Transforming Students' Lives through an Equitable Mathematics Approach: The Case of Railside School.

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Abstract

The low and inequitable mathematics performance of students in urban American high schools has been identified as a critical issue contributing to societal inequities. In an effort to better the field's understanding of equitable and successful teaching, we report results from a four-year longitudinal study of approximately 700 students as they progressed through three high schools. One of the findings of the study was the incredible success of "Railside" school, where students learned more, enjoyed mathematics more and progressed to higher mathematics levels. This paper presents large-scale evidence of these important achievements and provides detailed analyses of the ways that the Railside teachers brought them about, with a focus on the teaching and learning interactions within the classrooms.

Introduction

The low and inequitable mathematics performance of students in urban American high schools has been identified as a critical issue contributing to societal inequities (Moses & Cobb, 2001) and poor economic performance (Madison & Hart, 1990). Thousands of students in the United States and elsewhere, struggle through mathematics classes experiencing repeated failure as they attempt to understand and connect with abstract mathematical ideas. The question of how best to teach mathematics remains controversial and debates are dominated by ideology and advocacy (Rosen, 2001). It is critical that researchers gather more evidence on the ways that mathematics may be taught more effectively, in different settings and circumstances. This paper reports upon one study that may contribute to the growing portfolio of evidence that the field is producing.

In this paper we report upon a five-year longitudinal study of approximately 700 students as they progressed through three high schools. The study comprised a range of qualitative and quantitative research methods including assessments, questionnaires and interviews, conducted every year and over 600 hours of classroom observations. One of the findings of the study was the incredible success of one of the schools. At 'Railside' school students learned more, enjoyed mathematics more and progressed to higher mathematics levels. What made this result more important was the fact that Railside is an urban school on what locals refer to as the 'wrong' side of the tracks. Trains pass just feet away from the students' desks, interrupting lessons at regular intervals. Students come from homes with few resources and the population is culturally and linguistically diverse, with many language learners. At the beginning of high school the Railside students were achieving at significantly lower levels than the students at the other two more suburban schools in our study. Within two years the Railside students were significantly outperforming students at the other schools. The students were also more positive about mathematics, they took more mathematics courses and many more of them planned to pursue mathematics at college. In addition, achievement differences between students of different ethnic groups were reduced in all cases and were eliminated in most. By their senior year 41% of Railside students were taking calculus compared to around 27% of students in the other two schools. At Railside mathematics classes were calm and peaceful with a high work-rate and few behavioral problems, and the ethnic cliques that are evident in many schools did not form. In interviews the students told us that they learned to respect students from other cultures and circumstances through the approach used in their mathematics classes. The mathematics teachers at Railside achieved something important that many other teachers could learn from – they provided students from disadvantaged backgrounds a great chance of success in life and they taught them to enjoy mathematics and to include it as part of their futures. This paper will present evidence of these important achievements and report upon the ways that the teachers brought them about.

Some would argue that research studies should sample large numbers of schools so that outsiders can be comfortable that findings are generalizable. But one of the results of this multi-method study is the importance of particular *teaching and learning interactions* in the promotion of high and equitable attainment and we would argue that detailed, fine-grained analyses are needed to understand teaching and learning interactions. Furthermore, such detailed analyses give many insights into the ways in which other schools may bring about similar achievements. A number of large-scale studies of different schools enacting contrasting teaching approaches have shown no differences in achievement between students taught in very different ways (Riordon & Noyce, 2001; Thompson & Senk, 2001). This is not, we contend, because the teaching approaches do not matter but because differences in the ways approaches are enacted are not taken into account across large samples. Researchers need to look inside classrooms in order to provide analyses of teaching and learning interactions and to understand ways of countering low and inequitable achievement and participation in mathematics.

Research on Equitable Teaching

Students' opportunities to learn are significantly shaped by the curriculum used in classrooms and by the decisions teachers make as they enact curriculum and organize other aspects of instruction (Darling-Hammond, 1998). Studies that have monitored the impact of conceptually oriented mathematics materials, taught well and with consistency, have shown higher and more equitable results for participating students (see for example, Boaler, 1997, 2000; Briars & Resnick, 2000; Schoenfeld, 2002; Silver, Smith & Nelson, 1995). Such findings support a widely held belief that 'reform' curricula hold the potential for more equitable outcomes (Schoenfeld, 2002). But studies of reform-oriented curricula have also shown that such approaches can be difficult to implement and that such curricula are unlikely to counter inequities unless accompanied by particular teaching practices (Boaler, 2002a, 2002b; Lubienski, 2000). There are some indications that teachers' careful and explicit attention to the new ways in which students need to engage (Boaler, 2002a, 2002b; Corbett & Wilson, 1995) as well as social and cultural awareness and sensitivity (K. Gutiérrez, Baquedano-Lopez, & Tejeda, 1999; R. Gutiérrez, 1999) may be critical to the success of reform-oriented approaches.

The demands placed upon students in reform-oriented classrooms are quite different from those in more traditionally organized classrooms. Students need support in understanding the new ways in which they are expected to participate and in developing the new learning practices they need (Cohen & Ball, 2001; Corbett & Wilson, 1995). The need for teachers to explicitly attend to students' understanding of the ways they need to work is consistent with a broad research literature on formative assessment. Black & Wiliam's (1998) review of hundreds of assessment studies showed that formative assessment was a practice that produced significant learning gains and helped to close the gap between the performance of low and high attaining students (see for example, White & Frederiksen, 1998). The main tenets of formative assessment are that students must have a clear sense of the characteristics of high quality work, a clear sense of the place they have reached in their current work, and an understanding of some steps they can take to close this gap. The idea that careful attention needs to be paid to students' awareness of expected ways of working is also supported by the perspective of Delpit (1998). Lisa Delpit (1988) has argued that teachers must make explicit the unarticulated rules governing classroom interactions that support different schooling practices, and students must be given opportunities to master those ways of being, doing and knowing. To not support students in "code switching" (Heath, 1983) is to participate in perpetuating inequality.

Many researchers have documented the importance of cultural sensitivity and awareness among teachers. In some instances this has involved the design of curricular examples and schooling structures that build upon the cultural resources students bring to school. Lee (1995) for example, developed an English course which built upon African-American students' competence with social discourse (specifically, the practice of signifying), by focusing on song lyrics and using this as a bridge into the study of other poetry, discussions of literary interpretation, and as a basis for students' writing. She described this approach as "a model of cognitive apprenticing based on cultural foundations" (p. 162). This form of cognitive apprenticeship produced achievement gains in the experimental group that were over twice the gains of the control group. Tharpe and Gallimore (1988) worked with native Hawaiians in their Kamehameha Elementary Education Program (KEEP), designing the structure of the school day and classroom activities to be consonant with the students' home cultures. Their research on this program has consistently demonstrated learning gains for this traditionally disadvantaged group of children that meet or surpass the average gains of the population as a whole.

