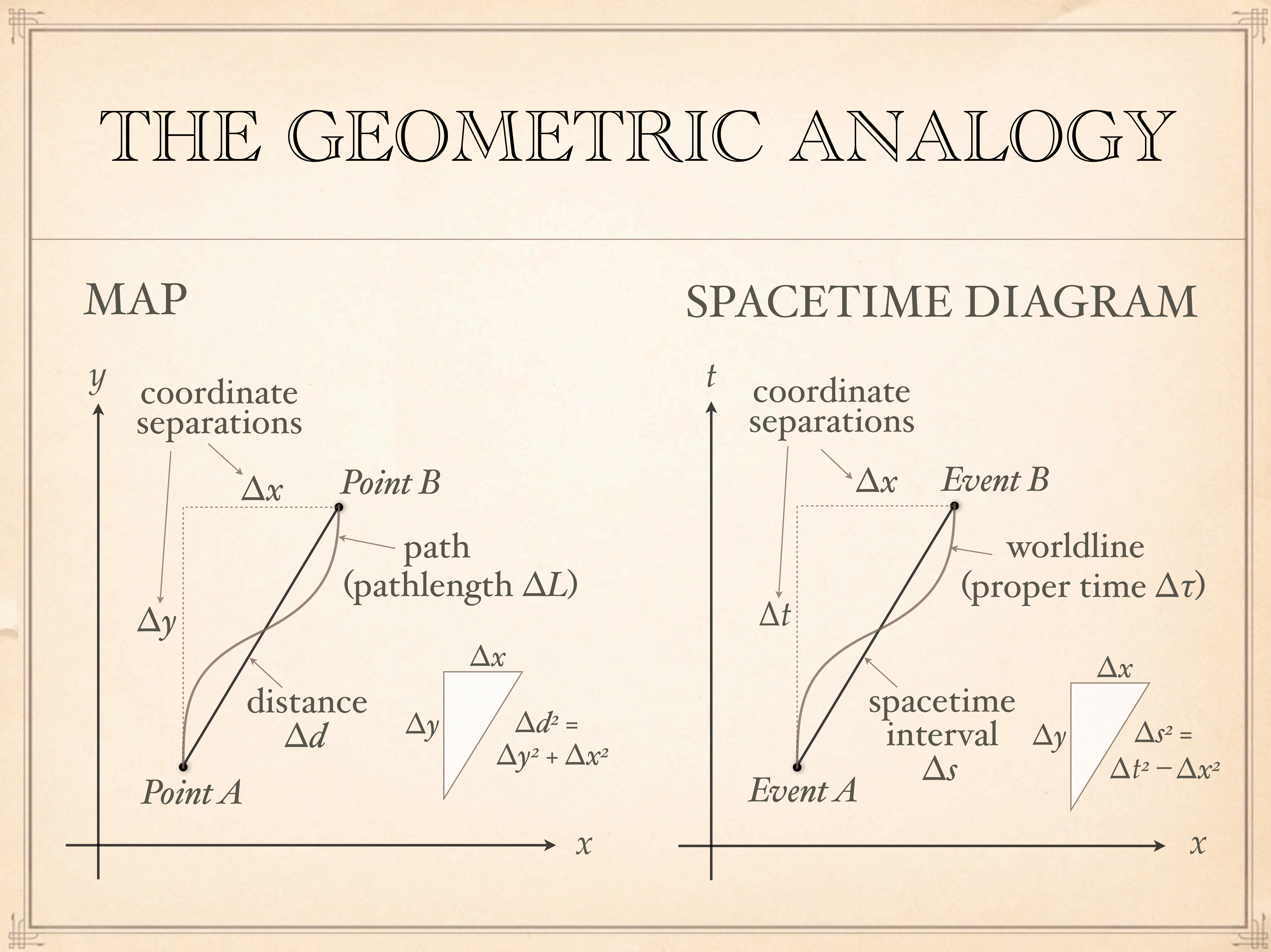
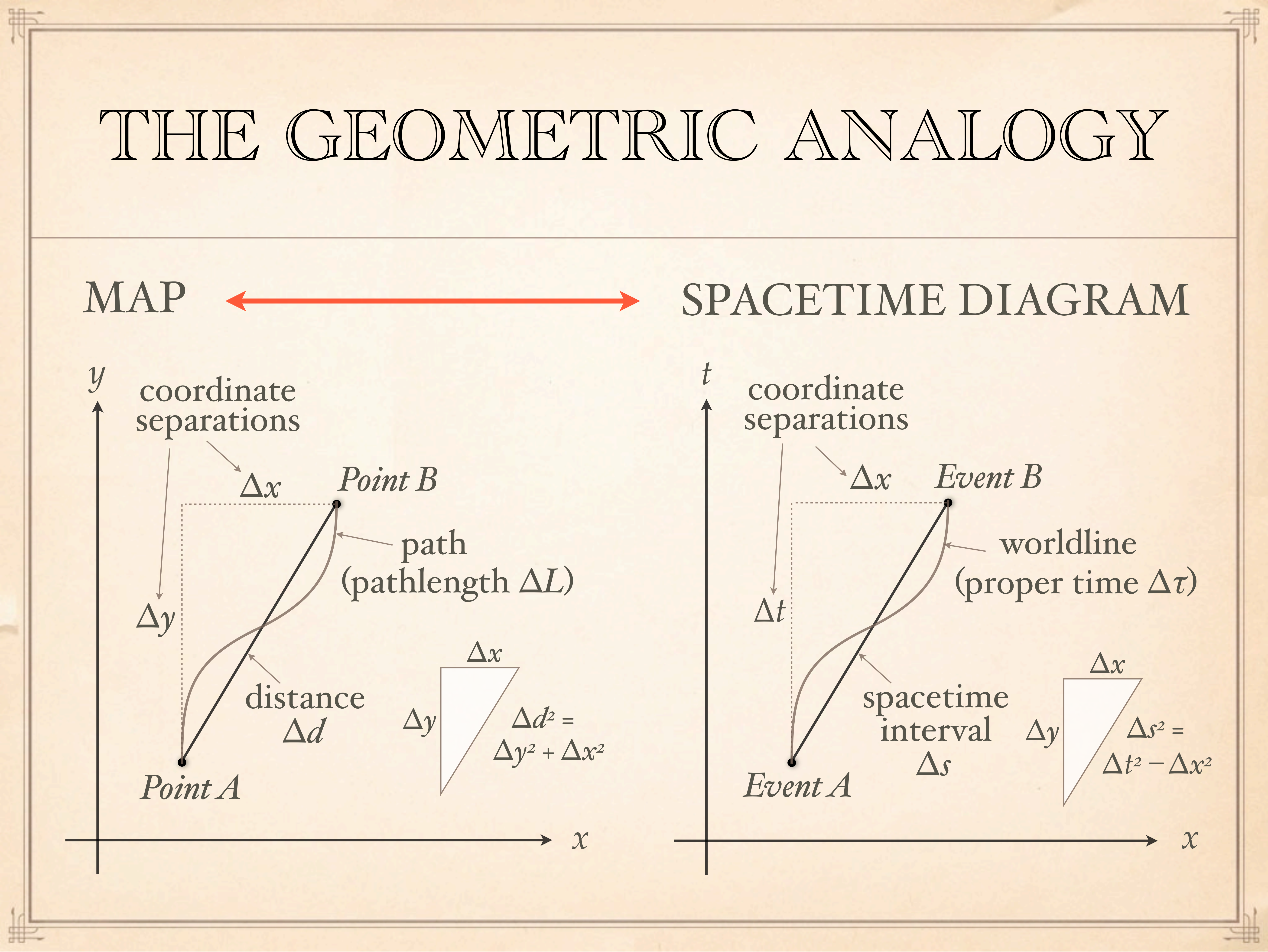
VISUAL AIDS FOR TEACHING SPECIAL RELATIVITY

AAPT Summer Meeting 2011, Omaha, NE

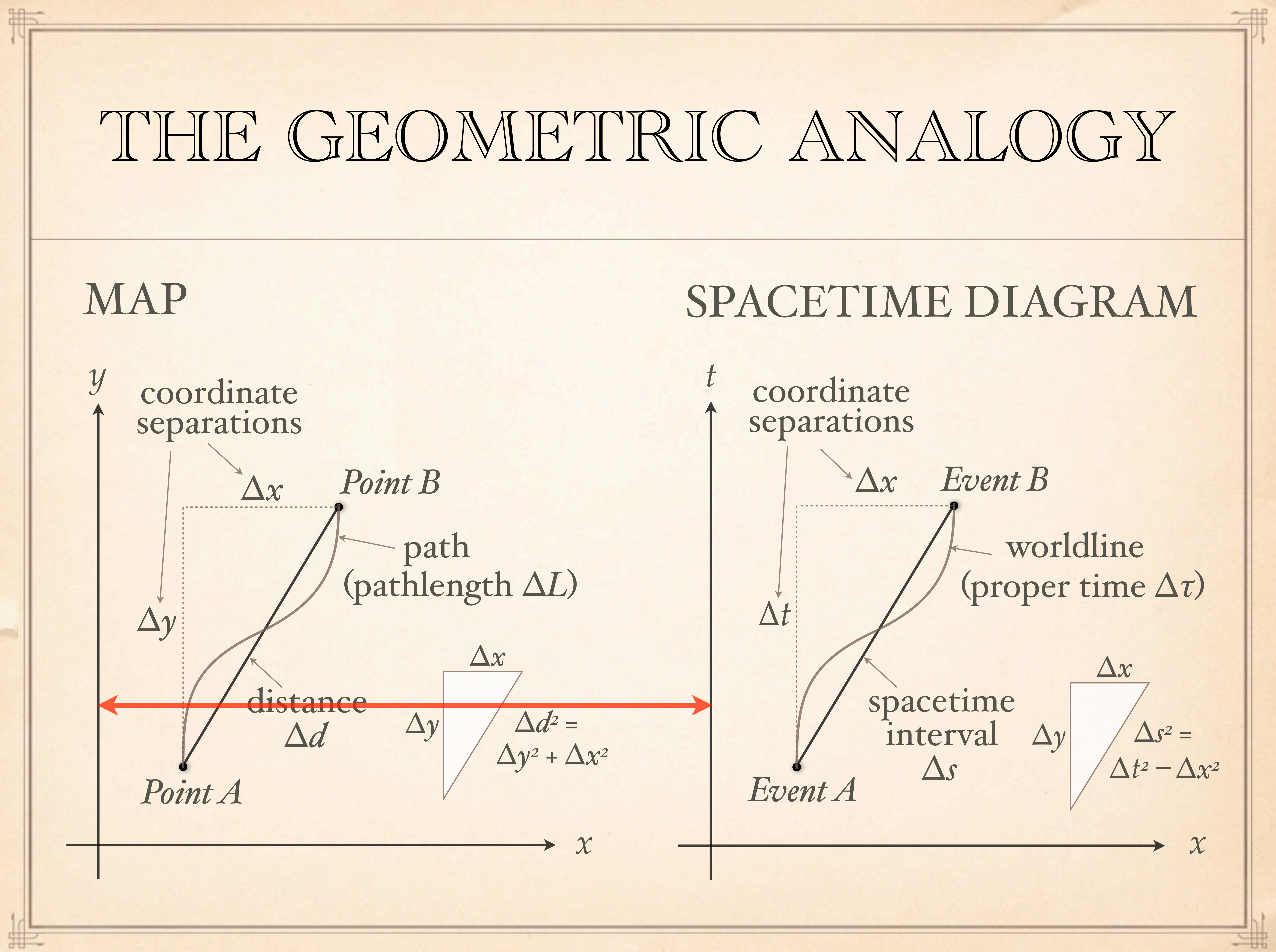
Thomas A. Moore, Pomona College



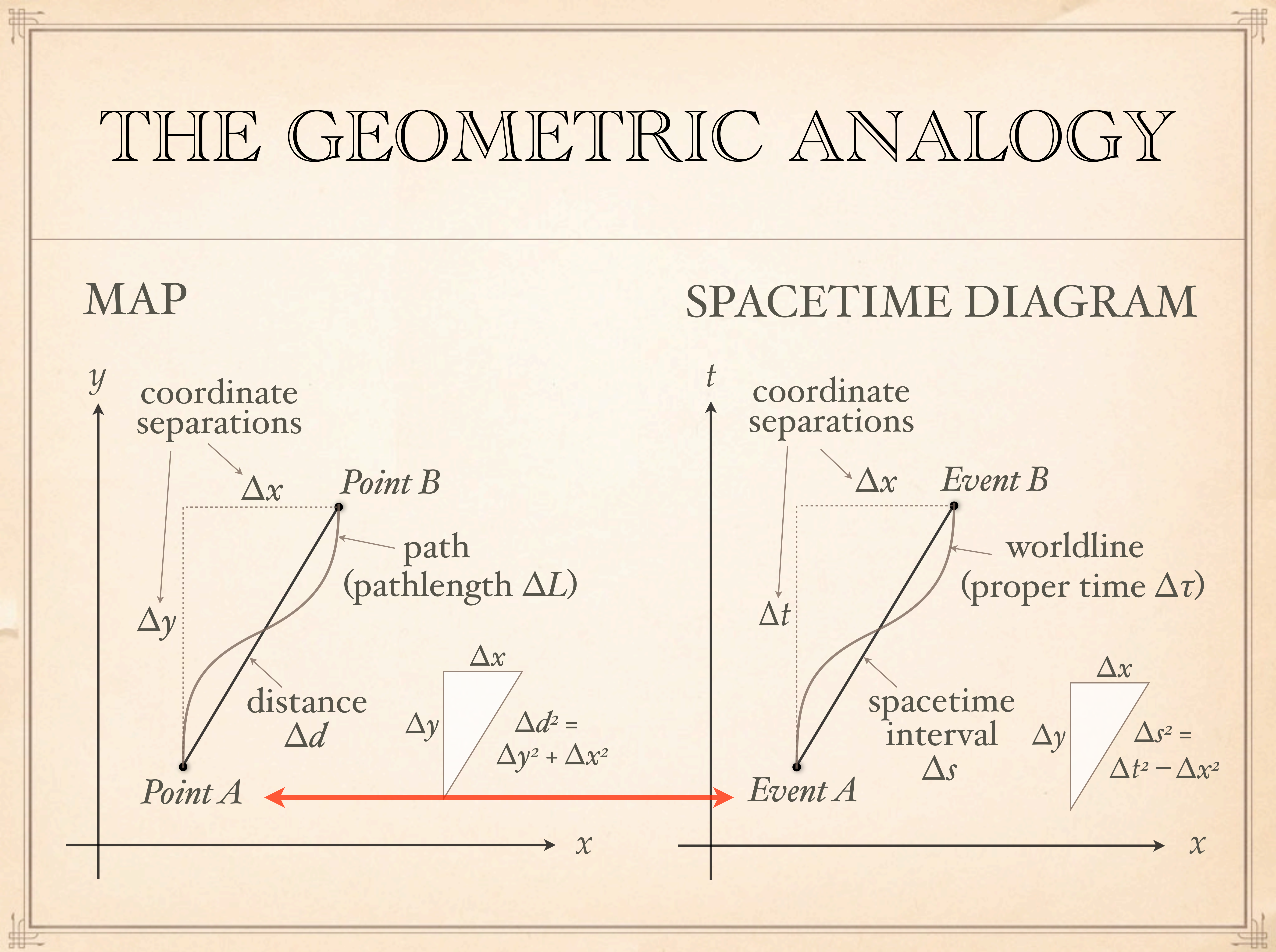
- In the 31 years of college teaching, SR ~ 25 times. What I present seems “well-known” but I know it’s not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial



- In the 31 years of college teaching, SR ~ 25 times. What I present seems "well-known" but I know it's not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial

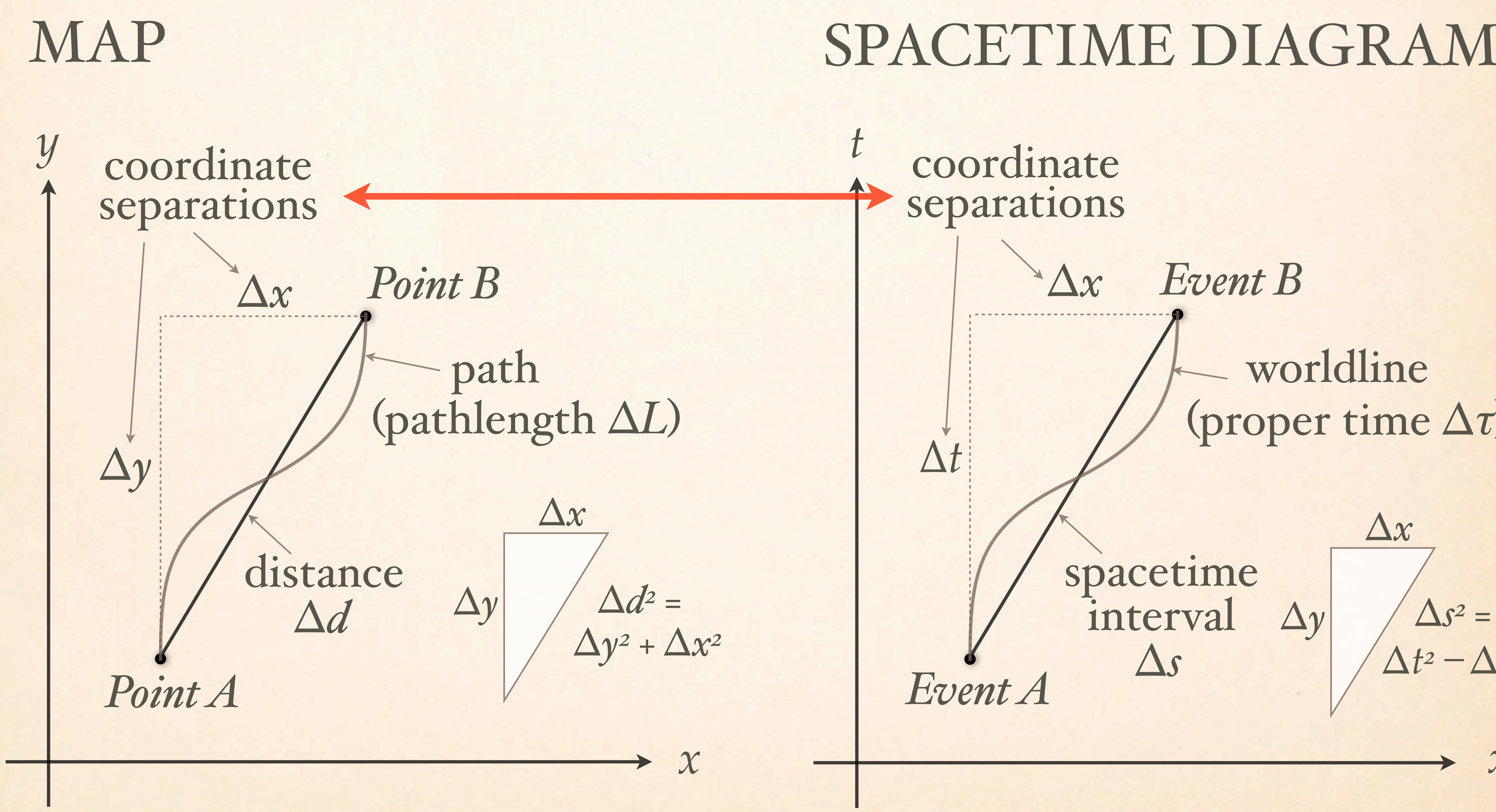


- In the 31 years of college teaching, SR ~ 25 times. What I present seems “well-known” but I know it’s not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial

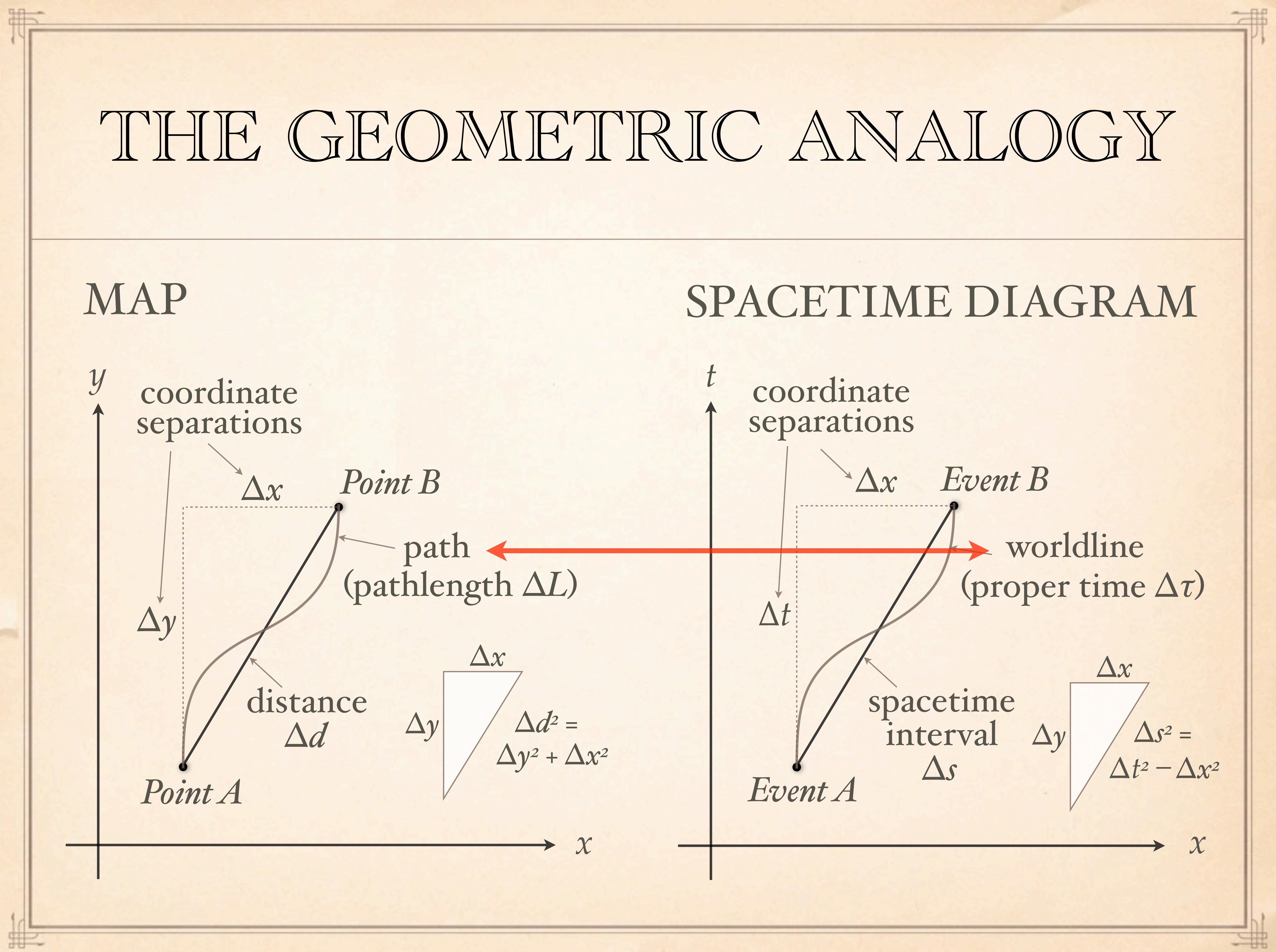


- In the 31 years of college teaching, SR ~ 25 times. What I present seems “well-known” but I know it’s not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial

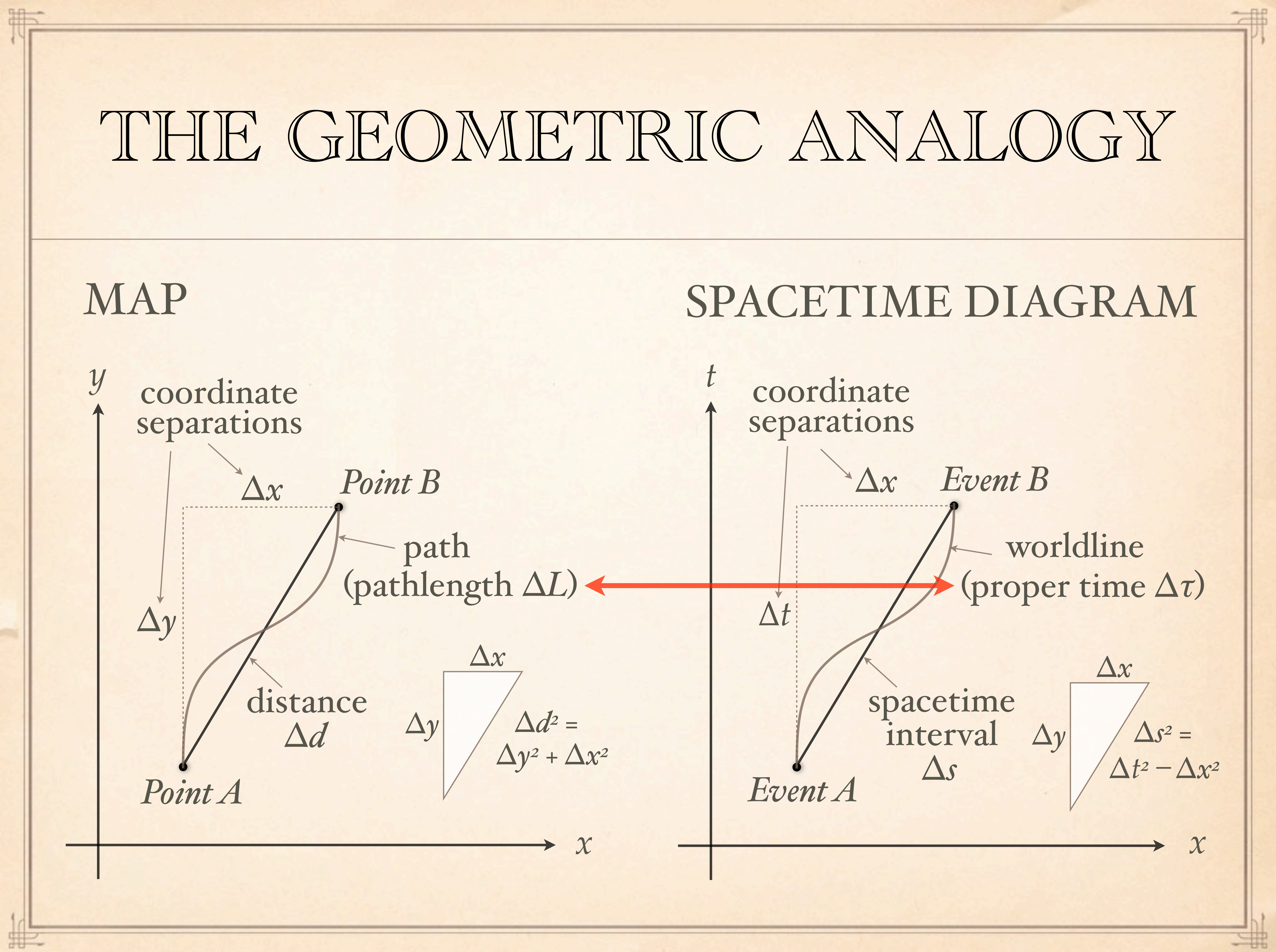
THE GEOMETRIC ANALOGY



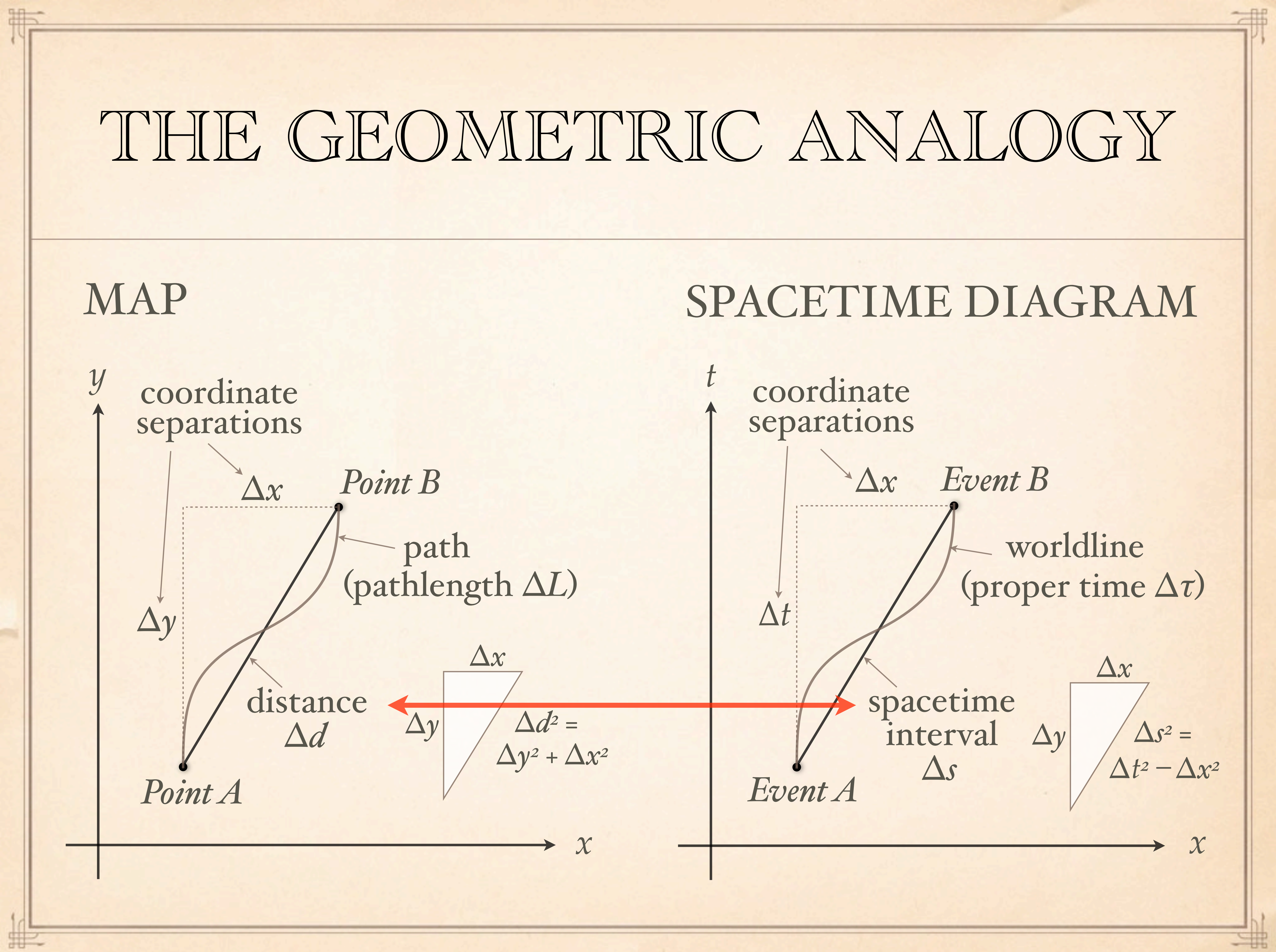
- In the 31 years of college teaching, SR ~ 25 times. What I present seems "well-known" but I know it's not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial



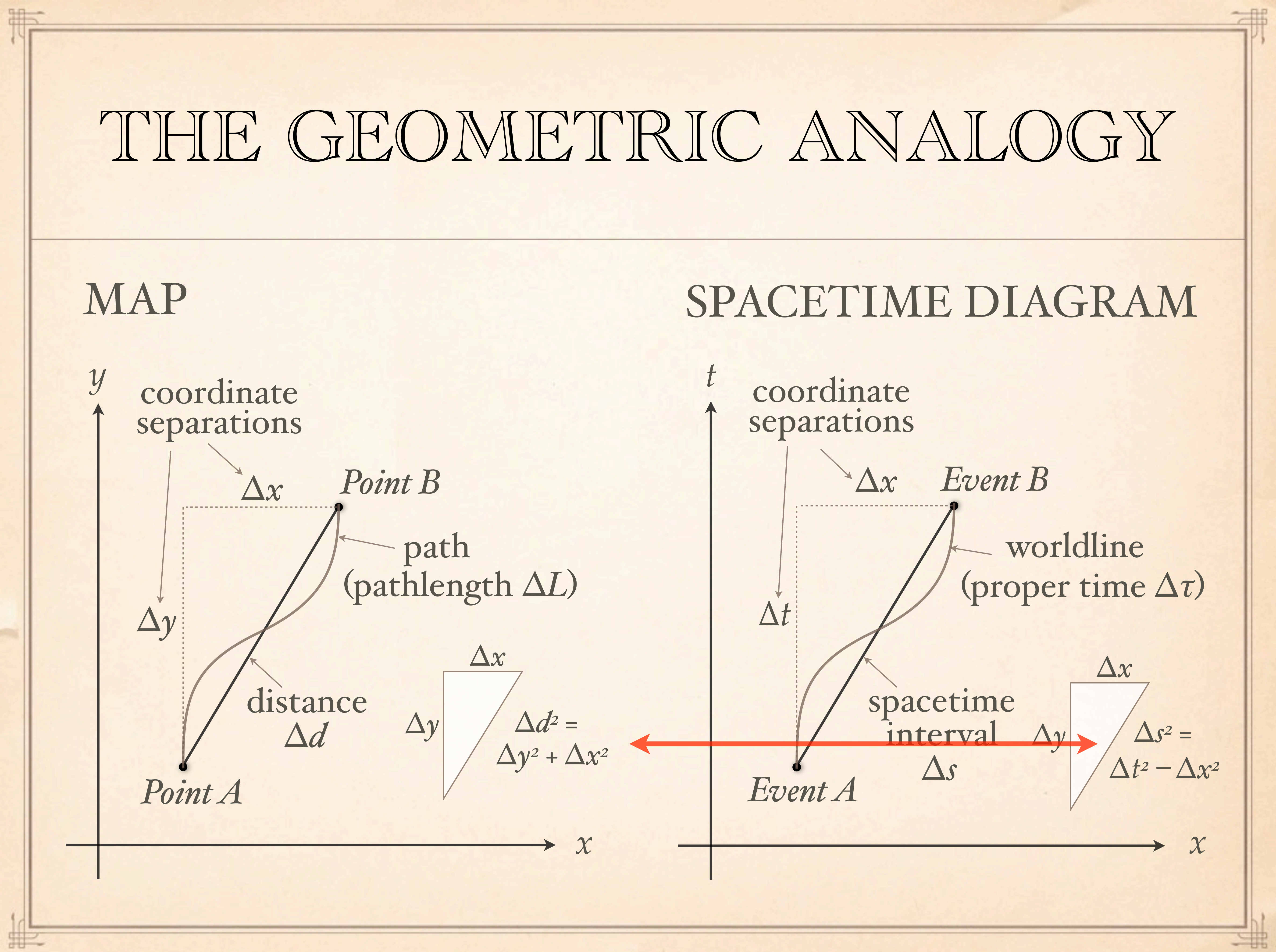
- In the 31 years of college teaching, SR ~ 25 times. What I present seems “well-known” but I know it’s not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial



