

## **British Combinatorial Newsletter No. 7 (October 2009).**

Remember this (7<sup>th</sup>) Newsletter aims to complement the Bulletin with some additional information about (e.g.) details of forthcoming meetings, summaries of recent movements of people, visitors, etc: records of “outreach” activities or recent breakthrough results in the subject: it might include a combinatorial problem or an occasional oddity. British Combinatorial Newsletters are produced at the start of the academic year (when the movements information is most useful to e.g. seminar organisers) and also at around the time of the Bulletin (in April) to let you know what is coming up over the Summer. They are on the BCB website at <http://www.essex.ac.uk/maths/BCB/newsletters.htm>

If you have material which you think might be suitable for inclusion, or suggestions as to how the newsletter should evolve, please contact the Editor, David Penman ([dbpenman@essex.ac.uk](mailto:dbpenman@essex.ac.uk)). The Editor reserves control of content.

## **Forthcoming regular meetings supported by the BCC.**

### **2<sup>nd</sup> Old Codger’s Meeting at Reading:**

This will take place at Reading on Wednesday **4 November 2009**. The speakers will be: Fred Holroyd (Open), Donald Keedwell (Surrey), David Larman (NYU and UCL), Curt Lindner (Auburn) and Carole Whitehead (Goldsmiths). The organiser is Anthony Hilton, and more details will be announced later. All interested (old or otherwise) are welcome to attend.

### **Open University Winter Combinatorics Meeting**

The **provisional** date for the next (11<sup>th</sup>) OU Winter Combinatorics meeting is **Wednesday 20 January 2010**. More details will appear at <http://wcm.open.ac.uk/> soon.

### **Oxford 1-day meeting in Combinatorics**

The **planned** date for the next Oxford 1-day meeting in Combinatorics is **Wednesday 17 March 2010**. More details will appear in due course at <http://people.maths.ox.ac.uk/scott/>

### **2 Linked One-day meetings at QMUL and LSE in May 2010.**

It is intended that there will again be two linked 1-day meetings at QMUL and LSE in May 2010. Details will be announced later.

### **PCC at QMUL 7-9 July 2010.**

The next PCC is at QMUL. The organisers will be Andy Drizen and John Faben. The

dates will be **7-9 July 2010**. (Note that it was suggested at one earlier stage that it might be somewhat earlier than that). A website has been set up for the event at <http://www.pcc2010.co.uk/> and it is possible to express interest in the event there. As usual, the aim is for research students to meet and discuss their research in a relaxed environment, to gain practice at presenting their research outside of their own department, and to meet pre-eminent researchers in their area. Students will be encouraged (but not required) to give a 20 minute talk about their research at the meeting. More details, including the invited speakers, will be announced later.

### **BCC2011**

The next (2011) BCC (the 23<sup>rd</sup>) will be at Exeter from **3-8 July 2011**. (Note these dates are somewhat earlier than was initially informally suggested). The Local Organiser is Robin Chapman. Further details will be available in due course.

You are reminded that the Editor maintains a mailing list for advertising other forthcoming UK meetings, Ph.D student level or above courses, etc. in combinatorics (broadly interpreted). Please email him if you would like to publicise such a meeting. Remember lists of forthcoming conferences in Combinatorics and related areas can be found at <http://www.maths.qmul.ac.uk/~pjc/bcc/conferences.html> or <http://www.math.uiuc.edu/~west/meetlist.html>

## **Movements.**

**Bath:** Dr. Bernd Sing is leaving to take up a lectureship at the University of the West Indies, Cave Hill, Barbados, from October 2009.

**Bristol:** Dr. Tim Riley has left to take up a new post at Cornell University, USA.

**Cambridge:** Dr. Boris Bukh (formerly at Princeton) has taken up a Herchel Smith Research Fellowship. He is interested in many aspects of combinatorics, especially geometric ones.

**Kent:** Dr. Bas Lemmens (formerly at Warwick) has taken up a Lectureship. He is interested in combinatorial aspects arising from his research on dynamical systems.

**Liverpool:** Dr. Rahul Savani (formerly at Warwick) has taken up a lectureship in the Economics and Computation Research Group in Computer Science. He is interested in game theory.

**QMUL:** Dr. Aidan Roy (formerly at the University of Calgary) has taken up a research post. He is interested in algebraic graph theory and quantum computing.

**Warwick:** Dr. Diana Piguet (formerly at Charles University, Prague) has been appointed to a post in Computer Science. She is primarily interested in extremal combinatorics and Ramsey theory. Dr. Bernard Ries (formerly at Columbia) has also taken up a post in the Business School: he is interested in graph theory, combinatorial

optimization, complexity theory and heuristics.

## Current and Forthcoming Combinatorial Visitors

**Aberystwyth** The following will be visiting during the academic year 2009/10:

**Prof. Jennifer D Key** will be visiting from mid-June 2010 to late August 2010. She is interested in finite geometries, designs, codes and groups, see <http://www.ces.clemson.edu/~keyj/>

**Prof. Mohan S Shrikhande** will be visiting Aberystwyth from 10th January to 4th March 2010. He is interested in various aspects of design theory.