In other instances researchers have found that teaching approaches are more equitable when teachers are sensitive to the cultural differences of their students, without necessarily basing curricular examples upon the students' cultures or aligning instruction with students' out-of-school practices. Rochelle Gutiérrez (1996, 1999, 2000) for example, found that mathematics departments committed to equity enhanced the success of students even when they did not speak the students' languages, nor did they design particular curricular examples to be culturally sensitive. They did, however, use innovative instructional practices and provided a rigorous and common curriculum for all students. Kris Gutiérrez (1995; Gutiérrez, Larson & Kreuter, 1995) documented the use of a "third space" by a teacher who was successful in supporting broad participation across a range of students. Often the only valid "space" for participation is one with a more formal, structured agenda that has been fairly well-defined by the teacher. A "third space" can be created when the teacher takes up a student's proposal or idea that, at least on the surface, might not seem to have a connection to the academic concepts or topics at hand. The creation of a third space allows students to influence the agenda and course of lessons, and allows the teacher to build upon students' prior experiences creating a classroom culture that supports a wider range of participation practices. Hand (2003) found support for the importance of this practice in her study of three high school teachers from Railside school (the focus of this article).

The pedagogies described by Lee, Tharpe and Gallimore, and K. Gutiérrez bridge an understood gap between students' out-of-school worlds and cultures and their experiences in school. Such practices leverage students' prior knowledge, ways of knowing, and experiences by offering opportunities for participation not afforded in many classrooms, and consequently increasing students' interest and access by facilitating their engagement in learning activities.

Ladson-Billing's (1994, 1995) description of 'culturally relevant teaching' also highlights the importance of teachers understanding culture and promoting a flexible use of students' local, national and global cultures. Ladson-Billings locates this dimension of teachers' work within a broad description of good teaching which includes such features as subject matter knowledge, pedagogical knowledge, notions of academic achievement, and assessment. While different researchers highlight the importance of being sensitive to culture (class and gender) and some show the effectiveness of approaches designed around the strengths students bring from home cultures, this aspect of teachers' and schools' work is just one among many dimensions involved in the promotion of equity.

Research on ability grouping also sheds light on the nature of teaching approaches that are more equitable. A consistent finding across studies on ability grouping is that students in lower groups are offered restricted curricular diets that severely limit their opportunities to learn (Boaler, 1997; Knapp, Shields & Turnbull, 1992; Oakes, 1985). Inequities are maintained or produced in schools as lower track classes, disproportionately populated by students of lower socio-economic status and ethnic minority students, are taught by less well qualified teachers and teachers who often have low expectations for their students (Oakes, 1985). Mixed ability approaches to teaching have consistently been shown to produce more equitable outcomes (Boaler, 1997; Cohen & Lotan, 1997; Linchevski & Kutscher, 1998).

Teachers carry out their work within a context created by the department, school, and district (McLaughlin & Talbert, 2001; Siskin, 1994; Talbert & McLaughlin, 1996). These contexts shape teachers' professional communities, establishing 'distinctive expectations for teachers' work and interactions with students" (McLaughlin & Talbert, 2001, p.10). Dimensions along which districts differ include the type and amount of resources provided, the community's expectations, and the organization and administrative structures. From these and other factors teachers derive a sense of purpose, value, professionalism, and collective enterprise (Talbert & McLaughlin, 1996). McLaughlin & Talbert (2001) found great variability in teachers' professional communities, even within the same school or same district. On one end of the spectrum, department cultures supported strong commitments to students and the development of innovative methods to meet their students' needs and support their learning. These departments often met regularly and discussed problems of practice. On the other end of the spectrum, less successful departments rarely discussed teaching matters and met infrequently. The less successful departments evinced little collegiality, and issues related to the success of their students were not taken up on the department level, rather they were left for individual teachers to address within the boundaries of their classrooms (see also Horn 2002, 2005). Rochelle Gutiérrez (1996) linked department culture and ways of working with equitable achievement. In her study of mathematics departments that were found to promote equity, she developed the Opportunity for Advancement (OFA) framework as a way of understanding how departments organize to support students' success, particularly among traditionally marginalized groups. The OFA comprised four components: a rigorous and common curriculum; innovative instructional practices (both noted above); active commitment to students; and commitment to a collective enterprise.

This range of studies collectively suggests that equity is encouraged when all students experience conceptually demanding curriculum, when they have access to high level curriculum, when teaching practices are sensitive to the needs of different students and when departments work collaboratively and receive support from schools and districts. We conducted our study of student learning in different schools with the knowledge that a multitude of schooling variables—ranging from district support and departmental organization to curricular examples and classroom interactions—could impact the learning of students and the promotion of equity. This helped direct our attention as we conducted a longitudinal, five-year study of the different factors impacting the mathematics learning of 700 high school students from different cultures, genders and social classes who were taught in very different ways. Our study centered upon the affordances of different curricula and the ensuing teaching and learning interactions in classrooms, and it also considered the role of broader school factors and the contexts in which the different approaches were enacted.

Description of the Study

The Schools and Students

The Stanford Mathematics Teaching and Learning Study was a five-year, longitudinal study. The study monitored students in three high schools with the pseudonyms: Greendale, Hilltop and Railside. These three schools are reasonably similar in terms of their size, and share the characteristic of committed and knowledgeable mathematics teachers. They differ in terms of their location and student demographics. (See Table 1.)

Railside High School, the focus of this analysis, is situated in an urban setting and has a diverse student population with students coming from a variety of ethnic and cultural backgrounds. Hilltop High School is situated in a more rural setting, and approximately half of the students are Latino and half white. Greendale High School is situated in a coastal community, with very little ethnic or cultural diversity (almost all students are white).

Table 1

Schools, Students & Mathematics Approaches

	Railside	Hilltop	Greendale
Enrollment (approx.)	1500	1900	1200
Study demographics	38% Latino/a 23% African Am. 20% White 16% Asian/Pac. Islanders 3% other ethnicities	57% White 38% Latino/a 5% other ethnicities	90% White 5% Latino/a 5% other ethnicities
ELL[1] students	25%	24%	0%
Free/reduced lunch	31%	23%	9%
Parent education, % college grads	23%	33%	37%
Mathematics curriculum approaches	Teacher designed reform-oriented curriculum, conceptual problems, groupwork	Choice between "traditional" (demonstration and practice, short problems) and IMP (group work, long, applied problems)	Choice between "traditional" (demonstration and practice, short problems) and IMP (group work, long, applied problems)

[1] ELL is English Language Learners

The three high schools were chosen because they enabled us to observe and study three different mathematics teaching approaches. Case selection then was *purposive* (Yin, 1994). Both

Greendale and Hilltop schools offered students (and parents) a choice between a traditional sequence of courses, taught using conventional methods of demonstration and practice, and an integrated sequence of courses in which students worked on a more open, applied curriculum called the Interactive Mathematics Program (Fendel, Fraser, Alper, & Resek, 2003), or "IMP." Students in IMP classes worked in groups and spent much more time discussing mathematics problems than those in the traditional classes. Railside school did not offer a choice and the approach they used was 'reform' oriented. The teachers worked collaboratively and they had designed the curriculum themselves, drawing from different 'reform' curriculum such as the College Preparatory Mathematics Curriculum (Sallee, Kysh, Kasimatis, & Hoey, 2000), or "CPM" and IMP. Mathematics was organized into the traditional sequence of classes — algebra followed by geometry, then advanced algebra and so on — but the students worked in groups on longer, more conceptual problems. Classes at Railside were heterogeneous as the school had de-tracked classes in previous years; classes at the other two schools were not. We monitored three approaches in the study - 'traditional' and 'IMP' (as labeled by the two schools) and the 'Railside approach.' However, as only one or two classes of students in Greendale and Hilltop chose the IMP curriculum each year there were insufficient numbers of students to include in our statistical analysis. The main comparison groups of students in the study were therefore approximately 300 students who followed the traditional curriculum and teaching approaches in Greendale and Hilltop schools and approximately 300 students at Railside who were taught using reform oriented curriculum and teaching methods. These two groups of students[1] provide an interesting contrast as they experienced the same courses, taught in very different ways.