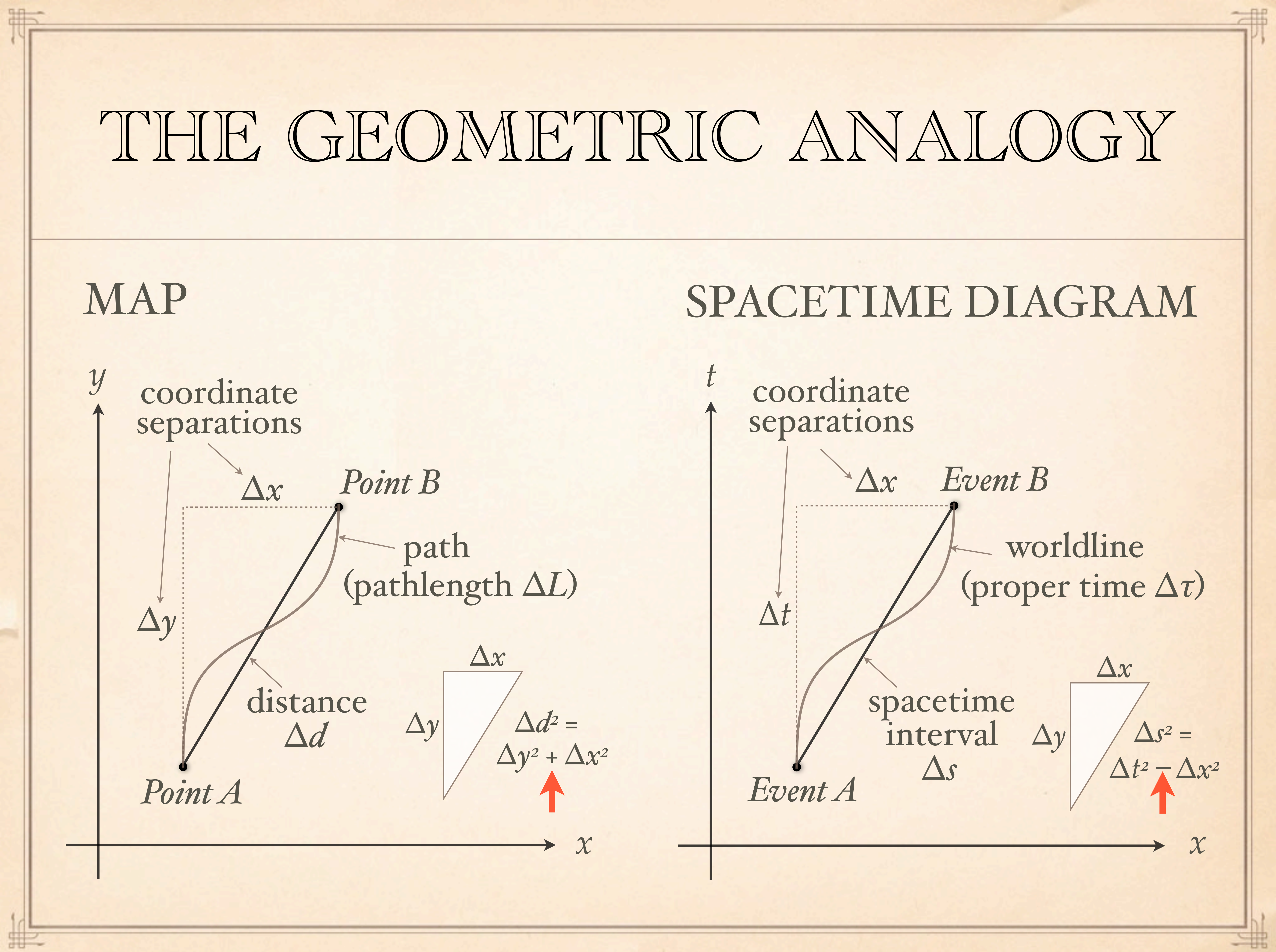
- In the 31 years of college teaching, SR ~ 25 times. What I present seems "well-known" but I know it's not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial



- In the 31 years of college teaching, SR ~ 25 times. What I present seems "well-known" but I know it's not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial

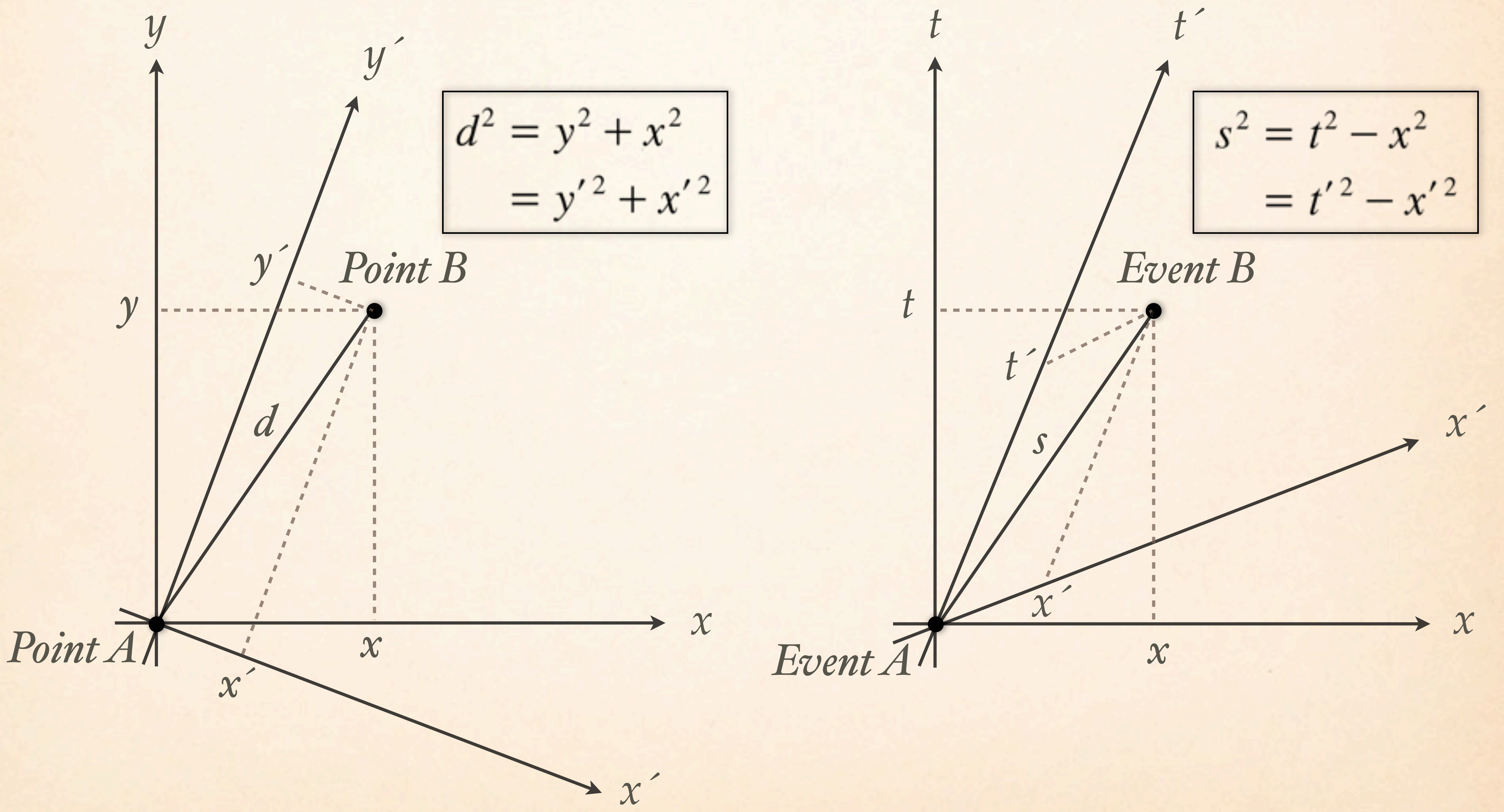


- In the 31 years of college teaching, SR ~ 25 times. What I present seems "well-known" but I know it's not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial



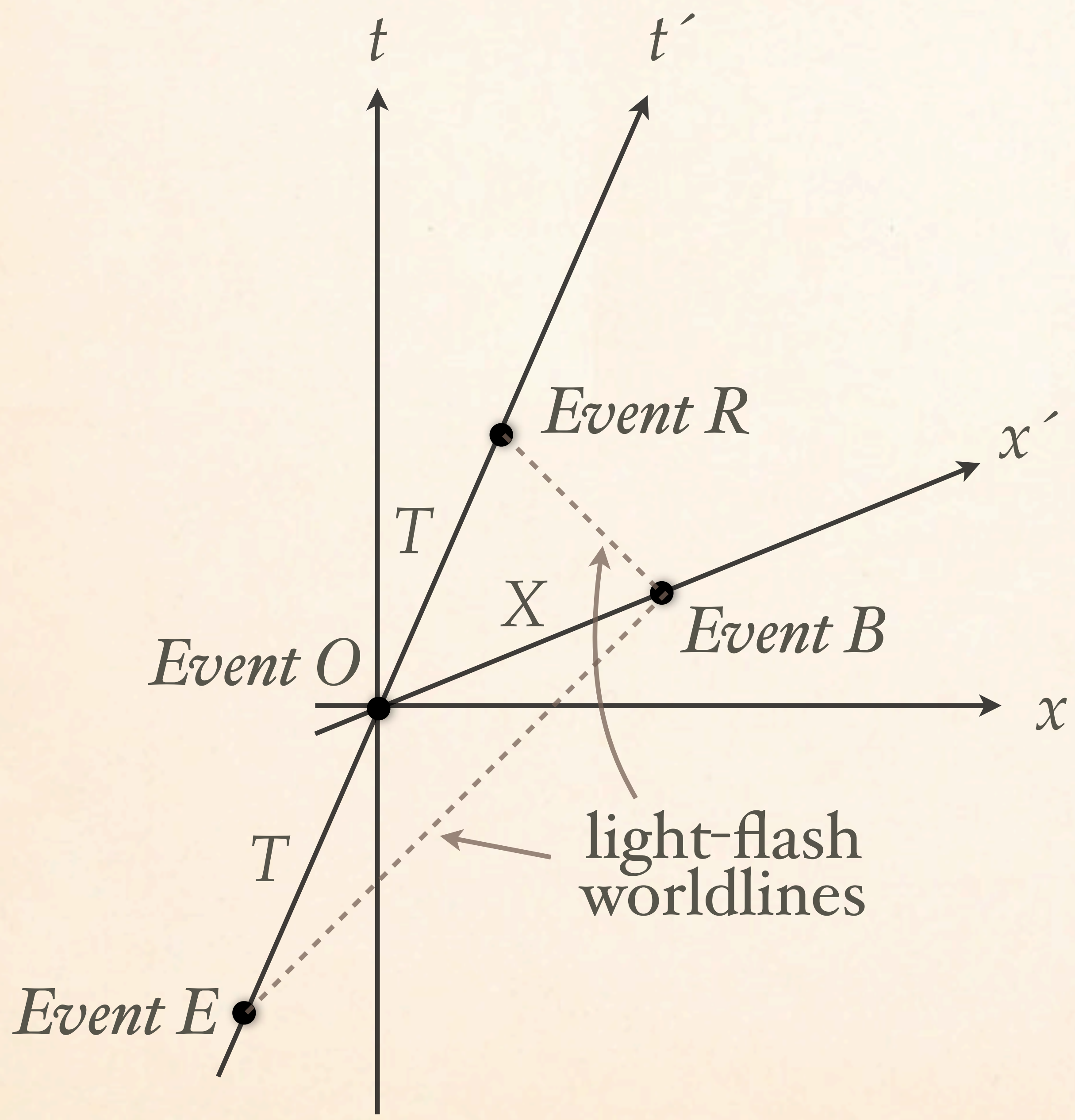
- In the 31 years of college teaching, SR ~ 25 times. What I present seems “well-known” but I know it’s not.
- In my experience, the *single most important thing you can do* to help your students is the **geometric analogy**.
- Analogy ultimately is between plane geometry and spacetime
- Map <--> Spacetime diagram (Click), coordinate axes <---> inertial reference frame (Click)
- (Understanding analogy is easiest if space and time coords have the same units -- Parable of the Surveyors)
- If axes are scaled the same, then light worldlines have slope ±1
- points <--> events (Click), coordinate separations <--> coordinate separations (Click)
- paths <---> worldlines, pathlength <--> proper time (Click)
- unique straight-line path: its pathlength is distance <--> ST interval (Click)
- connection between coordinates & this special number is Pythagorean Theorem <--> metric equation (Click)
- *difference* is the minus sign: crucial

TWO-OBSERVER DIAGRAM



- Another important (but more subtle) analogy is rotations <--> boosts
- In plane, we can construct a pair of rotated coordinate systems, read coords of *B* in both (drop ||s)
- The PT implies that *d* is system-independent
- In ST, we can similarly construct a pair of axes for IRFs in relative motion, read coords of B in both
- The metric eqn implies that *s* is frame-independent
- (Should be called Theory of Absolutivity!)
- Two-observer diagrams are a very powerful tool (graphical rep of LTEs)
- But to make them useful, students need to understand differences
    - why x' axis is not perpendicular to t' axis
    - why we have to drop parallels
    - how to calibrate the axes (and why we can't do it with a ruler)

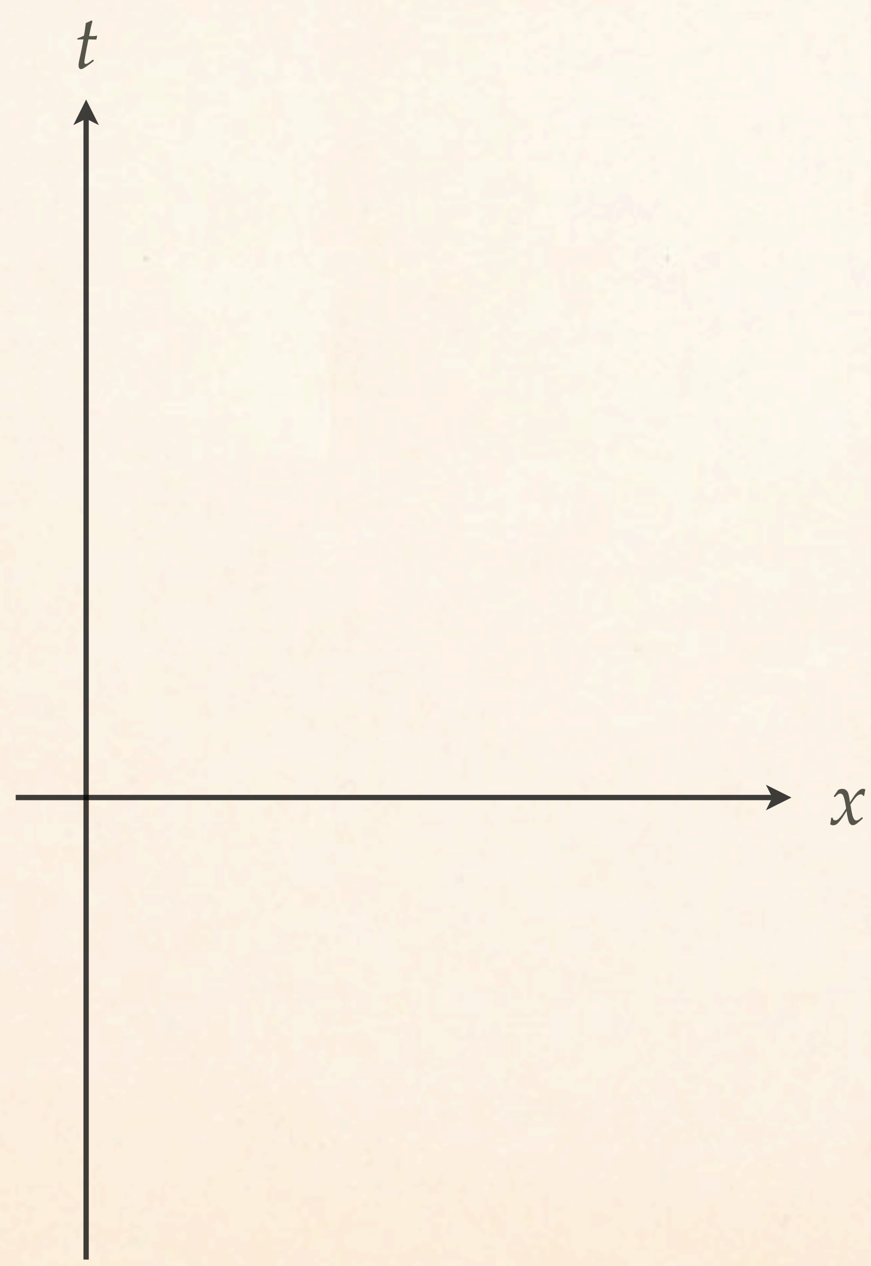
LOCATING THE X′ AXIS



- Events A and B are simultaneous in S'
- The x' axis therefore connects them
- Note also that $T = X$

- Let's work on the tilt of the x' axis first using a radar method (method from *Six Ideas*, Unit R)
- Definition of x' axis: line connecting all events that occur at $t' = 0$.
- To locate, imagine that primed observer emits a flash of light at E a time T before origin event O. At B it bounces off a mirror some distance X away in the primed frame, and returns at R a time T *after* O.
- Since the speed of light is 1 in all frames, the primed observer concludes B must have happened halfway between E and R, i.e. simultaneously with O.
- So x′ axis therefore must go through O and B, tilted at the angle shown.
- Note also that $T = X$, since light has gone $2X$ in time $2T$.
- Symmetry of triangle ORB implies slope of x' axis is inverse slope of t' axis.
- Why we need to drop parallels

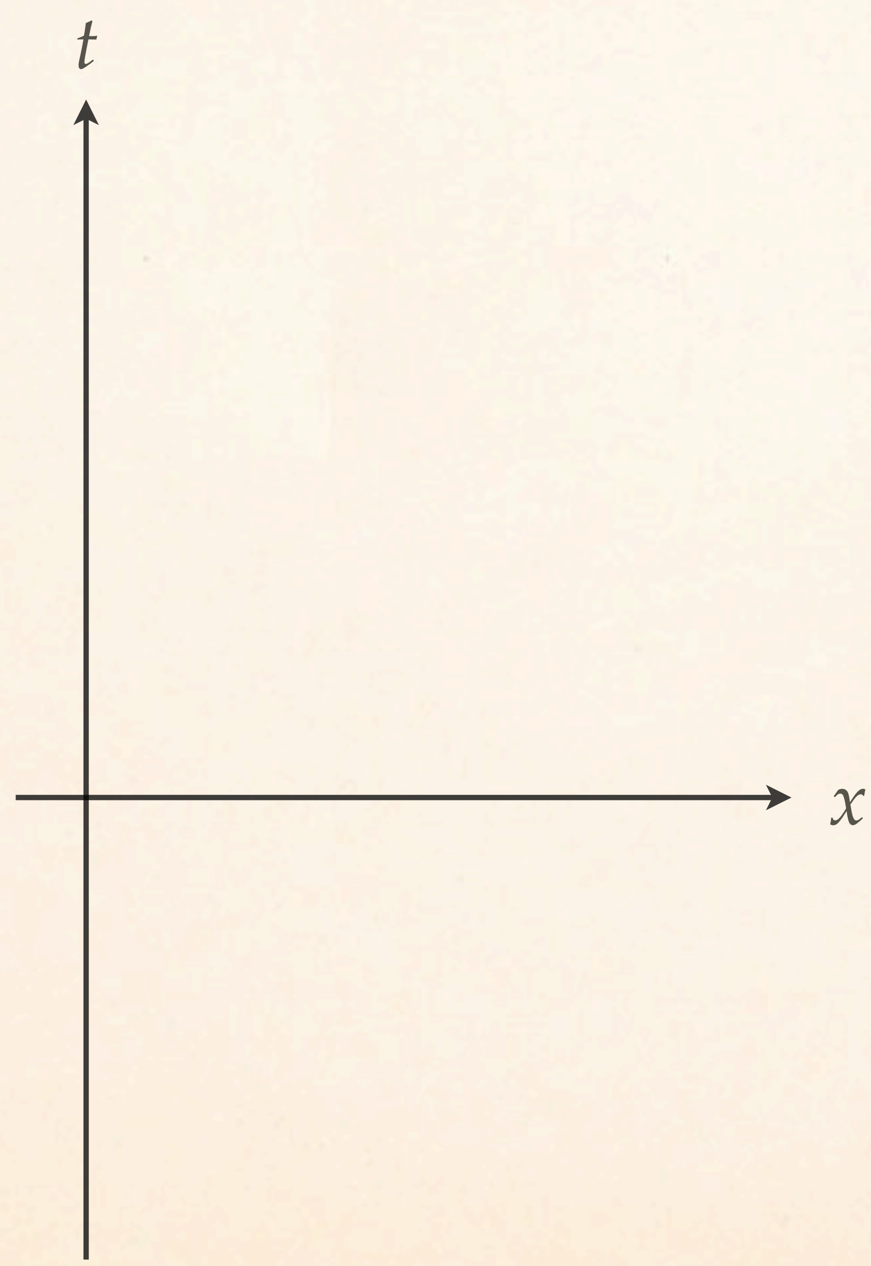
AN EXAMPLE PROBLEM



“Spacecraft problem” from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: “Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event R be Rainier erupting, and event H be Hood erupting. In the spaceship’s frame, does R occur before, after, or at the same time as H? Explain.”
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: R and H are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event R going toward Hood (i.e. in $+x$ direction) (Click)
- Which occurs first? H is below the x' axis, so H occurs before R. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

AN EXAMPLE PROBLEM

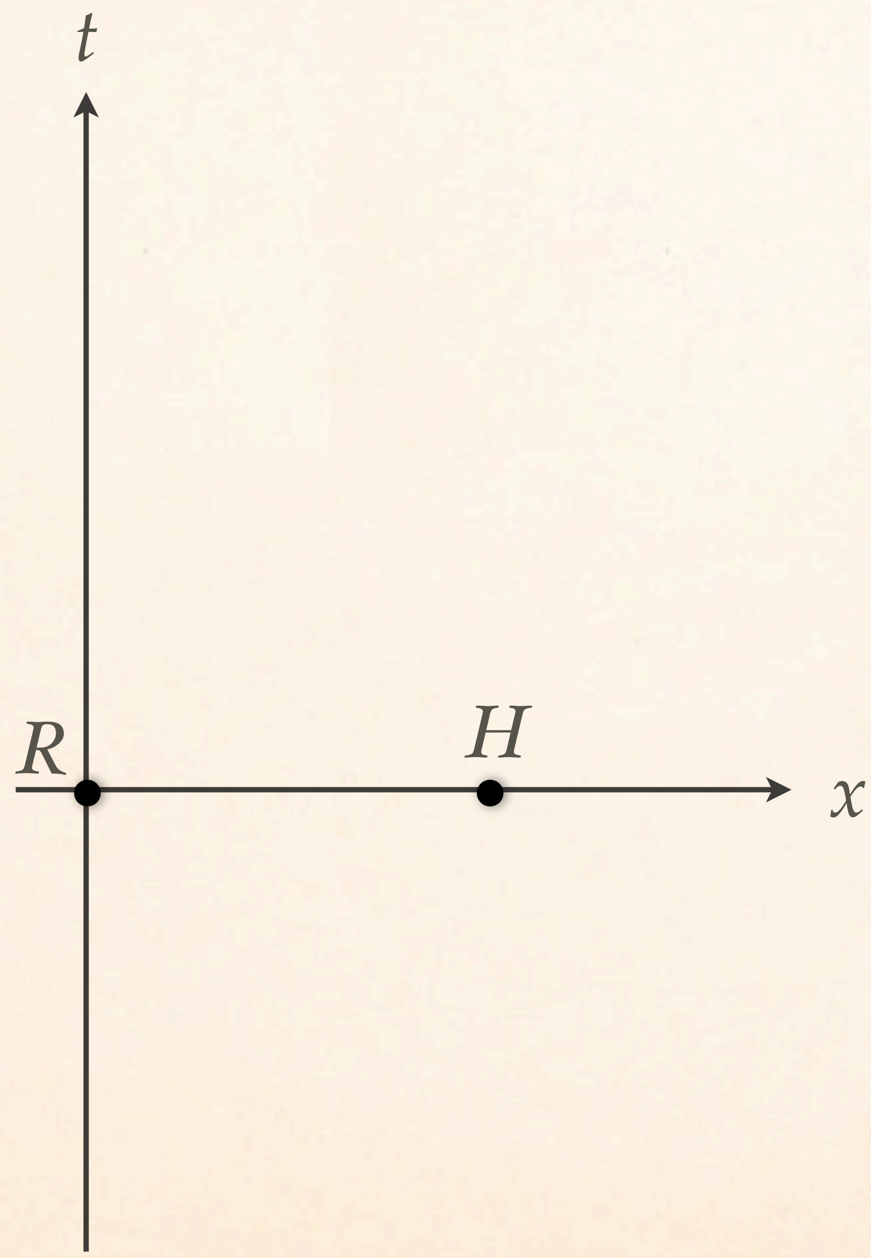


"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

After tutorials: 51%

- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: "Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event R be Rainier erupting, and event H be Hood erupting. In the spaceship's frame, does R occur before, after, or at the same time as H? Explain."
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: R and H are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event R going toward Hood (i.e. in $+x$ direction) (Click)
- Which occurs first? H is below the x' axis, so H occurs before R. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

AN EXAMPLE PROBLEM

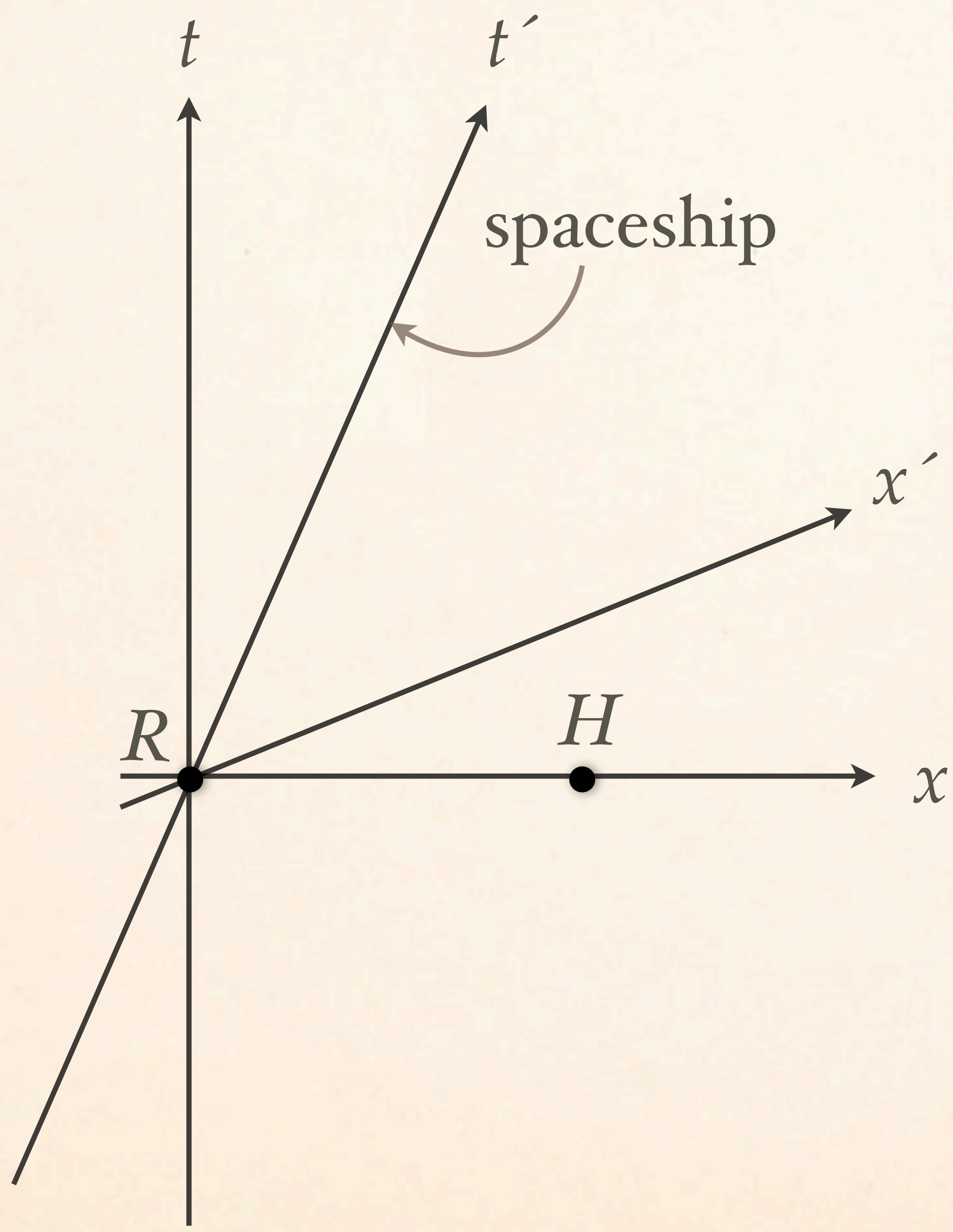


"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

After tutorials: 51%

- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: "Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event *R* be Rainier erupting, and event *H* be Hood erupting. In the spaceship's frame, does *R* occur before, after, or at the same time as *H*? Explain."
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: *R* and *H* are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event *R* going toward Hood (i.e. in +*x* direction) (Click)
- Which occurs first? *H* is below the x' axis, so *H* occurs before *R*. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

AN EXAMPLE PROBLEM

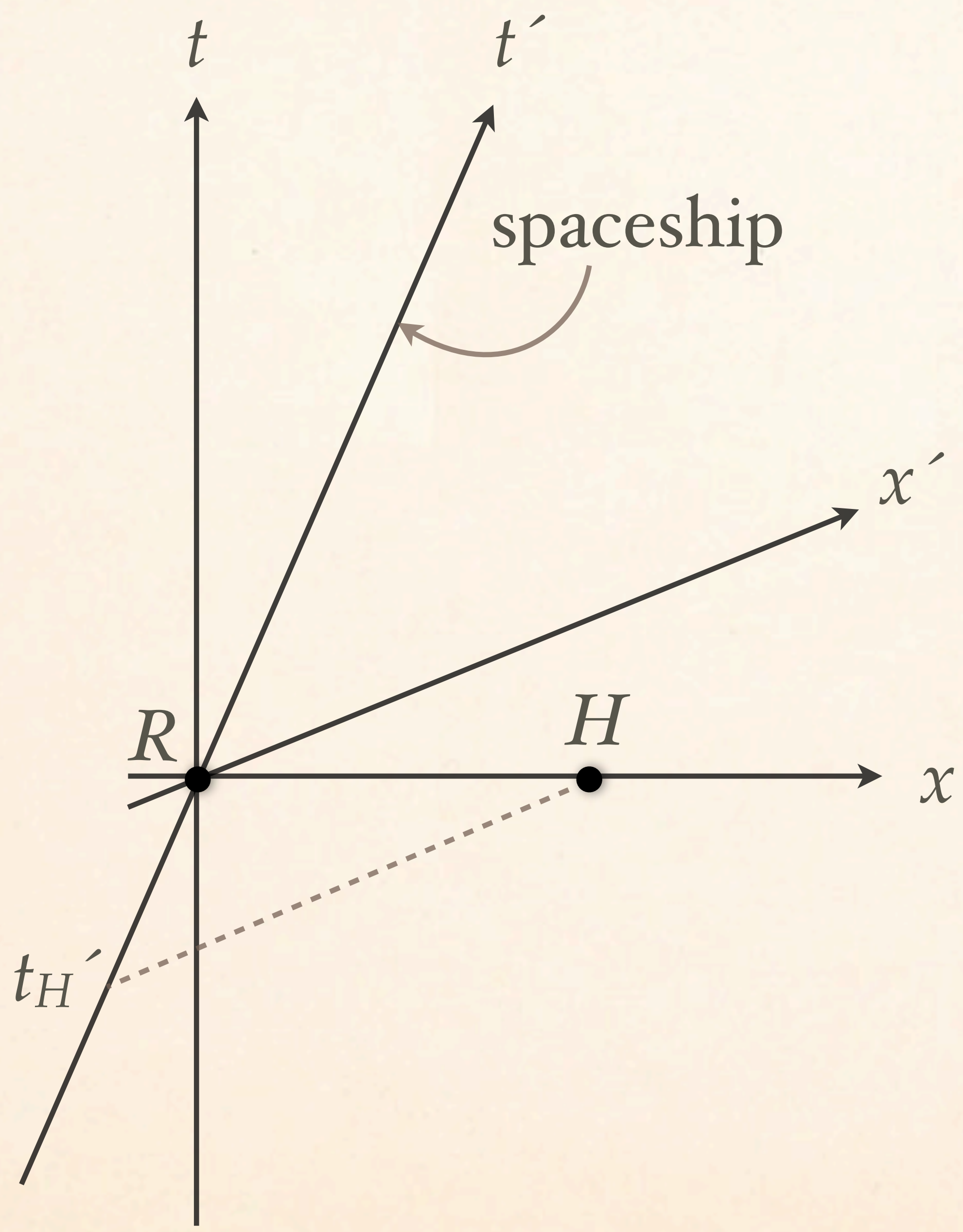


“Spacecraft problem” from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

After tutorials: 51%

- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: “Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event R be Rainier erupting, and event H be Hood erupting. In the spaceship’s frame, does R occur before, after, or at the same time as H? Explain.”
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: R and H are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event R going toward Hood (i.e. in $+x$ direction) (Click)
- Which occurs first? H is below the x' axis, so H occurs before R. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

