



# **Modelling and Simulation of Rigid and Flexible Multibody Systems in Modelica**

**Tutorial at the Modelica'2011 Conference**

**Dresden, March 20<sup>th</sup>, 2011**

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**Institute of Robotics and Mechatronics**



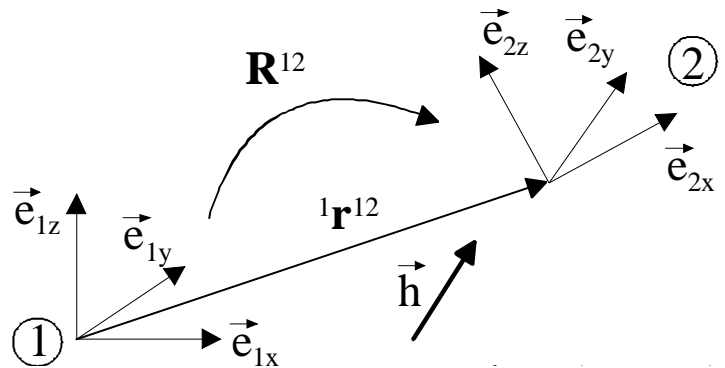
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- FE-Preprocessing
- FlexibleBodies Library extensions at this conference



# Modelica Multibody Basics: Orientation

## ➤ Coordinate systems and their orientation



```
import MultiBody.Frames;
Frames.Orientation R12;
Real h1[3] "h resolved in frame 1";
Real h2[3] "h resolved in frame 2";
equation
  h2 = Frames.resolve2(R12, h1); //or
  h1 = Frames.resolve1(R12, h2);
```

## ➤ Orientation object $R^{12}$

➤ describes orientation of coordinate system 2 wrt. 1

➤ holds

```
Real T[3, 3] "Transformation matrix from world frame to local frame";
SI.AngularVelocity w[3]
  "Absolute angular velocity of local frame, resolved in local frame";
```

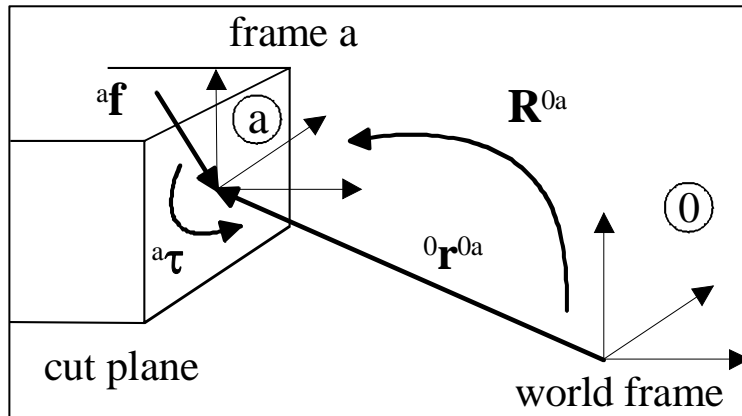
➤ may be computed using rotation angles or quaternions

➤ Multibody Lib. contains over 30 functions to operate on orientation objects

- Frames
- ▣ Orientation
- orientationConstraint
- angularVelocity1
- angularVelocity2
- resolve1
- resolve2
- resolveRelative
- resolveDyade1
- resolveDyade2
- nullRotation
- inverseRotation
- relativeRotation
- absoluteRotation
- planarRotation
- planarRotationAngle
- axisRotation
- axesRotations
- axesRotationsAngles
- smallRotation
- from\_nxy
- from\_nxz
- from\_T
- from\_T2
- from\_T\_inv
- from\_Q
- to\_T
- to\_T\_inv
- to\_Q
- to\_vector
- to\_exy
- length
- normalize
- axis

# Modelica Multibody Basics: Connectors I

- Connectors: the interface to connect components
  - Position is resolved in world frame
  - Forces and torques are resolved in local frame



```
connector Frame
  "Coordinate system fixed to the component with one cut-force and cut-torque (no icon)"
  import SI = Modelica.SIunits;
  SI.Position r_0[3]
    "Position vector from world frame to the connector frame origin, resolved in world frame";
  Frames.Orientation R
    "Orientation object to rotate the world frame into the connector frame";
  flow SI.Force f[3] "Cut-force resolved in connector frame" a;
  flow SI.Torque t[3] "Cut-torque resolved in connector frame";
end Frame;
```

non-flow !

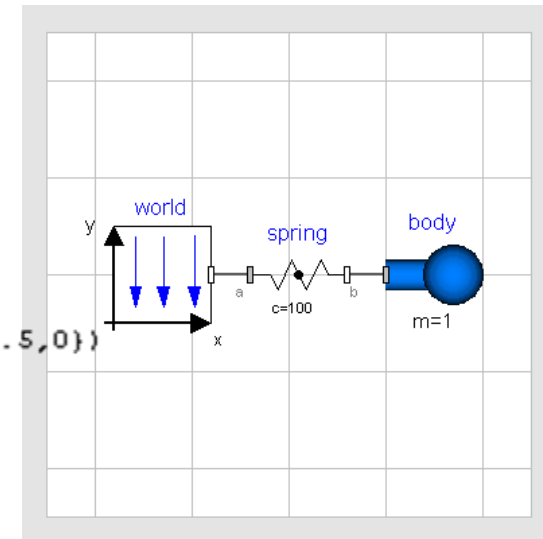
flow !



# Modelica Multibody Basics: Connectors II

## ➤ Connectors: how they work

```
model SpringMass
  inner Modelica.Mechanics.MultiBody.World world
    a;
  Modelica.Mechanics.MultiBody.Forces.Spring spring(c=100)
    a;
  Modelica.Mechanics.MultiBody.Parts.Body body(x_0_start={0,.5,0})
    a;
equation
  connect(world.frame_b, spring.frame_a) a;
  connect(spring.frame_b, body.frame_a) a;
end SpringMass;
```



## ➤ Modelica's general connections rules

- non-flow variables are set to be equal, i.e. frames coincide
  - since they represent „some kind of potential“
- flow variables sum to zero (Kirchhoff's current law)
  - since they represent time derivatives of preserved quantities
  - are consequently set to zero if connector is not connected to anything

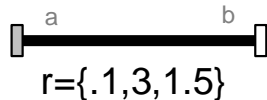
➤ see Modelica.UsersGuide.Connectors for a comparison of connectors in various domains

# Modelica Multibody Basics: Components I

## ➤ Kinematics:

- Component equations provide relations between connector variables on position level
- MultiBody.Parts.FixedTranslation  
i.e. fixed translation of frame\_b with respect to frame\_a
- Tool (e.g. Dymola) differentiates these equations twice for dynamics

### fixedTranslation

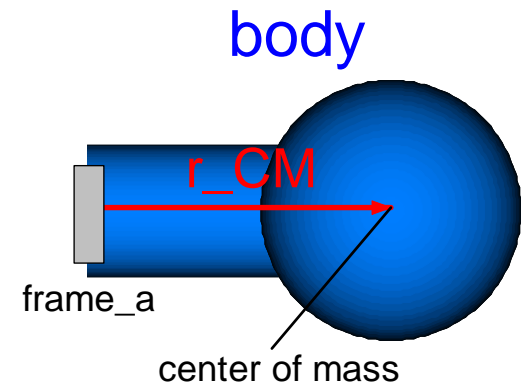


```
frame_b.r_0 = frame_a.r_0 + Frames.resolve1(frame_a.R, r);  
frame_b.R = frame_a.R;  
  
/* Force and torque balance */  
zeros(3) = frame_a.f + frame_b.f;  
zeros(3) = frame_a.t + frame_b.t + cross(r, frame_b.f);
```

# Modelica Multibody Basics: Components II

## ➤ Dynamics

- Newton-Euler equations
- MultiBody.Parts.Body

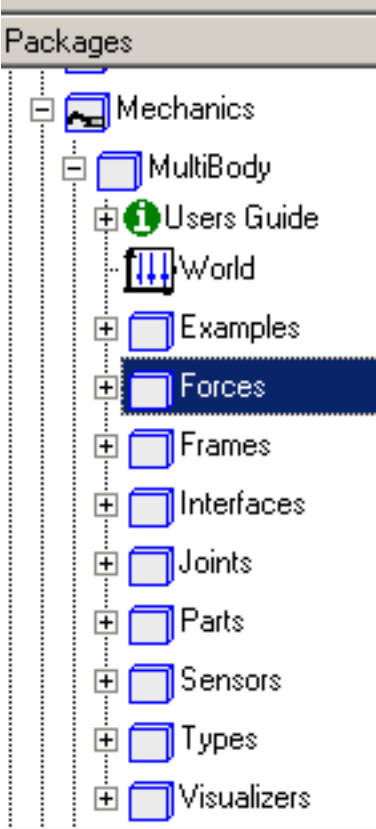


```
// import Modelica.Mechanics.MultiBody.Frames;
// translational kinematic differential equations resolved in local frame_a
v_a = Frames.resolve2(frame_a.R, der(frame_a.r_0));
a_a = der(v_a);

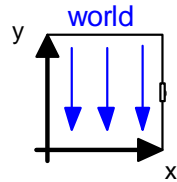
// rotational kinematic differential equations
w_a = Modelica.Mechanics.MultiBody.Frames.angularVelocity2(frame_a.R);
z_a = der(w_a);

// Newton/Euler equations with respect to center of mass
a_CM = a_a + cross(z_a, r_CM) + cross(w_a, cross(w_a, r_CM));
f_CM = m*a_CM;
t_CM = I*z_a + cross(w_a, I*w_a);
frame_a.f = f_CM;
frame_a.t = t_CM + cross(r_CM, f_CM);
```

