

Dirichlet kernel*

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The *Dirichlet kernel* D_n of order n is defined as

$$D_n(t) = \sum_{k=-n}^n e^{ikt}.$$

It can be represented as

$$D_n(t) = \frac{\sin\left(n + \frac{1}{2}\right)t}{\sin \frac{t}{2}}.$$

Proof: It is

$$\begin{aligned} \sum_{k=-n}^n e^{ikt} &= e^{-int} \frac{1 - e^{i(2n+1)t}}{1 - e^{it}} \\ &= \frac{e^{i\left(n + \frac{1}{2}\right)t} - e^{-i\left(n + \frac{1}{2}\right)t}}{e^{i\frac{t}{2}} - e^{-i\frac{t}{2}}} \\ &= \frac{\sin\left(n + \frac{1}{2}\right)t}{\sin \frac{t}{2}}. \quad \square \end{aligned}$$

The Dirichlet kernel arises in the analysis of periodic functions because for any function f of period 2π , the convolution of D_N and f results in the Fourier-series approximation of order n :

$$(D_N * f)(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(y) D_n(x - y) dy = \sum_{k=-n}^n \hat{f}(k) e^{ikx}.$$

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