Research Methods

Our study was a mixed-method, multi-case, longitudinal study (Yin, 1994) of the teaching approaches at three high schools in which we monitored the same students over four years of high school. Given our goal of understanding the highly complex phenomena of teaching and learning mathematics, we gathered a wide array of data, both qualitative and quantitative. Data was collected to inform our understanding of the teaching approaches and classroom interactions, students' views of mathematics, and student achievement. Each data source (lesson observations, interviews, videos, questionnaires, assessments) was analyzed individually using standard procedures of coding and / or statistical analysis. The findings from these multiple sources were then analyzed and understood in relation to one another, thus illuminating trends and themes across sources and affording the opportunity to triangulate the data. We were greatly aided in our analytic process by having a team of researchers. Each investigator brought an informed perspective that enhanced our discussions. The validity of emerging themes was agreed upon by the team which served to increase confidence in our analyses and findings (Eisenhart, 2002). In addition, although we focus on Railside school in this paper, data analysis for our other two school sites, Greendale and Hilltop, was concurrent. Constant comparison across cases (Glaser & Strauss, 1967, 1991) served to illuminate critical defining features and practices of each school, allowing us to capture subtle aspects of each learning environment that may have otherwise been overlooked. The analyses were shared with the teachers as a form of *member check* (Glesne & Peshkin, 1992), further enhancing the validity of the findings.

Classroom observations and teaching approaches.

[1] In the remainder of this paper we combine the students from Greendale and Hilltop that followed the traditional curriculum.

To monitor and analyze the teaching practices in the three schools we observed approximately 600 hours of lessons, many of which were videotaped. These lessons were analyzed in three different ways. First, we drew upon our observations from class visits and videotapes to produce 'thick descriptions' (Geertz, 2000) of the teaching and learning in the different classes. We also identified one or two focal teachers for each approach in each school, and developed analyses of their teaching, focusing on "teacher moves" that shaped students' engagement with mathematics and mathematical activity. These focal cases were based on classroom observations throughout the course and analyses of videos of lessons. Second, we conducted a quantitative analysis of time allocation during lessons. A mutually exclusive set of categories of the ways in which students spent time in class was developed, which included such categories as teacher talking, teacher questioning whole class, students working alone, and students working in groups. When agreement was reached on the categories, three researchers coded lessons until over 85% agreement was reached. We then completed the coding of over 55 hours of lessons, coding every 30-second period of time. This yielded 6,800 coded segments. We also recorded the amount of time that was spent on each mathematics problem in class. This coding exercise was only performed on Year 1 classes (traditional algebra, Railside algebra, and IMP 1) as it was extremely time intensive and we lacked the resources to perform the same analysis every year. Third, in addition to these qualitative and quantitative analyses of lessons, we performed a detailed analysis of the questions teachers asked students. This level of analysis fell between the qualitative and quantitative methods we had used and was designed in response to our awareness that the teachers' questions were an important indicator of the mathematics on which students and teachers worked (see Boaler & Brodie, 2004). Our coding of teacher questions was more detailed and interpretive than our coding of instructional time but it was sufficiently quantitative to enable comparisons across classes. Our coding of videos, along with the development of cases for focal teachers, provided a strong foundation for understanding differences in the approaches. These analyses also informed our design of interview questions and questionnaires which further informed the themes by which we analyzed the data.

Students' beliefs and relationships with mathematics.

We interviewed at least 60 students in every year of the study to consider their reported experiences and interpretations of mathematics class. This helped us to consider and analyze the ways the different approaches influenced students' developing relationships with mathematics (see also Boaler, 2002c). Students were typically interviewed in same-sex pairs and we sampled high and low achievers from each approach in every school taking care to interview students from different cultural and ethnic groups. We also administered questionnaires to all the students in the focus cohorts in Years 1, 2 and 3 of the study (when most students were required to take mathematics). The questionnaires combined closed, Likert-response questions with more open questions. The questionnaires asked students about their experiences in class, their enjoyment of mathematics, and their perceptions about the nature of mathematics and learning. Interviews and open responses to questionnaires were carefully coded by at least two researchers and Likert scale questionnaire items were analyzed using factor analysis. The observations, interviews and questionnaires combined to give us information on the teaching and learning practices in the different approaches and students' responses to them. Teachers from each approach were also interviewed at various points in the study although the teachers' perspectives on their teaching were not a major part of our analyses. The analyses of the Railside approach, presented in this paper, have been subject to a process of respondent validation with the mathematics department of the school approving the categories used and the analyses within them.

Student achievement data.

In addition to monitoring the students' experiences of the mathematics teaching and learning we assessed their understanding in a range of different ways. At the beginning and end of Years 1 and 2 and at the end of Year 3 we administered tests that were carefully written by the research team and considered by the teachers in each approach to make sure they fairly assessed each approach. The test at the beginning of high school was a test of middle school mathematics, as that was the mathematics students would be expected to know at that time. At the end of Year 1 we administered an algebra test. The test was designed to assess only algebraic topics that the students had encountered in common across the different approaches, and we used an equal proportion of question-types from each of the three teaching approaches. At the beginning of Year 2 we administered the same test, giving us a record of the achievement of all students starting Year 2 classes. At the end of Year 2 we wrote and administered a test of algebra and geometry, again focusing only upon content all students had met, using question-types from each approach that teachers from each approach reviewed, and repeated this process at the end of Year 3. In addition to these tests that matched, as closely as possible, the mathematical work students had met in the different approaches, we also designed and administered longer more applied problems that students were given to work on in groups. These problems were administered in Years 1, 2 and 3 and they were given to one class in each approach in each school, and the different groups were videotaped as they worked (see Fiori & Boaler, 2004). We also gathered data on the students' scores on state administered tests.

Results

The Teaching Approaches

Students in traditional classes at the two schools offering such an approach were taught using a 'traditional' approach – they sat individually, the teachers presented new mathematical methods through lectures and the students worked through short, closed problems. Our coding of lessons showed that approximately 21% of the time in algebra classes was spent with teachers talking to the students, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole class format. Approximately 48% of the time students were practicing methods in their books, working individually, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics problem was 2.5 minutes. Our focused analyses of the types of questions teachers asked, conducted with two of the teachers of traditional classes, showed that the vast majority of their questions were procedural (Boaler & Brodie, 2004). We classified teachers' questions into seven categories, from 325 minutes of teaching. This showed that 97% and 99% of the two teachers' questions in traditional algebra classes fell into the procedural category.

At Railside school the teachers posed longer, conceptual problems and combined student presentations with teacher questioning. Teachers rarely lectured and students were taught in heterogeneous groups. Our coding of time spent in classrooms showed that teachers lectured to classes for approximately 4% of the time. Approximately 9% of the time teachers questioned students in a whole class format. Approximately 72% of the time students worked in groups while teachers circulated the room helping students and asking them questions of their work, and students presented work for approximately 9% of the time. The average time spent on each mathematics problem was 5.7 minutes. Our focused analysis of the types of questions teachers asked, conducted with two of the Railside teachers over 352 minutes of teaching, showed that they asked many more varied questions than the teachers of traditional classes. Sixty-two percent were procedural, 17% conceptual, 15% probing, and 6% fell into other questioning categories (Boaler & Brodie, 2004). The broad range of questions they asked was typical of the teachers at Railside who deliberately and carefully discussed their teaching approaches, a practice which included sharing good questions to ask students, as will be described below. We conducted our most detailed observations and analyses in the first-year classes when students were taking algebra, but our observations in later years as students progressed through high school showed that the teaching approaches described above continued in the different mathematics classes the students took.