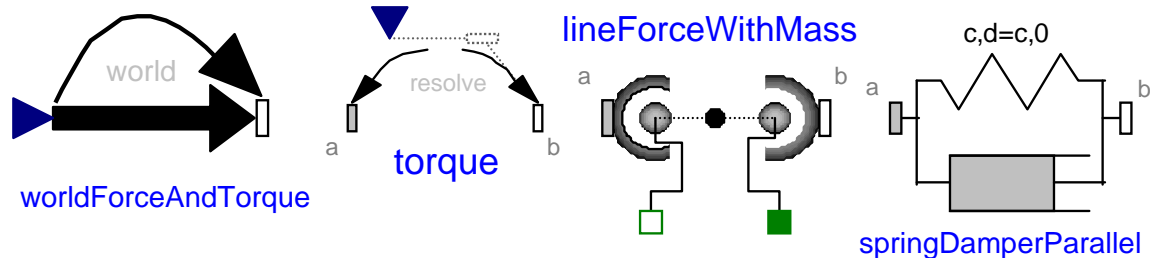
# Modelica Multibody Basics: Elementary Components I



- Modelica.Mechanics.MultiBody.World
  - defines inertial frame, gravity, animation defaults

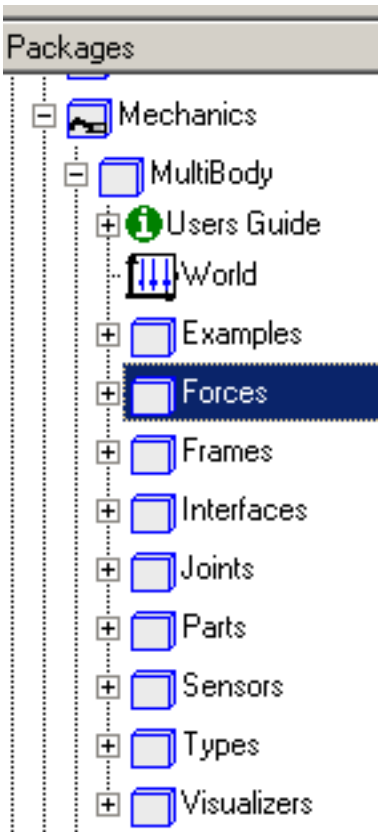


- Modelica.Mechanics.MultiBody.Forces
  - different resolution properties
  - interface to Real input functions and 1D mechanics
  - several spring/damper configurations

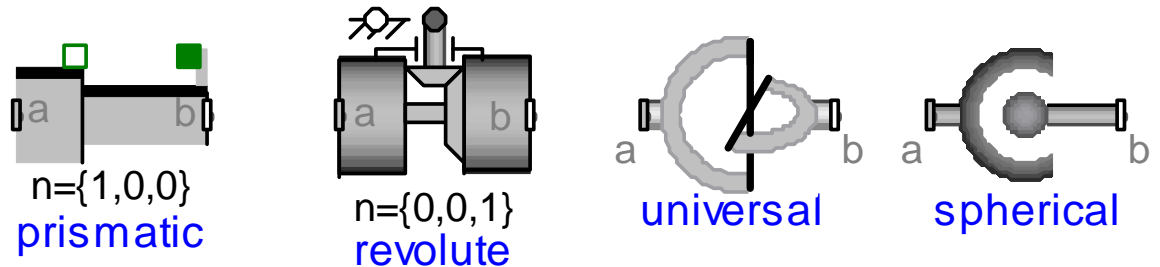




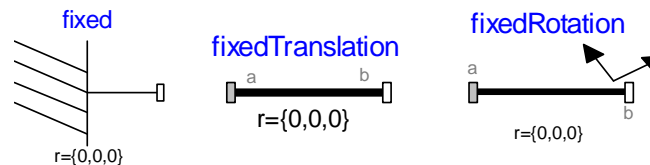
# Modelica Multibody Basics: Elementary Components II



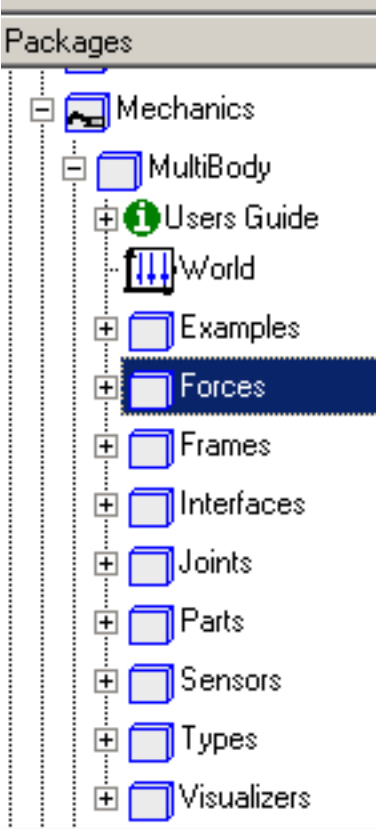
- Modelica.Mechanics.MultiBody.Joints
  - define specific degree of freedom
  - capability to set-up initial configuration
  - interface to/for 1D mechanics and rheonom motion
  - e.g.:



- Modelica.Mechanics.MultiBody.Parts
  - Fixed, FixedTranslation and FixedRotation

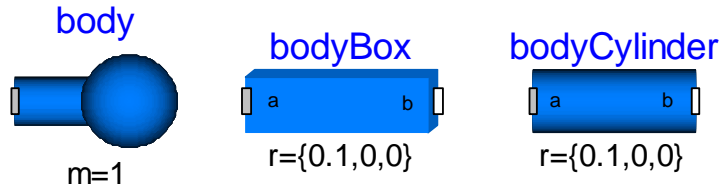


# Modelica Multibody Basics: Elementary Components III



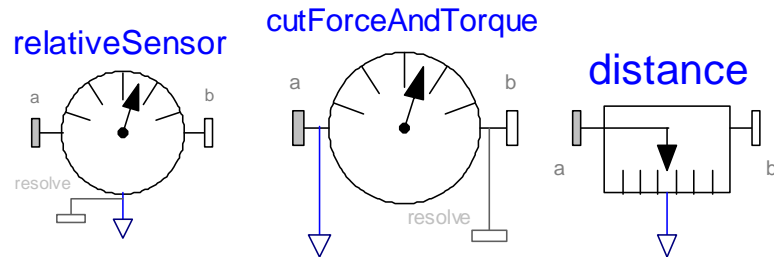
➤ Modelica.Mechanics.MultiBody.Parts

➤ Rigid bodies with predefined geometric shapes

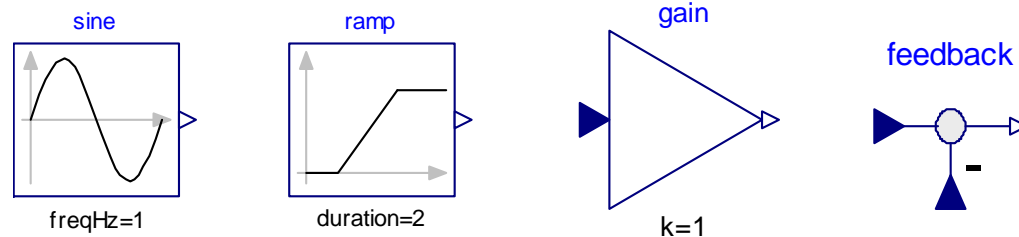


➤ Modelica.Mechanics.MultiBody.Sensors

➤ for control and validation purposes



➤ Modelica.Blocks.Sources + Modelica.Blocks.Math



# Modelica Multibody Basics: Analysis Methods

- Model check
- Experiment setup, translation and time simulation

The screenshot displays the Modelica IDE interface for a model named 'InversePendulum'. The top window shows the 'Messages - Dymola' panel with a successful check message: 'Check of Tutorial.exercises.InversePendulum: DAE having 1836 scalar unknowns and 1836 scalar equations. Check of Tutorial.exercises.InversePendulum successful.' The middle window shows the 'Experiment Setup' dialog with the following configuration:

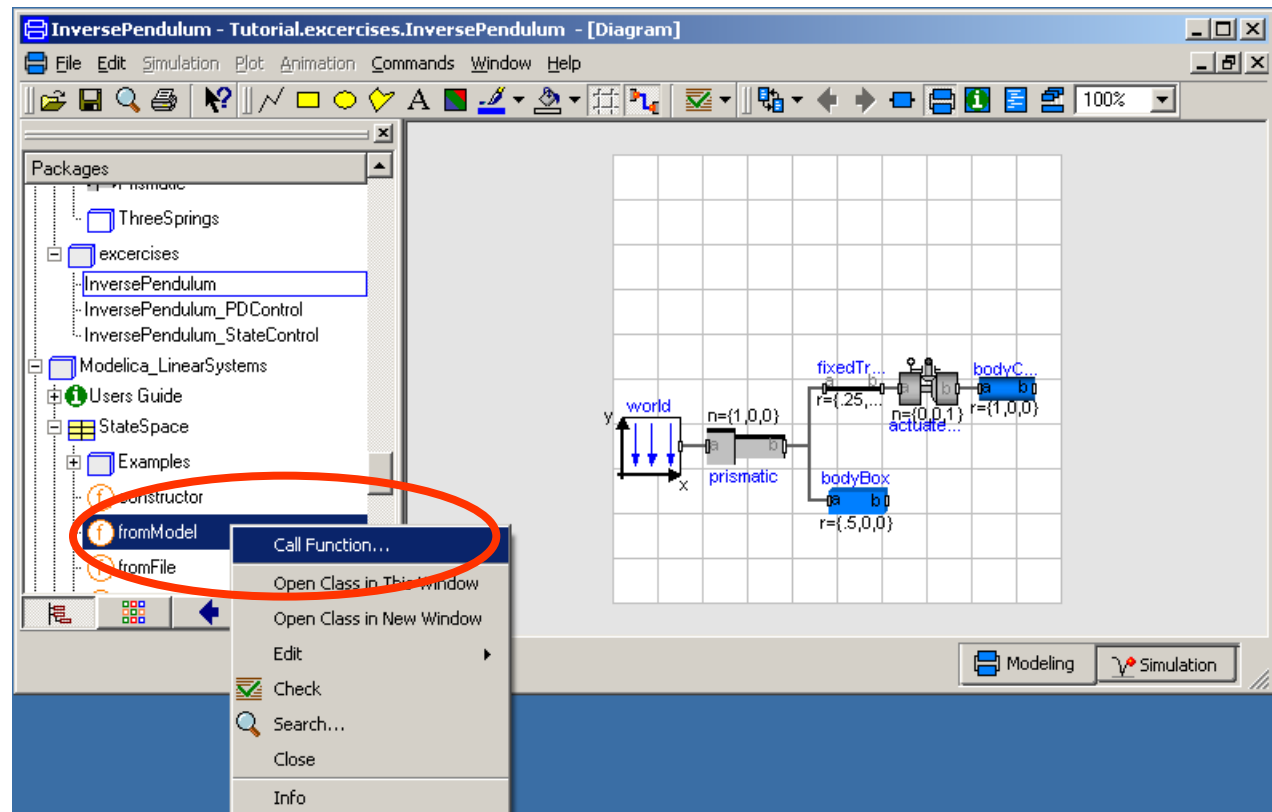
- Experiment Name: InversePendulum
- Simulation interval: Start time 0, Stop time 5
- Output interval: Number of intervals 500 (selected)
- Integration: Algorithm Dassl, Tolerance 0.0001, Fixed Integrator Step 0

The bottom window shows the 'Variables' table with the following structure:

Variables	Values	Unit	De
InversePendulum_StateControl 1			
InversePendulum 2			
world			
bodyBox			
bodyCylinder			
fixedTranslation			
actuatedRevolute			
prismatic			

# Modelica Multibody Basics: Analysis Methods

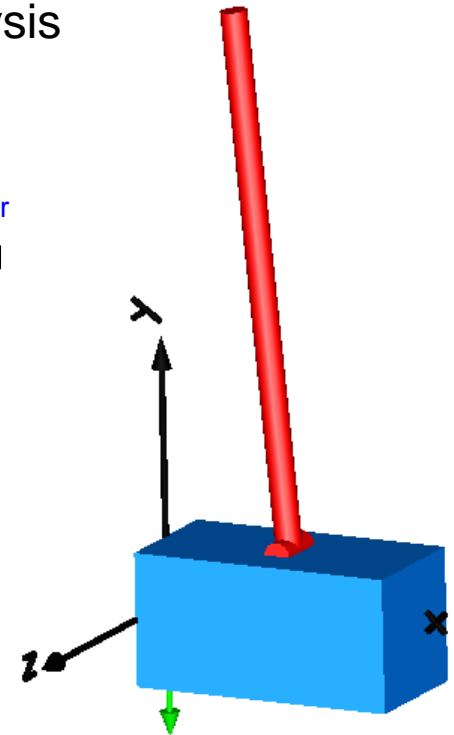
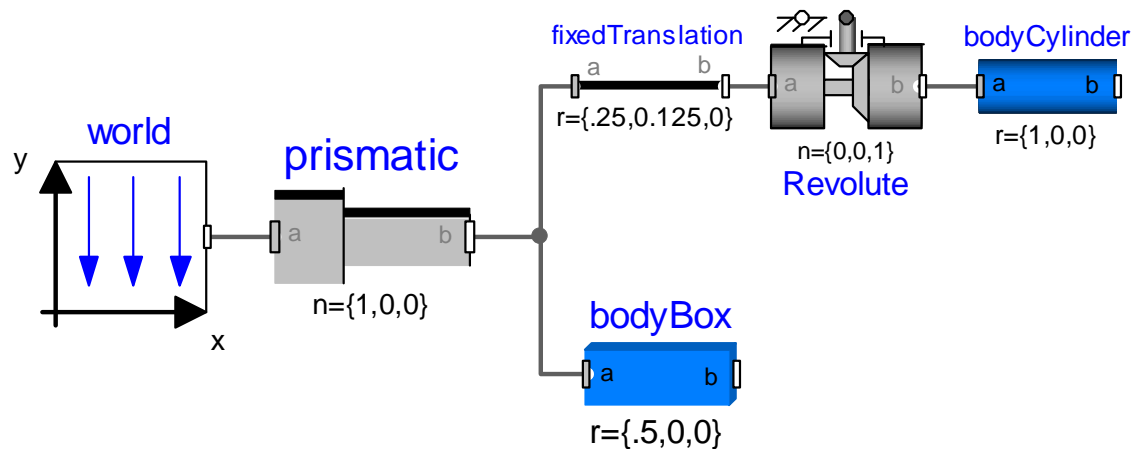
- Model check
- Experiment setup, translation and time simulation
- Eigenvalue analysis
  - Menu: File→Libraries→LinearSystems



# Example 1: Control of an inverse pendulum I

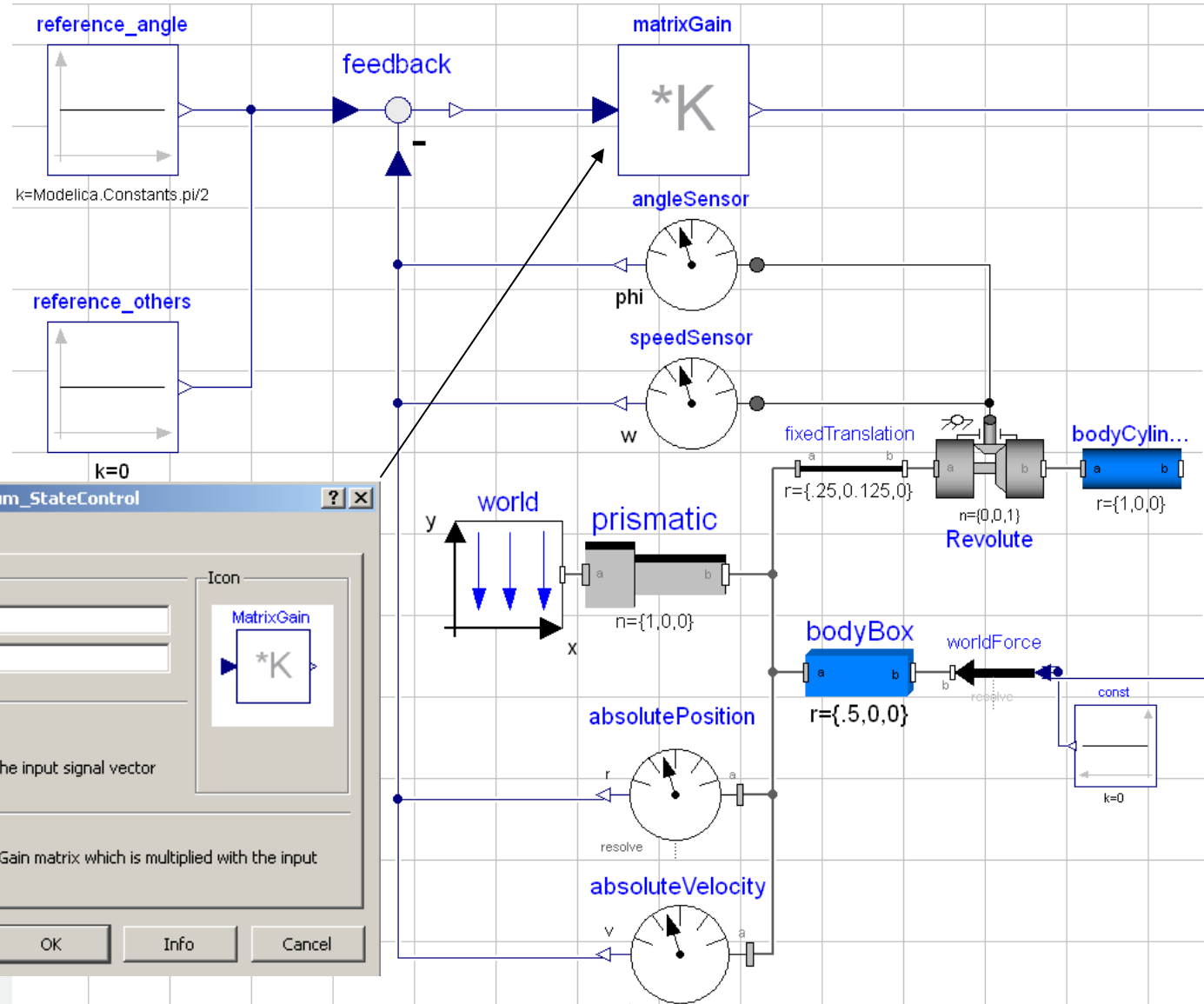
## ➤ Initial model

- Box: 0.5 x 0.25 x 0.25 m
- actuatedRevolute:  $\text{phi.start} = 95^\circ$ ,  $\text{fixed} = \text{true}$
- perform time simulation and eigenvalue analysis



# Exercise 1: Control of an inverse Pendulum II

## ➤ state space control



**matrixGain in IWES.Multibody.InversePendulum\_StateControl**

General | Add modifiers

Component

Name: matrixGain

Comment:

Model

Path: Modelica.Blocks.Math.MatrixGain

Comment: Output the product of a gain matrix with the input signal vector

Parameters

K: [5000, 1200, -100, -250] Gain matrix which is multiplied with the input

OK Info Cancel

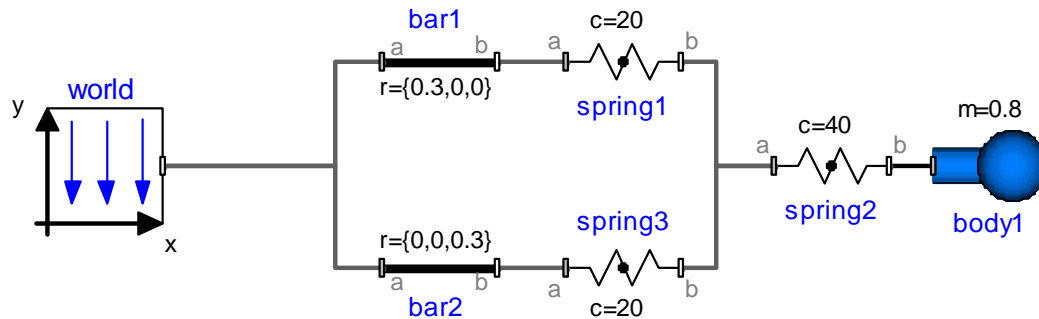
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# Modelica Multibody Advanced: State selection I

- Joints AND bodies have potential states
  - number of joints is independent from number of bodies
  - an assignment of joints to bodies is not mandatory
  - force elements may be connected to each other
  - e.g.:

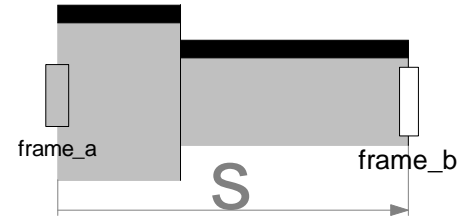


- here: body coordinates: position, quaternions and their derivatives are used as states

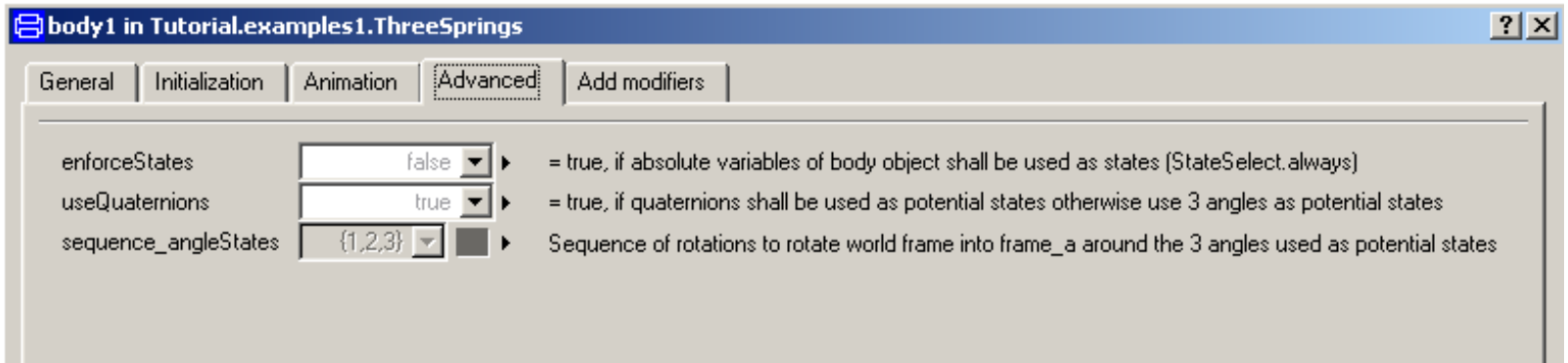


# Modelica Multibody Advanced: State selection II

- relative joint coordinates are used as states if possible
  - default: `stateSelect = StateSelect.prefer`
  - e.g. `Multibody.Joints.Prismatic`



```
final parameter Real e[3]=Modelica.Mechanics.MultiBody.Frames.normalize(n)
  "Unit vector in direction of prismatic axis n";
SI.Position s(stateSelect=if enforceStates then
  StateSelect.always else StateSelect.prefer)
  "Relative distance between frame_a and frame_b";
SI.Velocity v(stateSelect=if enforceStates then StateSelect.always else
  StateSelect.prefer) "First derivative of s (relative velocity)";
```

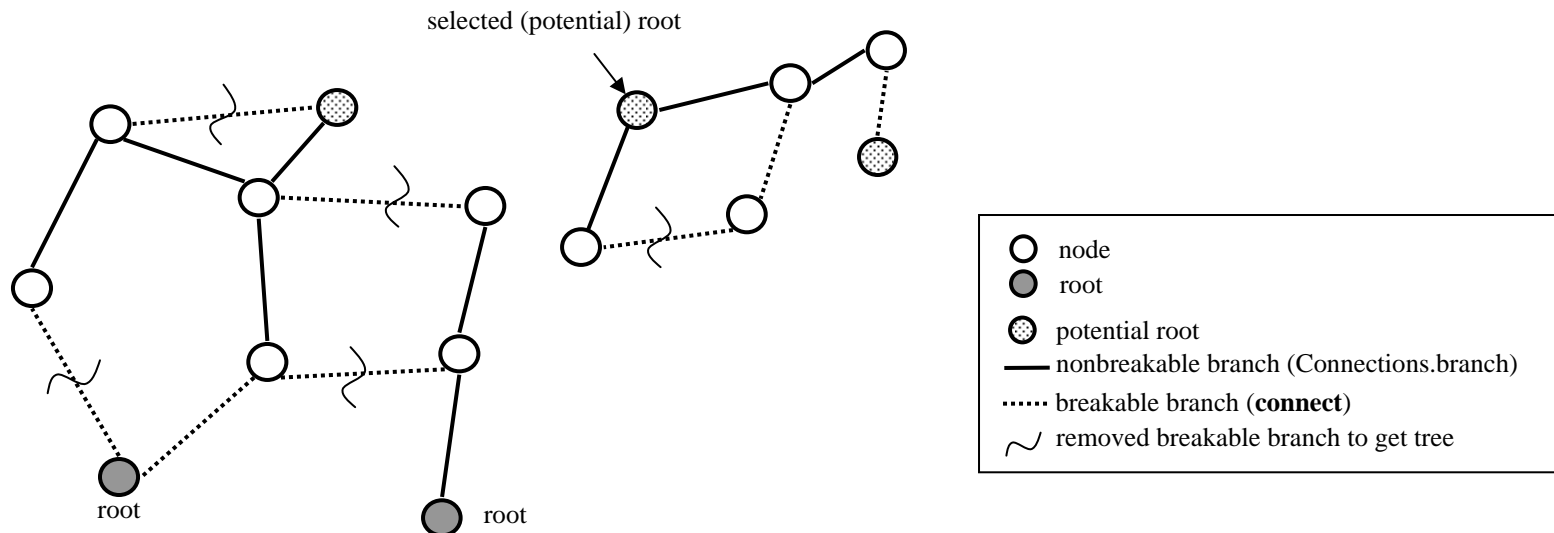


- Advanced user may influence state selection directly

# Modelica Multibody Advanced: Loops I

## ➤ Standard case

- no specific action by the user is required
- every connector is one node in the virtual connection graph
- roots of the virtual connection graph are found, e.g. world.frame\_b
- loops are virtually broken



# Modelica Multibody Advanced: Loops I

## ➤ Standard case

- no specific action by the user is required
- every connector is one node in the virtual connection graph
- roots of the virtual connection graph are found, e.g. world.frame\_b
- loops are virtually broken
- the related constraint equations are provided

⇒ DAE

$$0 = f(\dot{x}, x, y, t, \dots) \quad \dim(f) = \dim(x) + \dim(y)$$

- Equations are rearranged to get a sequence for model evaluation  
(**B**lock **L**ower **T**riangle-partitioning)

$$\begin{array}{l} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array} \begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix} \Rightarrow \begin{array}{l} f_2 \\ f_4 \\ f_3 \\ f_5 \\ f_1 \end{array} \begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

# Modelica Multibody Advanced: Loops I

## ➤ Standard case

- no specific action by the user is required
- every connector is one node in the virtual connection graph
- roots of the virtual connection graph are found, e.g. world.frame\_b
- loops are virtually broken
- the related constraint equations are provided