AN EXAMPLE PROBLEM

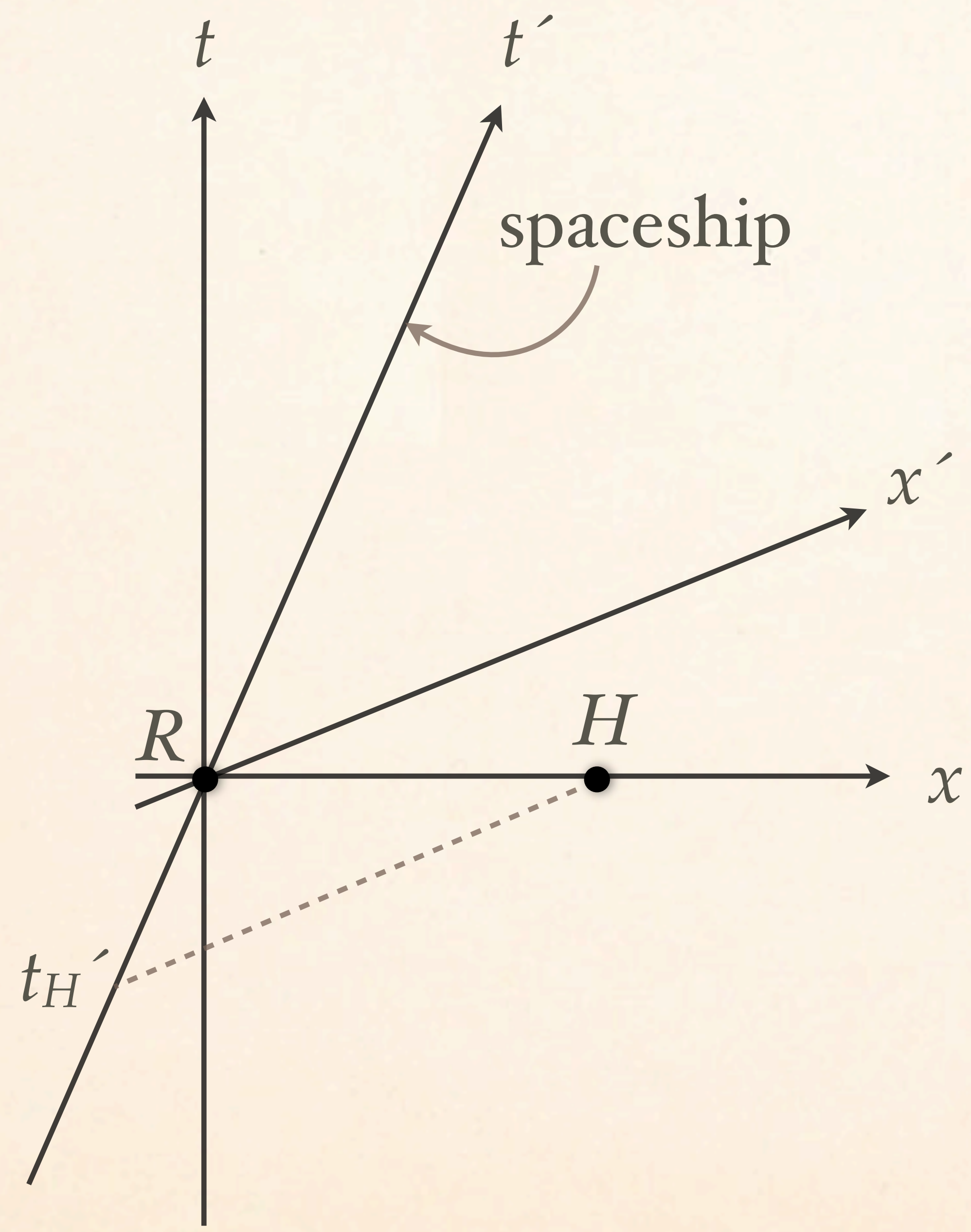


"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

After tutorials: 51%

- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: "Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event *R* be Rainier erupting, and event *H* be Hood erupting. In the spaceship's frame, does *R* occur before, after, or at the same time as *H*? Explain."
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: *R* and *H* are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event *R* going toward Hood (i.e. in $+x$ direction) (Click)
- Which occurs first? *H* is below the x' axis, so *H* occurs before *R*. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

AN EXAMPLE PROBLEM



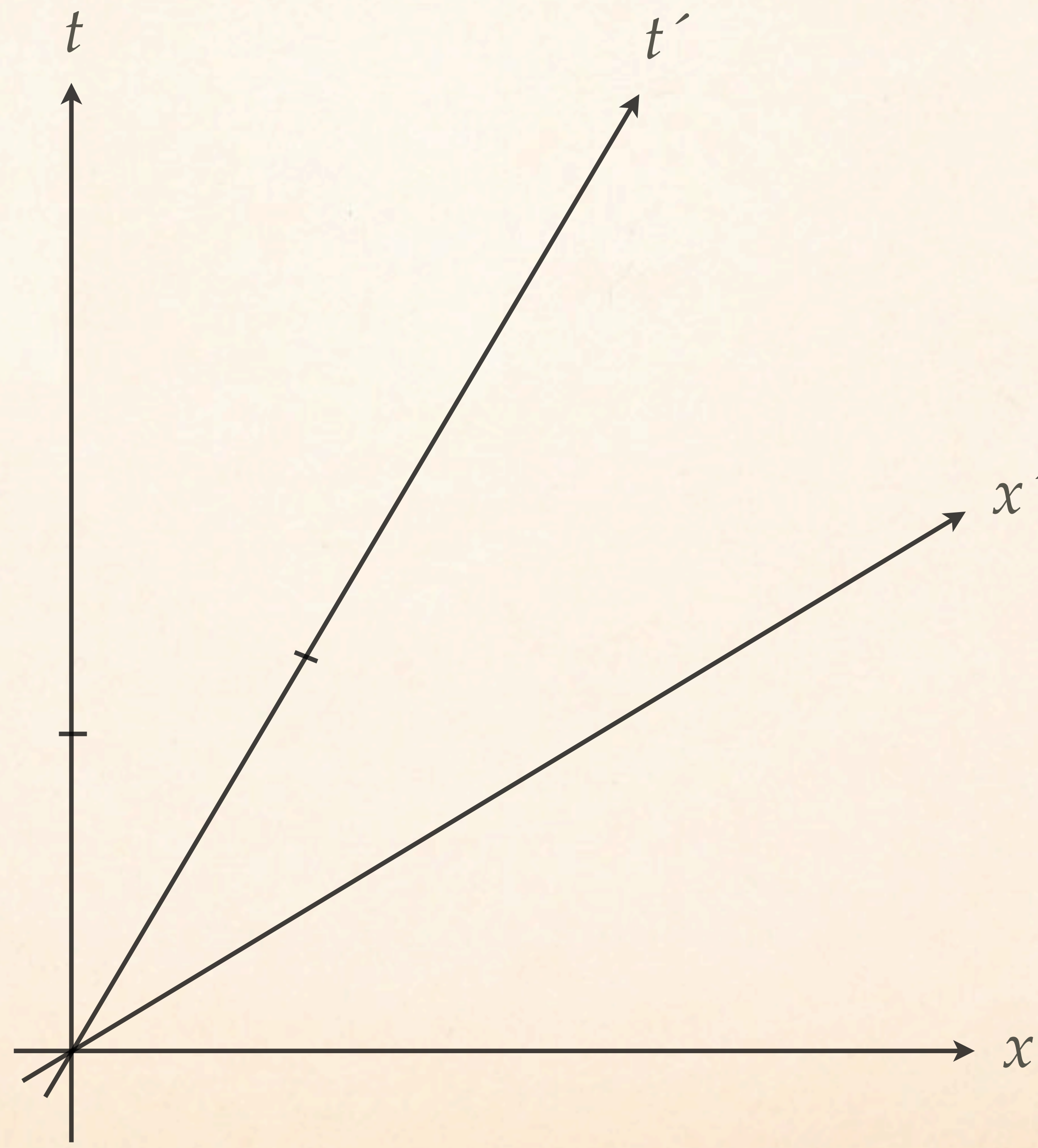
"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

After tutorials: 51%

In my intro class:
~ 80%
(but only after hint to "draw a spacetime diagram"!)

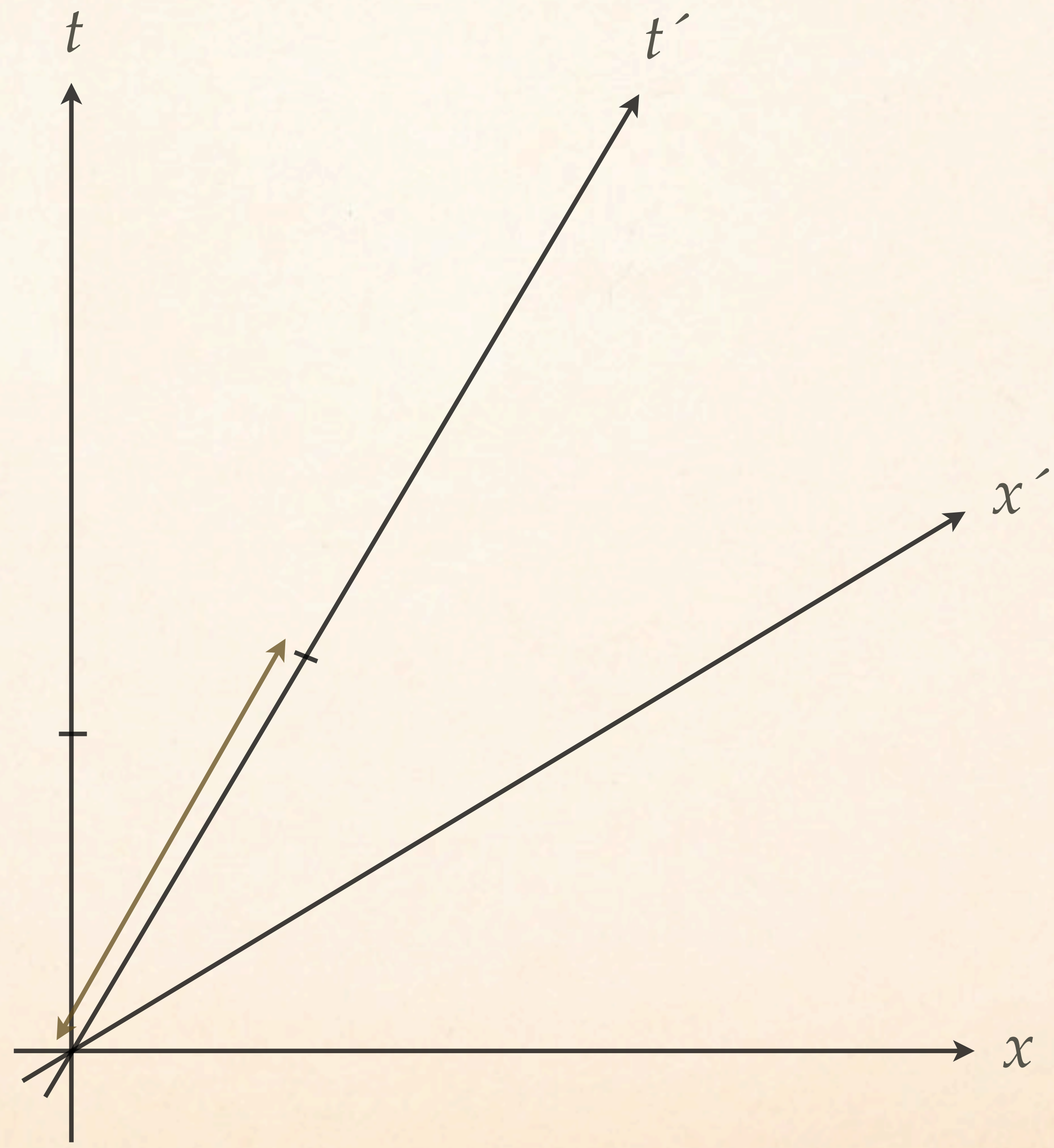
- Just this much is sufficient for helping students solve tough problems.
- Spacecraft problem from Scherr et al: "Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanos. A spacecraft flying at 3/5 the speed of light from Rainier to Hood is directly over Rainier when it erupts. Let event *R* be Rainier erupting, and event *H* be Hood erupting. In the spaceship's frame, does *R* occur before, after, or at the same time as *H*? Explain."
- Challenging (Click): Scherr et al. report event that *after* tutorials, only 51% of intro students got this right.
- But this is *easy* with a spacetime diagram: *R* and *H* are simultaneous in the ground frame (Click)
- Spaceship moving at 3/5 passes Rainier at event *R* going toward Hood (i.e. in +*x* direction) (Click)
- Which occurs first? *H* is below the *x′* axis, so *H* occurs before *R*. (Click)
- I gave this problem on an exam in my intro class (couple of years) (Click). About 80% (after hint).

CALIBRATING THE AXES



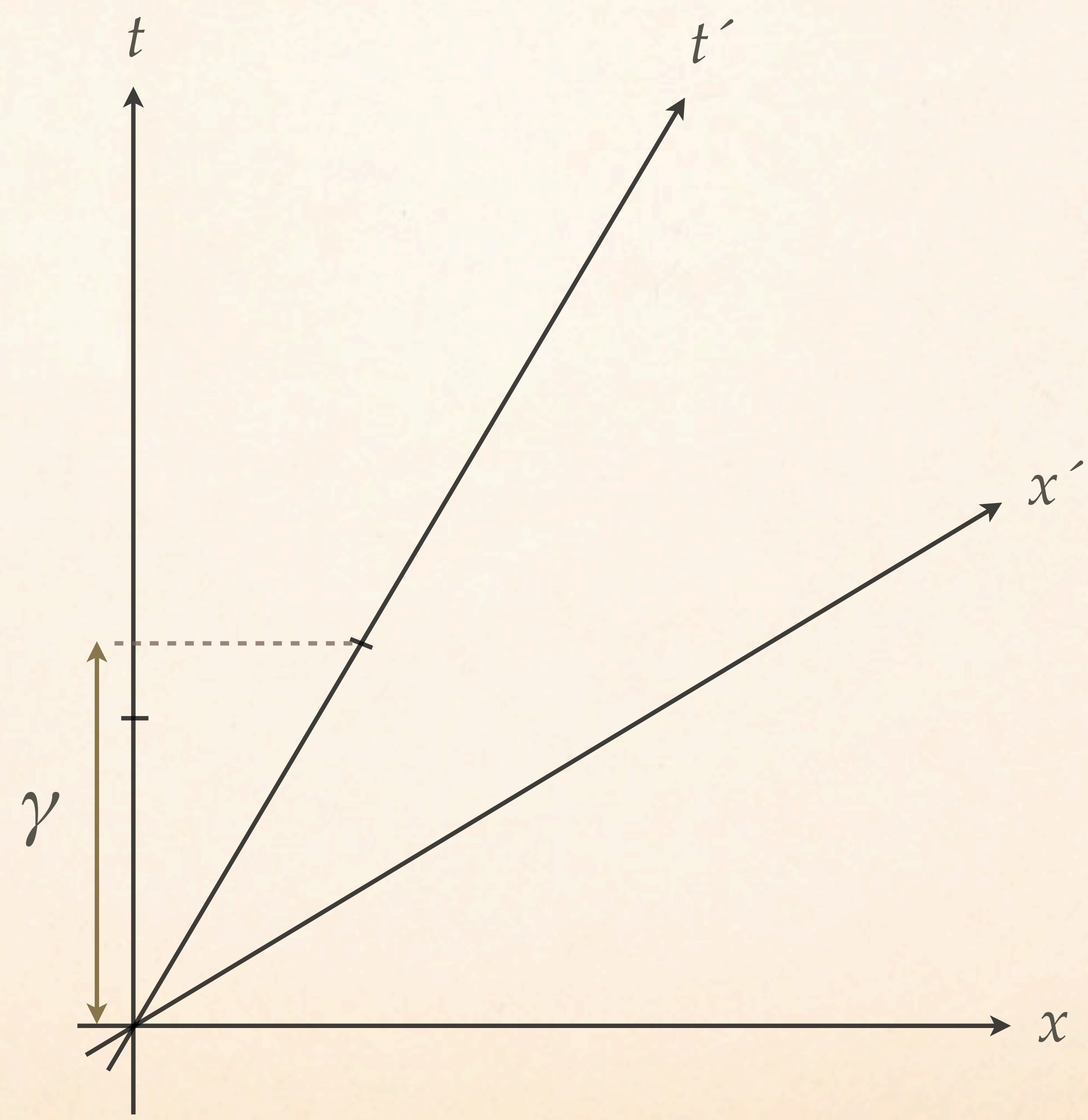
- To do more sophisticated calculations, we need to calibrate the axes of the primed frame
- In my time teaching SR, I have tried lots of methods
- My first approach was to derive a formula for the measured distance between marks ito beta (Click)
  - directly similar to how we'd calibrate rotated axes, but too abstract, not illuminating, tedious
- Next approach (*Six Ideas*) was to project the marks on the main axes (Click)
  - this distance is simply gamma, so better connection to LTEs, but still tedious
- Current approach: hyperbola graph paper (Click)
  - built on idea that axis marks have to be fixed spacetime interval from origin
  - emphasizes centrality of the metric equation
  - fast and easy (no calculations required)
  - Available online at the *Six Ideas* website.

CALIBRATING THE AXES



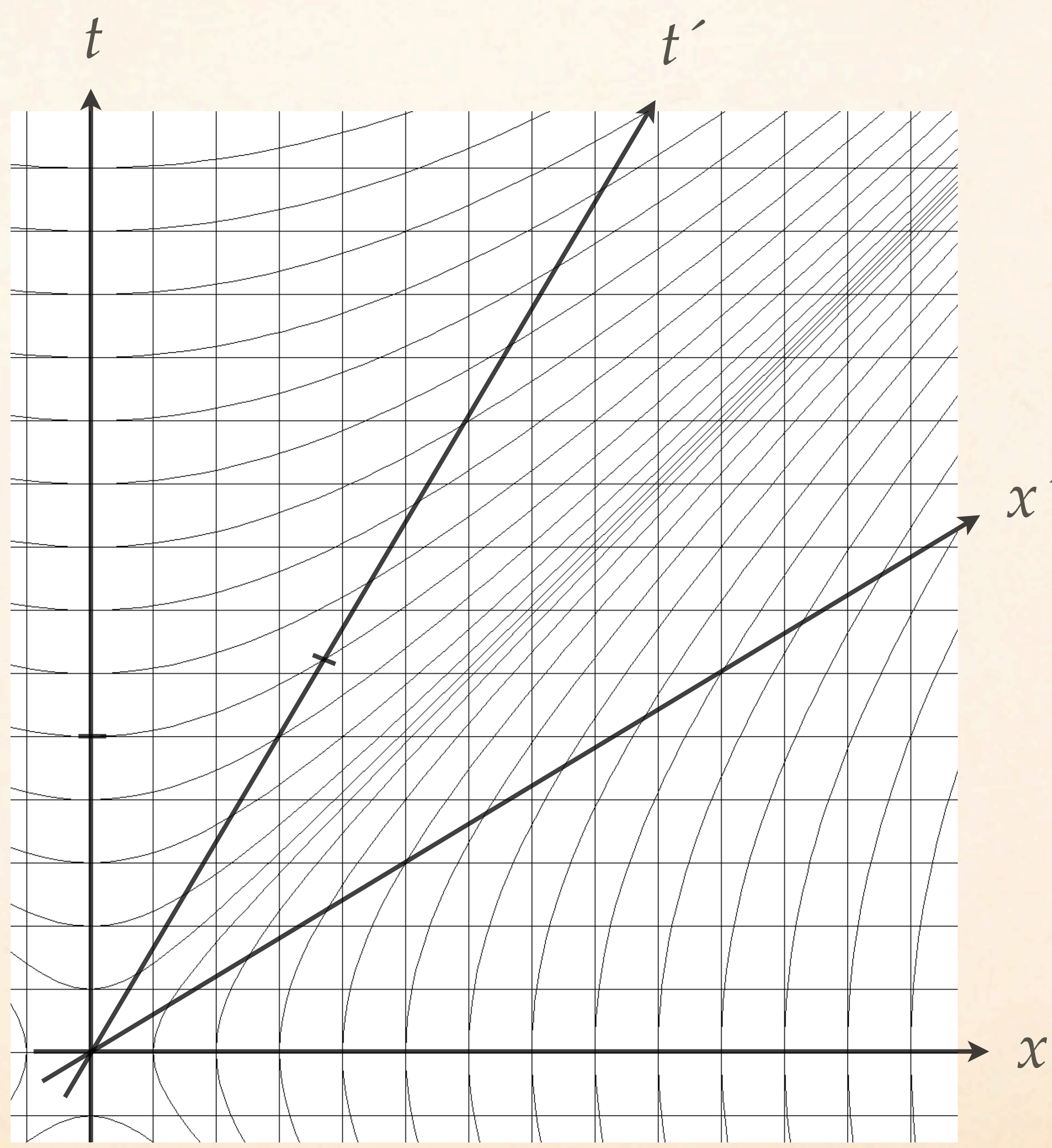
- To do more sophisticated calculations, we need to calibrate the axes of the primed frame
- In my time teaching SR, I have tried lots of methods
- My first approach was to derive a formula for the measured distance between marks ito beta (Click)
    - directly similar to how we'd calibrate rotated axes, but too abstract, not illuminating, tedious
- Next approach (*Six Ideas*) was to project the marks on the main axes (Click)
    - this distance is simply gamma, so better connection to LTEs, but still tedious
- Current approach: hyperbola graph paper (Click)
    - built on idea that axis marks have to be fixed spacetime interval from origin
    - emphasizes centrality of the metric equation
    - fast and easy (no calculations required)
    - Available online at the *Six Ideas* website.

CALIBRATING THE AXES



- To do more sophisticated calculations, we need to calibrate the axes of the primed frame
- In my time teaching SR, I have tried lots of methods
- My first approach was to derive a formula for the measured distance between marks ito beta (Click)
    - directly similar to how we'd calibrate rotated axes, but too abstract, not illuminating, tedious
- Next approach (*Six Ideas*) was to project the marks on the main axes (Click)
    - this distance is simply gamma, so better connection to LTEs, but still tedious
- Current approach: hyperbola graph paper (Click)
    - built on idea that axis marks have to be fixed spacetime interval from origin
    - emphasizes centrality of the metric equation
    - fast and easy (no calculations required)
    - Available online at the *Six Ideas* website.

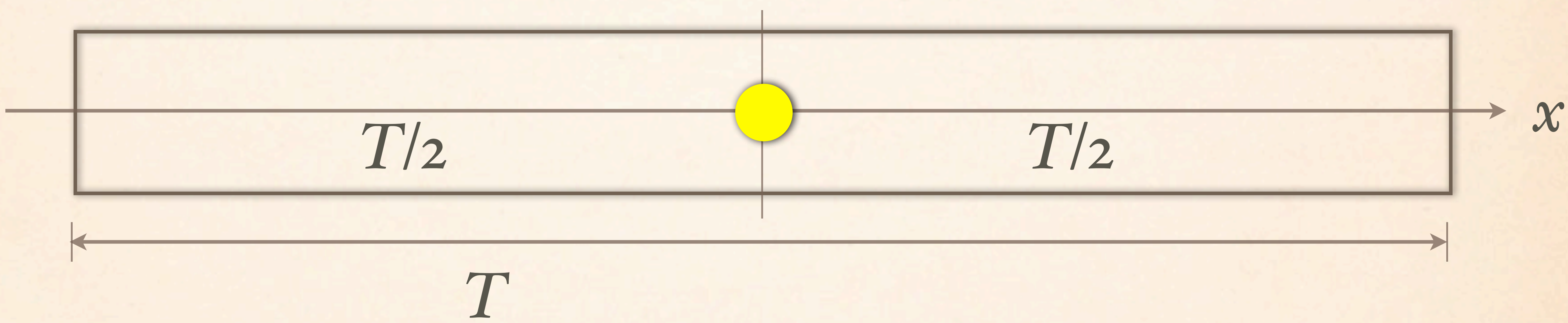
CALIBRATING THE AXES



- To do more sophisticated calculations, we need to calibrate the axes of the primed frame
- In my time teaching SR, I have tried lots of methods
- My first approach was to derive a formula for the measured distance between marks ito beta (Click)
    - directly similar to how we'd calibrate rotated axes, but too abstract, not illuminating, tedious
- Next approach (*Six Ideas*) was to project the marks on the main axes (Click)
    - this distance is simply gamma, so better connection to LTEs, but still tedious
- Current approach: hyperbola graph paper (Click)
    - built on idea that axis marks have to be fixed spacetime interval from origin
    - emphasizes centrality of the metric equation
    - fast and easy (no calculations required)
    - Available online at the *Six Ideas* website.

LIGHT CLOCK METHOD

(Scheme by Rob Salgado, Bowdoin College)

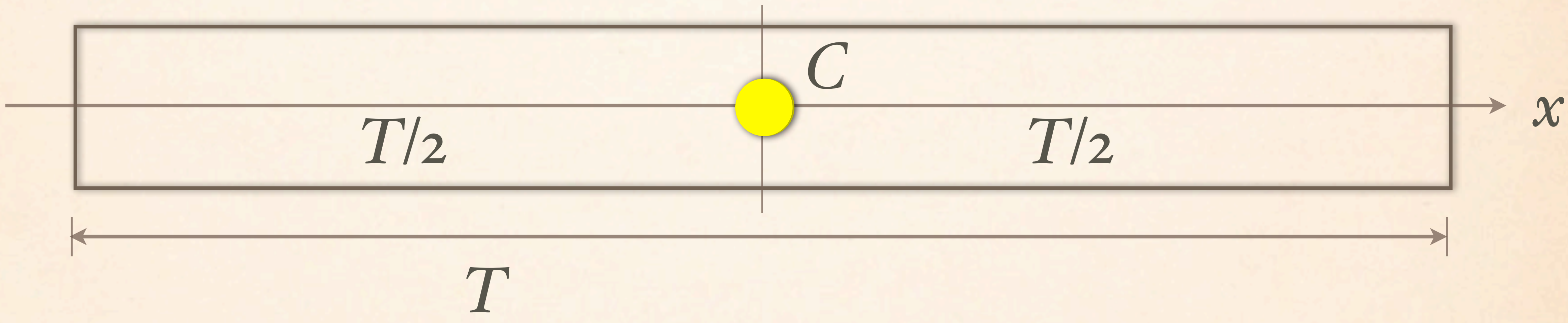


- Another very clever method developed by Rob Salgado, Boh-din College (private communication)
- Because it is an unusual approach, I am going to present it at some length
- Starts with a longitudinal light clock of length T sitting along the x axis.
- Two opposite-going light flashes are emitted at event E, reflect off the right and left ends at events R and L respectively, and cross again at event C. We can consider this one “tick” of the light clock (duration T).
- (Hit backarrow and space to repeat).

LIGHT CLOCK METHOD

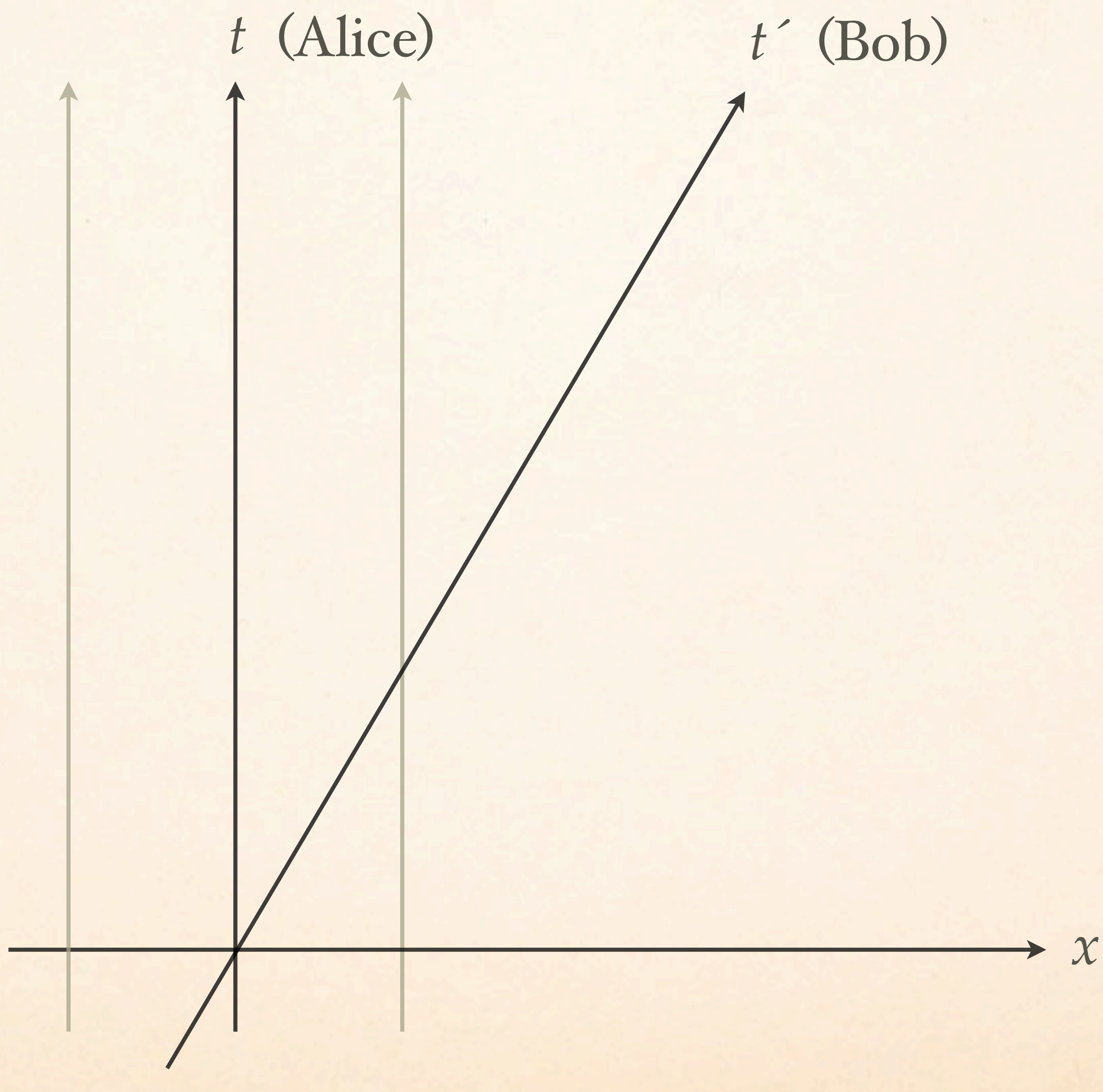
(Scheme by Rob Salgado, Bowdoin College)

Tick of duration T



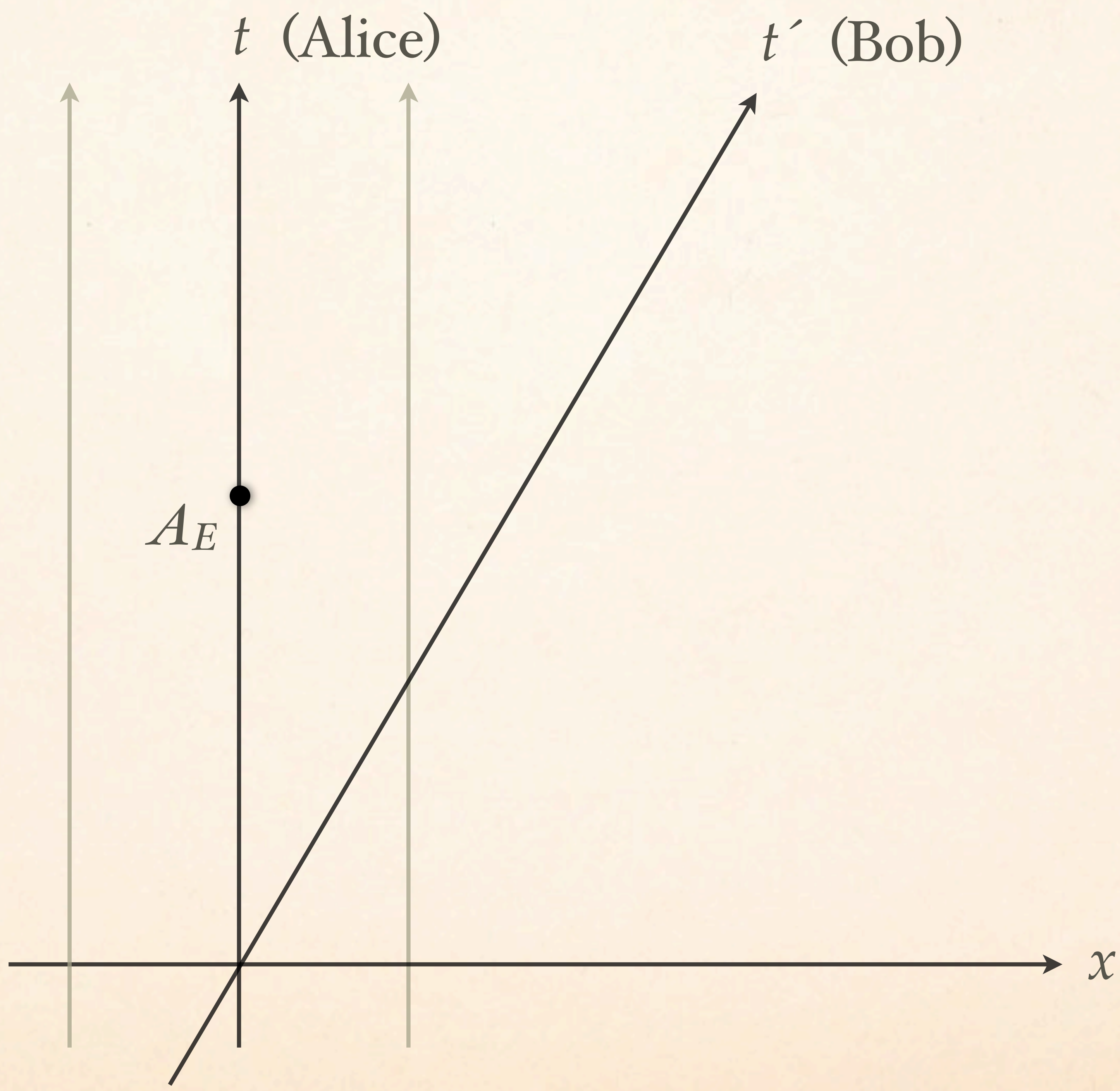
- Another very clever method developed by Rob Salgado, Boh-din College (private communication)
- Because it is an unusual approach, I am going to present it at some length
- Starts with a longitudinal light clock of length T sitting along the x axis.
- Two opposite-going light flashes are emitted at event E, reflect off the right and left ends at events R and L respectively, and cross again at event C. We can consider this one “tick” of the light clock (duration T).
- (Hit backarrow and space to repeat).

