# Early statistics in the Nordic countries – when did the Scandinavians slip behind the British?

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### **1. Introduction**

Trygve Haavelmo got the Nobel prize in economics (the prize in memory of Alfred Nobel) in 1989 – essentially for his 1944 treatise: "The probability approach to econometrics", where econometrics eventually was founded on probability and mathematical statistics and where simultaneous equation models were introduced. Haavelmo learned statistics from Frisch, and was influenced by Neyman. Ragnar Frisch was awarded the first Nobel Prize in economics in 1969, together with Jan Tinbergen. Frisch was one of the founders of the Econometric Society. His first papers, including his thesis from 1926, were on Thiele's semi-invariants. Where had Frisch learned statistics?

In the early 20s, Frisch read most of the then available literature on statistics. He studied the Scandinavians: Oppermann, Gram, Thiele, Steffensen and Charlier; continental writers like Bruns, von Mises, de Bruno and Levy; Russians like Chebycheff and Tshuprov; and most of all British statisticians: Karl Pearson, Burton, Yule, Edgeworth, Sheppard, Romanovsky and Fisher. Frisch thought then, apparently, that Britain was the most dynamic arena of statistics. He kept, however, on to the study of the semi-invariants, which were re-invented by Fisher and called cumulants, nearly 30 years later than fully developed and internationally published by Thiele.

By presenting my favourite Scandinavian statisticians and scientists with a statistical bend before 1926, I hope to argue that in statistics, Scandinavia was not behind Britain at the turn of the century. The men (they are all men) are presented chronologically, by year of their main contribution. Due to another contribution in the session, Thiele is given less space than he deserves. Hald (1998) presents his contributions, and also those of other mathematical statisticians like Oppermann, Gram and Charlier in full, and he shows how impressive the Copenhagen school was at the turn of the century. There are additional early Nordic statisticians and scientists that deserve mentioning. My selection is subjective, and surely debatable. I have no knowledge of statistics in the Balticum or in the Finnish language.

## 2. Short presentations

Eilert Sundt (1817-1875) contributed strongly to sociology, demography and ethnography in Norway. He was trained in theology. As a Sunday school teacher in a prison, Sundt was struck by the misery of tinkers that seasonally were jailed for the purpose of preparing them for confirmation. Sundt did fieldwork several months each year for some 10 years – on foot. He first wanted to study the condition of the tinkers, but met none on his first field trip. He saw, however, so much misery in the Norwegian countryside that he devoted his life to the study of the common man. He published nearly one book every year between 1850 and 1868; all written in a poetic style – hopefully not only about the common man but also for the same, but filled with masterfully tabulated statistical data. Some of the titles are: Account of the tinkers in Norway; On mortality in Norway; On marriage in Norway; On the state of morality in Norway; On temperance conditions in Norway. Sundt's work is characterised by good design and logic. His field trips had double tracks: gathering data for the current main project, and developing hypotheses and gathering pilot data for the next project. He stuck with his hypotheses, and tested them informally by comparing distributions, not only mean values. By cross-tabulation, also in three-way tables, Sundt was able to control for

confounders and to expose potential biases. Sundt did also use exemplary questionnaires to teachers and clergymen, and personal interviews. In the latter case, he kept the interviewer ignorant of the purpose of the study to prevent interviewer bias. Eilert Sundt wrote exclusively in Norwegian, and is not translated to any degree. Had Sundt been British, you would all have known his name.

<u>Ludvik Henrik Ferdinand Oppermann</u> (1817 – 1883) was a professor of German language, member of the Danish Parliament, forester, insurer – and mathematical statistician. Oppermann had a good grasp of the German tradition, from Gauss on. His 1863 paper under the signature "En Dilettant" presented the method of least squares concisely and elegantly, and turned the Gauss elimination method into a for-runner of the Gram-Schmidt orthogonalization. Oppermann was the first to use least squares to smooth mortality tables for use in insurance. He also used the empirical cumulative distribution function to check the assumption of normality.