⇒ DAE

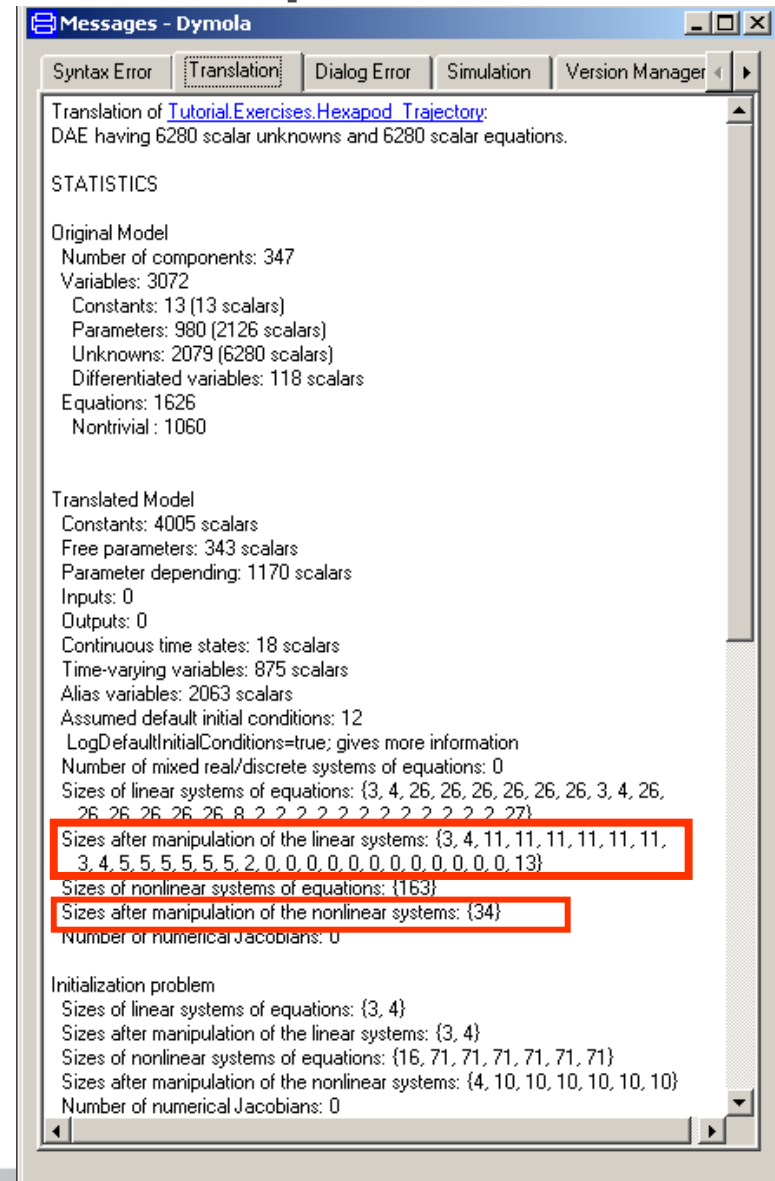
$$0 = f(\dot{x}, x, y, t, \dots) \quad \dim(f) = \dim(x) + \dim(y)$$

- Equations are rearranged to get a sequence for model evaluation (**B**lock **L**ower **T**riangle-partitioning)
- Equations to be differentiated are determined (Pantelides algorithm)
- superfluous potential states are deselected dynamically (dummy derivative method) ⇒ ODE:

$$\dot{x} = f(x, t, \dots)$$

# Modelica Multibody Advanced: Loops II

- review Translation Log in order to streamline simulation performance with model adjustments



```
Messages - Dymola
Syntax Error | Translation | Dialog Error | Simulation | Version Manager

Translation of Tutorial.Exercises.Hexapod.Trajectory:
DAE having 6280 scalar unknowns and 6280 scalar equations.

STATISTICS

Original Model
Number of components: 347
Variables: 3072
  Constants: 13 (13 scalars)
  Parameters: 980 (2126 scalars)
  Unknowns: 2079 (6280 scalars)
  Differentiated variables: 118 scalars
Equations: 1626
  Nontrivial: 1060

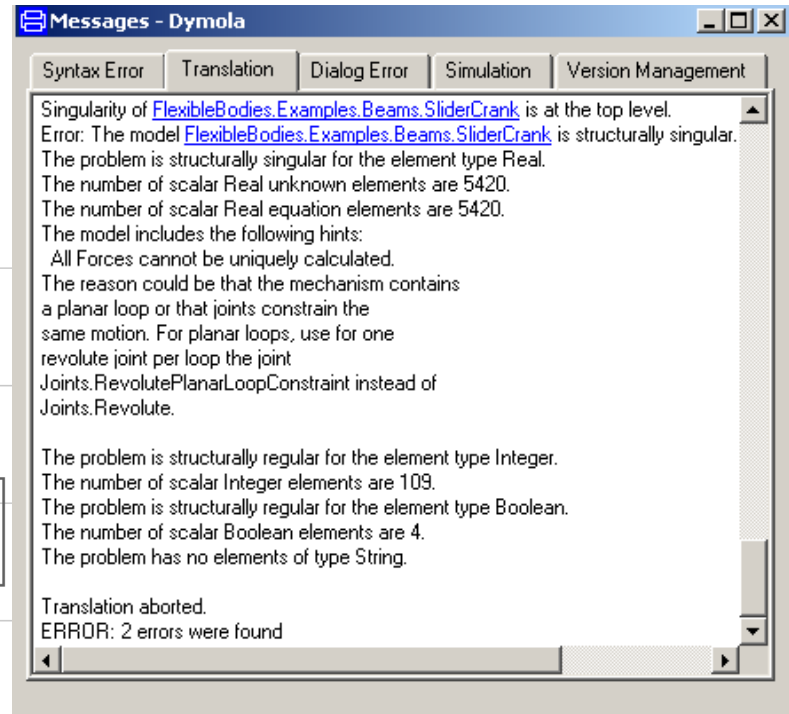
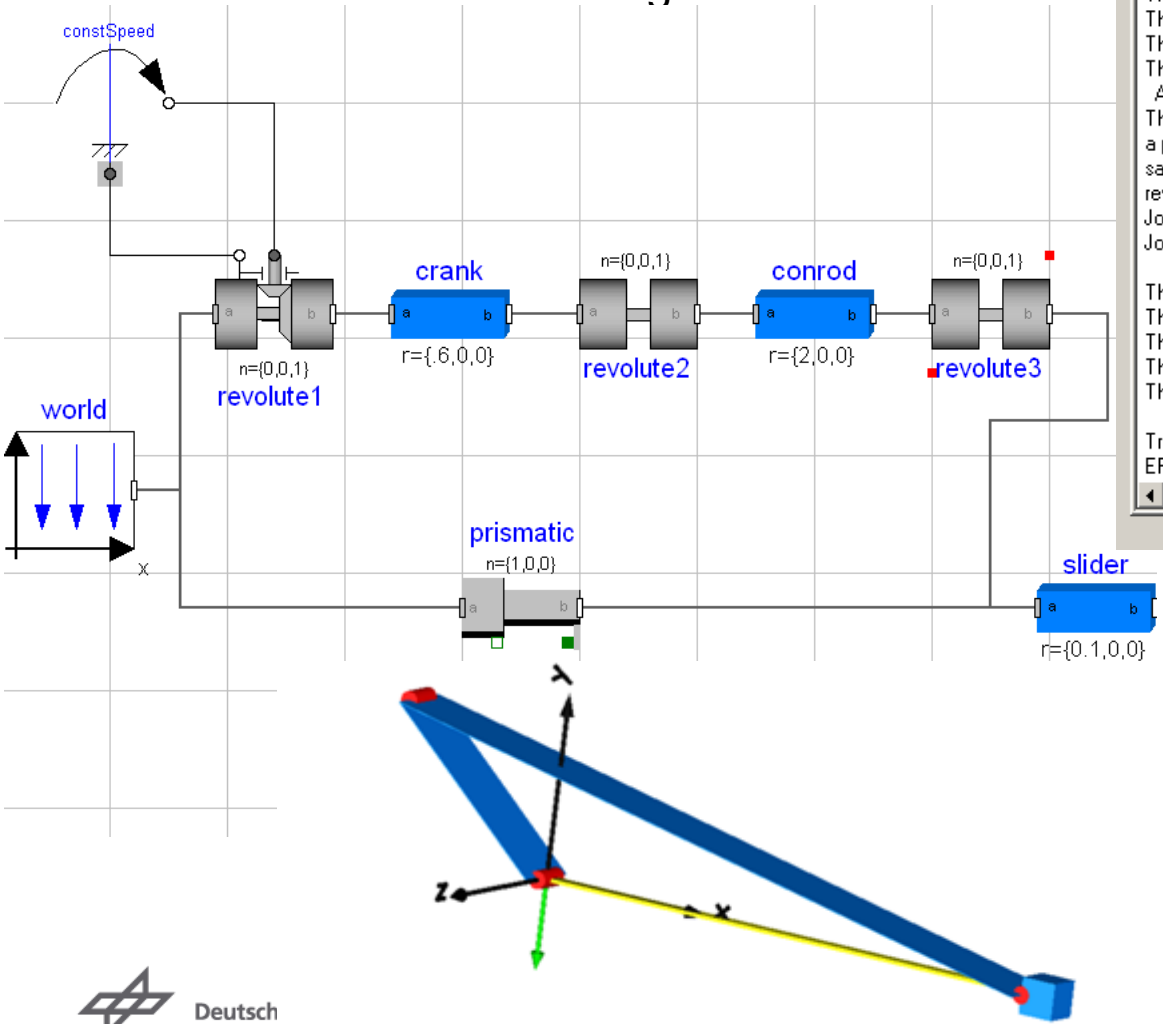
Translated Model
Constants: 4005 scalars
Free parameters: 343 scalars
Parameter depending: 1170 scalars
Inputs: 0
Outputs: 0
Continuous time states: 18 scalars
Time-varying variables: 875 scalars
Alias variables: 2063 scalars
Assumed default initial conditions: 12
LogDefaultInitialConditions=true: gives more information
Number of mixed real/discrete systems of equations: 0
Sizes of linear systems of equations: {3, 4, 26, 26, 26, 26, 26, 26, 3, 4, 26,
26, 26, 26, 26, 26, 8, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2}
Sizes after manipulation of the linear systems: {3, 4, 11, 11, 11, 11, 11, 11,
3, 4, 5, 5, 5, 5, 5, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 13}
Sizes of nonlinear systems of equations: {163}
Sizes after manipulation of the nonlinear systems: {34}
Number of numerical Jacobians: 0

Initialization problem
Sizes of linear systems of equations: {3, 4}
Sizes after manipulation of the linear systems: {3, 4}
Sizes of nonlinear systems of equations: {16, 71, 71, 71, 71, 71, 71}
Sizes after manipulation of the nonlinear systems: {4, 10, 10, 10, 10, 10, 10}
Number of numerical Jacobians: 0
```

