HEC should pay for the membership of at least one international professional society membership and subscription of at least two international journals for each active faculty member.
- iii) MPhil/PhD students publishing an international research paper should receive monetary incentives.
- iv) Course load for Professor, Associate Professor, Assistant Professor and Lecturer should be 3-credit hours, 6-credit hours 9-credit hours and 12-credit hours, respectively.
- v) For selection/promotion, teaching experience should be counted after 16 years of schooling.
- vi) Existing faculty members should be selected/promoted to higher grades according to existing criteria. New faculty members may be inducted according to revised criteria.
- vii) There should be equal salaries for Mathematics and IT teachers.
- viii) Mathematics research impact factor should be evaluated separately from other disciplines.
- ix) GRE (Mathematics) with a score of 50 percentile or above should be a must to get HEC scholarship.
- x) There should be a provision of teachers-training program (contents + pedagogical techniques) to prepare manpower for BS, MS and PhD programs proposed in this document.
- xi) In order to make these programs successful, faculty satisfaction is essential. Hence, a committee should be formed to formulate career-development protocols for the faculty.
- xii) Textbook writing by Pakistani experts to be encouraged, monetary incentives and sabbatical leave to bring out quality publications; young faculty members should be compiling their lecture notes as e-books, which could come out regular hard-copy books after they have been teaching the course for 5 consecutive years.
- xiii) Evaluation of thesis should be by experts in the field, who may reside in or outside Pakistan. The competence of expert is not to be determined by his country of residence, but the number of publications in the area during the last 5 years.
- xiv) Examination systems must also be modernized in order to maintain standards. A separate forum should chart out examination-conduct

procedures, pre- and post-examination exercises to conduct the examinations smoothly, with efficiency and effectiveness.

- xv) Four-Year BS Program in Mathematics should, also, be implemented in Engineering Universities.
- xvi) Discipline-wise national list of faculty members and their addresses, phone numbers, e-mails and list of recent publications be available on HEC website.