LIGHT CLOCK METHOD



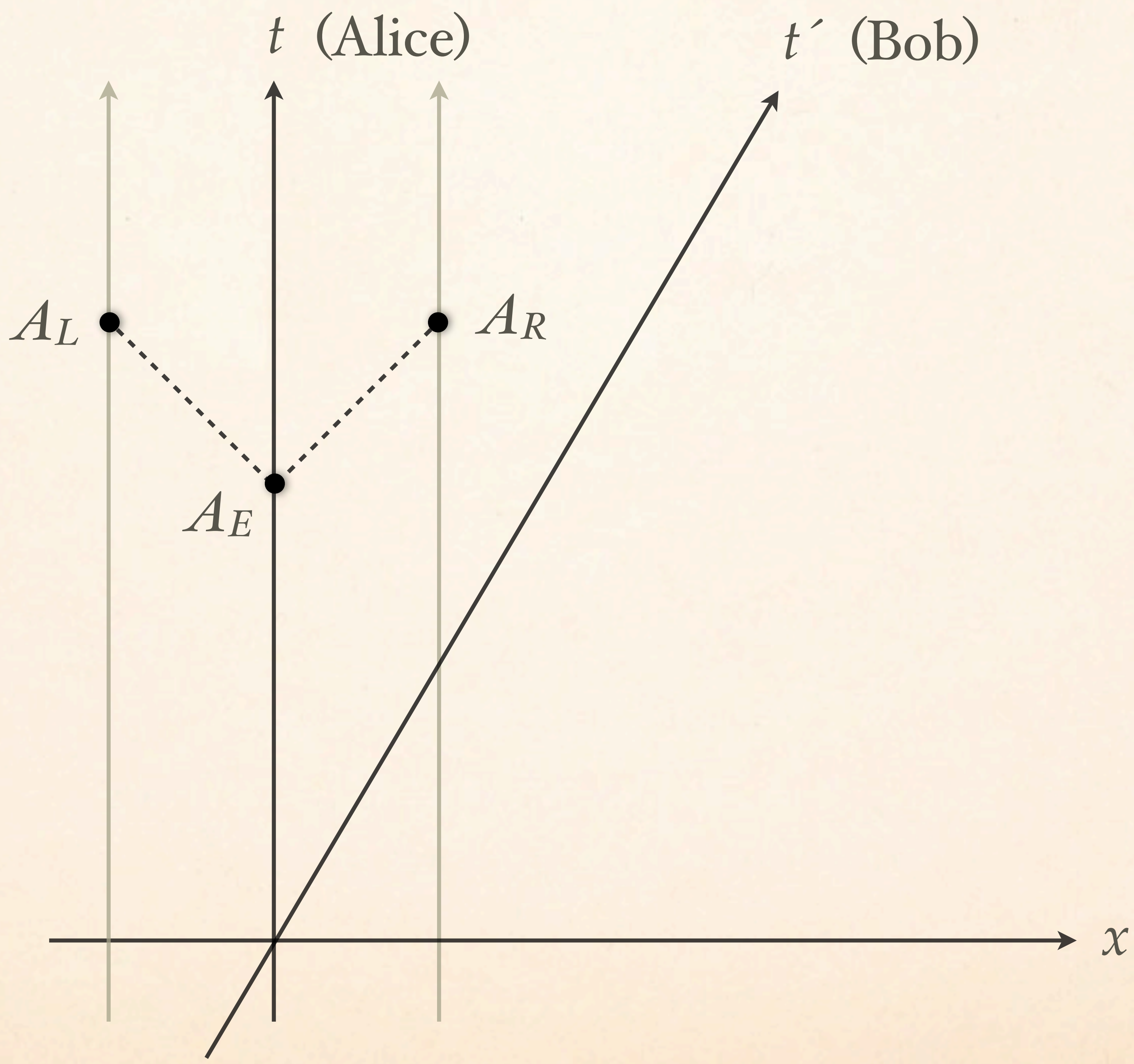
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



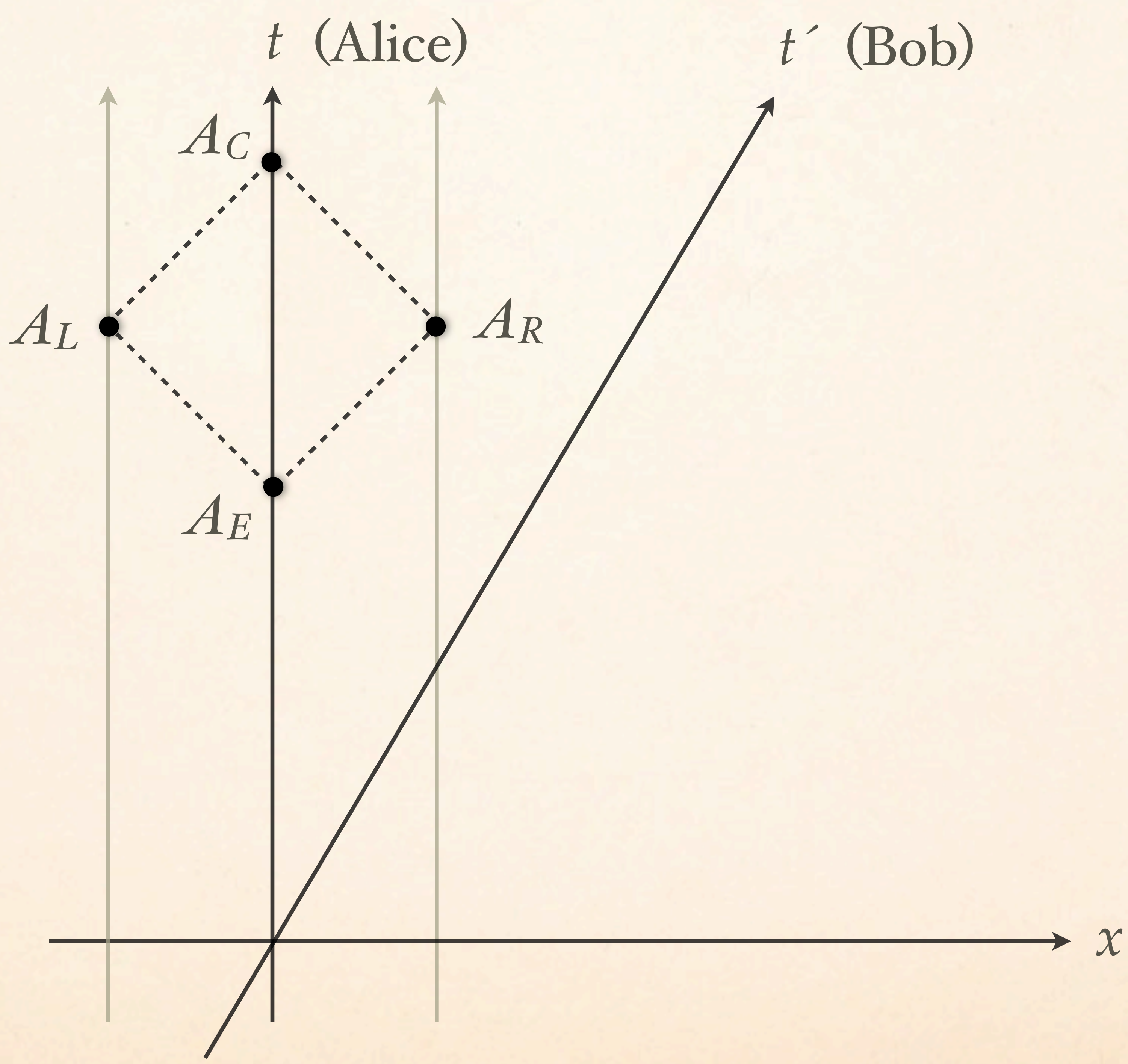
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



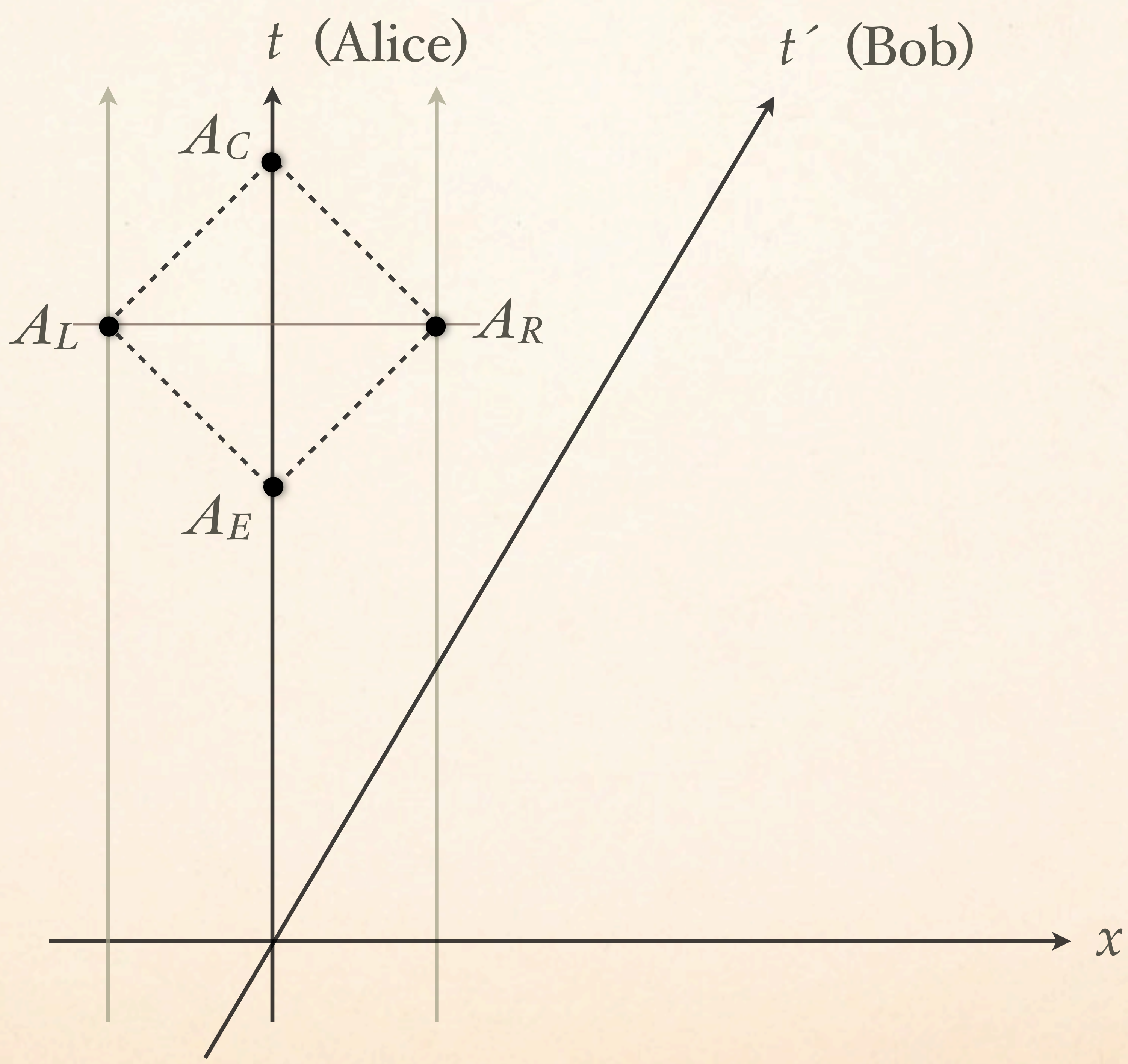
- Let’s see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice’s clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice’s frame (Ck) so define a line parallel to Alice’s x axis (Ck).
- This is what Salgado calls a “causal diamond” for Alice. Its size, both temporally and spatially, is determined by the light clock’s length, which we will call T.
- Now let’s see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob’s frame (Ck) so define a line that must be parallel to Bob’s x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob’s tick is also T. (Ck)
- How? Well, I bet you don’t know that Alice’s and Bob’s causal diamonds have the *same area*.

LIGHT CLOCK METHOD



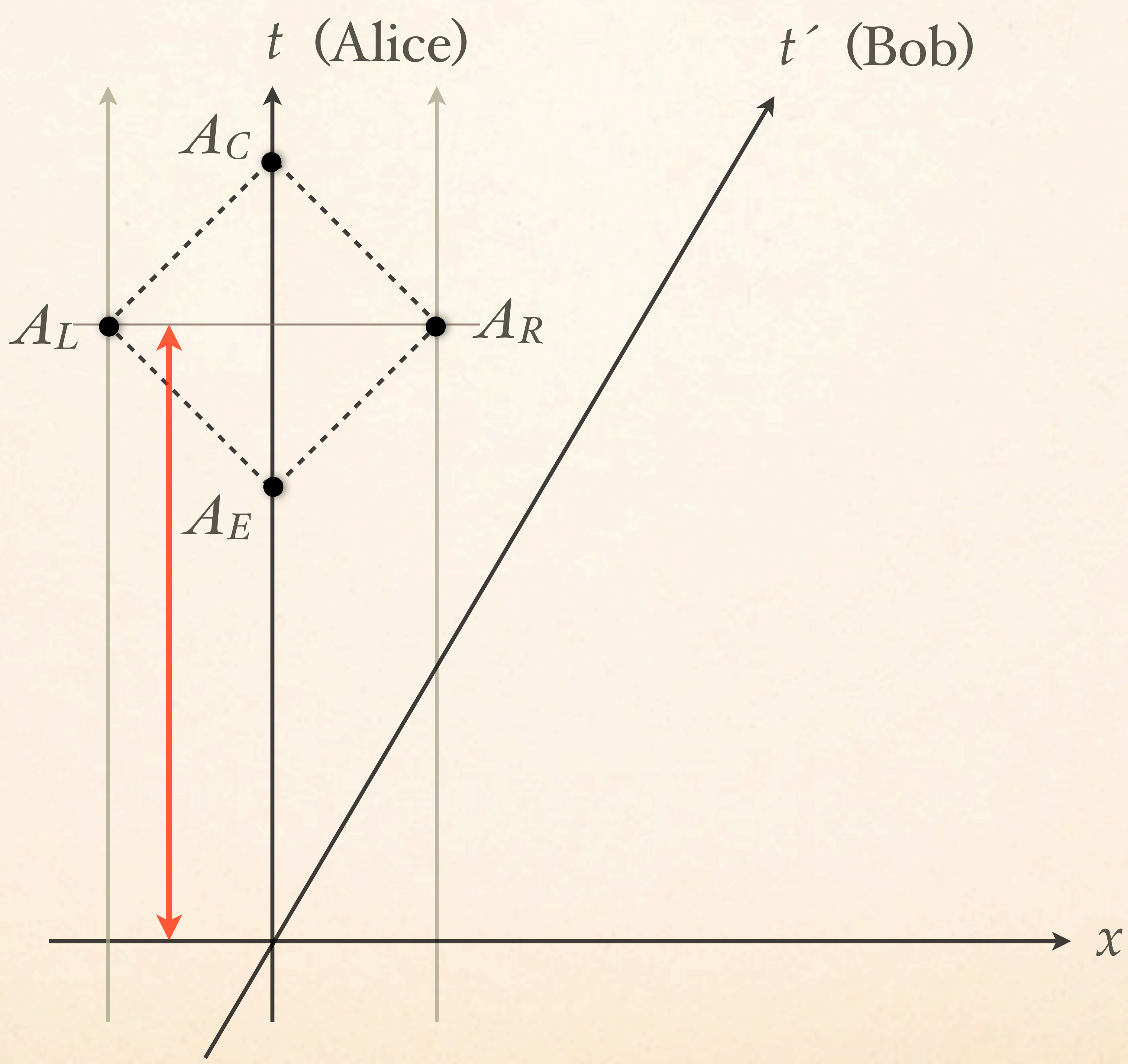
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



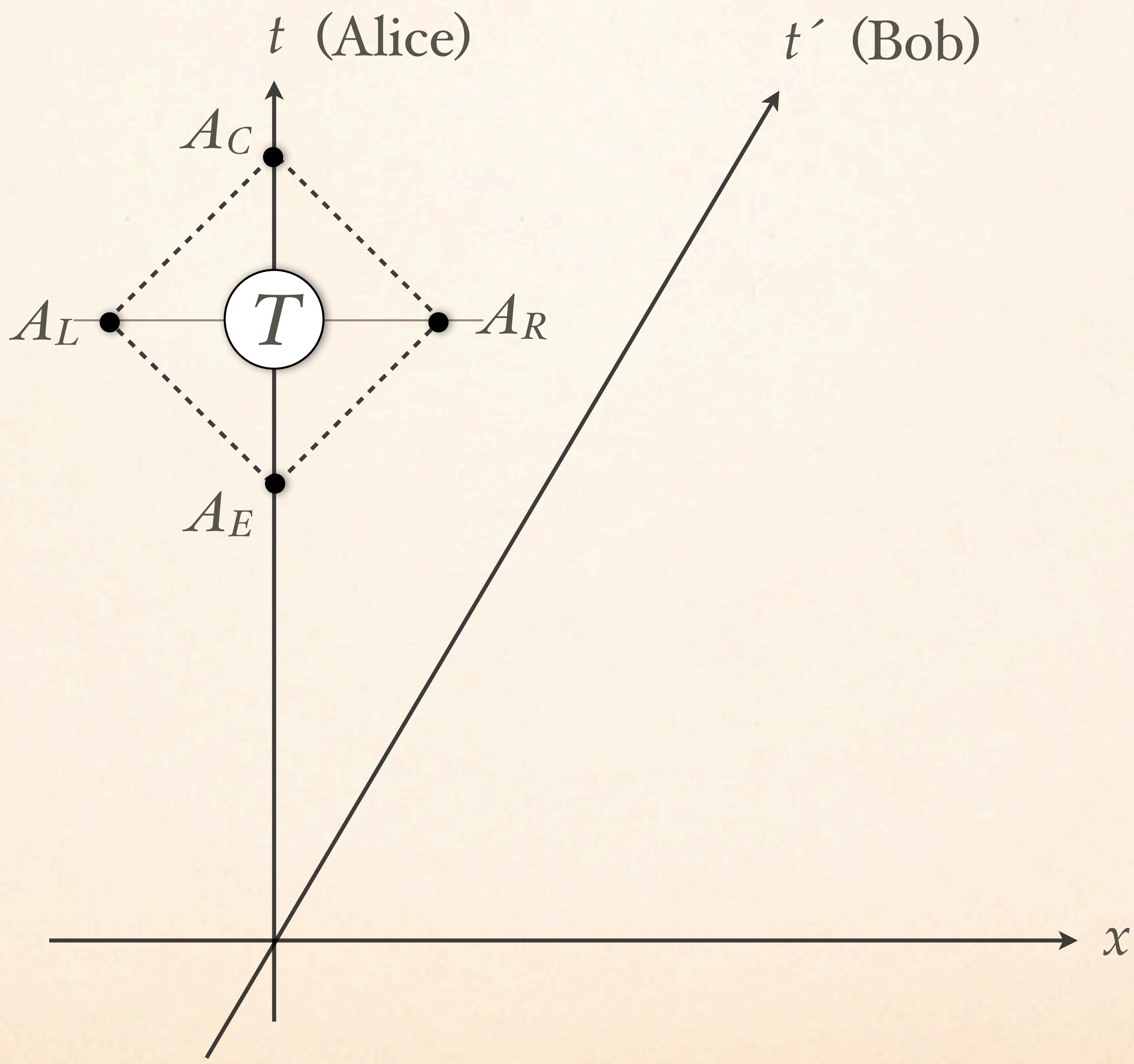
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



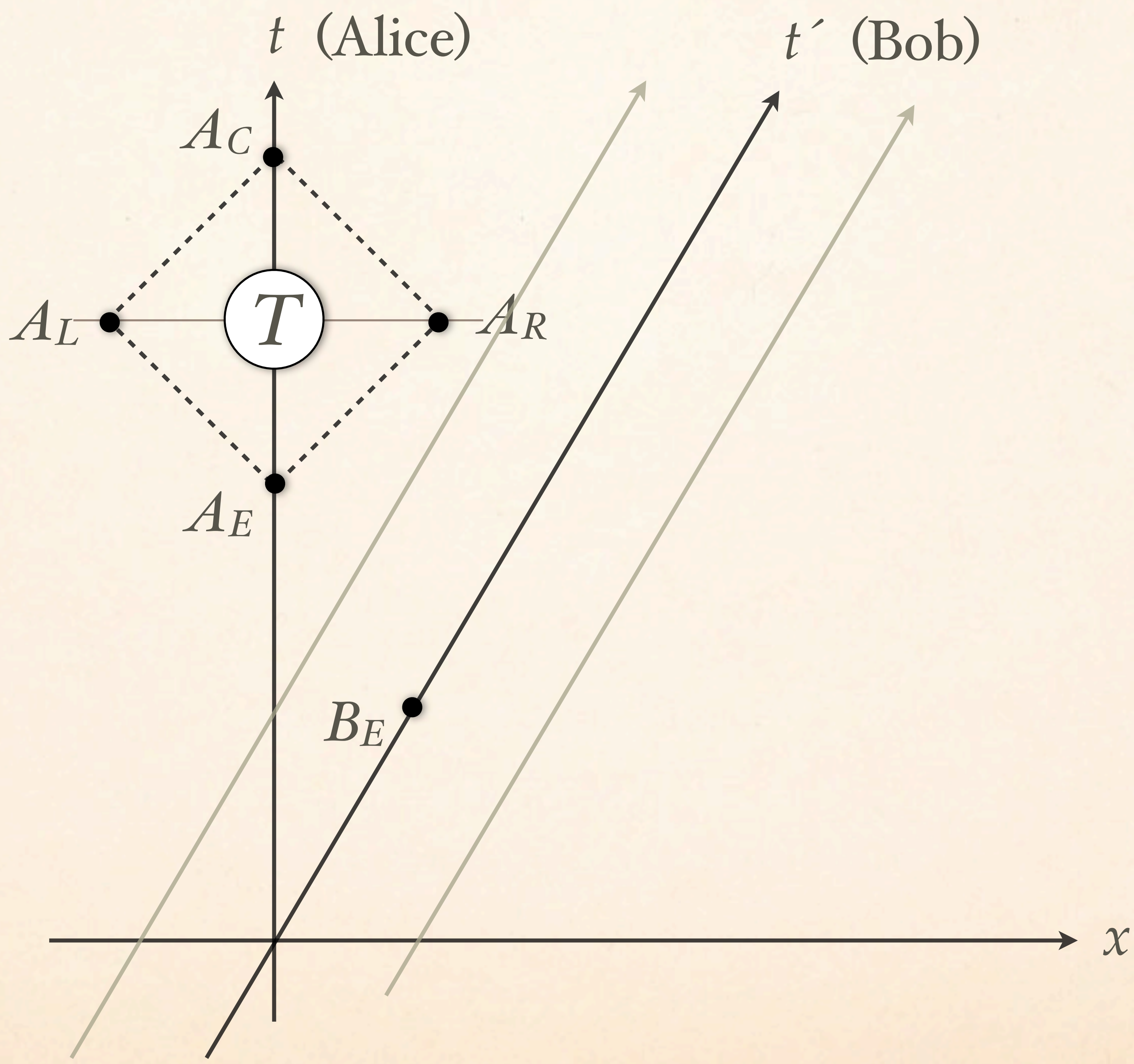
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



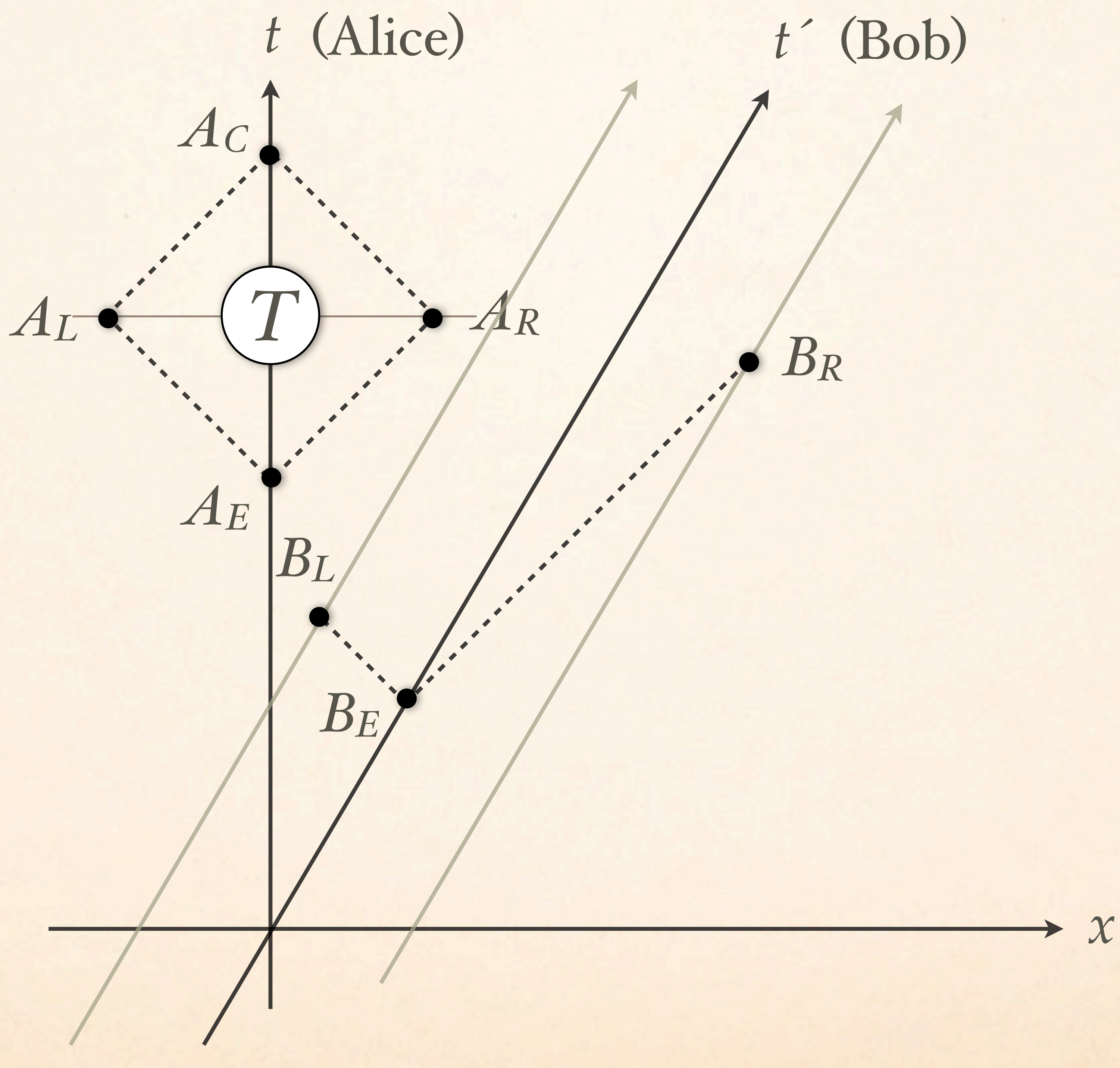
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's x' axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



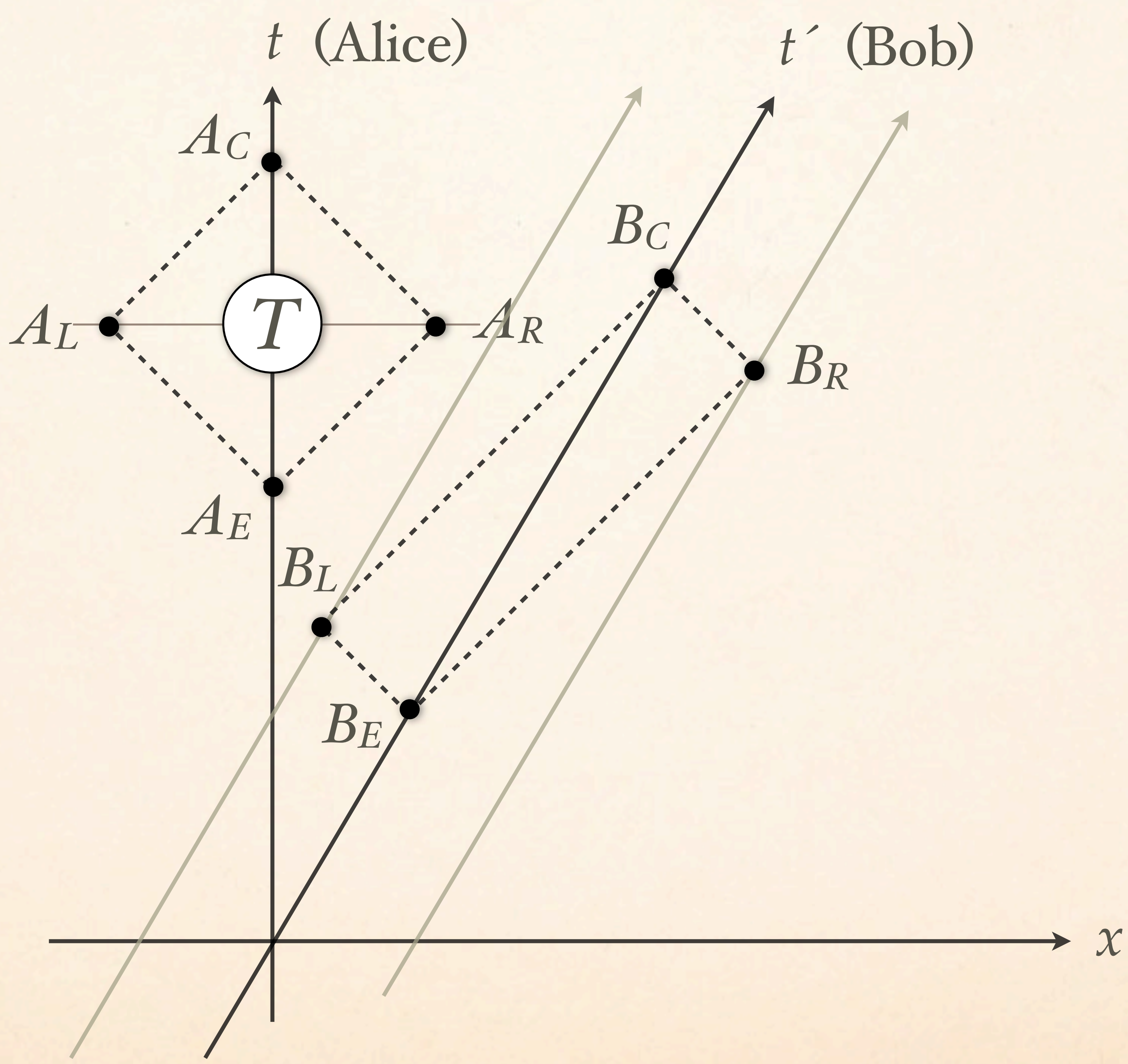
- Let’s see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice’s clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice’s frame (Ck) so define a line parallel to Alice’s x axis (Ck).
- This is what Salgado calls a “causal diamond” for Alice. Its size, both temporally and spatially, is determined by the light clock’s length, which we will call T.
- Now let’s see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob’s frame (Ck) so define a line that must be parallel to Bob’s $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob’s tick is also T. (Ck)
- How? Well, I bet you don’t know that Alice’s and Bob’s causal diamonds have the *same area*.