# Modelica Multibody Advanced: Loops III

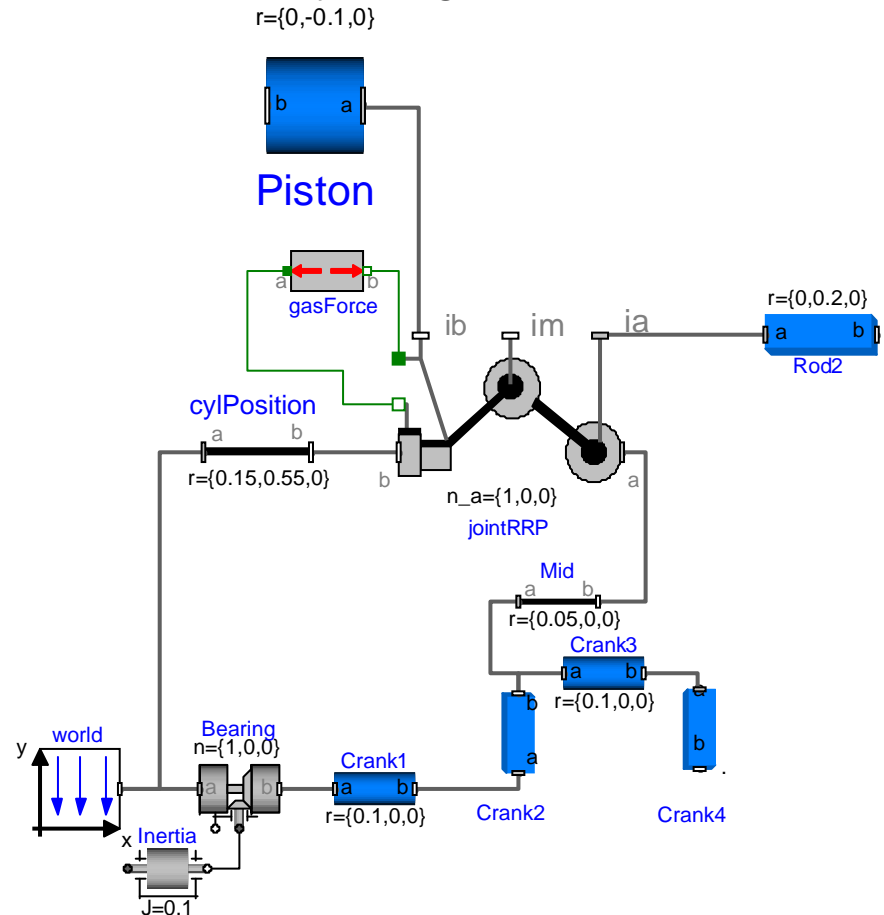
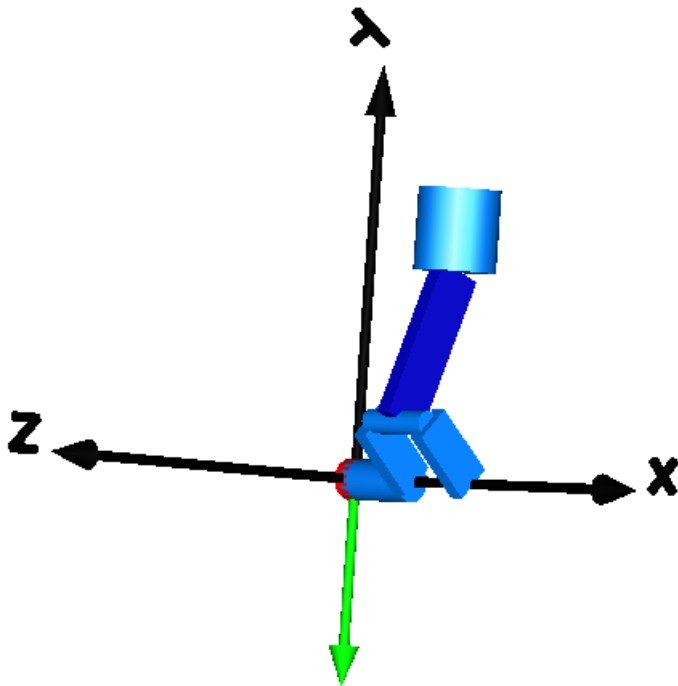
➤ Planar loops

➤ error message



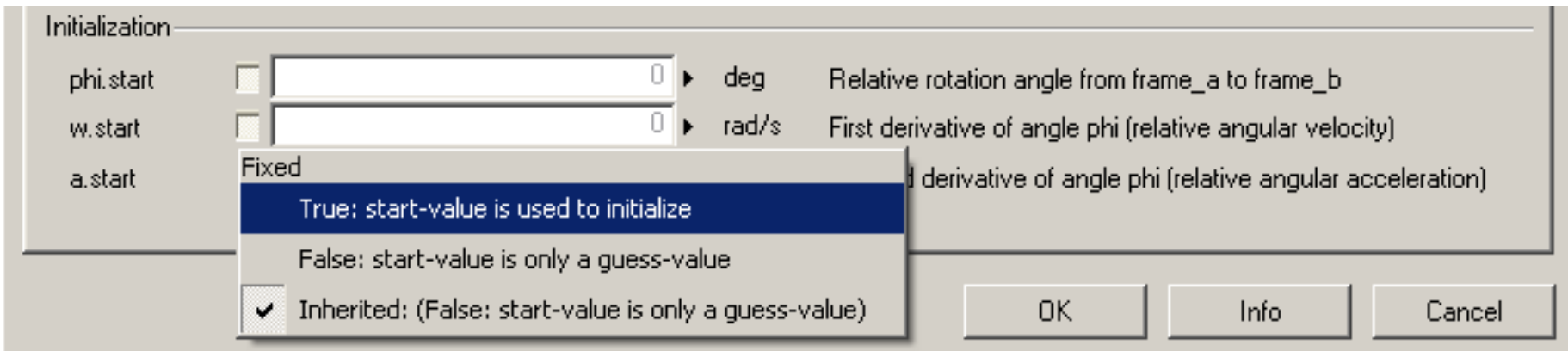
# Modelica Multibody Advanced: Loops IV

- Use of aggregated joint objects
  - to profit from analytical loop handling according to the „characteristic pair of joints“ method by the group of Prof. Hiller



# Modelica Multibody Advanced: Initialisation

- Initialisation default:
  - every state is assumed to be arbitrary unless otherwise provided
  - Newton solver starts with guess value zero in order to find consistent initial states unless otherwise provided
- If initialisation fails
  - determine, i.e. fix, characteristic variables/states in order to influence the system of equations to solve
  - provide „good“ guesses for initial states
  - be aware of singular positions, e.g. piston at bottom dead center
  - keep initialisation system consistent



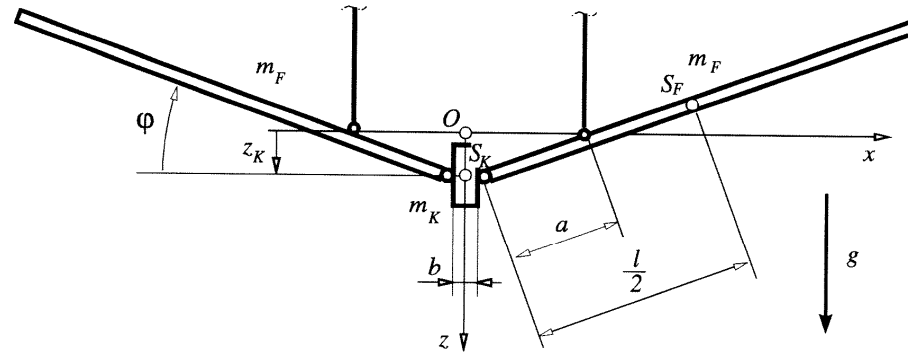


# Exercise 2: The Flying Gull

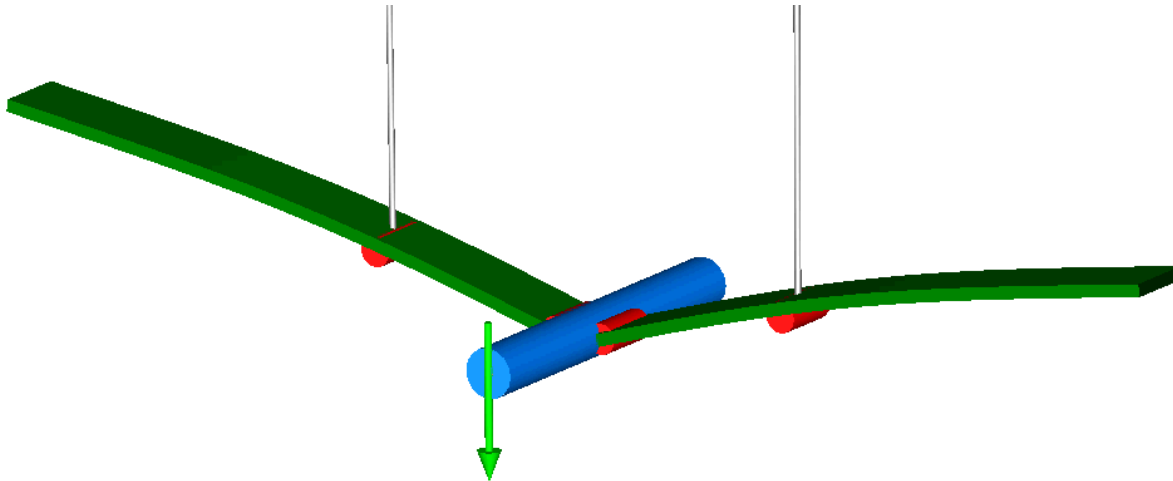
## Aufgabe nach Weidemann/Pfeiffer TM 94:

### Aufgabe 10

Ein beliebtes Kinderspielzeug ist die 'Fliegende Möwe'. Sie besteht aus zwei identischen Flügeln (schlanke, homogene Balken, jeweils Länge  $l$  und Masse  $m_F$ ), welche um die Längsachse der Möwe drehbar am Zentralkörper (Masse  $m_K$ , Schwerpunkt  $S_K$ ) aufgehängt sind. Die Breite  $b$  des Zentralkörpers sei vernachlässigbar klein. Die Möwe ist an zwei masselosen, sehr langen Fäden jeweils im Abstand  $a$  vom Zentralkörper so aufgehängt, daß die Aufhängepunkte immer auf der  $x$ -Achse des raumfesten  $x$ -,  $y$ -,  $z$ -Koordinatensystems (Ursprung  $O$ ) liegen. Zur Beschreibung des Systems dient neben der Auslenkung  $z_K$  des Zentralkörpers auch der Winkel  $\varphi$  der Flügel gegenüber einer Waagerechten.

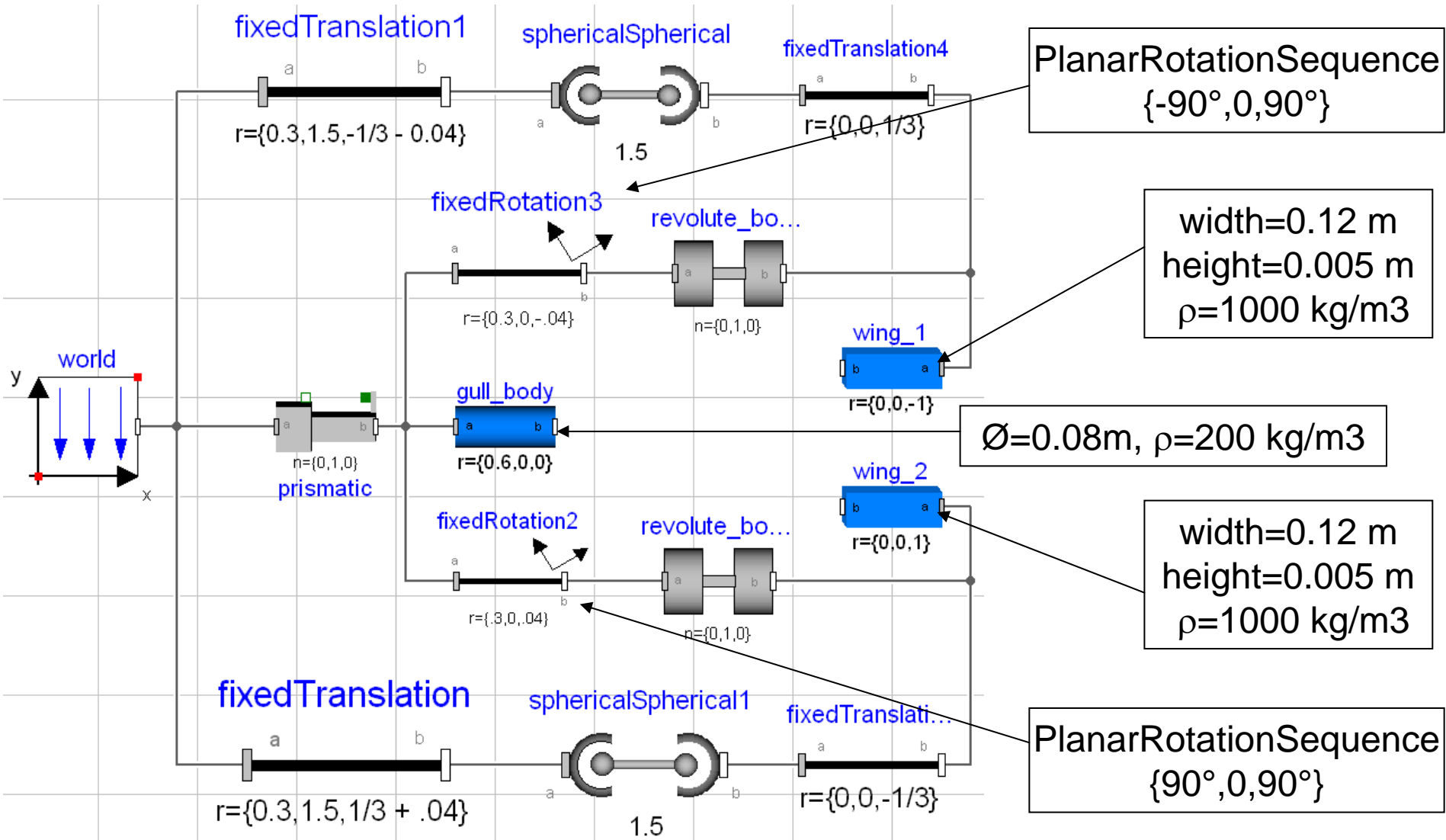


- Wie lautet die kinematische Abhängigkeit zwischen  $\varphi$  und  $z_K$  ?
- Wie groß ist die kinetische Energie des Gesamtsystems ?
- Wie groß ist die potentielle Energie des Gesamtsystems ?
- Wie lautet die Bewegungsgleichung für die Koordinate  $\varphi$  ?



➤ Look for the equilibrium position !

# Exercise 2: The Flying Gull I

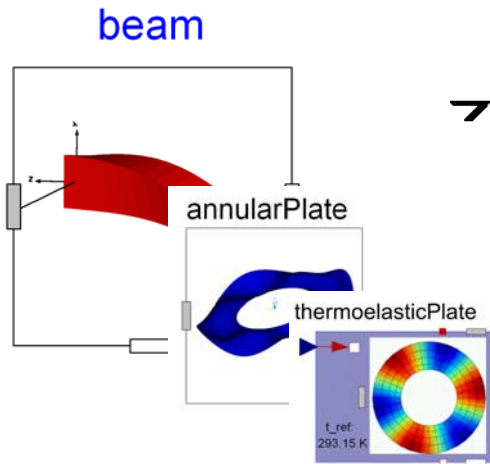


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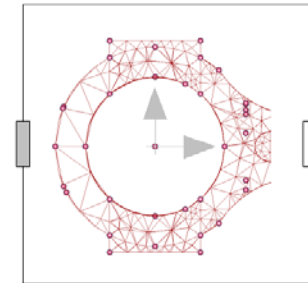
# The name of the game : 2 types of modelling elements



## What do they have in common ?

- Floating frame of reference approach
- Structure of equations of motion
- Data structure, so called SID (Standard-Input-Data: Wallrapp '94)

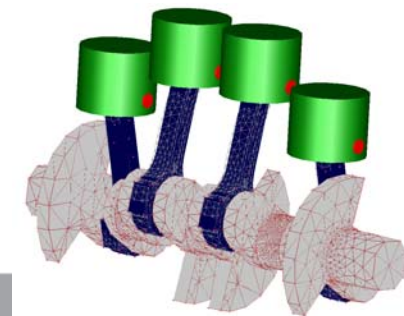
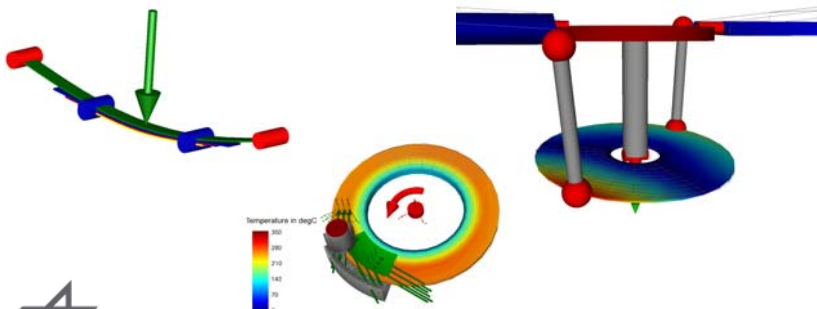
## modalBody



## In what do they differ ?

- Semi-analytical description implemented in Modelica
- Modelica generates SID
- Animation uses analytical description

- FEM-based body description (Abaqus-SID-interface, SIMPACK-FEMBS)
- Modelica reads externally generated SID file
- Modelica reads externally generated animation data (wavefront) file



# Theory: the equations of motion

➤ principle of virtual power

$$\int \delta \mathbf{v} (\mathbf{f} - \mathbf{a}) \, dm = 0$$

➤ equations of motion: here

$$\boldsymbol{\omega} := \boldsymbol{\omega}_R, \quad \tilde{\boldsymbol{\omega}} := \boldsymbol{\omega} \times$$

$$\begin{pmatrix} m\mathbf{I}_3 & & \text{sym.} \\ m\tilde{\mathbf{d}} & \mathbf{J} & \\ \mathbf{C}_t & \mathbf{C}_r & \mathbf{M}_e \end{pmatrix} \begin{pmatrix} \mathbf{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\mathbf{q}} \end{pmatrix} + \begin{pmatrix} 2\tilde{\boldsymbol{\omega}}\mathbf{C}_t^T\dot{\mathbf{q}} + \tilde{\boldsymbol{\omega}}\tilde{\boldsymbol{\omega}}\mathbf{d} \\ \mathbf{G}_r\dot{\mathbf{q}}\boldsymbol{\omega} + \tilde{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega} \\ \mathbf{G}_e\dot{\mathbf{q}}\boldsymbol{\omega} + \mathbf{O}_e\boldsymbol{\Omega} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{K}_e\mathbf{q} + \mathbf{D}_e\dot{\mathbf{q}} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_a \\ \mathbf{f}_\alpha \\ \mathbf{f}_q \end{pmatrix}$$

the generalized Newton-Euler-equations of motion of an unconstrained deformable body