<u>Jørgen Pedersen Gram</u> (1850 – 1916) of Copenhagen was an applied mathematician with interests in insurance and forestry. In his thesis of 1879, Gram picked up Oppermann's orthogonalization, and developed this to a theory of function approximation in  $L_2$  space. This Gram-Schmidt orthogonalization was applied to the derivatives of the normal density, and lead to the expansion, which is associated with Charlier. In his thesis, variance-stabilising transformations were used to get rid of differential weights in least squares. Later, Gram used his method to develop piece-wise cubic regression (1915). When studying the reproductivity of a forest, Gram used stratified sampling, and he developed the formula for optimal allocation in 1889. This result is usually ascribed to Neyman's 1934 paper.

<u>Thorvald Nicolai Thiele</u> (1838 – 1910) was a professor of astronomy in Copenhagen, with interests in insurance and mathematical statistics. He developed the Gauss - Oppermann - Gram approach to statistics further, with his half-invariants as the crowning tool. Thiele made a clear distinction between theoretical and empirical distributions, and he was interested in the sampling distribution of his estimators. He removed bias from his estimators. In his method of "summary criticism", Thiele partitioned the sums of squares in linear analyses, and used the two first moments of the chi-square distribution with the correct degree of freedom to assess the fit and to test effects. A two-way ANOVA was presented in his 1889 text. His more elementary and less interesting 1897 book, was translated to English in 1903.

Anders Nicolai Kiær (1838 – 1919) was the first director of Statistics Norway. As Quetelet in 1827, Kiær realised the value of survey sampling to study social issues, but in contrast to Quetelet he got several such studies carried out. Quasi-random survey sampling was employed from 1880. Kiær used the term "the representative method". Except for comparing the results from two independent samples, Kiær made no attempt to evaluate the uncertainty in results obtained by the representative method. No explicit probability model was employed, but how the sample was drawn was described in detail, and Kiær noted that "the sampling mechanism is so haphazard and random relative to the population that one may assume that it does function in the same way as a lottery". ISI was founded in 1885. Kiær was an active member, and he proudly hosted the fifth ISI meeting in Oslo in 1899. Kiær argued, however, in vain for his representative method at four sessions of the ISI from 1895 to 1903.

<u>Johannes A. G. Fibiger</u> (1867 – 1928) was a medical doctor in Copenhagen (Hrobjartsson et al 1998). In 1896-7, Fibiger conducted the first properly controlled clinical trial on record. At issue was the new serum treatment of diphtheria. His professor in Copenhagen had previously carried out a series of less controlled trials, but had not seen any positive effect of serum treatment. In the new trial, treatment allocation depended on the day of admittance. A rather complete protocol for the experiment was developed, followed, and fully reported. On alternating days, admitted patients with diphtheria bacteria were allocated to the standard treatment and to standard treatment *with* serum treatment. The primary response was survival. Secondary responses were also recorded, along with covariate information. Of the 239 patients in the serum treated group, 8 patients died, while 30 of the 245 patients in the control group died. Without formally testing the null hypothesis, Fibiger concluded, "no objection can be raised against the statistical significance of the numbers". Fibiger was aware of various pitfalls, and he listed four methodological features of his experiment: 1. Large

number of study units to provide a sufficient number of cases (dead); 2. Long study period; 3. "To compensate for seasonal variation in mortality, the study should last at least one year"; and 4. The treatment allocation ought to be impartial "to eliminate completely the play of chance and the influence of subjective judgement", hence the alternate day allocation. Fibiger received the Nobel Prize in Medicine in 1927, not for his groundbreaking clinical trial of serum treatment of diphtheria, but for his work on gastric cancer in rats.

Ernst Filip Oscar Lundberg (1876 – 1965) was managing director of an insurance company in Sweden. His early personal experiences with near collapse of insurance companies must have motivated Lundberg to study the probability of ruin. Another motivation was that mathematical risk theory was "an area of limited literature and in which it was not required to master the methodology of mathematical research too profoundly". In his 1903 thesis, Lundberg departed from the tradition of looking at the individual insurance policies, and took a collective view where he modelled the claims to come at stochastic points in time. Lundberg developed the marked Poisson process model: forward equations, operational time etc. He was particularly interested in the cumulative risk process, and developed asymptotics and Berry-Esséen type bounds for this process. For degenerate claim size distributions, Lundberg found the Poisson process as a simple solution to the forward equations. The theory was refined in later papers, with studies of extreme values and level crossings. In the Scandinavian actuarial community, Lundberg's work was regarded as too difficult but probably very important, and it lay dormant for some 30 years.