LIGHT CLOCK METHOD



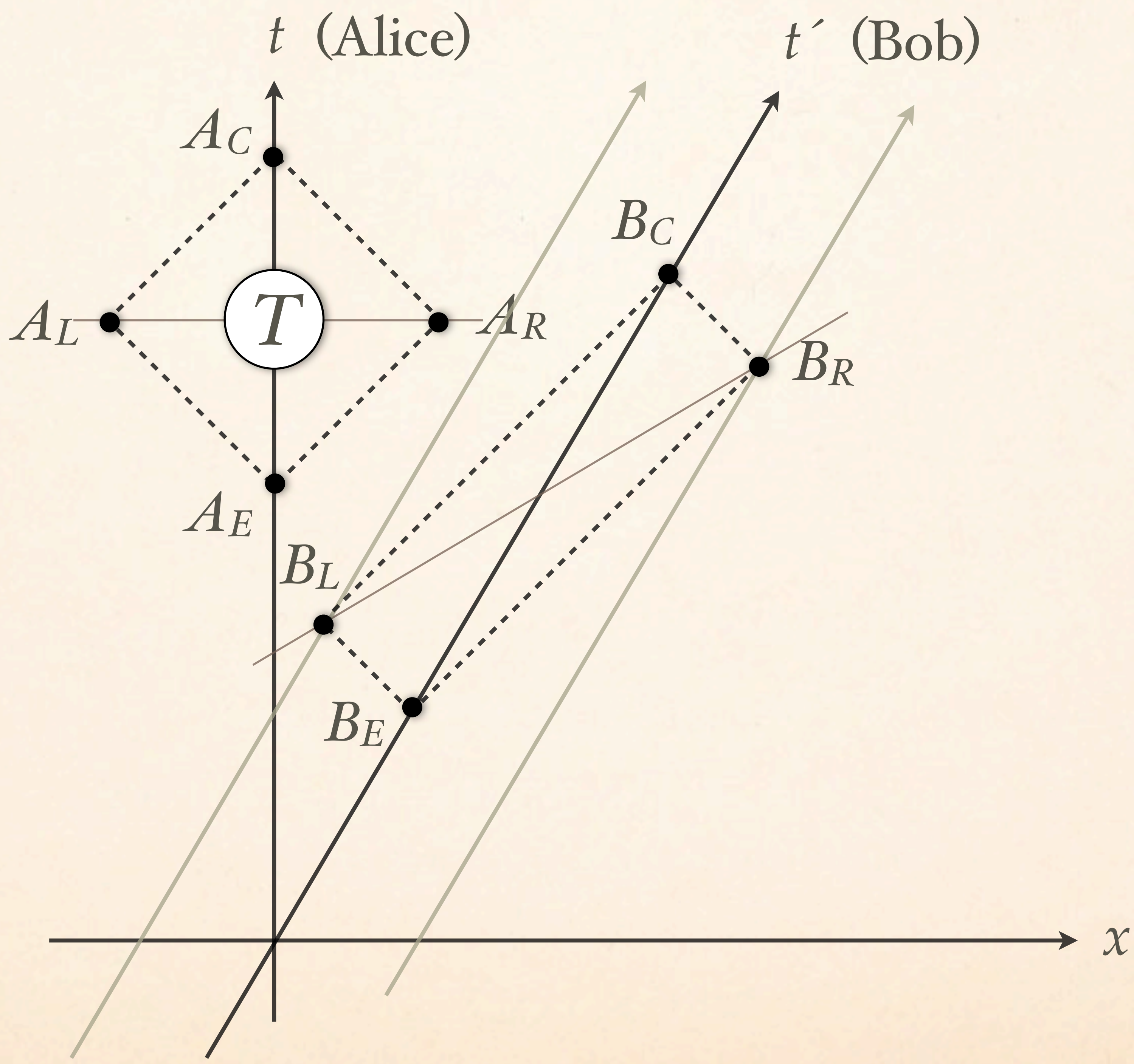
- Let’s see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice’s clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice’s frame (Ck) so define a line parallel to Alice’s x axis (Ck).
- This is what Salgado calls a “causal diamond” for Alice. Its size, both temporally and spatially, is determined by the light clock’s length, which we will call T.
- Now let’s see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob’s frame (Ck) so define a line that must be parallel to Bob’s $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob’s tick is also T. (Ck)
- How? Well, I bet you don’t know that Alice’s and Bob’s causal diamonds have the *same area*.

LIGHT CLOCK METHOD



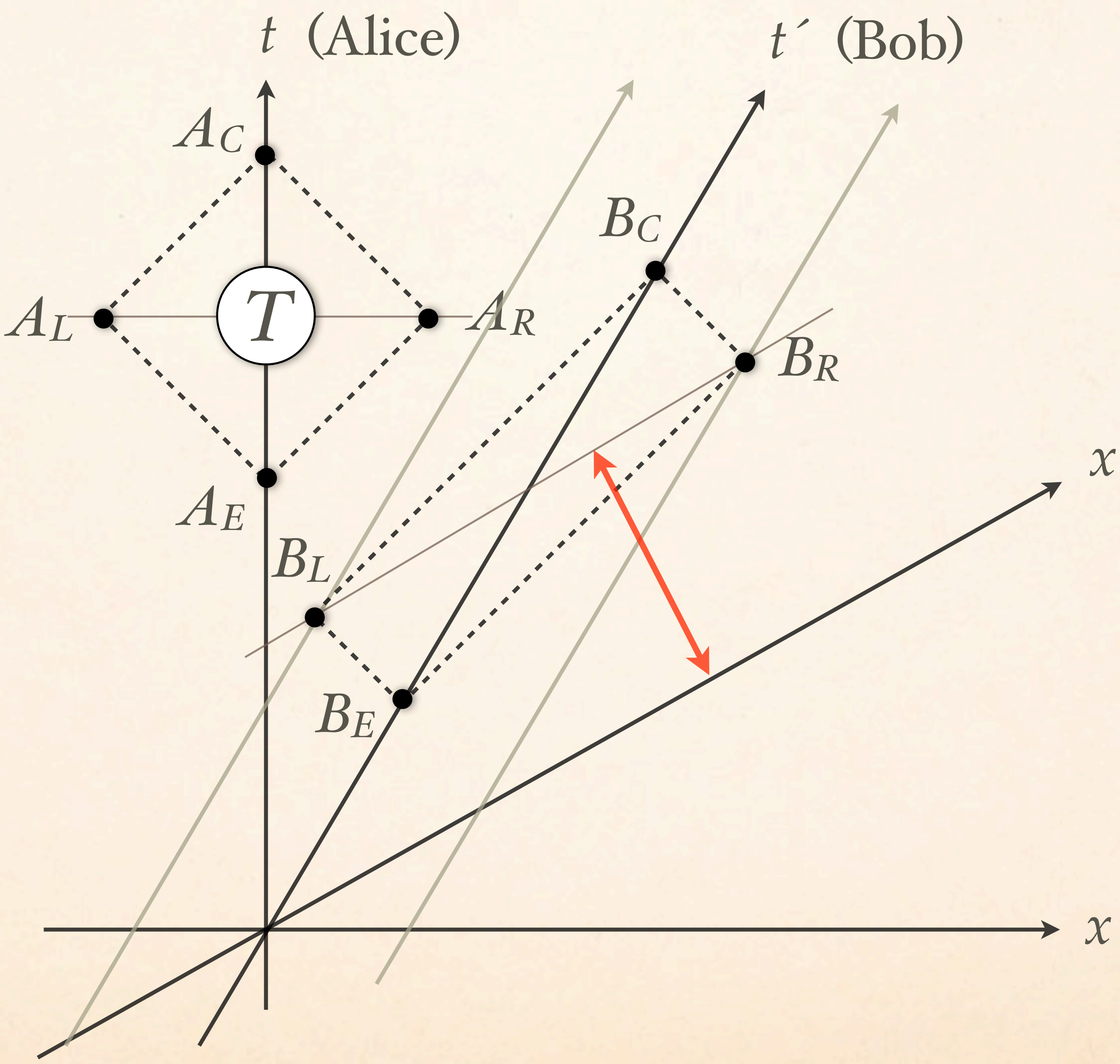
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



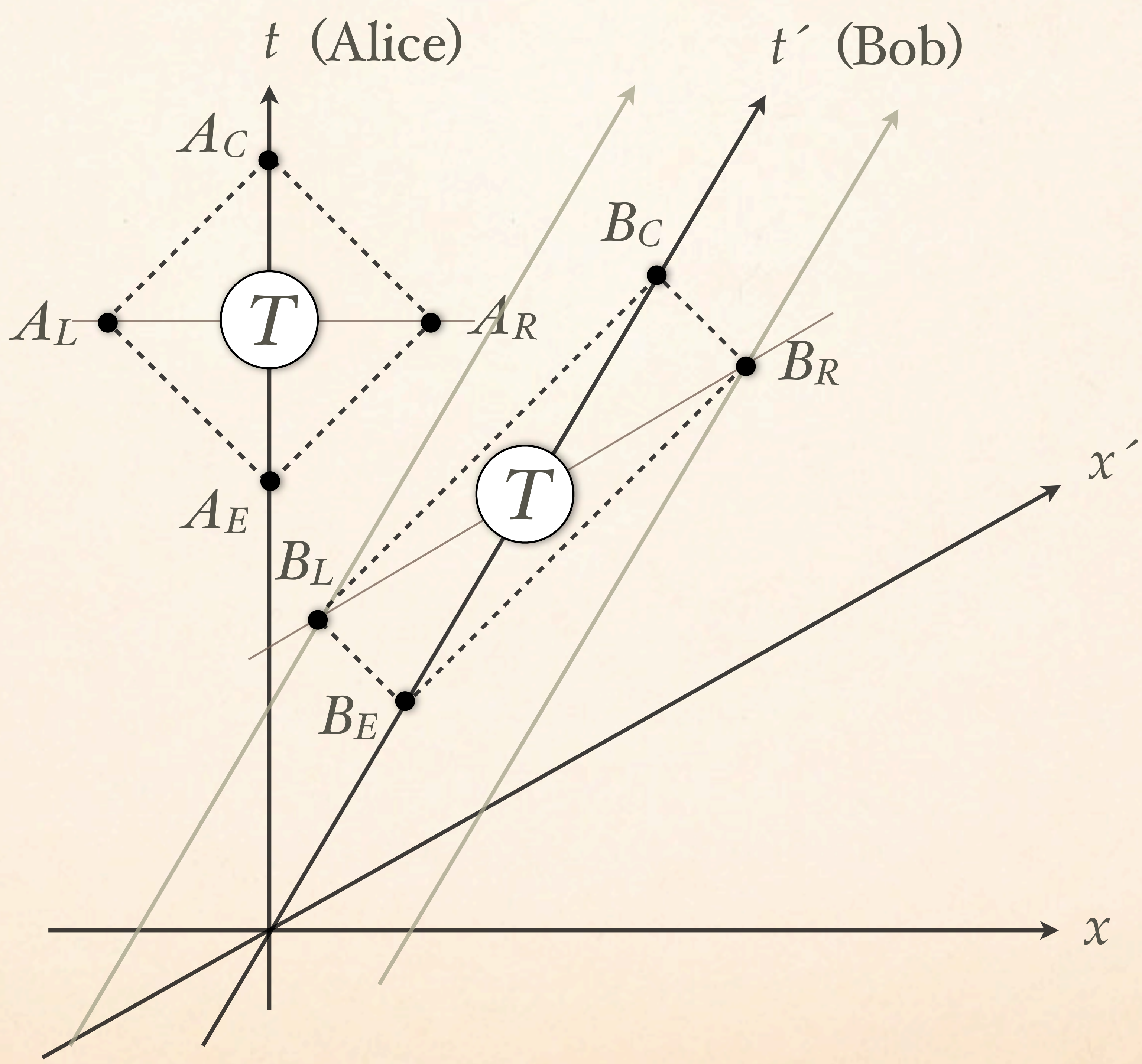
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

LIGHT CLOCK METHOD



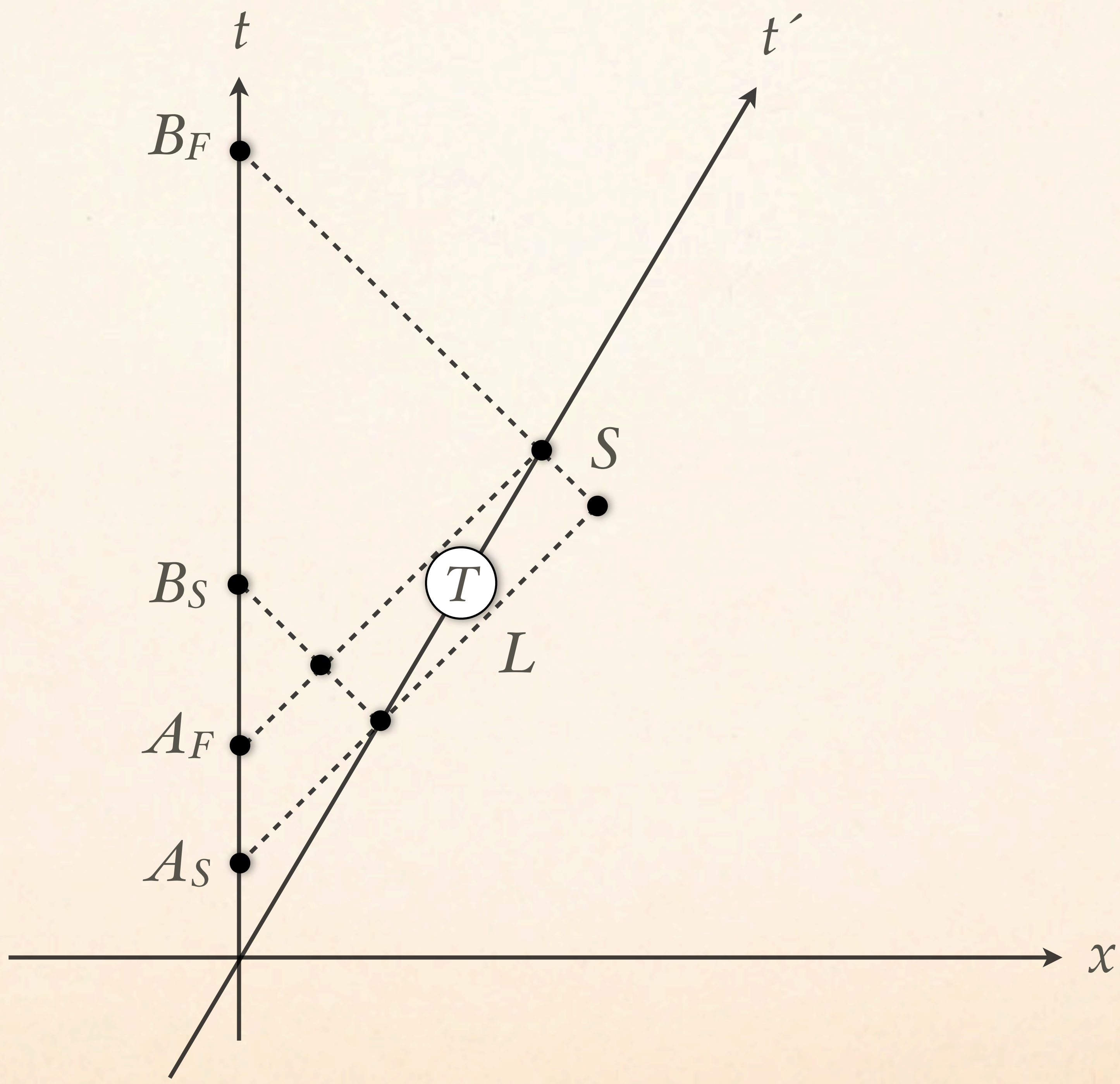
- Let’s see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice’s clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice’s frame (Ck) so define a line parallel to Alice’s x axis (Ck).
- This is what Salgado calls a “causal diamond” for Alice. Its size, both temporally and spatially, is determined by the light clock’s length, which we will call T.
- Now let’s see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob’s frame (Ck) so define a line that must be parallel to Bob’s $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob’s tick is also T. (Ck)
- How? Well, I bet you don’t know that Alice’s and Bob’s causal diamonds have the *same area*.

LIGHT CLOCK METHOD



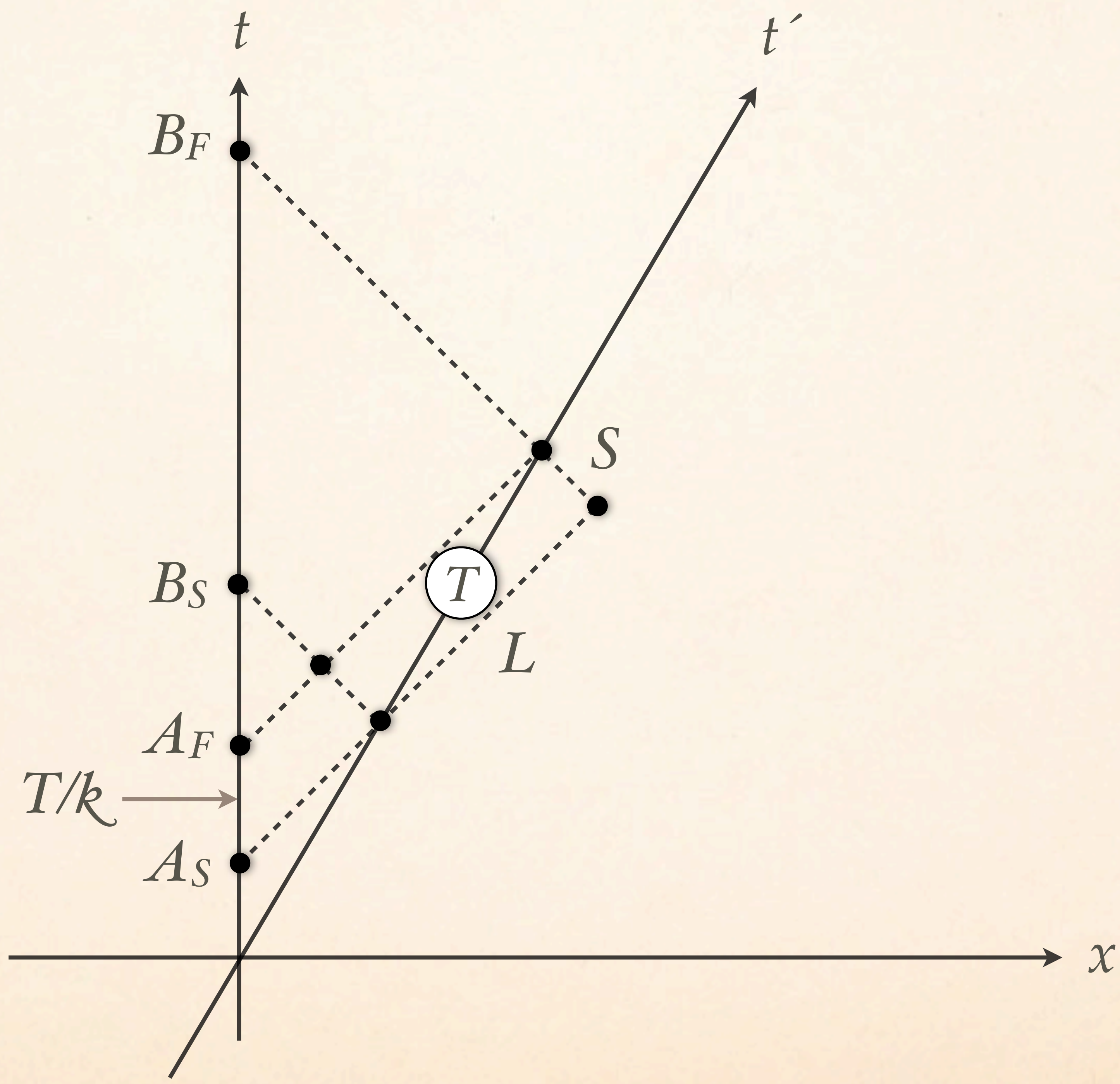
- Let's see how this looks on a ST diagram showing worldlines of two observers, Alice and Bob.
- The lighter lines are the worldlines of the mirrors at the ends of Alice's clock
- Emission event A_E happens (Ck), flashes travels to the mirrors (Ck) and return at event A_C (Ck).
- Note events A_R and A_L are simultaneous in Alice's frame (Ck) so define a line parallel to Alice's x axis (Ck).
- This is what Salgado calls a "causal diamond" for Alice. Its size, both temporally and spatially, is determined by the light clock's length, which we will call T.
- Now let's see what this diamond looks like for a Bob. (Ck) Again, the light lines are the worldlines of the clock ends and we have emission at event BE (Ck), reflection (Ck) and crossing (Ck).
- B_R and B_L are simultaneous in Bob's frame (Ck) so define a line that must be parallel to Bob's $x´$ axis (Ck).
- Scale determined by separation of mirror WLs. For calib, find right sep so that Bob's tick is also T. (Ck)
- How? Well, I bet you don't know that Alice's and Bob's causal diamonds have the *same area*.

DIAMOND AREAS



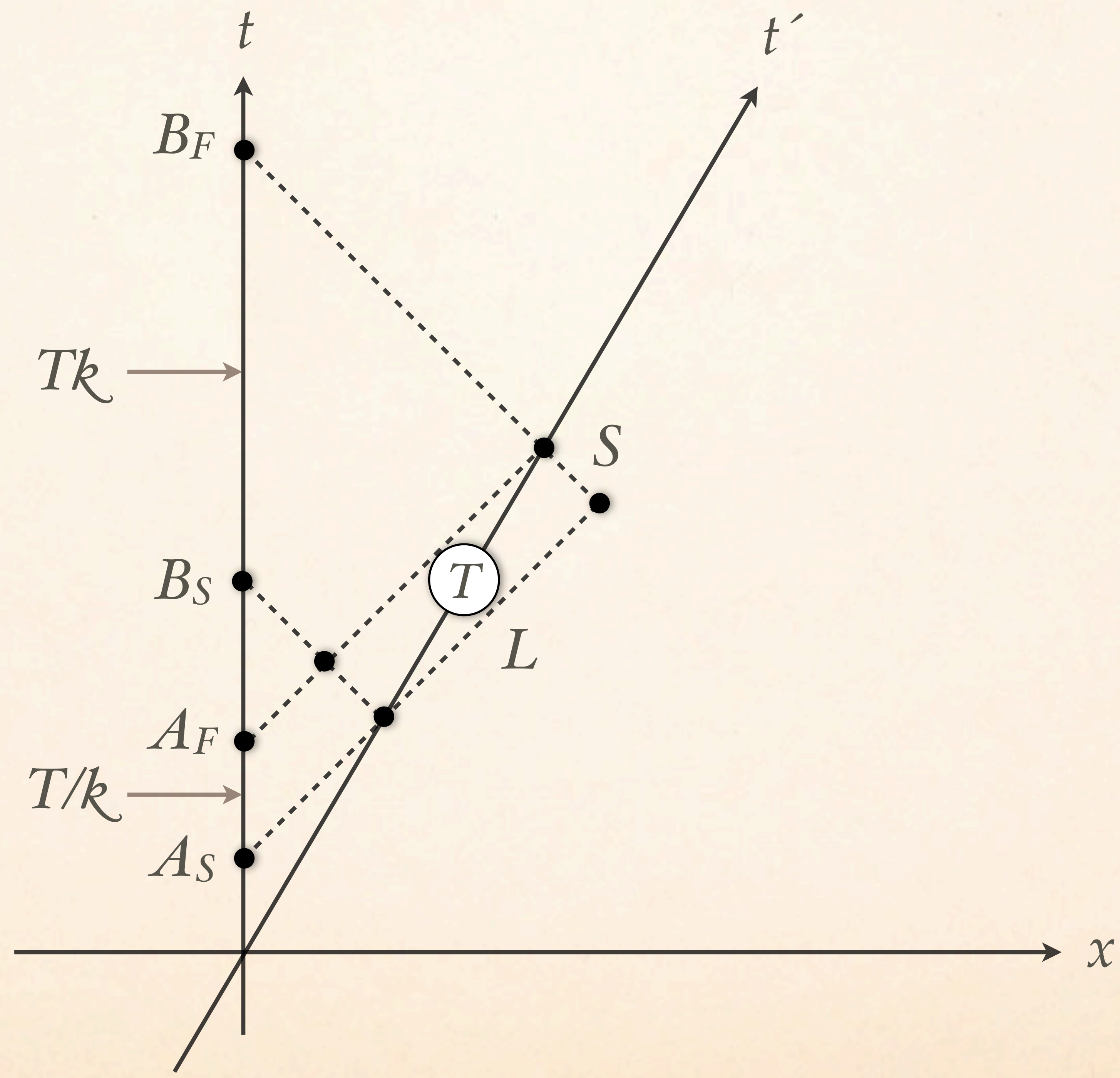
- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS



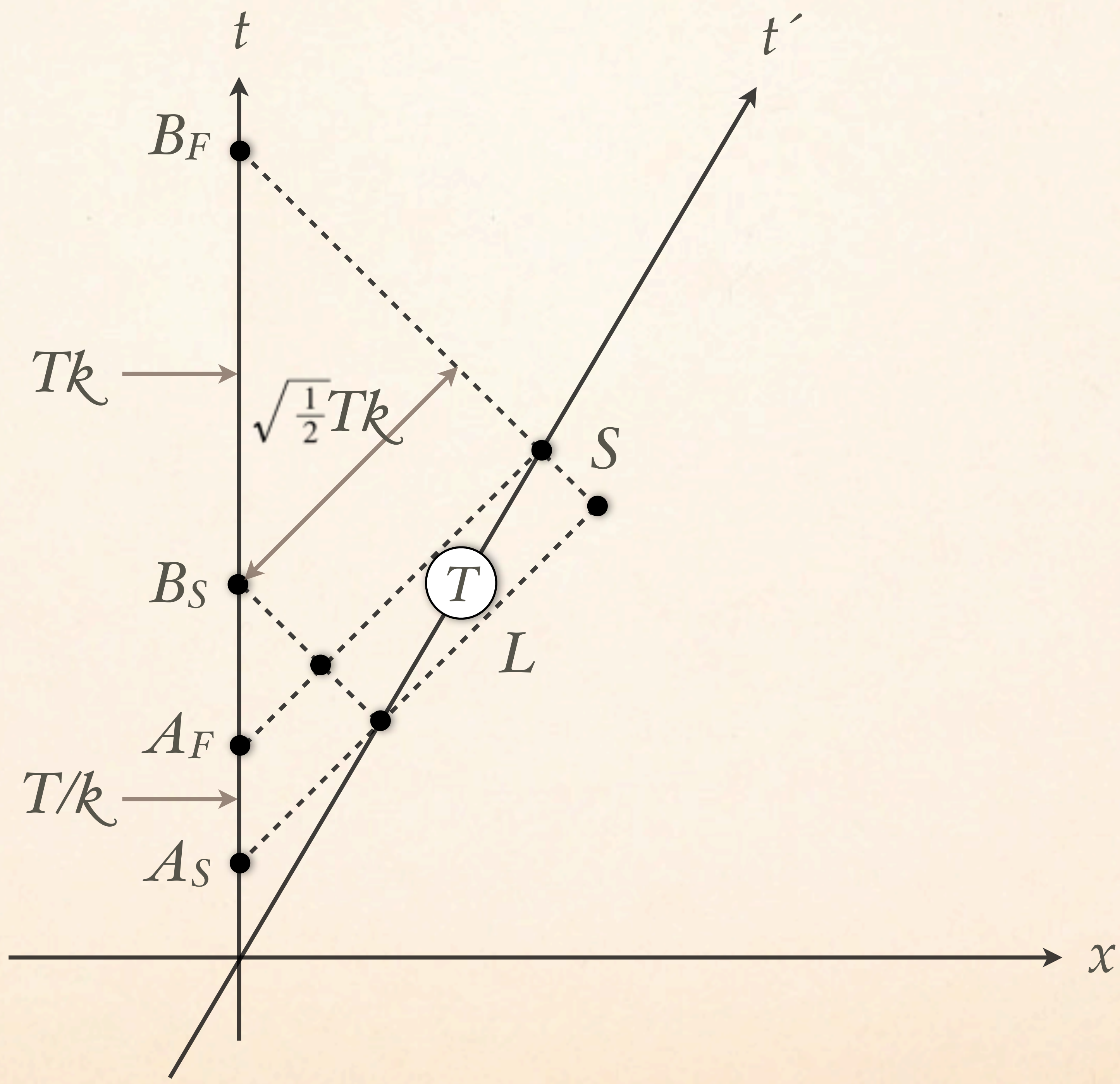
- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS



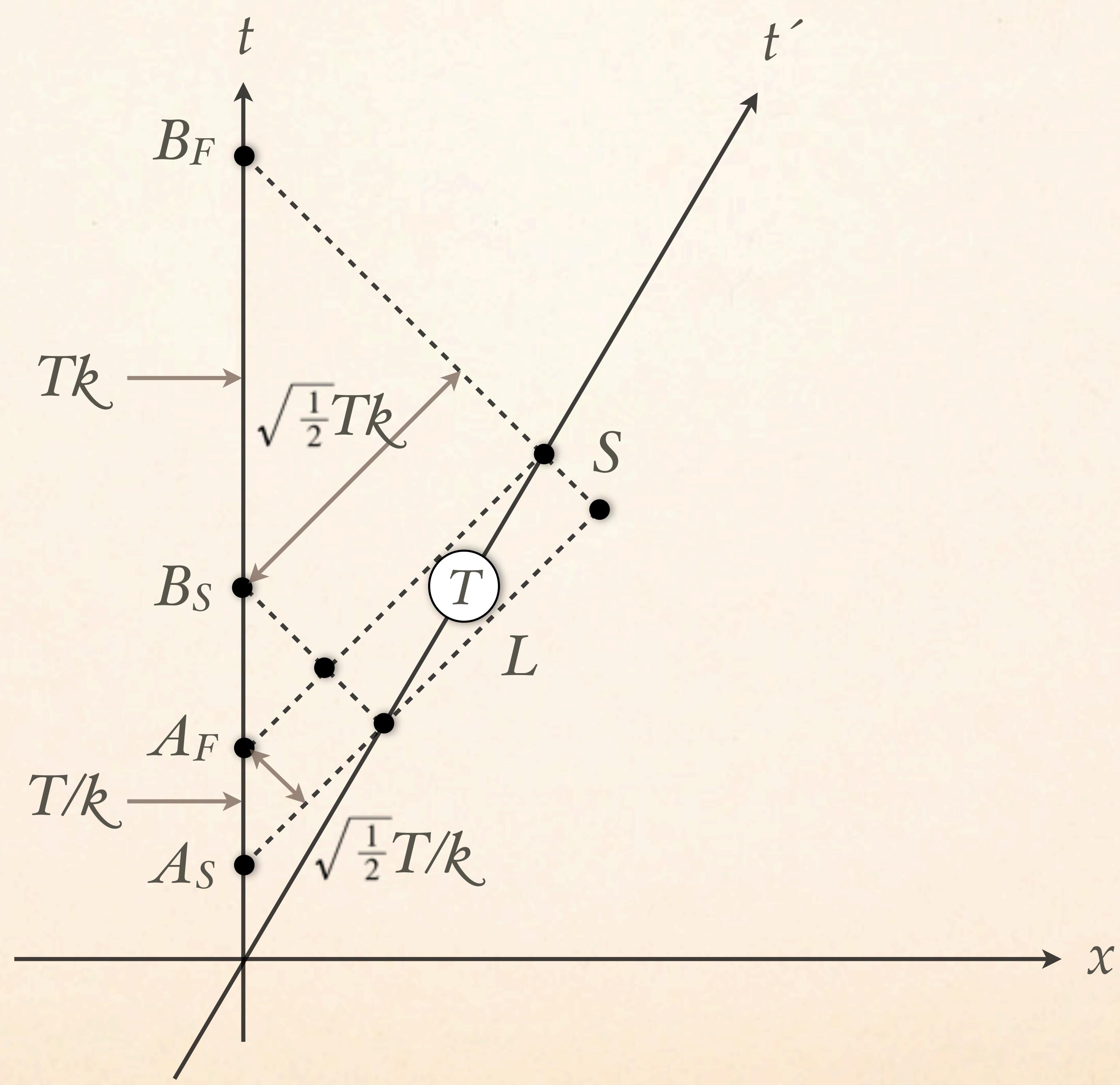
- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS



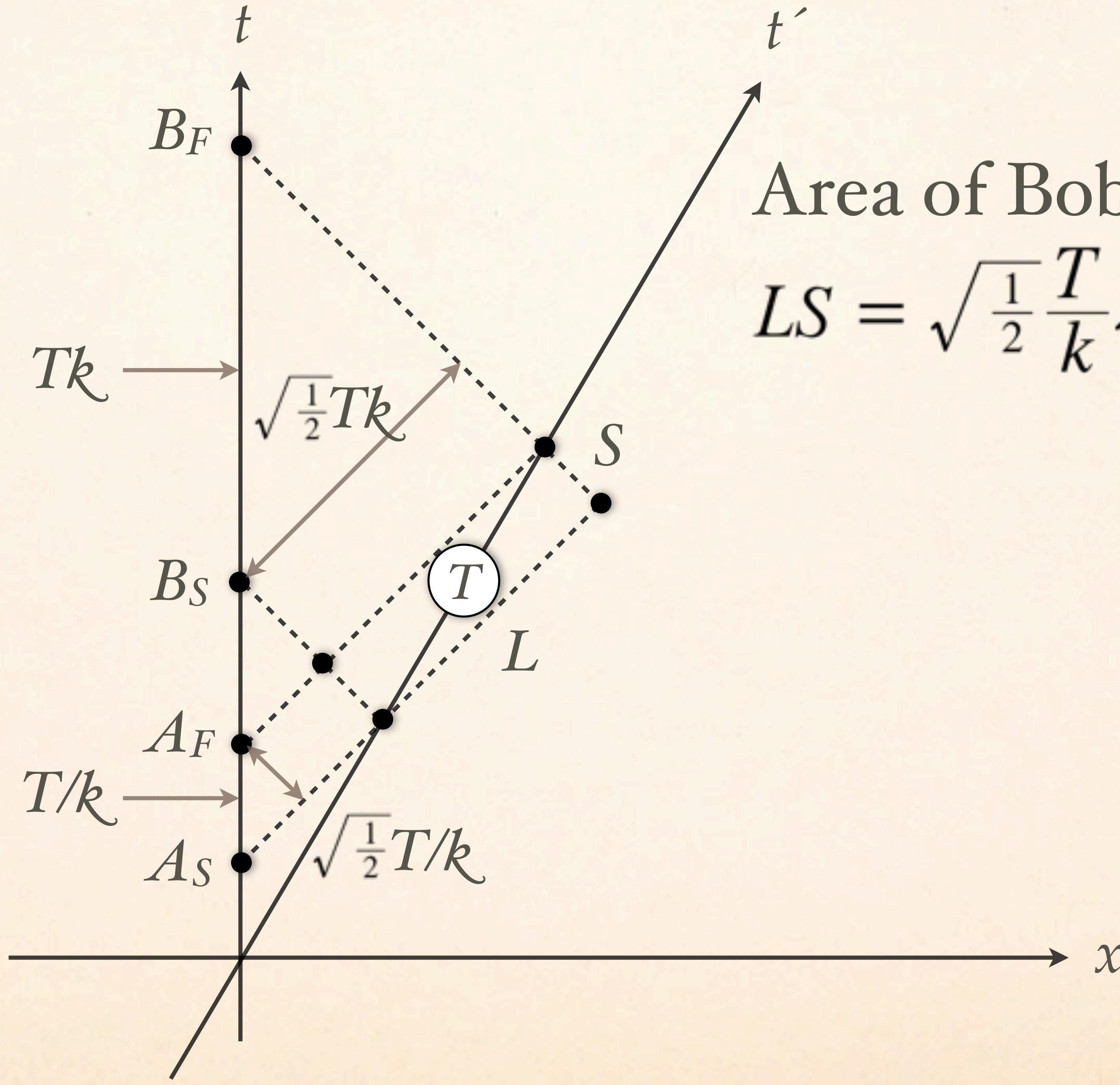
- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS



- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS

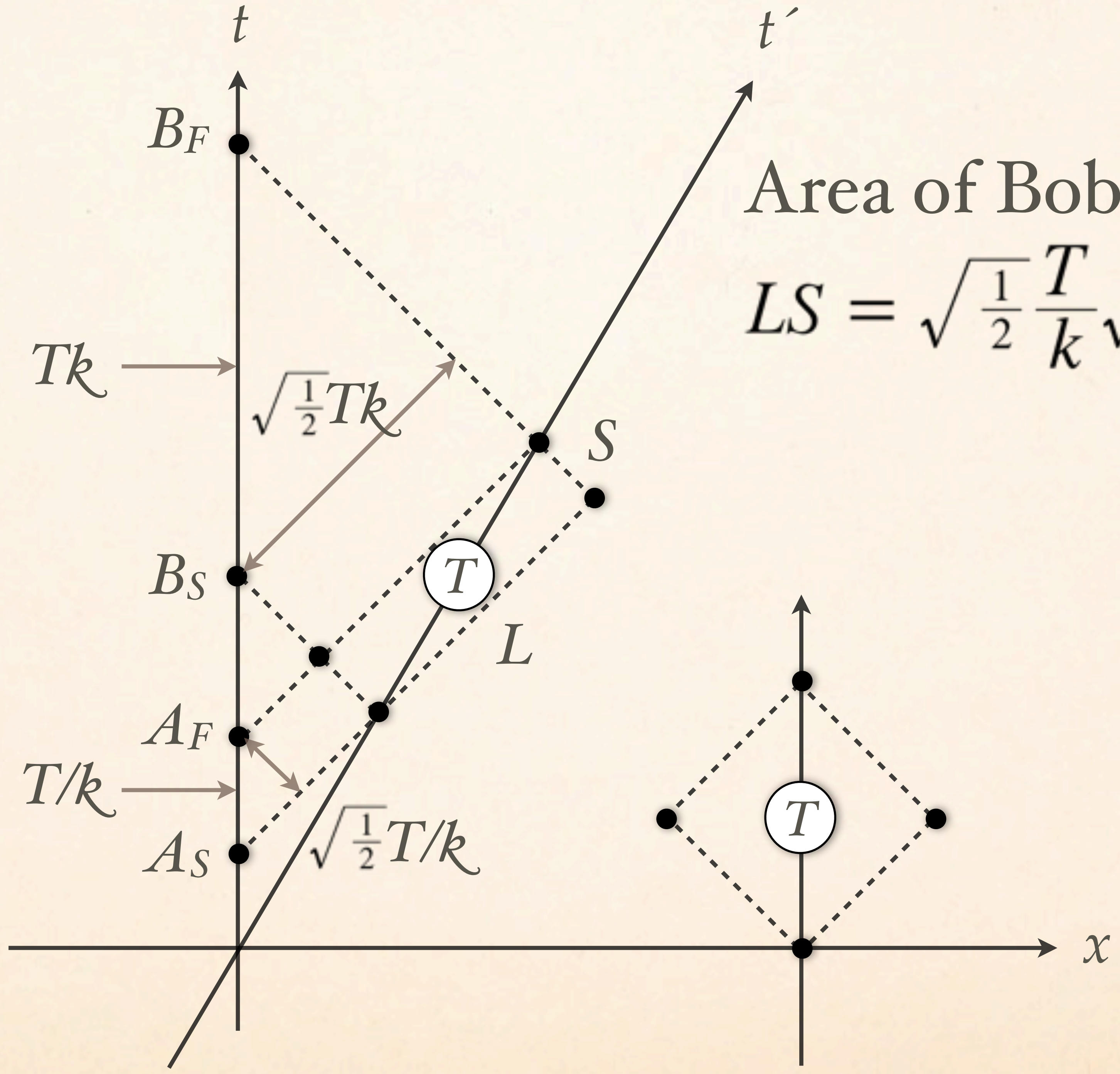


Area of Bob's diamond:

$$LS = \sqrt{\tfrac{1}{2}}\frac{T}{k}\sqrt{\tfrac{1}{2}}Tk = \tfrac{1}{2}T^2$$

- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS

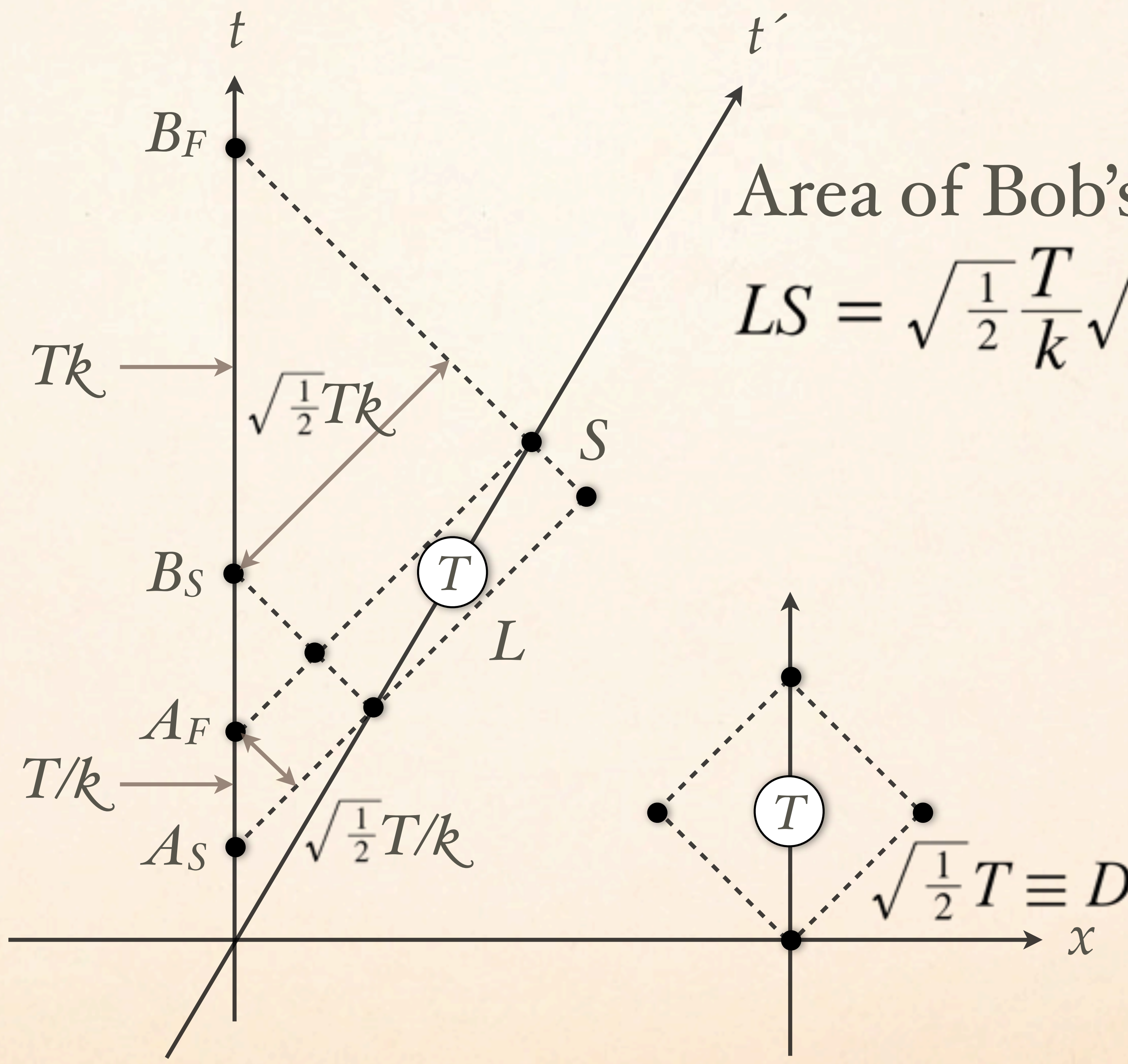


Area of Bob's diamond:

$$LS = \sqrt{\tfrac{1}{2}}\frac{T}{k}\sqrt{\tfrac{1}{2}}Tk = \tfrac{1}{2}T^2$$

- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS

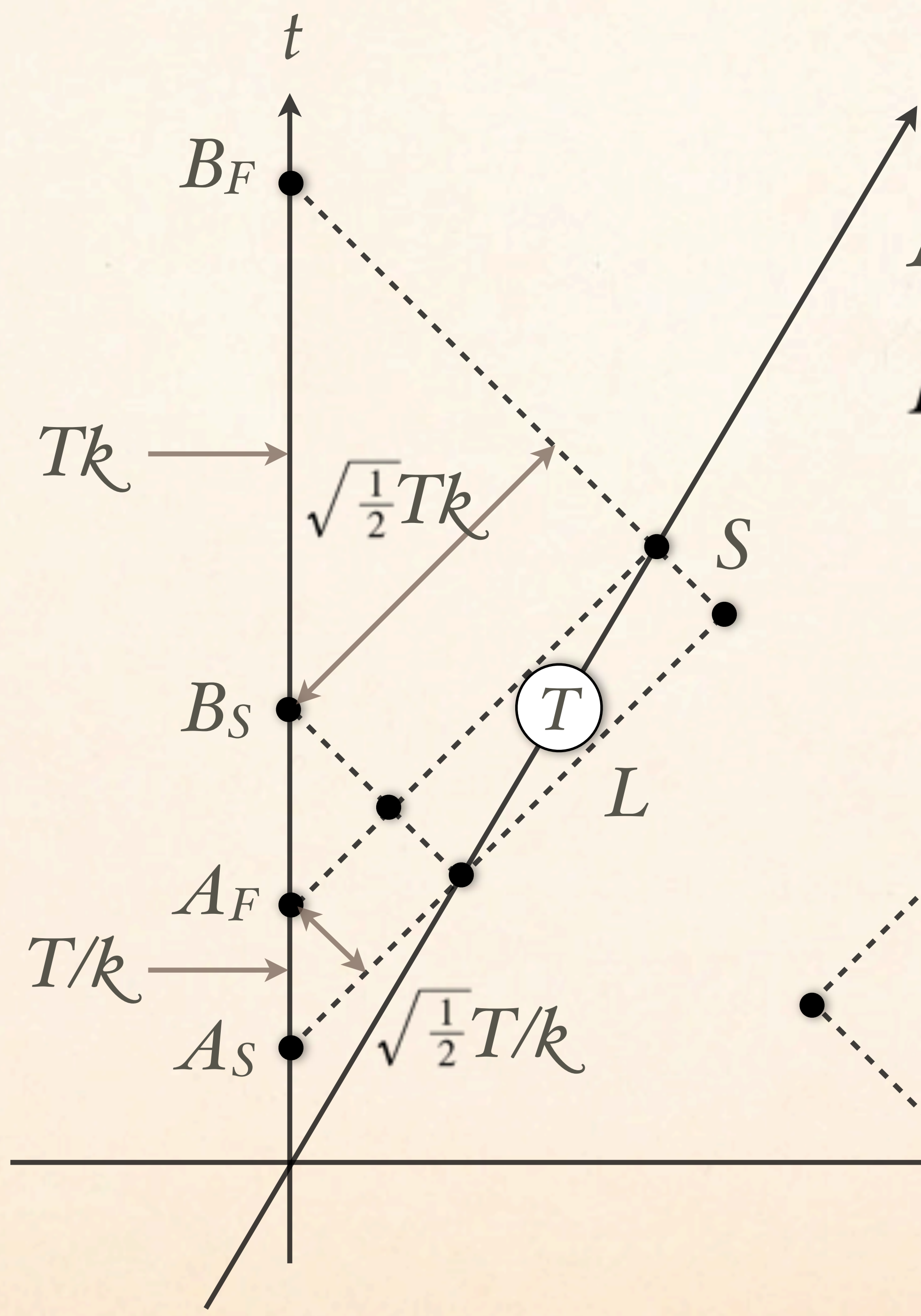


Area of Bob's diamond:

$$LS = \sqrt{\tfrac{1}{2}}\frac{T}{k}\sqrt{\tfrac{1}{2}}Tk = \tfrac{1}{2}T^2$$

- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

DIAMOND AREAS

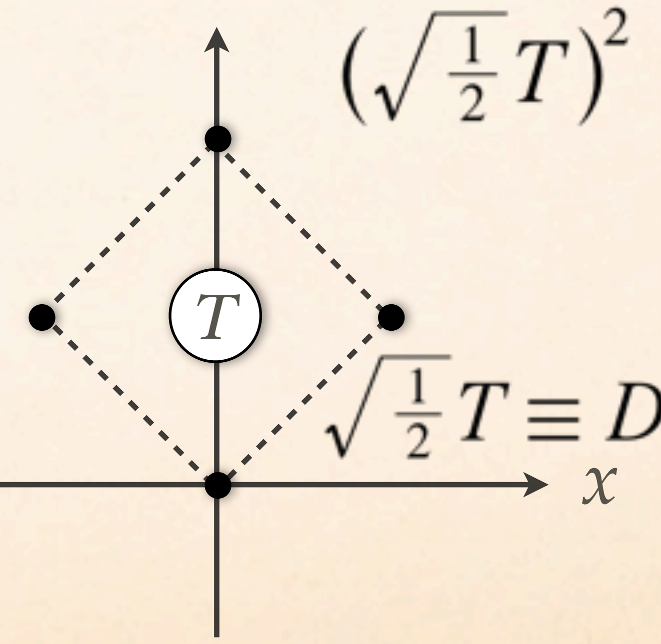


Area of Bob's diamond:

$$LS = \sqrt{\tfrac{1}{2}}\frac{T}{k}\sqrt{\tfrac{1}{2}}Tk = \tfrac{1}{2}T^2$$

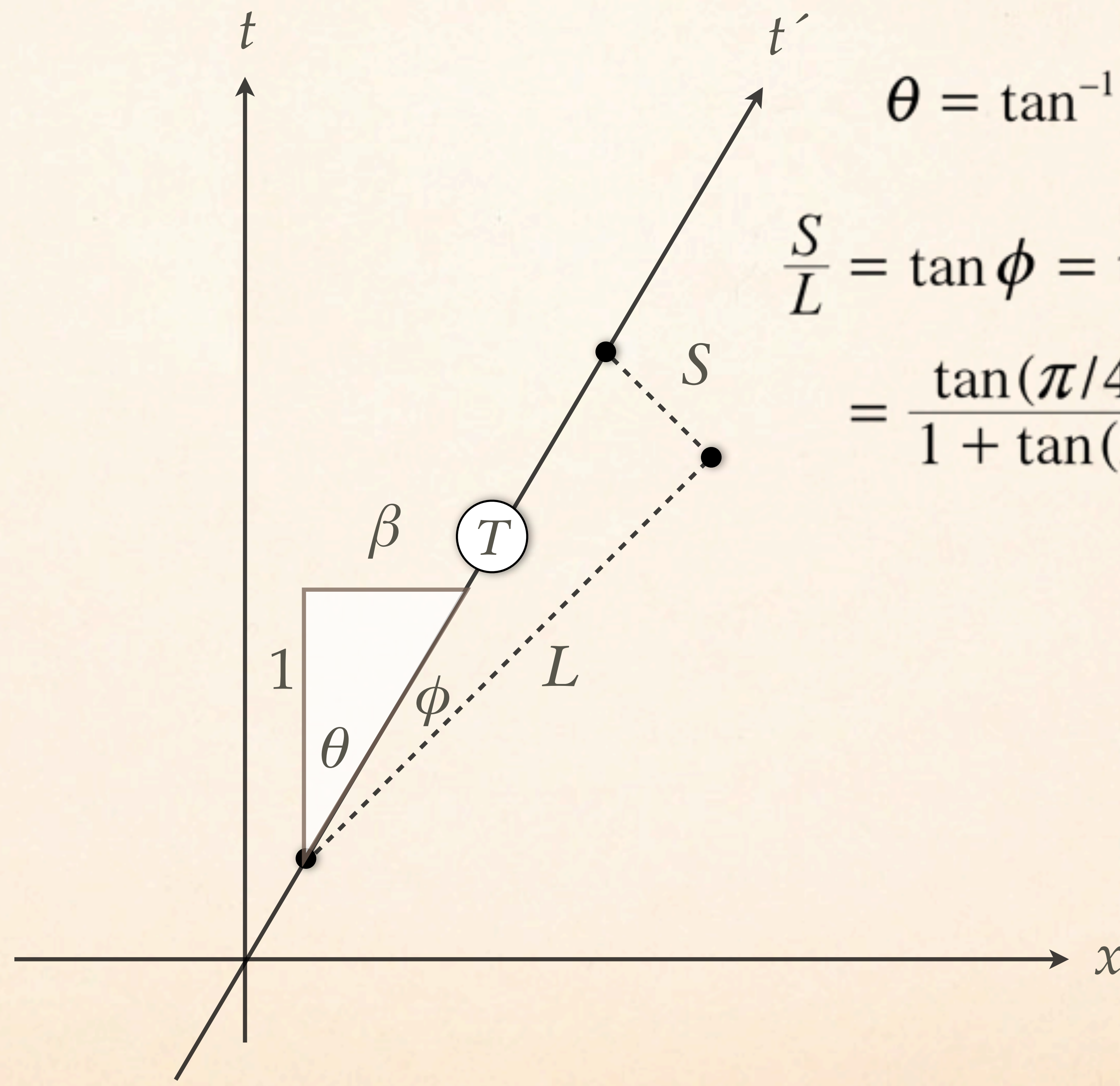
Area of Alice's diamond:

$$\left(\sqrt{\tfrac{1}{2}}T\right)^2 = \tfrac{1}{2}T^2 \equiv D^2$$



- To see this, imagine that Alice sends a pair of signals from events A_S and A_F that happen to reach the center of Bob's clock just as it begins and ends a "tick" of duration T (in Bob's frame). Salgado (knowing his audience) imagines this to be the start and finish of a TV program.
- Bob sees program to be redshifted by k, so if it lasts T in Bob's frame, it lasted T/k in Alice's frame. (Ck)
- Bob reflects back the program to Alice. Its start and finish arrive at her location at events B_S and B_F.
- By POR, Alice sees Bob's signals redshifted by *same* k, so if lasts T in Bob's frame, it lasts Tk in Alice's (Ck).
- We can see that the long side of Bob's diamond L = root(1/2)Tk (Ck), short side S = root(1/2)T/k (Ck), so its area is LS = (1/2)T^2, irrespective of Bob's velocity (Ck).
- Since this is independent of velocity, it should be the same as for Alice, but let's check. Here's Alice's causal diamond (Ck). If the diagonal is T, each side is D = root(1/2)T (Ck), so area is (1/2)T^2 (Ck): check!
- So the areas for *all* observers' causal diamonds are the same. (pause)

S IN TERMS OF D



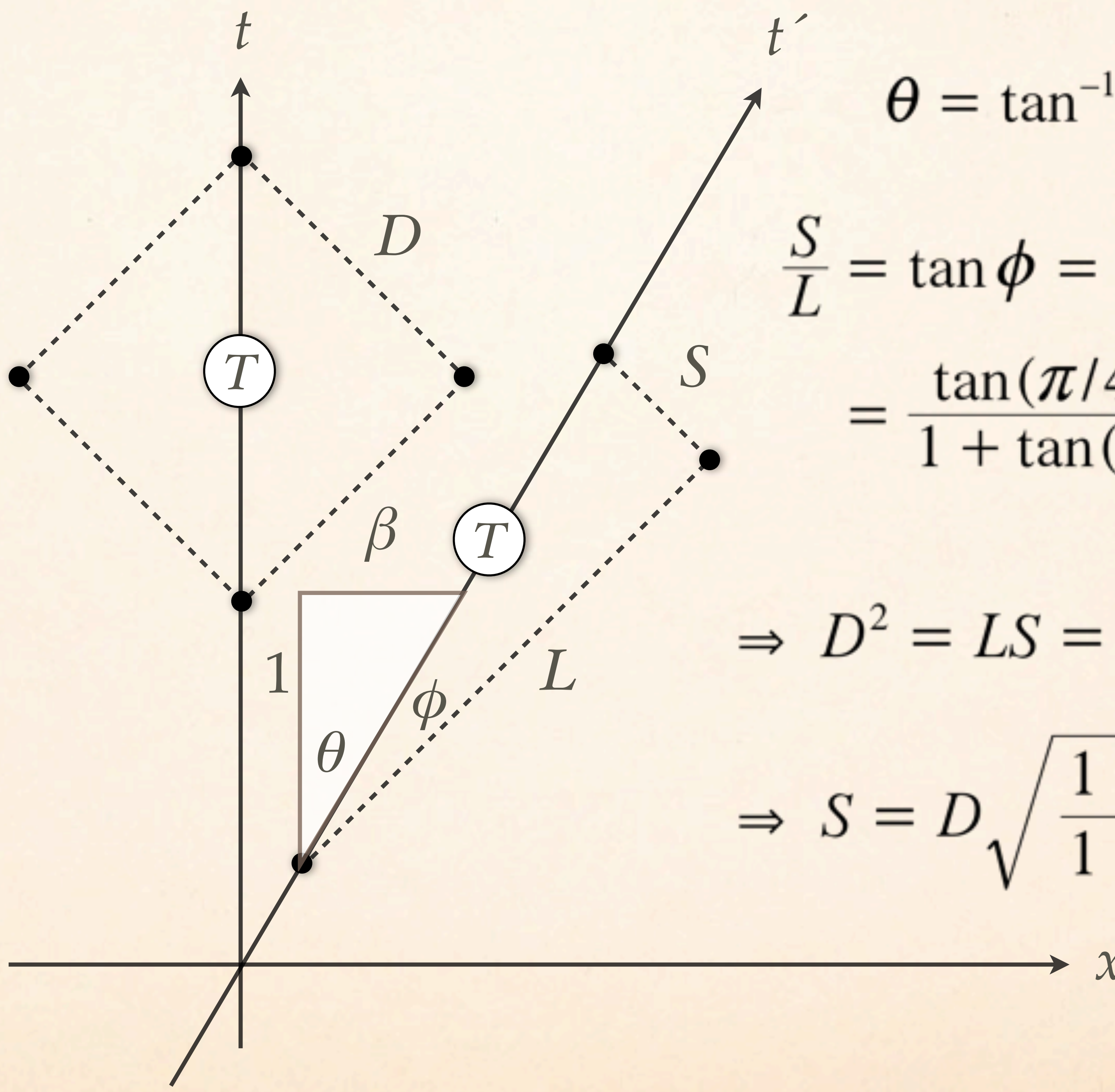
$$\theta = \tan^{-1}\beta; \quad \phi = \frac{\pi}{4} - \theta$$

$$\frac{S}{L} = \tan\phi = \tan\left(\frac{\pi}{4} - \theta\right)$$

$$= \frac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\tan\theta} = \frac{1-\beta}{1+\beta}$$

- To determine the absolute size of Bob's causal diamond, we need to know how the relative sizes of L and S are related to Bob's boost β. The inverse slope of Bob's worldline is β so θ is $\tan^{-1}\beta$. Not also that ϕ, the angle between Bob's WL and the long leg of his diamond, is 45°–θ. So $S/L = \tan\phi = 1–\beta/1+\beta$ by a simple trig id.
- Recalling that Alice's diamond has legs D by definition (Ck), and that the diamonds have the same area, we see after a bit of simple math that $S = D$sqrt$(1–\beta/1+\beta)$. We have now completely determined Bob's diamond.

S IN TERMS OF D



$$\theta = \tan^{-1}\beta; \quad \phi = \frac{\pi}{4} - \theta$$

$$\frac{S}{L} = \tan\phi = \tan\left(\frac{\pi}{4} - \theta\right)$$

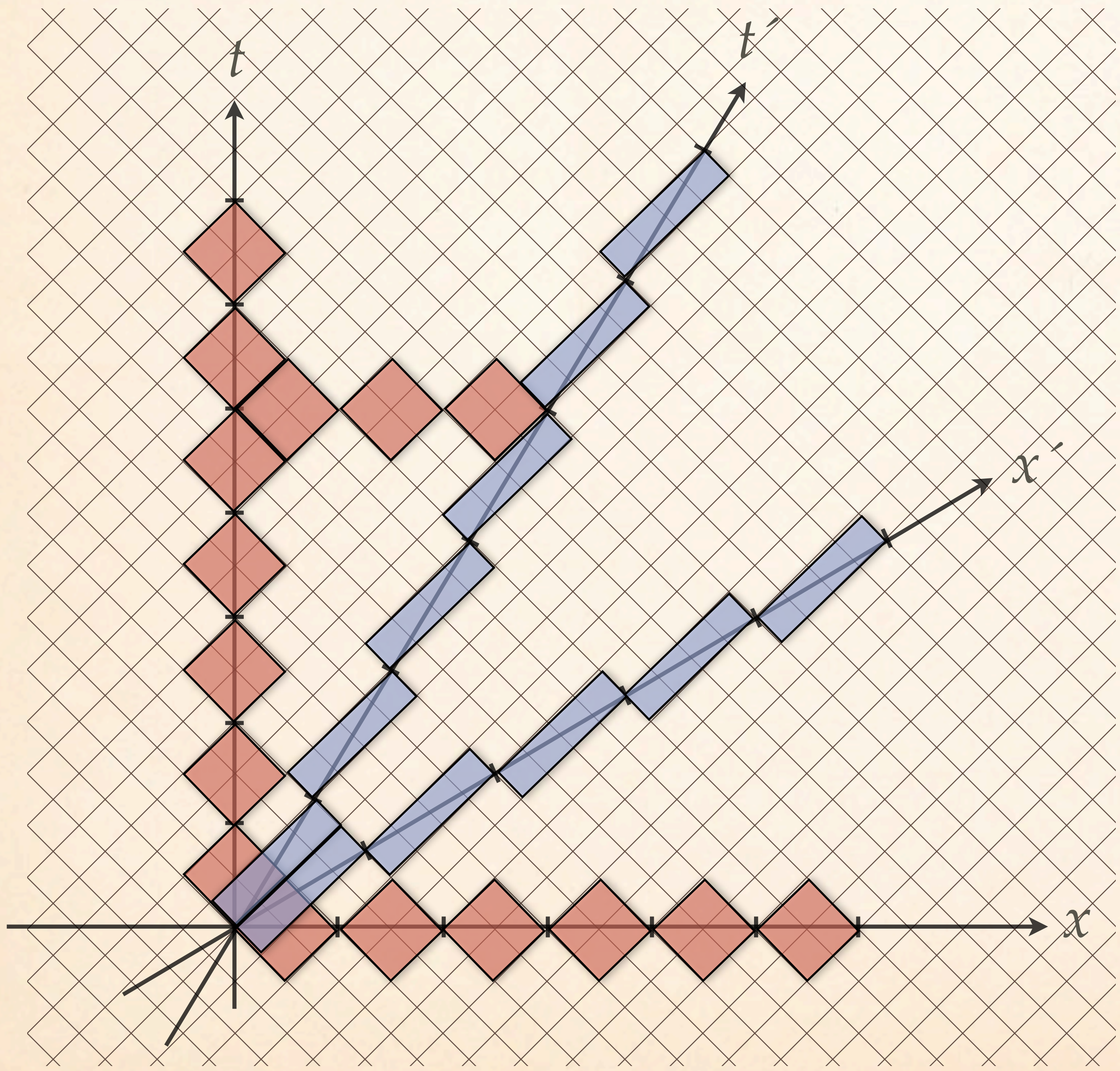
$$= \frac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\tan\theta} = \frac{1-\beta}{1+\beta}$$

$$\Rightarrow D^2 = LS = S^2\frac{L}{S} = S^2\left(\frac{1+\beta}{1-\beta}\right)$$

$$\Rightarrow S = D\sqrt{\frac{1-\beta}{1+\beta}}$$

- To determine the absolute size of Bob's causal diamond, we need to know how the relative sizes of L and S are related to Bob's boost β. The inverse slope of Bob's worldline is β so θ is $\tan^{-1}\beta$. Not also that ϕ, the angle between Bob's WL and the long leg of his diamond, is $45° - \theta$. So $S/L = \tan\phi = 1 - \beta / 1 + \beta$ by a simple trig id.
- Recalling that Alice's diamond has legs D by definition (Ck), and that the diamonds have the same area, we see after a bit of simple math that $S = D\text{sqrt}(1 - \beta / 1 + \beta)$. We have now completely determined Bob's diamond.