Student achievement and attainment

As noted above, at the beginning of high school we gave all students who were starting algebra classes in the three schools a test of middle school mathematics. The numbers of students included in this paper represent all students who started high school in algebra at Greendale, Hillside and Railside and who completed our tests, and not those who were placed in higher or lower level classes.[2] At Railside all incoming students were placed in algebra as the school employed heterogeneous grouping. Comparisons of means indicated that at the beginning of Year 1, the students at Railside were achieving at significantly lower levels than students at the two other schools starting the traditional approach ($t = -9.141$, $p < 0.001$, $n= 658$), as can be seen in Table 2. The relatively low performance of the Railside students is not atypical for students in urban, low-income communities. At the end of Year 1 we gave all students a test of algebra to measure what students had learned over the year. Comparisons of means showed no significant differences between the scores of students in the two approaches – the students in traditional classes were still scoring at higher levels but the differences were not significant ($t= -2.04$, $p =0.04$, $n=637$). Thus the Railside students were able to achieve at comparable levels after a year of algebra teaching, despite starting the course at significantly lower achievement levels. At the end of Year 2 we gave students a test of algebra and geometry, reflecting the content the students had been taught over the first two years of school. By the end of Year 2 Railside students were significantly outperforming the students in the traditional approach ($t = -8.304$, $p <0.001$, $n = 512$). There are fewer students in the geometry classes in Railside because the timetable was structured so that students could take geometry in second, third or fourth years whereas students in the other schools needed to take it in year two of the study (as will be described in the next section). The students in geometry classes at Railside did not represent a selective group; they were of the same range as the students entering Year 1[3].

Table 2
Assessment Results

	Traditional	Railside

[2] In both cases we only include students who gave permission to be in the study, approximately 87% of the eligible students.
[3] The test was given to all students in geometry classes. These were not the exact same students as those in Year 1 as some students chose to take geometry in a later year and some older students joined the classes, who had not been in our algebra sample. Thus, we performed analyses that included only those students who were in Year 1 and Year 2, so the students who took algebra and geometry and who took all three tests (pre-test, end of Year 1 test, end of year 2 test). The results from this smaller number of students and also showed that the Railside students started at lower levels and ended at significantly higher levels.

	Mean score	Std Deviation	n	Mean score	Std. Deviation	n	t (level of significance)
Y1 Pre-test	22.23	8.857	311	16.00	8.615	347	-9.141 (*p*<0.001)
Y1 Post-test	23.90	10.327	293	22.06	12.474	344	-2.040 (*p*=0.04)
Y2 Post-test	18.34	10.610	313	26.47	11.085	199	-8.309 (*p*<0.001)

Railside was also extremely successful at reducing the achievement differences among groups of students belonging to different ethnic groups at the school. Table 3 shows significant differences between groups at the beginning of the ninth-grade year, with Asian, Filipino, and White students each outperforming Latino and Black students (p<.001).

Table 3
Railside Year 1 Pre-test Results by Ethnicity

Ethnicity	n	Mean	Median	Std. Dev
Asian	27	22.41	22	8.509
Black	68	12.28	12	6.286
Hispanic/Latino	103	14.28	12	7.309
Filipino	23	21.61	22	8.289
White	51	21.20	21	9.362

At the end of Year 1, only one year after the students started at Railside, there were no longer significant differences between the achievement of white and Latino students, nor Filipino students and Latino and Black students. The significant differences that remained at that time were between white and Black students and between Asian students and Black and Latino students (ANOVA F=5.208; df=280; p=0.000). Table 4 shows these results.

Table 4
Railside Year 1 Post-test Results by Ethnicity

Ethnicity	n	Mean	Median	Std. Dev
Asian	27	29.44	30	12.148
Black	68	18.21	16.50	10.925
Hispanic/Latino	103	21.31	21	11.64
Filipino	23	26.65	26	10.504
White	51	26.69	28	13.626

In subsequent years the only consistent difference that remained was the high performance of Asian students who continued to significantly outperform Black and Latino students, but differences between White, Black and Latino students disappeared. Achievement differences between students of different ethnicities at the other schools remained.

Student Perceptions and Relationships with Mathematics

In addition to the high achievement of the students, the students at Railside also enjoyed mathematics more. In questionnaires given to the students each year the Railside students were always significantly more positive. For example, in the Year 3 questionnaire students were asked to finish the statement: 'I enjoy math in school' with one of four time options: all of the time, most of the time, some of the time, or none of the time. Twenty-nine per cent of students in traditional classes (n=318) chose all or most of the time, compared with 54% of students from Railside (n=198) which is a significant difference ($t = 4.758$; $df = 286$; $p<0.001$). In addition, significantly more Railside students agreed or strongly agreed with the statement ' I like math', with 74% of Railside students responding positively, compared with 54% of students in traditional classes ($t = -4.414$ $df=220.77$; $p<0.001$). Other years produced similar results: 71% of Railside students in Year 2 classes (n =198) for example reported enjoying 'math class' compared with 46% of students in traditional classes (n=318) ($t = -4.934$; $df=444.62$; $p<0.001$).

By the end of year 4, 41% of Railside seniors were enrolled in advanced classes of pre-calculus and calculus, compared with 30% of seniors at Hilltop and 23% of seniors at Greendale[4]. There were no gender differences in performance in any of the tests we gave students at any level, and young women were well represented in higher mathematics classes. They made up 50% of students in the advanced classes at Hilltop, 48% at Greendale and 59% at Railside. In Year 4 we conducted interviews with 105 students in the three different approaches. Most of the students were seniors and they were chosen to represent the breadth of attainment displayed by the whole school cohort. These interviews were coded and students were given scores on the categories of *interest, authority*, *agency* and *future plans for mathematics*. The categories of authority and agency (Holland, Lachiotte, & Cain, 1998) were ones that emerged as important as students in the different approaches varied in the extent to which they believed they had authority or that they could work with agency (see Boaler & Gresalfi, in preparation). Significant differences emerged in all of these categories with the students at Railside being significantly more interested in mathematics ($\chi^2 = 12.806$, df 2, $p = 0.002$, n= 67) and believing they had significantly more authority ($\chi^2 = 29.035$, df 2, $p = 0.000$, n= 67) and agency ($\chi^2 = 22.650$, df 2, $p = 0.000$, n= 63). Importantly, *all* of the students interviewed at Railside intended to pursue more mathematics courses compared with 67% of students from the traditional classes and 39% of Railside students planned a future in mathematics compared with 5% of students from traditional classes ($\chi^2 = 18.234$, df 2, $p = 0.000$, n= 65).

The students at Railside school enjoyed mathematics more than students taught more traditionally, they achieved at higher levels, and achievement differences between students of different ethnic and cultural groups were lower than those at the other schools. In addition the teachers and students achieved something that Boaler (2004) has termed 'relational equity'. In studying equity most researchers look for reductions in achievement differences for students of different ethnic and cultural groups and genders when tests are taken. But Boaler has argued that another goal for equity is the creation of classrooms in which students learn to treat each other equitably, showing respect for students of different cultures, genders and social classes. Schools are places where students will learn relations that they are likely to replicate in society, making equitable relations an important goal. It is not commonly thought that mathematics classrooms are places where students should learn about societal respect but students at Railside reported that they learned to value students who came from very different backgrounds to themselves because of the approach of their mathematics classes, as we will describe shortly.