<u>Carl Wilhelm Ludwig Charlier</u> (1862 – 1934) was professor of astronomy in Lund. Without referring to his colleagues in Copenhagen, Charlier published in 1905 the expansion developed by Gram and discussed at length by Thiele. Charlier used Fourier series methods. As was popular at the time, Charlier fitted distribution models to a variety of statistical data.

<u>Agner Krarup Erlang</u> (1878 – 1929) was hired by Jensen (Jensen's inequality) to apply probability methods to telephony in Copenhagen. In 1909, Erlang used the Poisson process and other related models to develop queuing theory. Erlang was particularly interested in the stationary aspects of his processes. His method of "statistical equilibrium" was used efficiently to find waiting time distributions etc.

Johan Hjort (1869 – 1948) was professor of marine biology in Oslo. To fishermen, the great fluctuations in their catches is a problem, while to marine biologists the cause of variation was one of the great challenges early in the century. According to the migration hypothesis, the abundance of fish was practically unlimited, but due to variation in the migration pattern, catches would fluctuate. Another hypothesis was that fertile females were fished and consequently the production of eggs was hampered. Fish hatching was proposed as a solution to the problems, both with respect to harvest quantity and variability. The proponents of cod hatching were pressed to conduct experiments to prove their case. These proponents understood testing in this way, and concluded that hatching indeed improved matters. Hjort and his colleagues (see Smith 1994) insisted on the experiment being controlled, and took a more sceptical approach in the interpretation. They actually argued convincingly that the proponents had capitalised on natural variability and over-interpreted the data in the favour of the hatching hypothesis. Hjort knew that enormous numbers of eggs were produced by each female, and that only a very small fraction of the eggs would develop to a cacheable fish. He also knew the variability from year to year of the environment for these eggs, larvae and juveniles, and developed the variable year class hypothesis. The idea of using demographic concepts like cohort, mortality etc was new in fisheries. Hjort had, however, developed a life insurance program for fishermen and took the demographic and statistical way of thinking to fisheries. To test his hypothesis, he needed methods to age the individual fish. Methods to count the year-rings on the scales on herring were developed and validated for ageing. Samples of size 25 were collected from as many schools of herring as possible over the years 1907 to 1913, and the yearly age distribution of mature herring was estimated. When putting his yearly age distributions on the same graph, a clear picture emerged. The herring stock was mainly made up of two strong year classes, one from 1899 and one from 1904. The distribution was bi-modal for the years 1907 to 1910, and then uni-modal from 1911 to 1913. The modes moved beautifully one year

up for each year, with the class of 1899 basically disappearing at the age of 11. Hjort had a clear concept of hypothesis testing: "Could these results be due to randomness ..., or are they due to a general law?". His variable cohort hypothesis was tested by excellent and convincing descriptive graphics. He did not use any probability. Hjort grouped his data by area, and found the same picture (a strong 1904 year class) in all areas. He also gathered data for 1914, and the age-distribution for that year was as predicted from his hypothesis. Hjort's conclusion in 1914 was that few fish survive in many years, and in contrast many fish survive in a few years. This is still the picture, and our methods to assess fishery resources and to manage fisheries are to this day based on the variable year class hypothesis.

<u>Jarl Waldemar Lindeberg</u> (1876 – 1932) was reader of mathematics in Helsinki. In 1920 he developed his version of the central limit theorem. What we know as Kendall's  $\tau$  was developed and he found the two first moments of its sampling distribution. Lindeberg used line transect methods in forestry, and when determining the necessary number of transect to obtain a sufficiently precise confidence interval, he seems to have rediscovered the Student distribution in 1926.

## **3.** Conclusion

In theoretical and applied statistics and in probability, the Scandinavians matched, in my view, Galton and his contemporaries. Galton died in 1911. However, with Fisher on the British scene, and with the Copenhagen school drying up, the centre of statistics shifted definitely to Britain.

#### References

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#### RESUMÉ

Les statisticiens et les scientifiques orientés à la statistique à la Scandinavie sont presentés brièvement: Sundt, Oppermann, Gram, Thiele, Kiær, Fibiger, Lundberg, Charlier, Erlang, Hjort, et Lindeberg. On pose la question que les statisticiens scandinaves, en particulier les danois, sont étés au niveau des anglais au temps de Galton, mais avec Fisher, les scandinaves sont tombés derrière.