CALIBRATING WITH DIAMONDS



For $\beta = 3/5$,

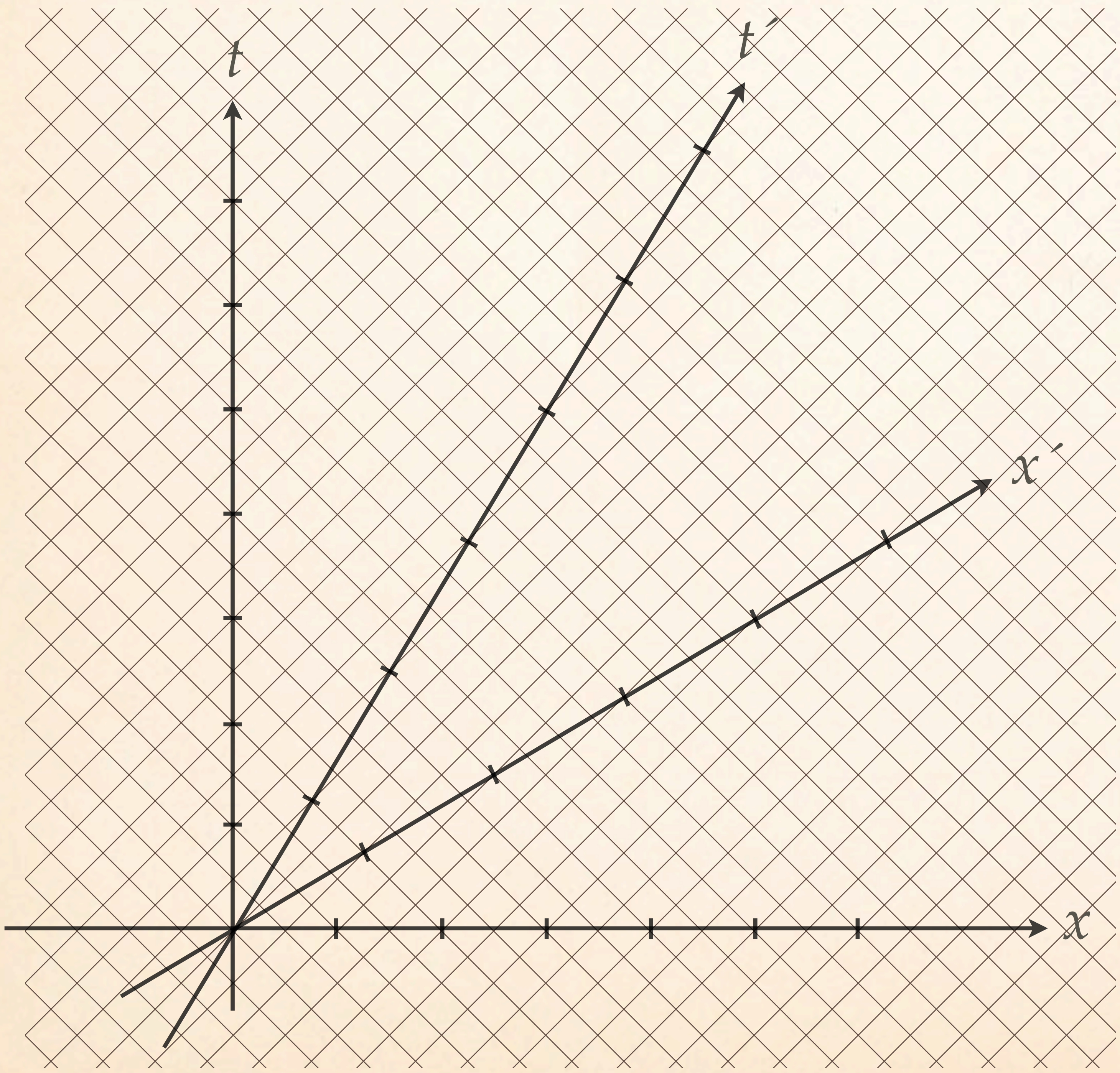
$$S = D\sqrt{\frac{1-\beta}{1+\beta}}$$
$$= D\sqrt{\frac{2/5}{8/5}} = \frac{1}{2}D$$

Works best for Pythagorean triples:

$$\beta = \frac{7}{25}, \frac{5}{13}, \frac{3}{5}, \frac{4}{5}, \frac{12}{13}, \frac{24}{25}, \text{etc.}$$

- To use Salgado's method for locating and calibrating, we use ordinary graph paper rotated 45°
- In the case where $\beta = 3/5$, S is simply $(1/2)D$. So if we draw Alice's diamonds (red) as 2 units by 2 units, Bob's diamonds (blue) are 1 unit by 4 units. Diamonds measure out corresponding tick marks on both axes.
- Also since the left-to-right diagonal of each diamond connects simultaneous events, we can use a string of diamonds to mark out and calibrate the x and x' axes as well (these diamonds are for light clocks laid out end-to-end along the spatial x direction of each frame).
- This method works best for Pythagorean triples (so that S/D is rational).
- So now we have calibration: end of Salgado's method (Ck). How does it compare to hyperbola method?
    - Pros: Very physical. Also locates the x' axis, so fewer steps. *Everything* else follows (even the metric!). Connects well to POR & constancy of speed of light. Uses readily available graph paper.
    - Cons: Doesn't emphasize metric. Requires calculation. Doesn't work with all speeds.

CALIBRATING WITH DIAMONDS



For $\beta = 3/5$,

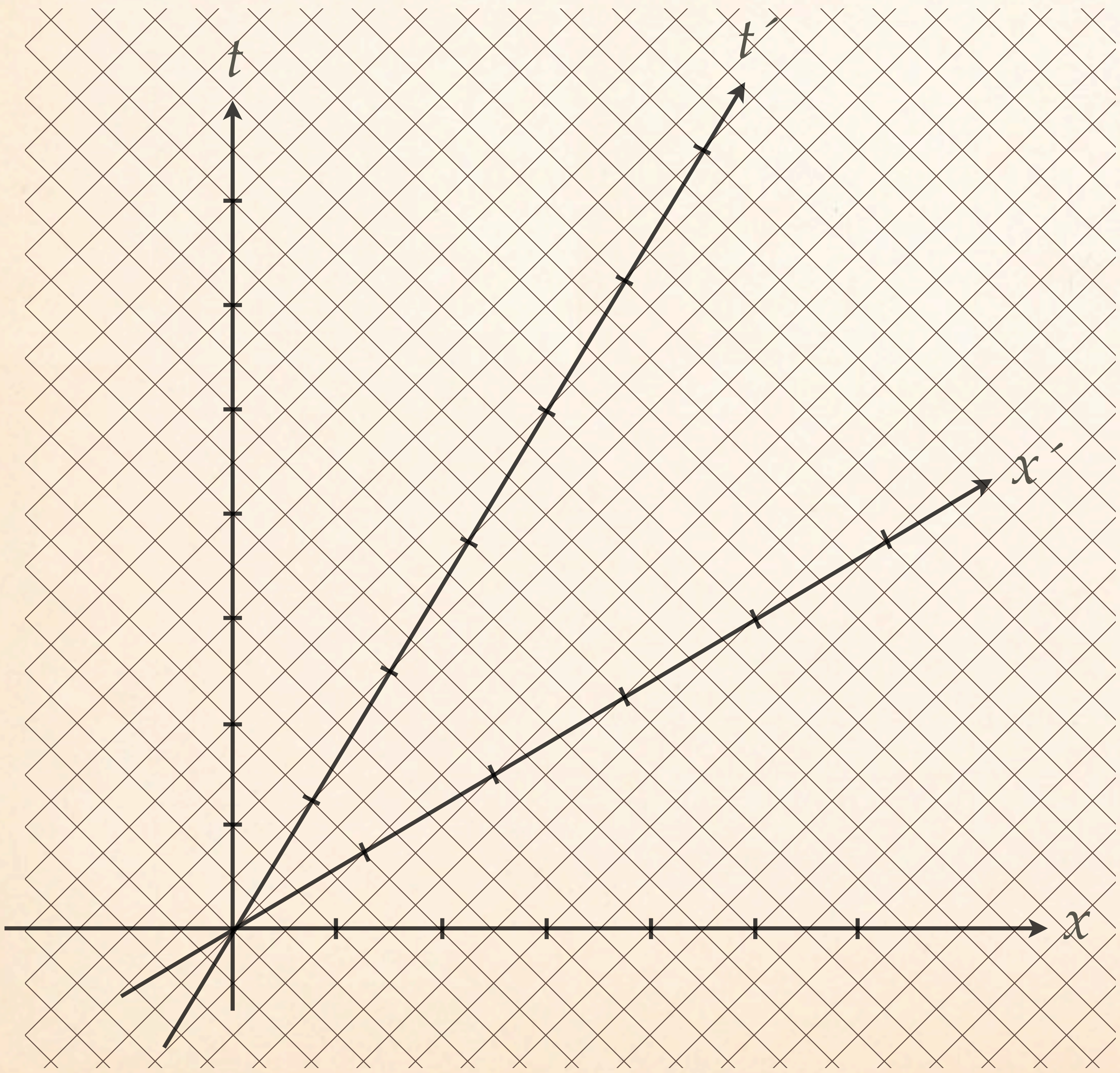
$$S = D\sqrt{\frac{1-\beta}{1+\beta}} = D\sqrt{\frac{2/5}{8/5}} = \frac{1}{2}D$$

Works best for Pythagorean triples:

$$\beta = \frac{7}{25}, \frac{5}{13}, \frac{3}{5}, \frac{4}{5}, \frac{12}{13}, \frac{24}{25}, \text{etc.}$$

- To use Salgado's method for locating and calibrating, we use ordinary graph paper rotated 45°
- In the case where $\beta = 3/5$, S is simply $(1/2)D$. So if we draw Alice's diamonds (red) as 2 units by 2 units, Bob's diamonds (blue) are 1 unit by 4 units. Diamonds measure out corresponding tick marks on both axes.
- Also since the left-to-right diagonal of each diamond connects simultaneous events, we can use a string of diamonds to mark out and calibrate the x and x' axes as well (these diamonds are for light clocks laid out end-to-end along the spatial x direction of each frame).
- This method works best for Pythagorean triples (so that S/D is rational).
- So now we have calibration: end of Salgado's method (Ck). How does it compare to hyperbola method?
    - Pros: Very physical. Also locates the x' axis, so fewer steps. *Everything* else follows (even the metric!). Connects well to POR & constancy of speed of light. Uses readily available graph paper.
    - Cons: Doesn't emphasize metric. Requires calculation. Doesn't work with all speeds.

CALIBRATING WITH DIAMONDS



For $\beta = 3/5$,

$$S = D\sqrt{\frac{1-\beta}{1+\beta}}$$

$$= D\sqrt{\frac{2/5}{8/5}} = \frac{1}{2}D$$

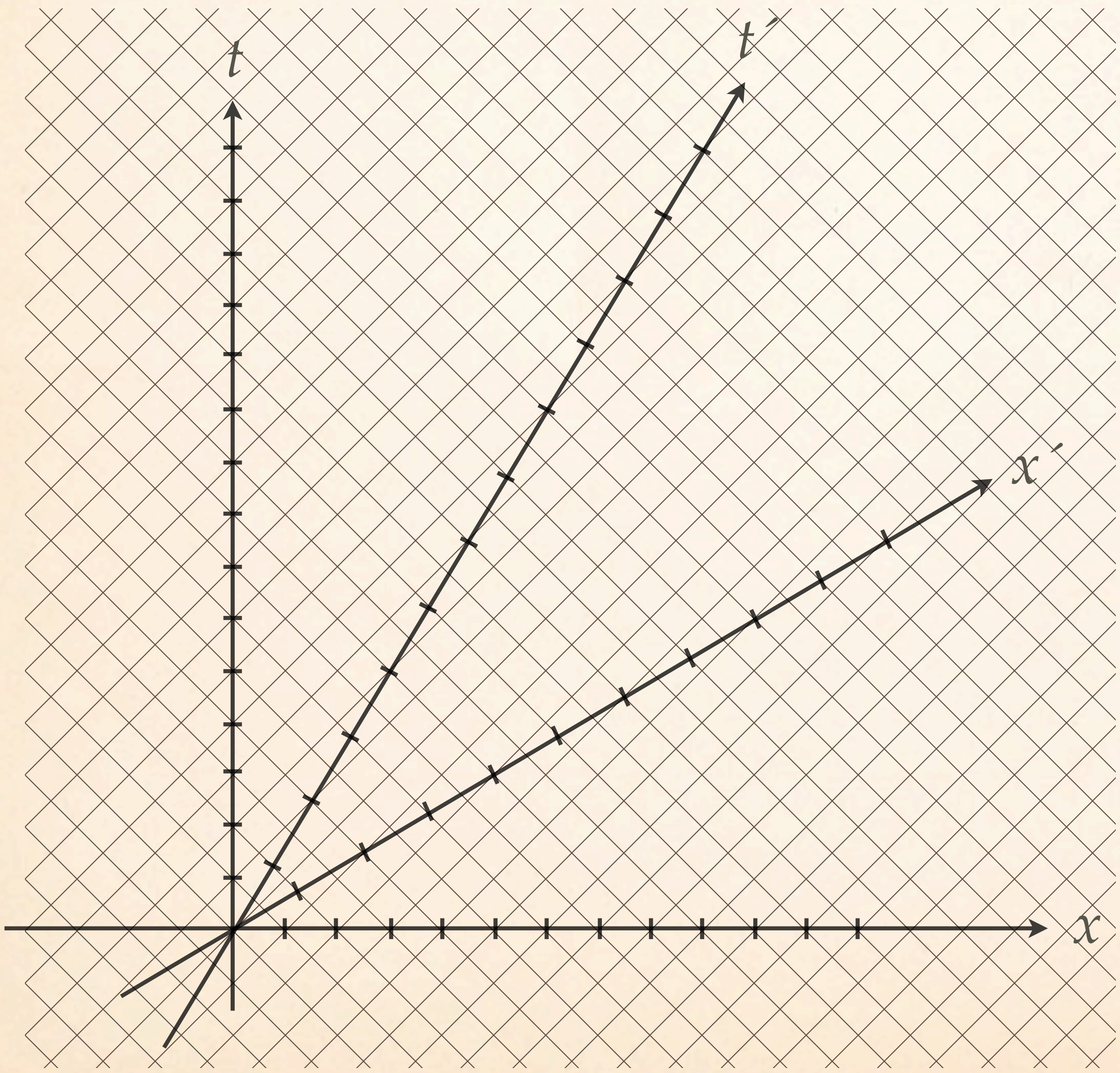
Works best for Pythagorean triples:

$$\beta = \frac{7}{25}, \frac{5}{13}, \frac{3}{5}, \frac{4}{5}, \frac{12}{13}, \frac{24}{25}, \text{etc.}$$

(End of Salgado's method)

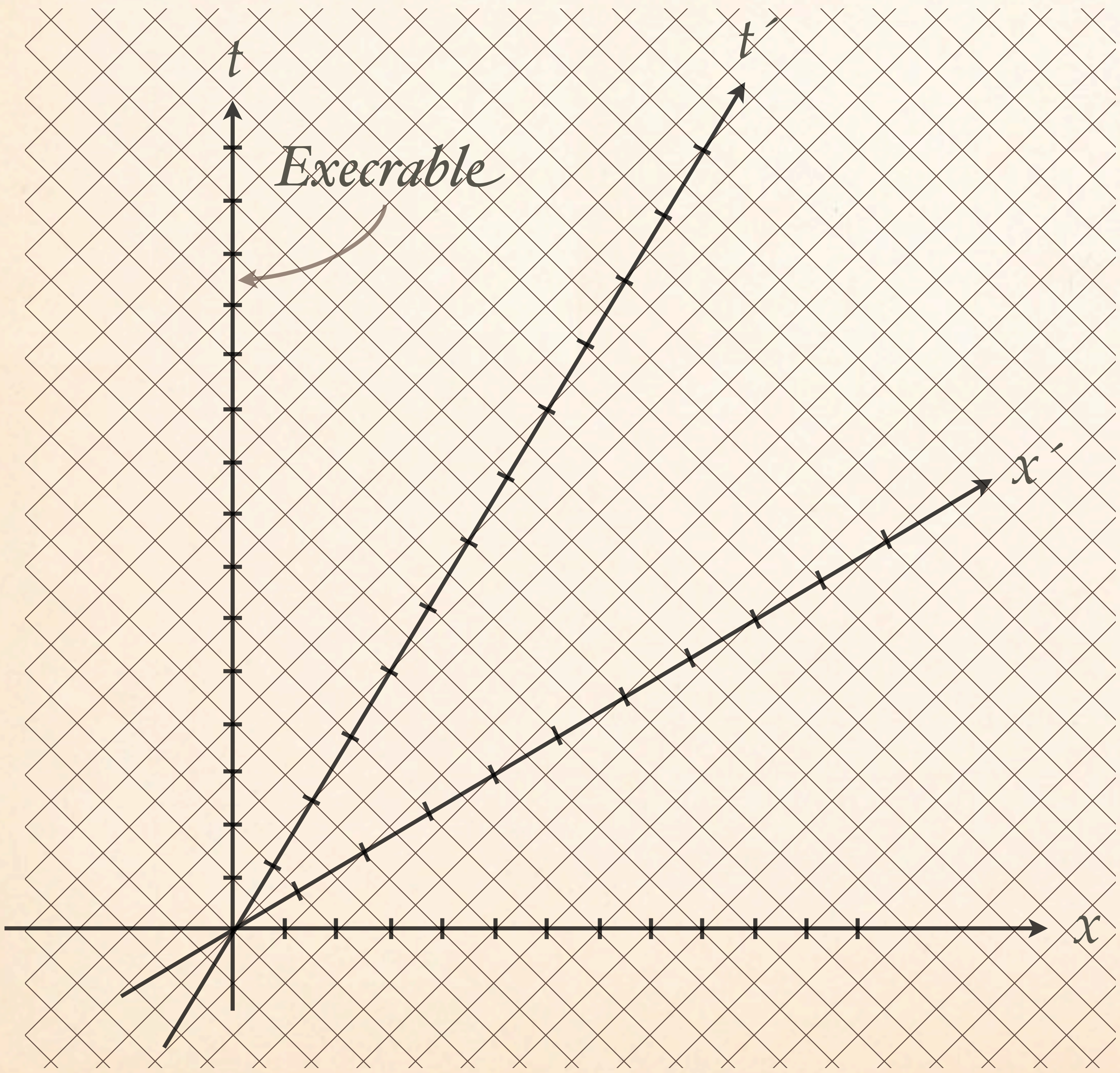
- To use Salgado's method for locating and calibrating, we use ordinary graph paper rotated 45°
- In the case where $\beta = 3/5$, S is simply $(1/2)D$. So if we draw Alice's diamonds (red) as 2 units by 2 units, Bob's diamonds (blue) are 1 unit by 4 units. Diamonds measure out corresponding tick marks on both axes.
- Also since the left-to-right diagonal of each diamond connects simultaneous events, we can use a string of diamonds to mark out and calibrate the x and x' axes as well (these diamonds are for light clocks laid out end-to-end along the spatial x direction of each frame).
- This method works best for Pythagorean triples (so that S/D is rational).
- So now we have calibration: end of Salgado's method (Ck). How does it compare to hyperbola method?
    - Pros: Very physical. Also locates the x' axis, so fewer steps. *Everything* else follows (even the metric!). Connects well to POR & constancy of speed of light. Uses readily available graph paper.
    - Cons: Doesn't emphasize metric. Requires calculation. Doesn't work with all speeds.

AN EXAMPLE PROBLEM



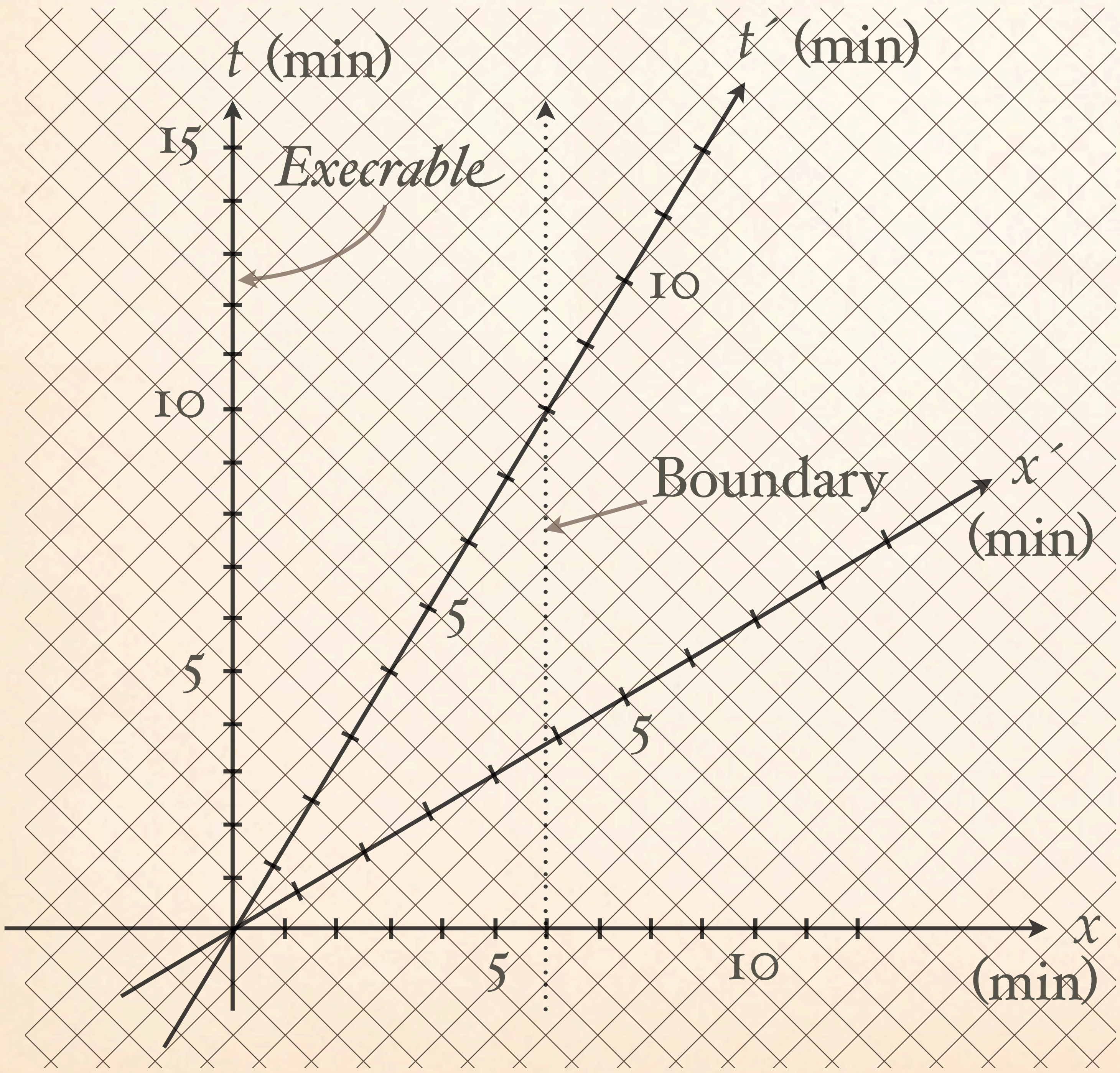
- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the $+x$ direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event A (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event C (Ck). Some time later the Klingons cross the boundary: this is event D (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C{}' = 10$ min (Ck), clearly *after* event D at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



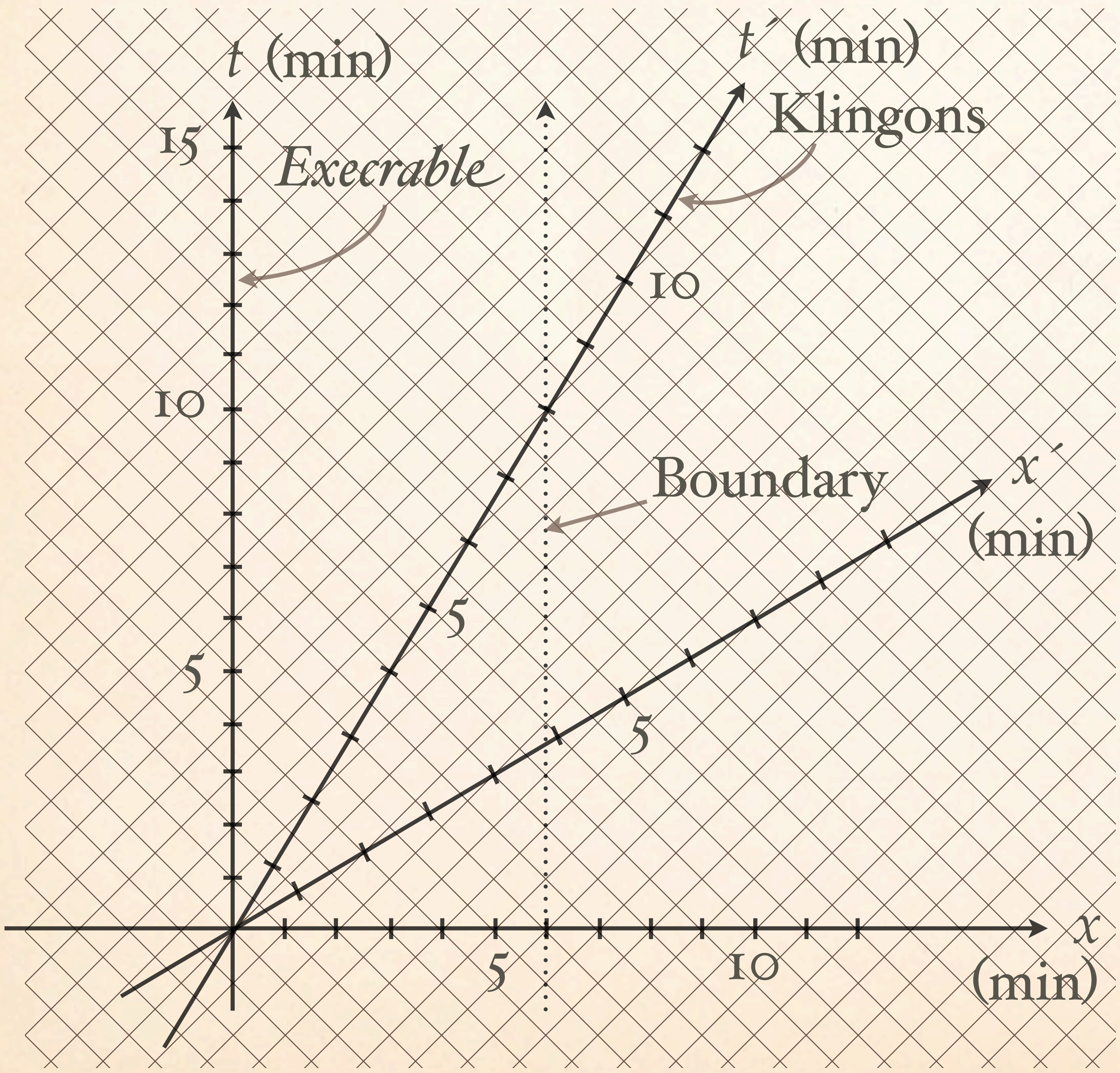
- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the $+x$ direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event A (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event C (Ck). Some time later the Klingons cross the boundary: this is event D (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event D at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



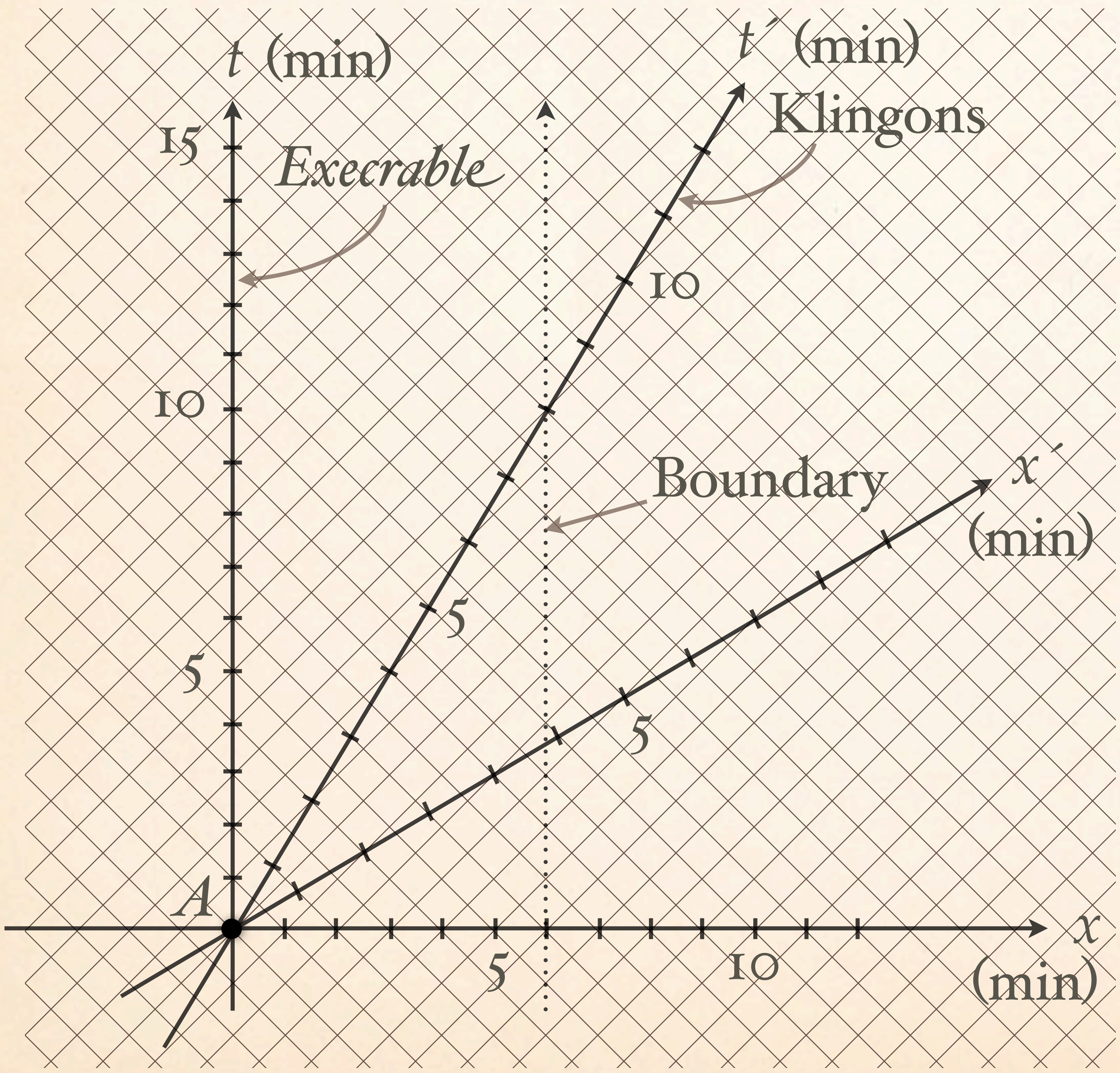
- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C{'} = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



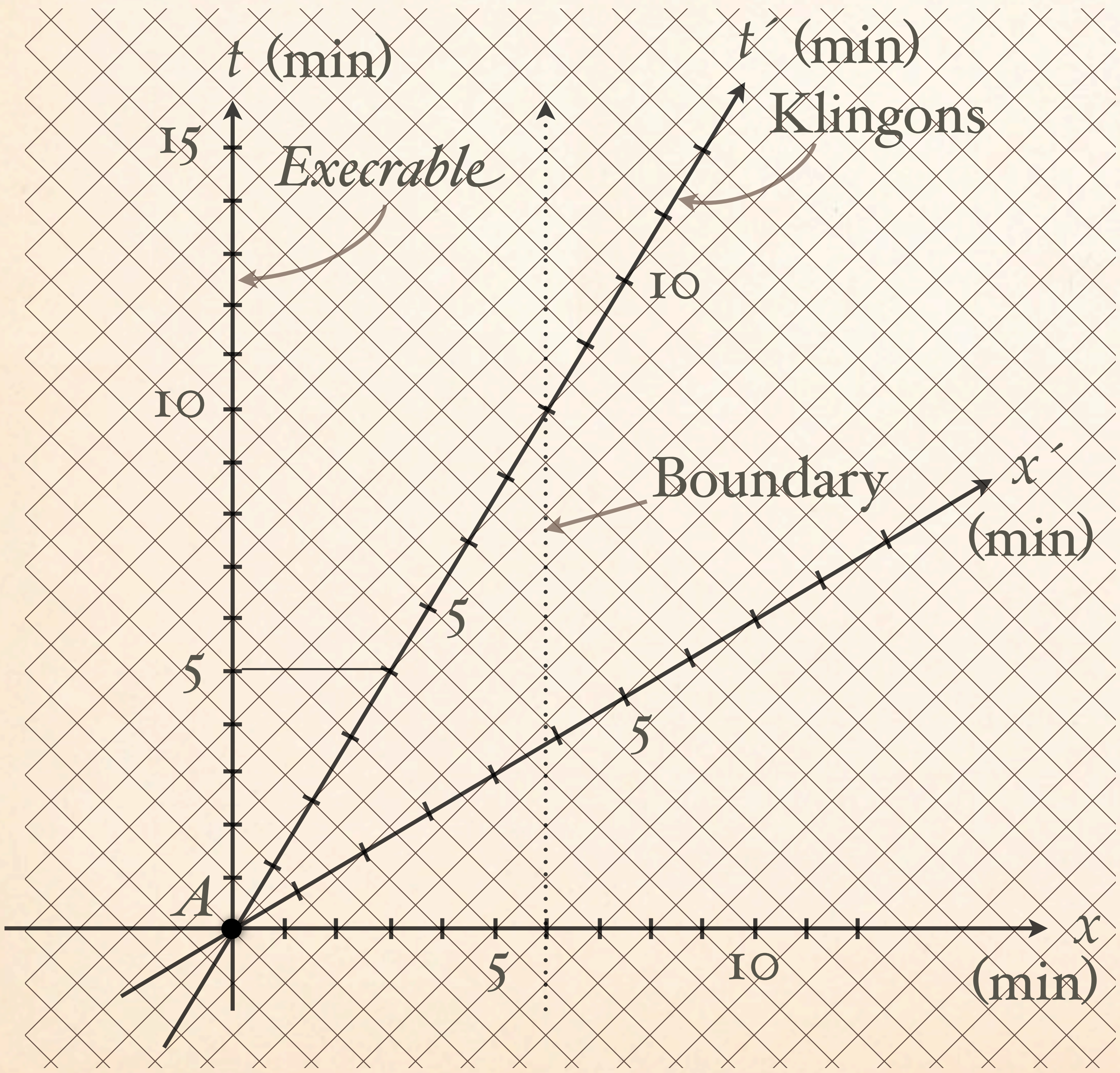
- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event *C* in the primed frame, drop a parallel from C to the t' axis. We see that $t_C{'} = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



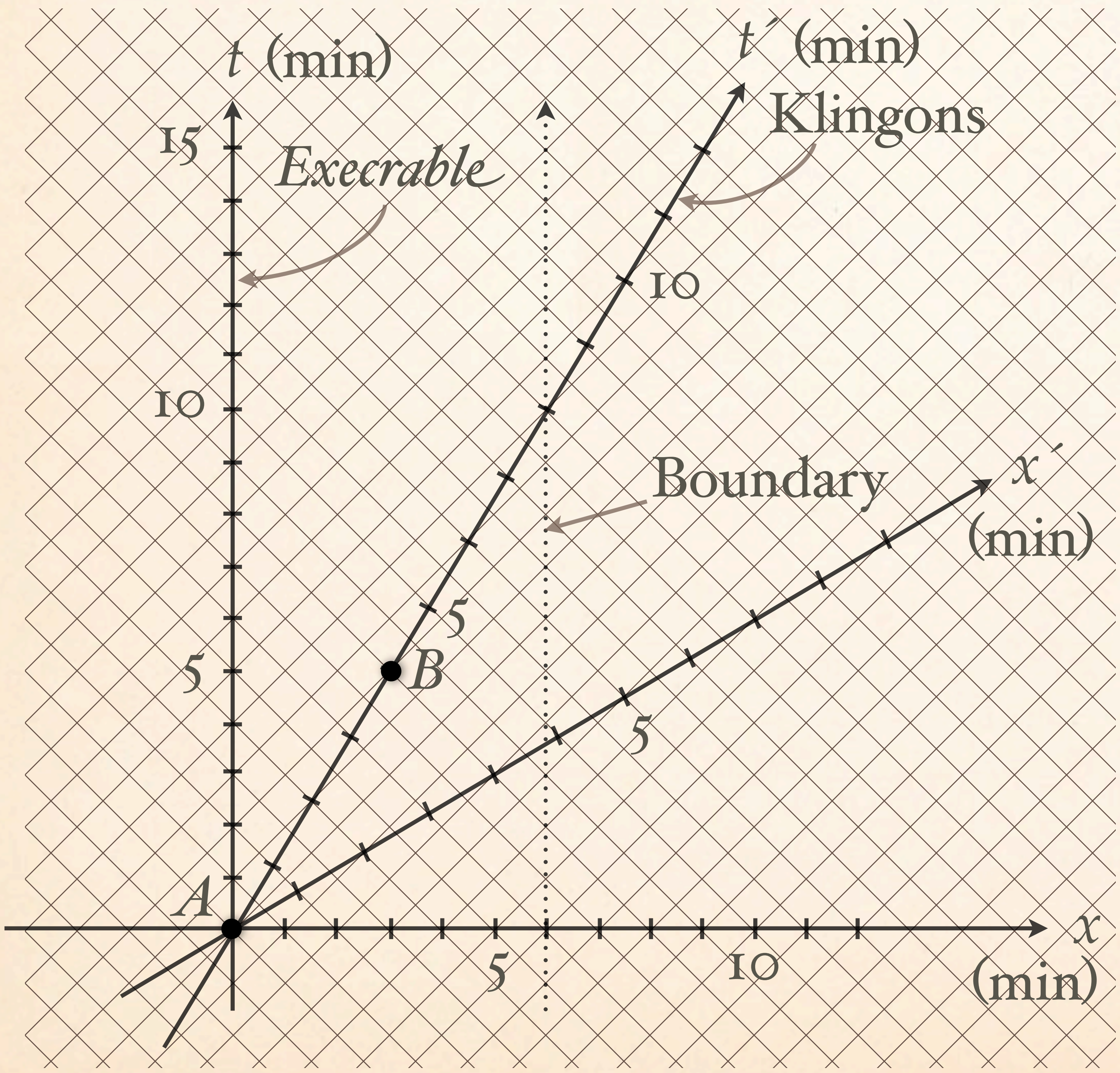
- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the $+x$ direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event A (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event C (Ck). Some time later the Klingons cross the boundary: this is event D (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event D at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the $+x$ direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event A (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event C (Ck). Some time later the Klingons cross the boundary: this is event D (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event D at 8 min (Ck). So they get off on a technicality.