**Open University** The following will be visiting at various times during 2009/10:

**Prof. Bruce Richter** (Waterloo, Canada) will be visiting during September-October 2009. He is interested at present in various aspects of topological graph theory, see <http://www.math.uwaterloo.ca/~brichter/pubs/publications.html>

**Prof. Diane Donovan** (University of Queensland, Australia) will be visiting during October-November 2009. She is interested in combinatorial structures such as designs and Latin squares, especially critical and defining sets and trades, see <http://www.maths.uq.edu.au/~dmd/research.html>

**Prof. Dan Archdeacon** (University of Vermont, USA) will be visiting during January-June 2010. He is interested in graph theory and combinatorics, especially topological graph theory. See <http://www.emba.uvm.edu/~archdeac/#research> for more details.

## Recent Ph.D. theses in Combinatorics.

Again, not more accurate than the information I receive: “recent” may be ill-defined.

**Bath:** Adam Kinnison has been awarded a Ph.D for a thesis on random walks on Galton-Watson trees. Peter Mörters was his supervisor.

**Brunel:** Nicole Eggemann was awarded her Ph.D for a thesis entitled “Some applications of graph theory”. Steve Noble was her supervisor and the examiners were Colin McDiarmid (Oxford), Colin Cooper (KCL) and Ilia Krasikov (Brunel).

## General News.

**QMUL:** Dr. Peter Keevash has been awarded the European Prize in Combinatorics at the recent EUROCOMB conference in Bordeaux. The prize is designed to recognise “excellent contributions in Combinatorics, Discrete Mathematics and their

Applications” by young (under 35) EU-resident researchers. More details, and a list of previous prize winners, can be found at <http://eurocomb09.labri.fr/pmwiki/pmwiki.php/Main/Prize>

## Unsolved Problem(s).

This is at least partly a spin-off from various discussions at the Newton Institute workshop on Combinatorics and Statistical Mechanics last year. Further reading on it can be found in for example an article on Peter Cameron’s webpage prepared for the study group at QMUL: <http://www.maths.qmul.ac.uk/~pic/csgnotes/alchrom1.pdf>

Recall that the chromatic polynomial of a graph  $G$  is the function  $P_G(k)$  which counts the number of proper  $k$ -colourings of  $G$ . One easily checks it is indeed a polynomial in  $k$ . A chromatic root is a complex number which is a root of some chromatic polynomial. Obviously such a number is an algebraic (as opposed to transcendent) number: further, as the coefficients of a chromatic polynomial are integers, it is an algebraic integer. The question (which was initially asked by David Wallace, Director of the Institute) is, crudely speaking: which algebraic integers are chromatic roots?

For a trivial example, every non-negative integer is a chromatic root as a complete graph on  $n$  vertices has zero proper colourings with  $m$  colours for any  $m < n$ . On the other hand, the fact that the signs of coefficients of a chromatic polynomial alternate in sign (and that at least one of them is non-zero....) implies that there are no negative roots. With a bit more work, it can be shown (exercise...) that there are no chromatic roots in  $(0,1)$ . With non-trivially more work, it can be shown that there are no roots in  $(1, \frac{32}{27})$ . (However, chromatic roots are dense in  $[\frac{32}{27}, \infty)$ , and by a result of Alan Sokal are also dense in the complex plane). A (slightly dated now) survey on this topic by Bill Jackson can be found at <http://arxiv.org/pdf/math/0205047v2>. Of course additional information about a graph’s structure can give extra restrictions on its chromatic roots: for example, a result by Carsten Thomassen says that a graph with a Hamilton path cannot have a non-integer chromatic root less than a certain surd approximately equal to 1.29: this is quite a bit bigger than  $32/27$ .

On the other hand, to emphasise our ignorance, at one point in the meeting, Dave Wagner noted that he was not aware of any chromatic polynomial which had  $i$  (as in the square root of  $-1$ ) as a root. I don’t think we have such an example yet. Nor, according to Peter Cameron’s article, do we know if  $3 + \frac{\sqrt{5}-1}{2}$  is a chromatic root. We do know that the algebraic integer  $\frac{\sqrt{5}-1}{2}$  is *not* a root of any chromatic polynomial (as it is in  $(0,1)$ ), and Tutte showed some time ago that the “Beraha number”  $\frac{\sqrt{5}+3}{2}$  is not a chromatic root.

Small graphs can mislead in these kinds of problems: for example, for all graphs with nine or fewer vertices, the real part of the roots has to be negative. But of course the Sokal result about density of roots in the complex plane implies that this cannot be true in general.

Here are two particular problems (both in Peter Cameron's article) to get your teeth into. I believe I am right in saying that neither of them is solved yet, though the first question is known to have the answer "yes" when  $\alpha$  is in a quadratic number field. It seems to be the consensus that both questions are likely to have the answer "yes".

Question 1. Suppose  $\alpha$  is an algebraic integer. Is it true that there is a natural number  $n$  such that  $\alpha + n$  is a chromatic root?

Question 2. Suppose  $\alpha$  is a chromatic root. Then for every natural number  $n$ ,  $n\alpha$  is a chromatic root.