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[4] This percentage includes all seniors at Greendale and Hilltop, whether they attended the 'traditional' or IMP classes. At this time we have been unable to separate the students from IMP but as they were few in number this would not affect the reported percentage greatly.

Because of the challenges of accessing data that are held in district offices for particular students who are in our study (rather than the whole school) we are unable to report anything beyond school scores for the students on state administered tests. Despite this limitation, these school-level data are interesting to examine and raise some important issues with respect to testing and equity, as some measures of the Railside students' performance (i.e., our tests, district tests and California Standards test of algebra) were much more positive than others (i.e., CAT 6 and indicators of adequate yearly progress (AYP)).

Railside school is part of a two high school district and the district administered a district wide mathematics test to students at both schools. The other school has fewer students who are language learners, from ethnic minority groups and from lower socio-economic homes. In addition to scoring at high levels on our tests the Railside students scored at significantly higher levels on the district test than students at the other school, but the other school students scored at higher levels on the standardized tests administered by the state. Another curriculum-aligned test is the California standards test, taken by students who had completed algebra. These scores show similar performance for all three schools with the Railside students scoring at higher levels than the other two schools (see Table 5). Forty-nine percent of Railside students scored at or above the basic level, compared to 33% at Greendale and 41% at Hilltop.

Table 5: California Standards Test, Algebra, 2003. Percent of students attaining given levels of proficiency.

	Greendale	Hilltop	Railside
n	125	224	188
Advanced	0	0	1
Proficient	6	13	15
Basic	27	28	33
Below basic	55	43	36
Far below basic	12	15	15

In contrast, Hilltop and Greendale scored at higher levels on the standardized tests, as seen in Tables 6 and 7.

Table 6: CAT 6, 2003, STAR, Grade 11 (Year 3): Percent of students at or above 50th percentile

	Railside	Hilltop	Greendale
n	341	436	257
Reading	37	56	74
Language	32	54	70
Mathematics	40	52	71

Table 7: AYP (adequate yearly progress): Percent of students 'proficient' at language arts and mathematics

	Language	Mathematics	Difference	"Similar schools" average difference
Railside	33.0	31.7	1.3	12.8
Hilltop	60.3	51.1	9.2	10.5
Greendale	72.3	57.8	14.5	11.9

There were many reasons for the students' lower performance we contend, most importantly the cultural and linguistic barriers provided by the state tests. The correlation between students' scores on the language arts and mathematics sections of the AYP tests, across the whole state of California is a staggering 0.932 for 2004. This data point provides a strong indication that the mathematics tests were testing language as much as mathematics. This argument could not be made in reverse as the language tests do not contain mathematics, but the mathematics tests are tested through language. Indeed the students at Railside reported that the standardized tests provided language barriers that our tests did not and they used unfamiliar terms and contexts that provided cultural bias (see also Boaler, 2003). Tables 6 and 7 also show interesting relations between mathematics and language as the Greendale and Hilltop school students were more successful on tests of reading and language arts, a trend that held across the state, but the Railside students were as or more successful on mathematics. Another result that is interesting to note is that 37% more White students scored at or above the 50^{th} percentile than Latino students at Hilltop (the only other sizeable group of ethnic minority students in the study) on the CAT 6. At Railside the difference between the same two groups was only 9%. The data in Tables 5-7 seem to show the inability of the mathematics tests to capture the mathematical understanding of the Railside students, an interpretation that is slightly mediated by the fact that the Railside students were performing at higher levels on the mathematics portions of the tests than might be expected. The test data also need to be approached with caution as the cohort we followed were the last year who did not need to pass the high school exit examination in order to graduate and the STAR tests that are used in California were taken by only a small proportion of students.

Analyzing the Sources of Success

Part I. The Department, Curriculum and Timetable

Railside school has an unusual mathematics department. Twelve of the thirteen teachers work collaboratively, spending vast amounts of time designing curriculum, discussing teaching decisions and actions, and generally improving their practice through the sharing of ideas. A study conducted by Horn on the ways in which the department collaborated, found that the teachers spent around 650 minutes a week planning, individually and collectively (their paid work week provides 450 minutes of preparation time) (Horn, 2002). Unusually for the United States, the mathematics department strongly influences the recruitment and hiring of teachers, enabling the department to maintain a core of teachers with shared philosophies and goals. The teachers share a strong commitment to the advancement of equity and the department has spent many years working out a coherent curriculum and teaching approach that teachers believe enhances the success of all students. The mathematics department has focused in particular upon the introductory algebra curriculum that all students take when they start the school. The algebra course is designed around key concepts with questions from various published curriculum such as CPM, IMP and a textbook of activities that use algebra LabGear™ (Picciotto & Wah, 1994). A theme of the algebra and subsequent courses is multiple representations, and students are frequently asked to represent their ideas in different ways, using "math tools" such as words, graphs, tables and symbols. In addition, connections between algebra and geometry are emphasized even though the two areas are taught in separate courses. Railside follows a practice of 'block scheduling' and lessons are 90 minutes long, with courses taking place over half a school year, rather than a full academic year. In addition, the introductory algebra curriculum that is generally taught in one course in US high schools is taught in the equivalent of two courses at Railside. The teachers have spread the introductory content over a longer period of time partly to ensure that the foundational mathematical ideas are taught carefully with depth and partly to ensure that particular norms – both social and socio-mathematical (Yackel & Cobb, 1996) – are carefully established. The fact that mathematics courses are only half a year long at Railside may appear unimportant but in fact this organizational decision has a profound impact upon the students' opportunities to take higher-level mathematics courses. In most North American high schools mathematics classes are one year long and they begin with algebra. This means that students cannot take calculus unless they are advanced, as the typical sequence of courses is algebra, geometry, advanced algebra then pre-calculus. If a student fails a course at any time they are knocked out of that sequence and have to retake the course, further limiting the level of content they will reach. At Railside the students could take two mathematics classes each year. This meant that students could fail classes, start at lower levels, and / or choose not to take mathematics in a particular term and still reach calculus. This relatively simple scheduling decision is part of the reason that significantly more students at Railside took advanced levels classes at school than students in the other two schools.

Another important difference between the classes in the three schools we studied was the heterogeneous nature of Railside classes. Whereas incoming students in Greendale and Hilltop could enter geometry or could be placed in a remedial class, such as 'math A' or 'business math', all students at Railside entered algebra classes. The department is deeply committed to the practice of mixed ability teaching and to giving all students equal opportunities for advancement. The teachers at Railside strive to ensure that good teaching practices are shared, one way in which this is achieved is through something that the department calls "following." The co-chairs structure teaching schedules so that a new teacher can stay a day or two behind a more experienced teacher, allowing the new teacher to observe lessons and activities during their daily preparation period before they try to adapt it for their classrooms (Horn, 2002, 2005).

The teachers at Railside have worked together over the past decade to develop and implement a curriculum that affords multiple points of access to the mathematics and comprises a variety of cognitively demanding tasks. It is worth noting that the curriculum is organized around units that have a unifying theme such as "What is a linear function?" This differs markedly from more standard textbooks where the units are organized around algebraic and other mathematical techniques (e.g., graphing linear functions; factoring polynomials). This organization of the Railside curriculum provides thematic coherence across a set of activities, which affords students the opportunity to make connections and affords teachers the opportunity to highlight and teach for those connections.