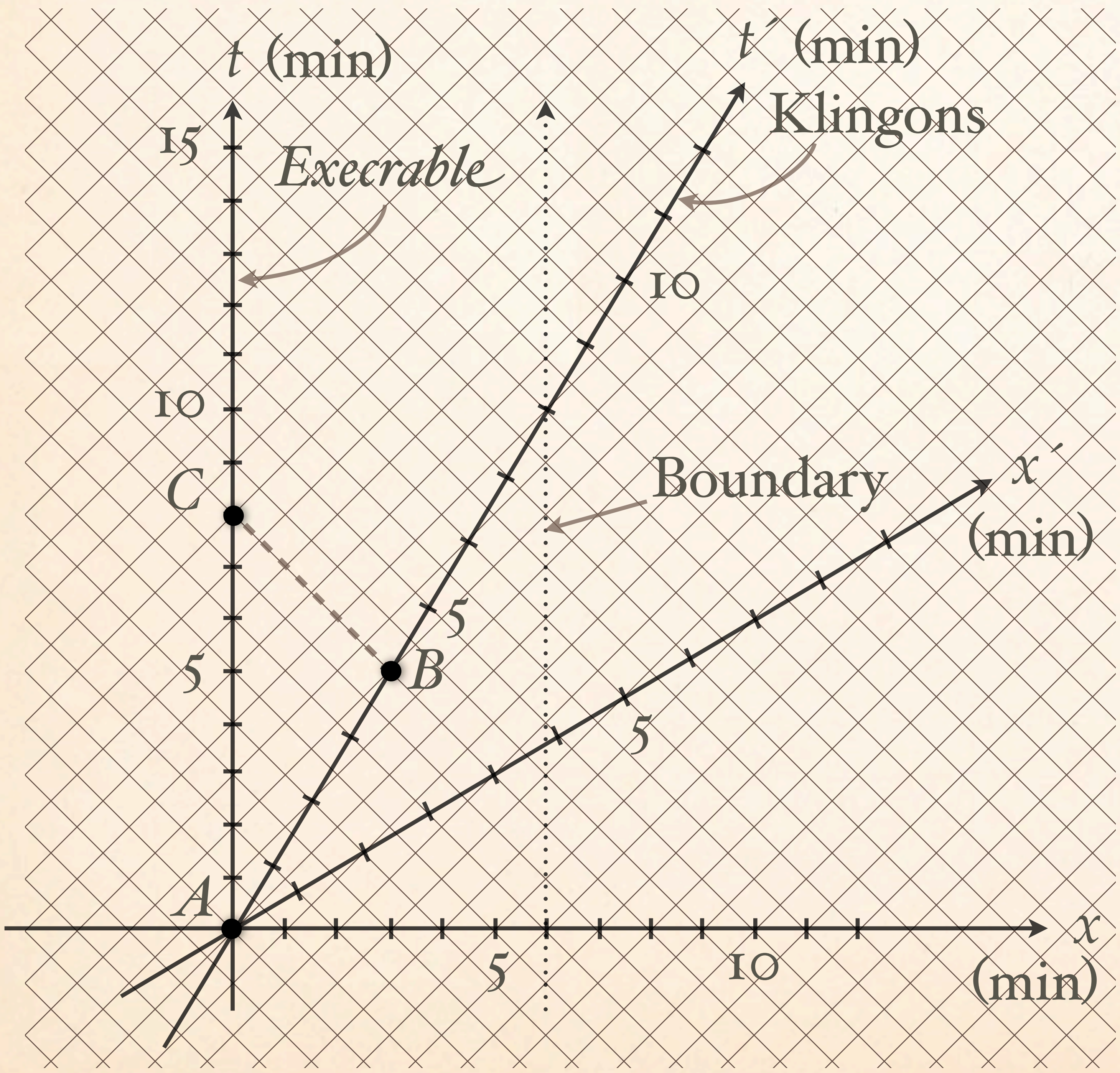
AN EXAMPLE PROBLEM



• Once we have calibration via either method, we can easily do quite complex problems.

• Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

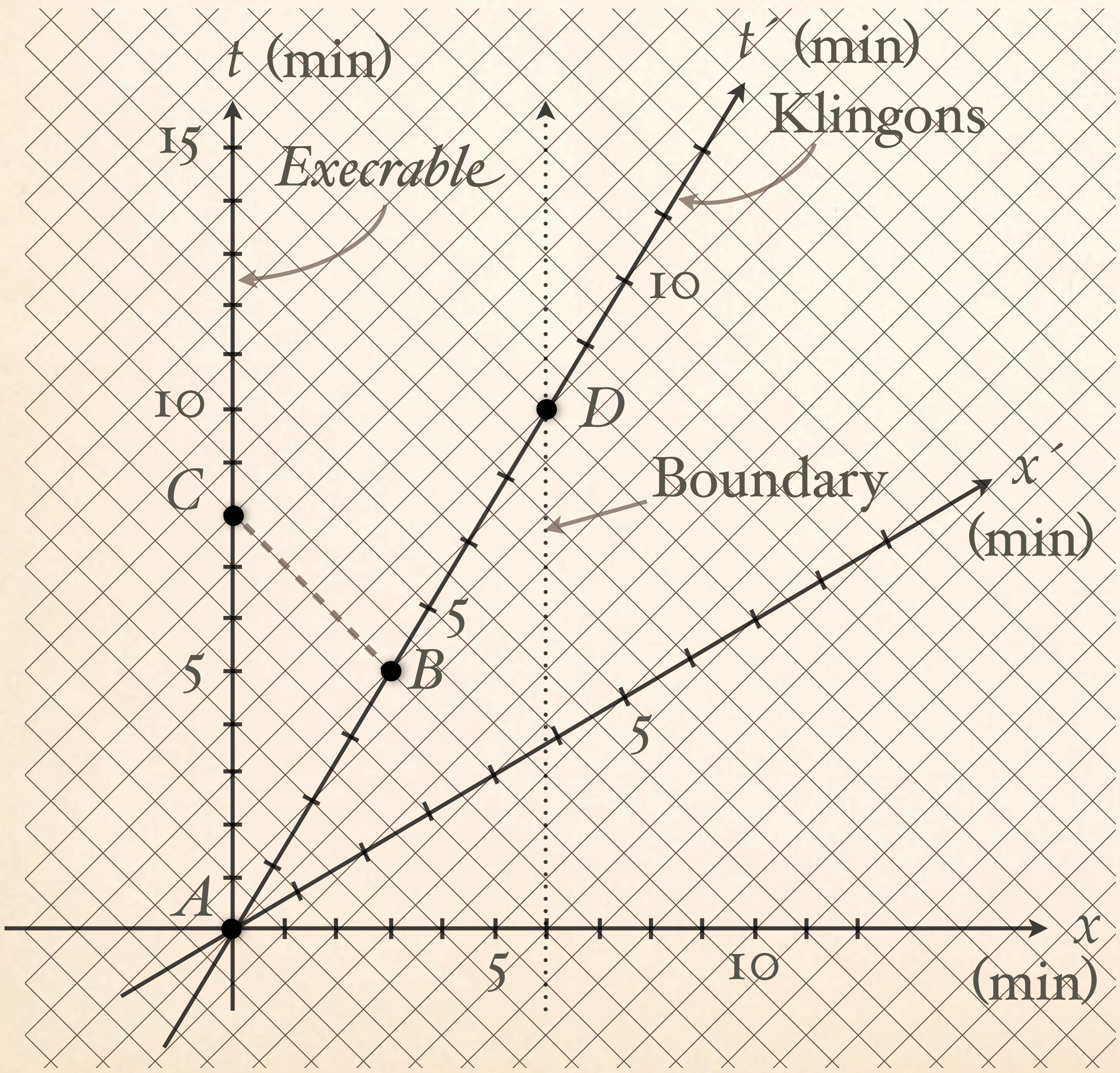
AN EXAMPLE PROBLEM



• Once we have calibration via either method, we can easily do quite complex problems.

• Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C{}' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

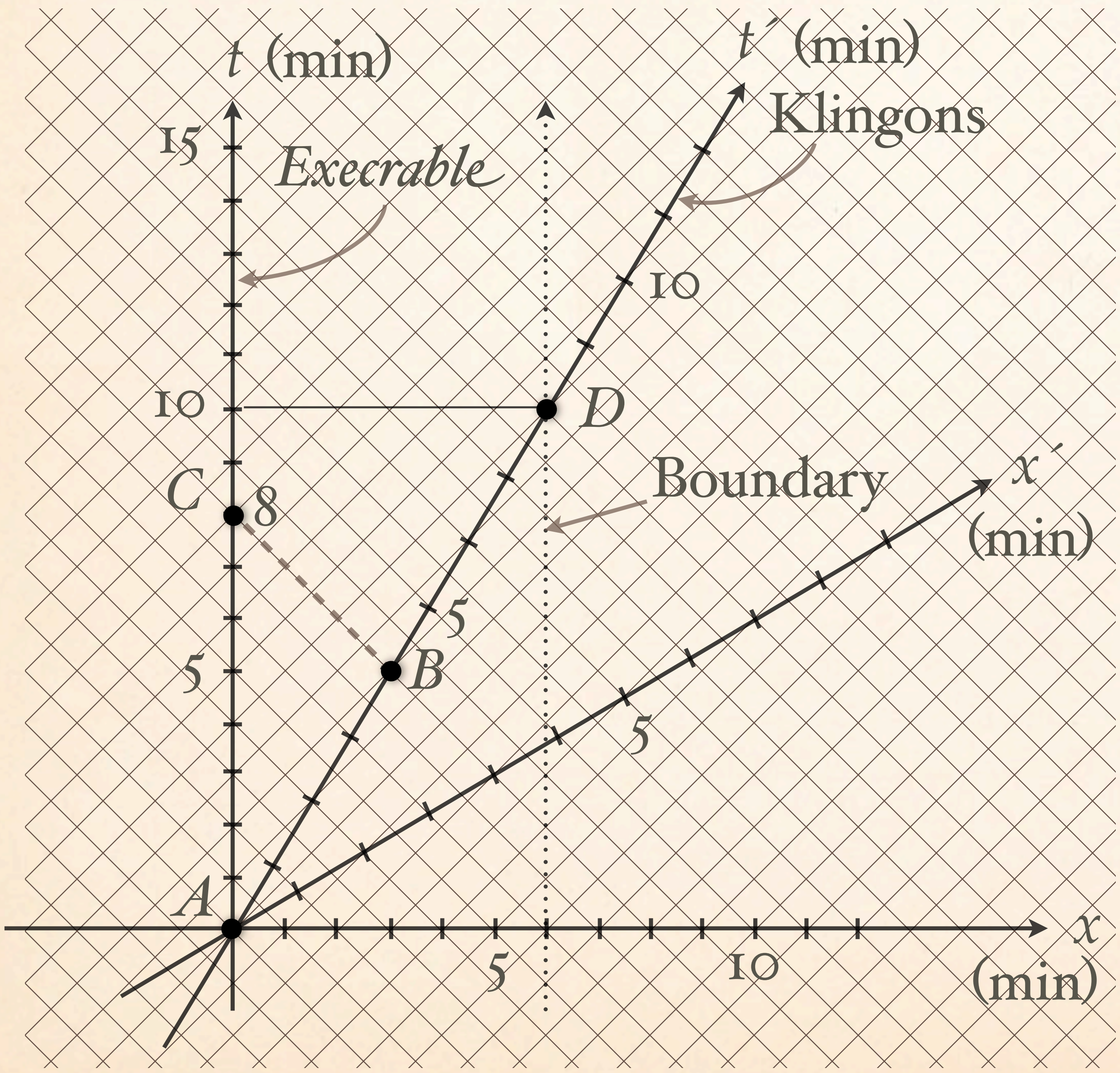
AN EXAMPLE PROBLEM



• Once we have calibration via either method, we can easily do quite complex problems.

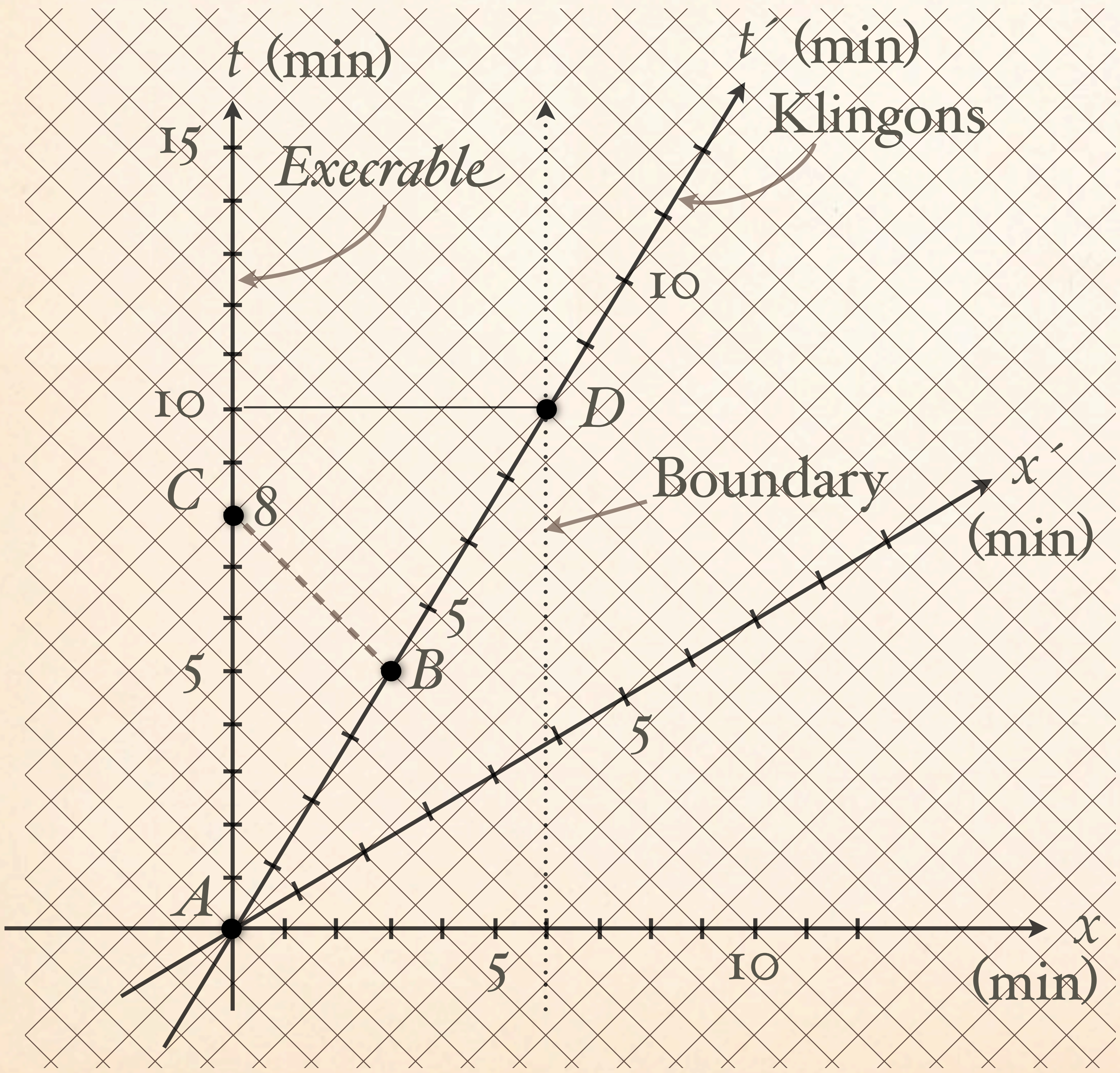
• Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C{}' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

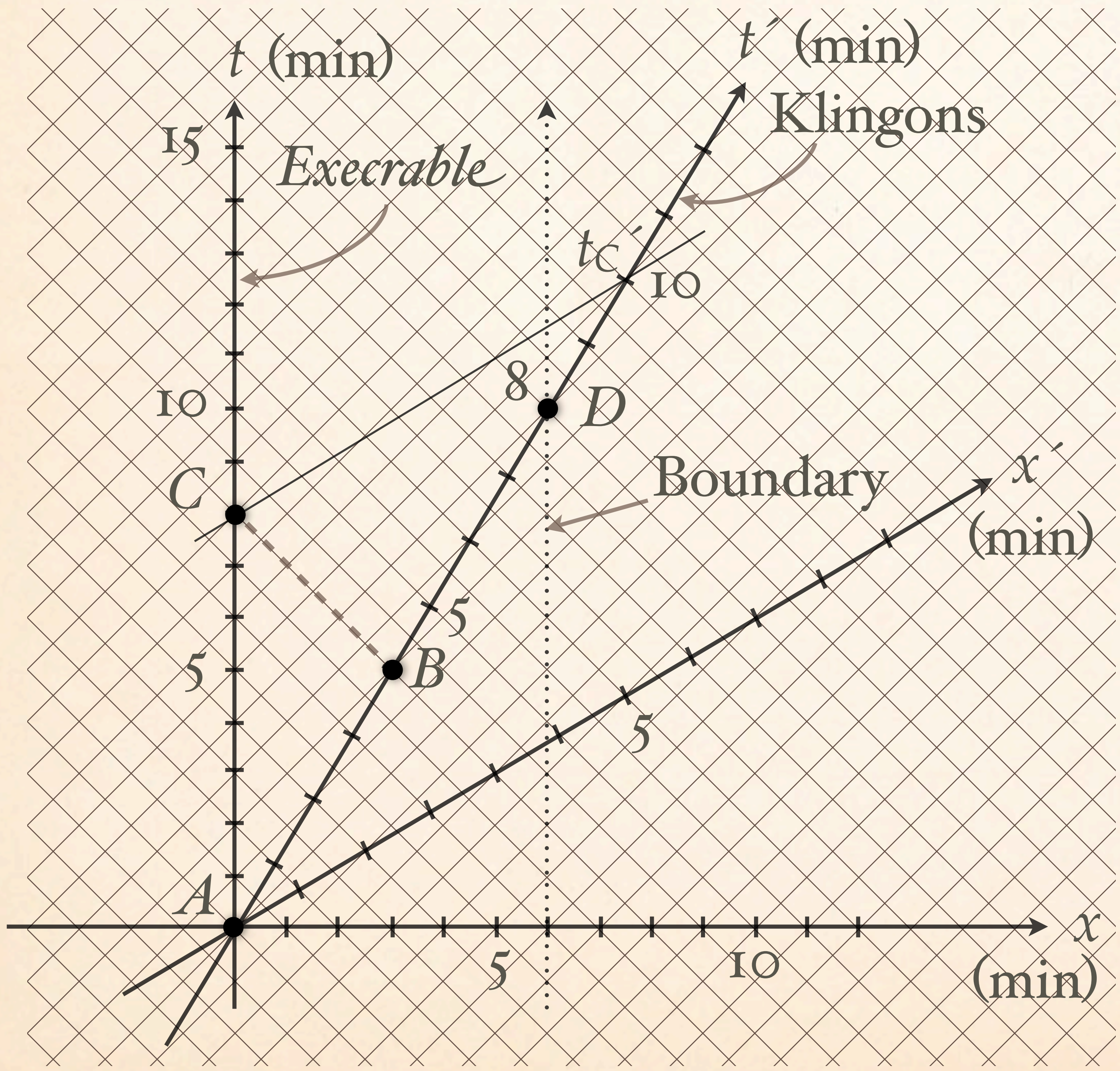
AN EXAMPLE PROBLEM



What is the time order of events *C* and *D* in the Klingons' frame?

- Once we have calibration via either method, we can easily do quite complex problems.
- Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM

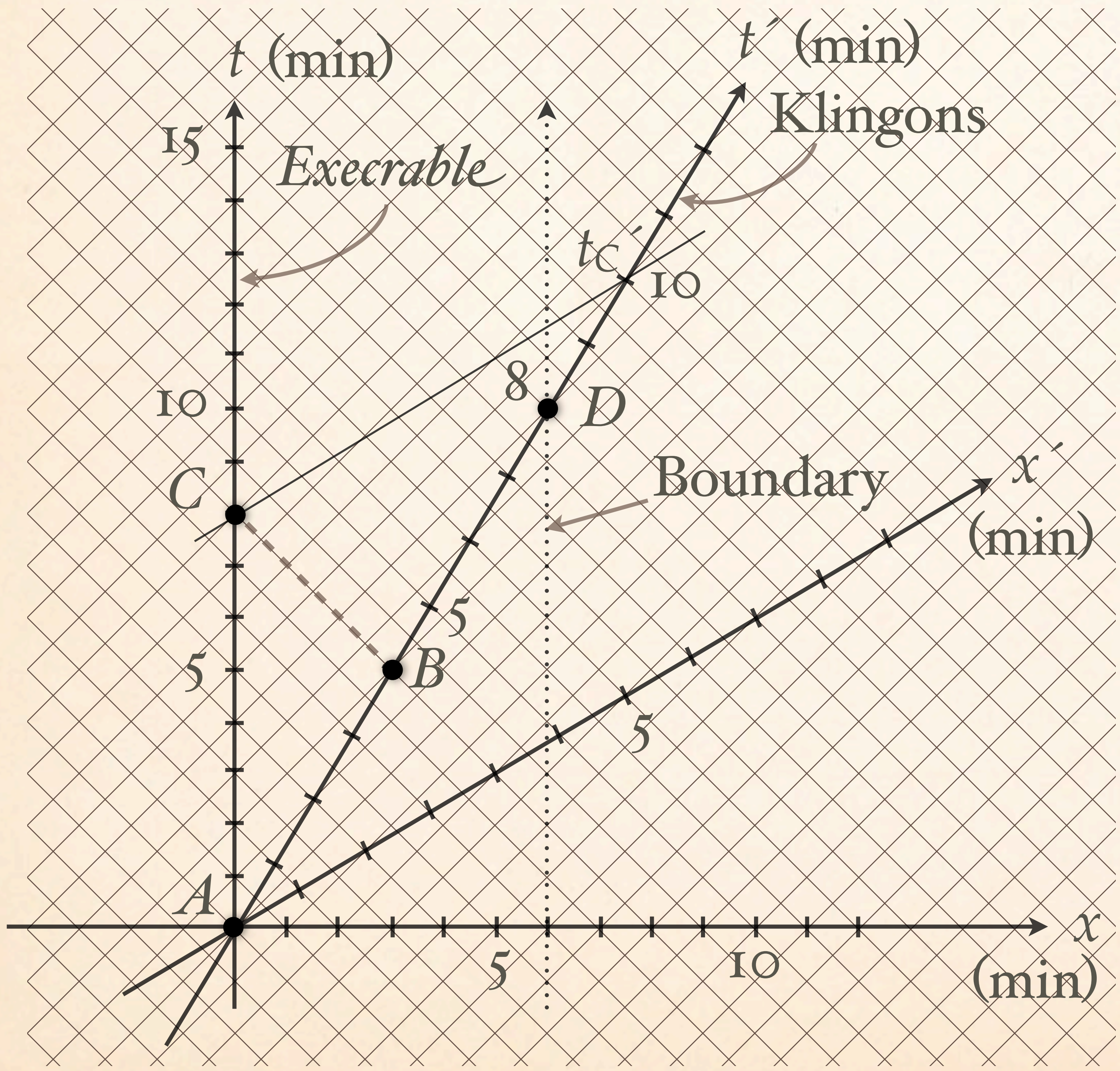


What is the time order of events *C* and *D* in the Klingons' frame?

• Once we have calibration via either method, we can easily do quite complex problems.

• Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the $+x$ direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event C in the primed frame, drop a parallel from C to the t' axis. We see that $t_C' = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

AN EXAMPLE PROBLEM



What is the time order of events *C* and *D* in the Klingons' frame?

Answer: Event *C* happens *after* Event *D* (in the Klingon frame)

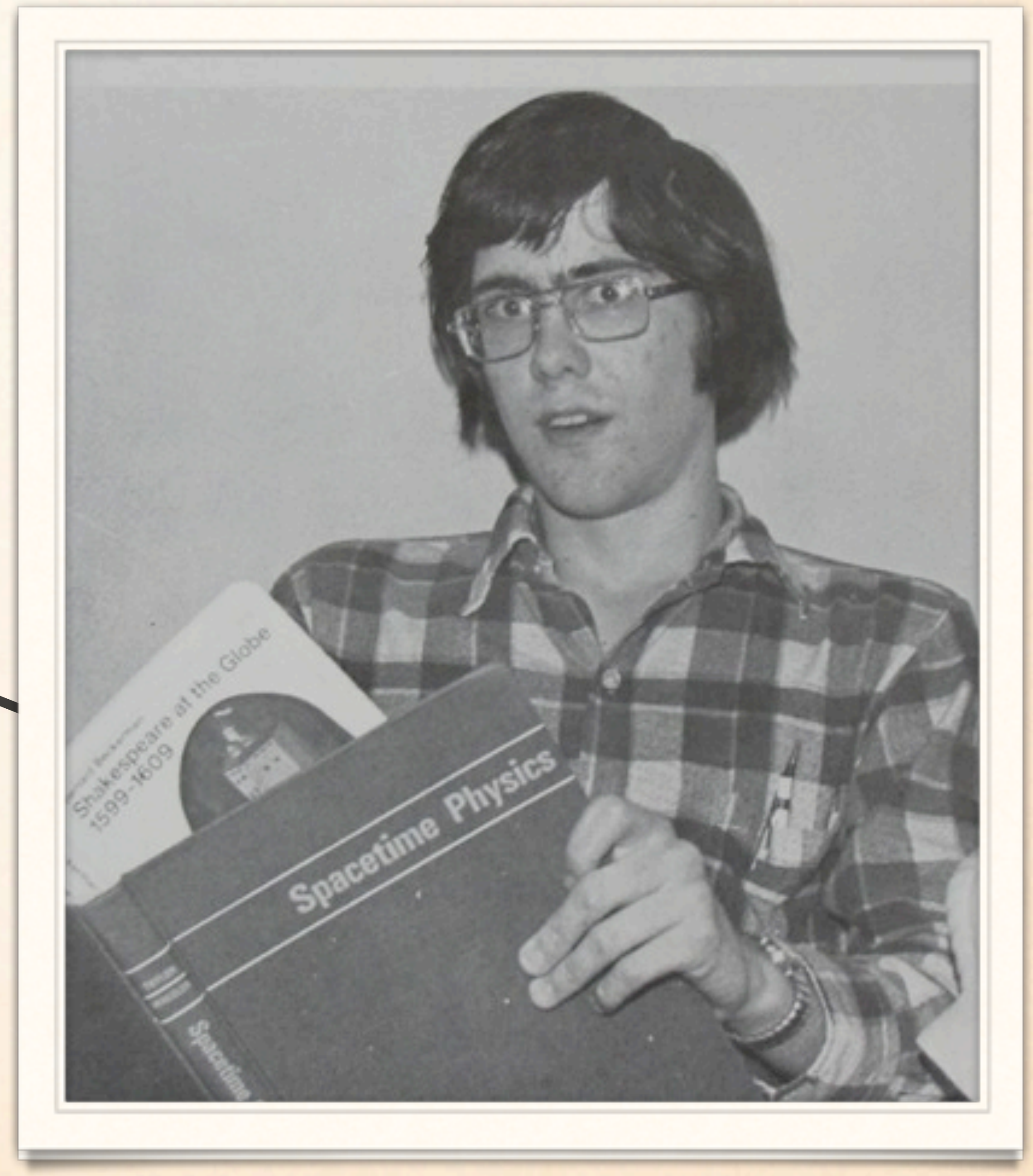
• Once we have calibration via either method, we can easily do quite complex problems.

• Example from unit R: "The Federation starship *Execrable* is floating in Federation territory (Ck) at rest relative to the border of Klingon space, which is 6 min away in the +*x* direction (Ck). Suddenly, a Klingon warship, flies past the *Execrable* in the direction of the border at a speed of 3/5 (Ck). Call this event *A* (Ck), and let this define the zero of time in both frames. After 5 min according to the Execrable's clocks (Ck), the Klingons emit a parting disrupter blast: call this event B (Ck). The disrupter beam travels at the speed of light back to the *Execrable* and disables it: this is event *C* (Ck). Some time later the Klingons cross the boundary: this is event *D* (Ck). How much later is in the *Execrable's* frame? [We can read from the diagram that $t_C = 8$ min, $t_D = 10$ min, so 2 min later.] Now, the Klingon-Federation treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up for adjudication, the Klingon's claim that they are within the letter of the law, because in their reference frame, the damage occurred *after* they had crossed safely back into Klingon territory. Are they correct?" So here is the task (Ck): what's the answer (pause). We can read the answer directly from the diagram. To find the time of the damage event *C* in the primed frame, drop a parallel from C to the *t*′ axis. We see that $t_C´ = 10$ min (Ck), clearly *after* event *D* at 8 min (Ck). So they get off on a technicality.

THANKS!

Thanks to Rob Salgado
and Edwin Taylor!

(Me as a Carleton College
senior in 1976 with
Spacetime Physics)



tmoore@pomona.edu
www.physics.pomona.edu/sixideas/
pages.pomona.edu/~tmoore/grw/

- Thanks to Rob Salgado for giving me something great to talk about and whose paper I hope you will see soon in print.
- I'd also like to thank Edwin Taylor for changing my life in so many ways that it is hard to recount them all, but whose book *Spacetime Physics* determined the trajectory of my academic life).
- Thank you all for listening!