As they developed the curriculum, the department placed a strong emphasis on creating problems that satisfy the criterion of "groupworthy." Groupworthy problems are those that "illustrate important mathematical concepts, allow for multiple representations, include tasks that draw effectively on the collective resources of a group, and have several possible solution paths" (Horn, 2005, p. 22). A very important feature of the curriculum that teachers use, that would not be seen in the curriculum materials, is the act of asking follow up questions. For example, when students find the perimeter of a figure with side lengths represented algebraically, as 10x + 10, the teacher asks a student in each group, "Where's the 10?" requiring that students relate the algebraic equation to the figure. Although the tasks provide a set of constraints and affordances (Greeno & MMAP, 1997), it is in the implementation of the tasks that the learning opportunities are realized (Stein, Smith, Henningsen & Silver, 2000). Teachers' questions significantly shaped the course of implementation. The question of "Where's the 10?" for example was not written on the students' worksheets, but was part of the curriculum, as teachers agreed the follow up questions they would ask of students.

Research studies in recent years have pointed to the importance of school and district contexts in the support of teaching reforms. Such support is undoubtedly important but Railside is not a case of a district or school that initiated or mandated reforms. The reforms put in place by the mathematics department were supported by the school and were in line with other school reforms but they were driven by the passion and commitment of the mathematics teachers in the department. The school, in many ways, provided a demanding context for the reforms, not least because they had been managed by five different principals in the six years we were there, and they had been labeled an 'under-performing' school by the state because of low state test scores. The department, under the leadership of two strong and politically astute co-chairs, fought to maintain their practices at various times and worked hard to garner the support of the district and school, and while the teachers felt well supported at the end of our study Railside does not represent a case of a reforming district encouraging a department to engage in new practices. Rather, Railside is a case of an unusual, committed and hard working department that continues to grow in strength through its teacher collaborations and work.

Part II. Groupwork and 'Complex Instruction'

Many mathematics departments in the US employ group work but few are able to report the success of the Railside students or such high rates of work, as groups do not always function well, with some students doing more of the work than others, and some students being excluded or choosing to opt out. At Railside the teachers employed additional strategies to make group work successful. They adopted an approach called 'complex instruction' designed by Elizabeth Cohen and Rachel Lotan (Cohen, 1994; Cohen & Lotan, 1997) for use in all subject areas. The system is designed to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group *interactions*. The approach includes a number of recommended practices that the school employs that we highlight below.

Multidimensional classrooms.

In many mathematics classrooms there is one practice that is valued above all others – that of executing procedures (correctly and quickly). The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, whilst others sink to the bottom with most students knowing where they are in the hierarchy created. Such classrooms are unidimensional – the dimensions along which success is presented are singular. A central tenet of the complex instruction approach is what the authors refer to as 'multiple ability treatment'. This 'treatment' is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different 'abilities'. Teachers should explain to students that "no one student will be 'good on all these abilities' and that each student will be 'good on at least one'" (Cohen & Lotan, 1997, p. 78). Cohen and Lotan provide theoretical backing for their 'multiple-ability treatment' using the notion of multidimensionality (Rosenholtz & Wilson, 1980; Simpson, 1981).

At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. This was achieved, in part, by having more open problems that students could solve in different ways. The teachers valued different methods and solution paths and this enabled more students to contribute ideas and feel valued. But multiple solution paths were not the only contributions that were valued by teachers. When we interviewed the students and asked them 'what does it take to be successful in mathematics class?' they offered many different practices such as: asking good questions, rephrasing problems, explaining well, being logical, justifying work, considering answers, and using manipulatives. When we asked students in 'traditional' classes what they needed to do in order to be successful they talked in much more narrow ways, usually saying that they needed to concentrate, and pay careful attention. The students at Railside regarded mathematical success much more broadly than students in the traditional classes, and instead of viewing mathematics as a set of methods that they needed to observe and remember, they regarded mathematics as a way of working with many different dimensions. The different dimensions that students believed to be an important part of mathematical work were valued in the teachers' interactions with students and the grading system.

Not surprisingly, multidimensionality has implications for curriculum, as the nature of the tasks implemented must be such that they support multiple approaches and a varied set of learning practices. Indeed, the teachers at Railside spent a great deal of time developing "groupworthy problems," discussed in Part I (Horn, 2002) which supported their work as they strove to support multidimensional classrooms.

The multidimensional nature of the classes at Railside was an extremely important part of the increased success of students. Put simply, *when there are many ways to be successful, many more students are successful.* Students are aware of the different practices that are valued and they feel successful because they are able to excel at some of them. Teachers at other schools may not encourage practices outside of procedure execution because they are not needed in state tests, but the fact that teachers at Railside valued a range of practices and more students could be successful in class made students feel more confident and positive about mathematics. This probably enhanced their success on though our tests assessed a more narrow range of mathematical work.

The following comments given by students in interviews give a clear indication of the multidimensionality of classes:

> *Back in middle school the only thing you worked on was your math skills. But here you work socially and you also try to learn to help people and get help. Like you improve on your social skills, math skills and logic skills (Janet, Y1)*

> *J: With math you have to interact with everybody and talk to them and answer their questions. You can't be just like "oh here's the book, look at the numbers and figure it out.*
> *Int: Why is that different for math?*
> *J: It's not just one way to do it (…) It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: "why does it work"? (Jasmine, Y1)*

It is not common for students to report that mathematics is more 'interpretive' than other subjects. The students at Railside recognized that helping, interpreting and justifying were critically valued practices in mathematics classes.

One of the practices that we found to be particularly important in the promotion of equity was justification. At Railside students were required to justify their answers at almost all times. There are many good reasons for this – justification is an intrinsically mathematical practice (RAND, 2002; Martino & Maher, 1999), but this practice also serves an interesting and particular role in the promotion of equity. Many teachers struggle to deal with the wide range of students who attend classes, particularly in introductory classes such as high school algebra, which include students who are motivated with a wealth of prior knowledge as well as those who are less motivated and / or lack basic mathematical knowledge. At Railside school classes had a remarkably wide gap, but the teachers embraced the diversity they encountered and one practice that helped them support the learning of all students was justification. The practice of justification made space for mathematical discussions that might not otherwise be afforded. Particularly given the broad range of students' prior knowledge, receiving a justification that satisfied the individual was important in that the explanation was catered to the needs of the individual, and mathematics that might not otherwise be addressed was brought to the surface.

The following two students give further indication of the role of justification in helping different students:

> *Int: What happens when someone says an answer?*
> *A: We'll ask how they got it*
> *L: Yeah because we do that a lot in class. (…) Some of the students – it'll be the students that don't do their work, that'd be the ones, they'll be the ones to ask step by step. But a lot of people would probably ask how to approach it. And then if they did something else they would show how they did it. And then you just have a little session! (Ana & Latisha, Y3)*

It is noteworthy that these two students did not describe students as slow, dumb or stupid, as other students in our study did; they talked only about students 'that don't do their work'.

The following boy was achieving at lower levels than other students and it is interesting to hear him talk about the ways he was supported by the practices of explanation and justification:

> *Most of them, they just like know what to do and everything. First you're like "why you put this?" and then like if I do my work and compare it to theirs. Theirs is like super different 'cos they know, like what to do. I will be like – let me copy, I will be like "why you did this? And then I'd be like: "I don't get it why you got that." And then like, sometimes the answer's just like, they be like "yeah, he's right and you're wrong" But like – why? (Juan, Y2)*

Juan also differentiates between high and low achievers without referring to such adjectives as 'smart' or 'fast', instead saying that some students 'know what to do'. He also makes it very clear that he is helped by the practice of justification and that he feels comfortable pushing other students to go beyond answers and explain 'why' their answers are given. At Railside the teachers carefully prioritized the message that each student has two important responsibilities – both to help someone who asks for help, but also to ask if they need help. Both are important in the pursuit of equity, and justification emerged as an important practice in the learning of a wide range of students.

Roles.

A large part of the success of the teaching at Railside came from the complex, interconnected system in each classroom in which students were taught to take responsibility for each other and all students were encouraged to contribute equally to tasks. When students were placed into groups they were given a particular role to play, such as 'facilitator', 'team captain', 'recorder / reporter' or 'resource manager' (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function. At Railside the teachers emphasized the different roles at frequent intervals, stopping, for example, at the start of class to remind 'facilitators' to help people check answers or show their work. Students changed roles at the end of each unit of work. The teachers reinforced the status of the different roles and the important part they played in the mathematical work that was undertaken. These roles, and students' engagement with mathematics that was supported by taking them on, contributed to the complex interconnected system that operated in each classroom; a system in which everyone had something important to do and all students learned to rely upon each other.

Assigning competence.

An interesting and subtle approach that is recommended within the complex instruction literature is that of 'assigning competence'. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole class setting. For example, during a classroom observation at Railside a quiet Eastern European boy muttered something in a group that was dominated by two happy and excited Latina girls. The teacher who was visiting the table immediately picked up on it saying "Good Ivan, that is important." Later when the girls offered a response to one of the teacher's questions he said, 'Oh that is like Ivan's idea, you're building on that'. He raised the status of Ivan's contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. Cohen (1994) recommends that if student feedback is to address status issues, it must be public, intellectual, specific and relevant to the group task (p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work, and the specific dimension means that students know exactly what the teacher is praising. This practice is linked to the multidimensionality of the classroom which values a broad range of practices and forms of participation. The practice of 'assigning competence' demonstrated the teachers' commitment to equity and to the principle of showing what different students could do in a multifaceted mathematical context.

Teaching students to be responsible for each other's learning.

A major part of the equitable results attained at Railside came from the serious way in which teachers taught students to be responsible for each other's learning. Many schools employ group work which, by its nature, brings an element of responsibility, but Railside teachers went beyond this to encourage the students to take the responsibility very seriously. In previous research on approaches that employ groupwork, students generally report that they prefer to work in groups and they list different benefits, but the advantages usually relate to their own learning (see Boaler, 2000, 2002a, 2002b). At Railside students also talked about the value groupwork added to their own learning, but their descriptions were distinctly reciprocal as they also voiced a clear concern for the learning of their classmates. For example:

Int: do you prefer to work alone or in groups?

A: I think it'd be in groups, 'cause I want, like people that doesn't know how to understand it I want to help them. And I want to, I want them to be good at it. And I want them to understand how to do the math that we do. (Amado, Y1)

Students talked about their enjoyment of helping others and the value in helping each other:

It's good working in groups because everybody else in the group can learn with you, so if someone doesn't understand – like if I don't understand but the other person does understand they can explain it to me, or vice versa, and I think it's cool. (Ana & Latisha, Y3)

One unfortunate but common side effect of some classroom approaches is that students develop beliefs about the inferiority or superiority of different students. In our other classes students talked about other students as smart and dumb, quick and slow. At Railside the students did not talk in these ways. This did not mean that they thought all students were the same, they did not, but they came to appreciate the diversity of classes and the different attributes that different students offered:

Everybody in there is at a different level. But what makes the class good is that everybody's at different levels so everybody's constantly teaching each other and helping each other out. (Zane, Y2)

The students at Railside not only learned to value the contributions of others, they also developed a responsibility to help each other.

One way in which teachers nurtured a feeling of responsibility was through the assessment system. Teachers graded the work of a group by, for example, rating the quality of the conversations groups had. The teachers also occasionally gave group tests, which took several formats. In one version students worked through a test together, but the teachers graded only one of the individual papers and that grade stood as the grade for all the students in the group. A third way in which responsibility was encouraged was through a practice of asking one student in a group to answer a follow up question after a group had worked on something. If the student could not answer the question the teacher would leave the group to have more discussion and return to ask the same student again. In the intervening time it was the group's responsibility to help the student learn the mathematics they needed to answer the question. This move of asking one member of a group to give an answer and an explanation, without help from their groupmates, was a subtle practice that had major implications for the classroom environment. In the following interview extract the students talk about this particular practice and the implications it holds:

Int: Is learning math an individual or a social thing?
G: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.
B: I think both - because individually you have to know the stuff yourself so that you can help others in your group work and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella & Bianca, Y2)

The students in the extract above make the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicated an interesting social orientation that became instantiated through the mathematics approach, saying that the purpose in knowing individually was not to be better than others but so "you can help others in your group". There was an important interplay between individual and group accountability in the Railside classrooms.

The four practices described– of multidimensionality, group roles, assigning competence and encouraging responsibility are all part of the complex instruction approach. We now review three other practices in which the teachers engaged that are also critical to the promotion of equity. These relate to the challenge and expectations provided by the teachers.

Part III. Challenge and Expectations

High cognitive demand.

The Railside teachers held high expectations for students and presented all students with a common, rigorous curriculum to support their learning. The cognitive demand that was expected of all students was higher than other schools partly because the classes were heterogeneous and no students were precluded from meeting high-level content. Even when students arrived at school with weak content knowledge well below their grade level, they were placed into algebra classes and supported in learning the material and moving on to higher content. Teachers also enacted a high level of challenge in their interactions with groups and through their questioning. Importantly the support that teachers gave to students did not serve to reduce the cognitive demand of the work, even when students were showing signs of frustration. The reduction of cognitive demand is a common occurrence in mathematics classes when teachers help students (Stein, et al., 2000). At Railside the teachers were highly effective in interacting with students in ways that supported their continued thinking and engagement with the core mathematics of the problems.

The students at Railside became aware that the teachers demanded high levels of mathematical work high and they came to appreciate that demand. When we interviewed students and asked them what it took to be a good teacher, many of them mentioned the high demand placed upon them, for example:

> *She has a different way of doing things. I don't know, like she won't even really tell you how to do it. She'll be like, 'think of it this way'. There's a lot of times when she's just like – 'well think about it' – and then she'll walk off and that kills me. That really kills me. But it's cool. I mean it's like, it's alright, you know. I'll solve it myself. I'll get some help from somebody else. It's cool. (Ana & Latisha, Y3)*

The following students, in talking about the support teachers provided, also referred to their push for understanding:

> *Int: What makes a good teacher?*
> *J: Patience. Because sometimes teachers they just zoom right through things. And other times they take the time to actually make sure you understand it, and make sure that you actually pay attention. Because there's some teachers out there who say: 'you understand this?' and you'll be like "yes" But you really don't mean yes you mean no. And they'll be like "OK" And they move on. And there's some teachers that be like – they <u>know</u> that you don't understand it. And they know that you're just saying yes so that you can move on. And so they actually take the time out to go over it again and make sure that you actually got it, that you actually understand this time. (John, Y2)*

The students' appreciation of the teachers' demand was also demonstrated in our questionnaires. One of the questions started with the stem: 'When I get stuck on a math problem, it is most helpful when my teacher …'. This was followed by answers such as 'tells me the answer' 'leads me through the problem step by step' and 'helps me without giving away the answer'. Students could respond to each on a four-point scale (SA, A, D, SD). Almost half of the Railside students (47%) *strongly* agreed with the response: "Helps me *without* giving away the answer," compared with 27% of students in the 'traditional' classes at the other two schools (n= 450, t = -4.257; df = 221.418; p<0.001).

Effort over ability.

In addition to the actions in which teachers engaged, challenging through difficult questions that maintained a high cognitive demand, the teachers also gave frequent and strong messages to students about the nature of high achievement in mathematics, continually emphasizing that it was a product of hard work and not of innate ability. The teachers kept reassuring students that they could achieve anything if they put in the effort. This message was heard by students and they communicated it to us in interviews, with absolute sincerity, for example:

> *To be successful in math you really have to just like, put your mind to it and keep on trying – because math is all about trying. It's kind of a hard subject because it involves many things. (…) but as long as you keep on trying and don't give up then you know that you can do it. (Sara, Y1)*

In the Year 3 questionnaires we offered the statement "Anyone can be really good at math if they try" At Railside, 84% of the students agreed with this, compared with 52% of students in the traditional classes (n= 473, t = -8.272; df = 451; p<0.001). But the students did not only come to believe that they could be successful. They developed an important practice that supported them in that – the act of persistence. It could be argued that persistence is one of the most important practices to learn in school – one that is strongly tied to success in school as well as in work and life. We have many indications in our data that the Railside students developed considerably more persistence than the other students. For example, as part of our assessment data we give students long, difficult problems to work on for 90 minutes in class, which we videotaped. The Railside students were more successful on these problems, partly because they would not give up on them and they continued to try to find methods and approaches even when they had exhausted many.

When we asked in questionnaires: 'How long (in minutes) will you typically work on one math problem before giving up and deciding you can't do it?' The Railside students gave responses that averaged 19.4 minutes, compared to the 9.9 minutes averaged by students in traditional classes (n=438, t = -5.641; df = 142.110; p< 0.001). This response is not unexpected given that the Railside students worked on longer problems in class but it also gives some indication of the persistence students are learning through the longer problems they experience.

In the following interview extract the student links this persistence to the question asking and justification highlighted earlier:

> *A: Because I know if someone does something and I don't get it I'll ask questions. I'm not just going to keep going and not know how to do something.*
> *L: And then if somebody challenges what I do then I'll ask back and I'll try to solve it. And then I'll ask them: "Well how d'you do it?" (Ana & Latisha, Y3)*

Clear expectations and learning practices.

The final aspect of the teachers' practice we will highlight relates to the expectations they offered the students. In addition to stressing the importance of effort the teachers were very clear about the particular ways of working in which students needed to engage. Cohen & Ball (2001) describe ways of working that are needed for learning as 'learning practices'. For example, the teachers would stop the students as they were working and talking and point out valuable ways in which they were working. In one observation we witnessed one of the Railside teachers, Guillermo, helping a boy called Arturo. Arturo said he was confused, so Guillermo told him to ask a specific question; as Arturo framed a question he realized what he needed to do and continued with his thinking. Arturo decided the answer was '550 pennies' but then stopped himself saying 'no, wait, that's not very much'. At that point Guillermo interrupted him saying:

> Wait, hold on a second, two things just happened there. Number one is, when I said what is the exact question? You stopped to ask yourself the exact question and then suddenly you had ideas. That happens to a lot of students, if they're confused, the thing you have to do is say, "OK what am I trying to figure out?" Like exactly, and like say it. So say it out loud or say it in your head but say it as a sentence. That's number one and number two, then you checked out the answer and you realized the answer wasn't reasonable and that is *excellent* because a lot of people would have just left it there and not said "What, 500 pennies? That's not very much." (Guillermo, math department co-chair)

The teachers also spent time before projects began setting out the valued ways of working, encouraging students to, for example, pick 'tricky' examples when writing a book (that is one of the projects they completed) as they would "show off" the mathematics that they knew. The teachers communicated very clearly to students which learning practices would help them achieve (see also Boaler, 1997, 2002b).

Conclusion

Railside is not a perfect place - the teachers would like to achieve more in terms of student achievement and the elimination of inequities, and they rarely felt satisfied with the achievements they had made to date, despite the vast amounts of time they spent planning and working. But research on urban schools and the experiences of mathematics students in particular tells us that the achievements at Railside are extremely unusual. There were many features of the approach at Railside that combined to produce important results, not only did the students achieve at significantly higher levels, but the differences in attainment between students of different ethnic groups were reduced in all cases and disappeared in some. Additionally the students learned to behave in a calm and respectful manner and they explained to us that they learned to value students from different cultures, classes and genders because of their mathematics approach.

In this paper we have attempted to convey the work of the teachers in bringing about the reduction in inequalities as well as general high achievement that they achieved. In doing so we hope also to have given a sense of the complexity of the relational and equitable system that they had in place. People who have heard about the achievements of Railside have asked for their curriculum so that they may use it, but whilst the curriculum plays a part in what is achieved at the school it is only one part of a complex, interconnected system. At the heart of this system is the work of the teachers, and the numerous different equitable practices in which they engaged. The Railside students learned through their mathematical work that alternate and multidimensional solutions were important which led them to value the contributions of the people offering such ideas. This was particularly important at Railside as the classrooms were multicultural and multilingual. It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues or read diverse forms of literature in English or Social Studies classes. We propose that all subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject removed from responsibilities of cultural or social awareness, has an important contribution to make. The discussions at Railside were often abstract mathematical discussions and the students did not learn mathematics through special materials that were sensitive to issues of gender, culture, or class. But through their mathematical work, the Railside students learned to appreciate the different ways that students saw mathematics problems and learned to value the contribution of different methods, perspectives, representations, partial ideas and even incorrect ideas as they worked to solve problems. As the classrooms became more multidimensional, students learned to appreciate and value the insights of a wider group of students from different cultures and circumstances.

Indeed the act of considering different mathematical ideas in the solving of problems coupled with a learned respect for other students promoted a respect for and understanding of different viewpoints that transcended the mathematics classroom. The following student was asked what she thought about the conversations between the students when they considered each other's different methods. Her answer reflects the connections we are describing between considering different methods and learning to consider the points of views of others in life:

> *I think it helps, because it helps with learning to get out of your comfort zone, cause whenever you learn, you're not always going to learn the exact way, so to be able to learn different types of ways, if someone interprets something the way they do, and then you look at it and you're like: "oh look at this", and you see it <u>their</u> ways, you never know later on when you might have to change your interpretation or something. So it allows you to come out of like your comfort zone.* (Ayana, Y4)

The equitable relationships that Railside students developed were only made possible by a conception of mathematics that valued the contribution of different insights, methods and perspectives in the collective solving of particular problems with particular solutions. This outcome seems extremely important. The work of the mathematics teachers at Railside school, and the equitable, multidimensional mathematics approach at Railside, meant that many students achieved good grades and test scores, and that they learned to respect students from different backgrounds and cultures. It gave students access to mathematical careers, higher-level jobs and more secure financial futures, in effect transforming their lives for the better. The fact that the teachers were able to achieve this through a multidimensional approach in a broader political context in which unidimensional mathematics work and test performance is all that is valued (Becker & Jacob, 2000) may give other teachers hope that working for equity and mathematical understanding against the constraints the system provides is both possible and worthwhile.

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