➤ SID structure: definition of file format to file volume integrals

$$\mathbf{C}_r, \mathbf{C}_t, \mathbf{J}, \mathbf{M}_e, \mathbf{K}_e, \mathbf{D}_e, \mathbf{G}_r \dots$$



# Theory: 2nd order beam theory

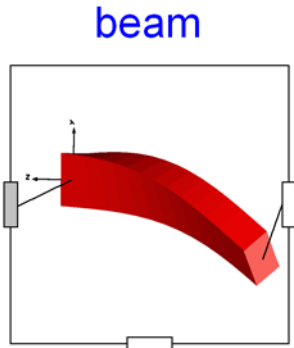
- Bending in **xy-** und **xz-plane**, **torsion** and **lengthening**

$$\mathbf{u}(x, t) = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \int_0^x v'^2 + w'^2 dx \\ -\int_0^x \int_0^{\bar{x}} \theta w'' d\bar{x} d\bar{x} + \int_0^x u'v' d\bar{x} \\ \int_0^x \int_0^{\bar{x}} \theta v'' d\bar{x} d\bar{x} + \int_0^x u'w' d\bar{x} \end{pmatrix}$$

- e.g. for bending in xy-plane:  $v(x, t) = \Phi_v(x)q_v(t)$
- analytical solutions of the eigenvalue problem of the Euler-Bernoulli-beam

$$\Phi_i = \begin{pmatrix} \cosh(\tau_i x) \\ \sinh(\tau_i x) \\ \cos(\tau_i x) \\ \sin(\tau_i x) \end{pmatrix}^T \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}_i \quad \mathbf{u}(c, t) = \Phi(c) \mathbf{q}(t) + \frac{1}{2} \begin{pmatrix} \mathbf{q}^T \Phi_x \\ \mathbf{q}^T \Phi_y \\ \mathbf{q}^T \Phi_z \end{pmatrix} \mathbf{q}$$

# FlexibleBodies Library: Beam Menu I



data in FlexibleBodies.Examples.Demo

General Add modifiers

Component

Name data

Comment

Model

Path FlexibleBodies.Interfaces.BeamData

Comment

Parameters

crossSection I\_beam

I 1.58 m length of beam

rho 7850 kg/m3 mass density

E 2.1e11 N/m^2 Young's modulus

G  $E/(2*(1 + 0.3))$  N/m^2 Shear modulus

xsi {5} specification of

Eigenmodes

bending\_xy ryConditionB=FlexibleBodies.Types.BoundaryCondition

bending\_xz ryConditionB=FlexibleBodies.Types.BoundaryCondition

torsion ryConditionB=FlexibleBodies.Types.BoundaryCondition

lengthening ryConditionB=FlexibleBodies.Types.BoundaryCondition

Parameters

crossSection I\_beam

I 1.58 m

rho 7850 kg/m3

E 2.1e11 N/m^2

G  $E/(2*(1 + 0.3))$  N/m^2

xsi {5}

Eigenmodes

bending\_xy

T\_beam

general

Icon

BeamData

I\_beam crossSection

I\_beam

Description

I-profile cross section

Inputs

width .02 m outer contour dimension in y-direction (along flange)

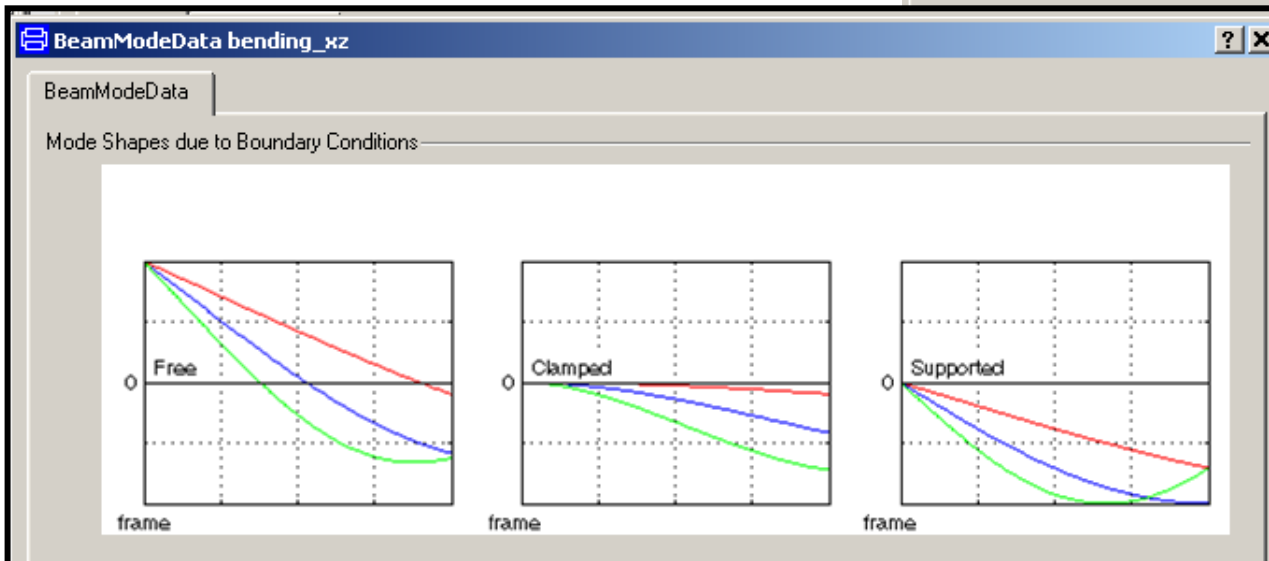
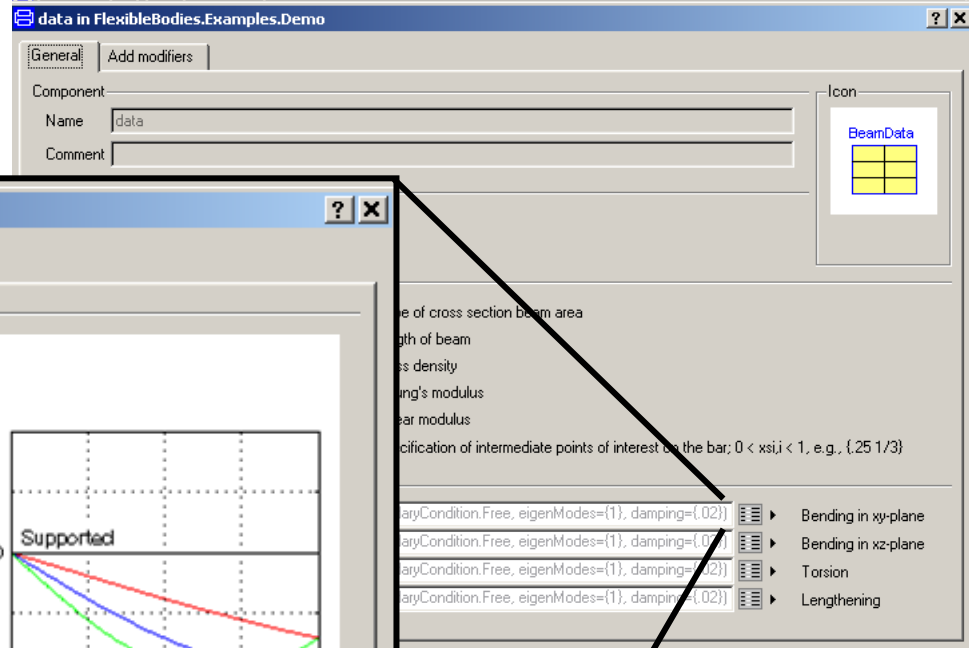
t\_bar 0.001 m thickness of central bar

height .03 m outer contour dimension in z-direction

t\_flange 0.001 m thickness of flanges

OK Info Close

# FlexibleBodies Library: Beam Menu II



Boundary Conditions

at frame\_a  Free  Clamped  Supported ▾

at frame\_b  Free  Clamped  Supported ▾

Mode Numbers

eigenModes {1,2,4} ▾

damping {0.01,0.03,0.02} ▾

Ordinal numbers of eigen modes to be used (e.g. 1,2,4)

Damping of eigen modes

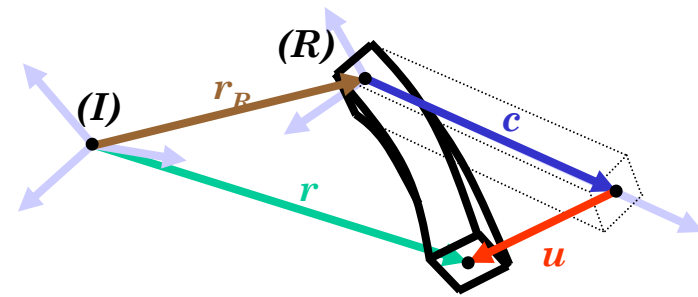
$$\Phi_i = \begin{pmatrix} \cosh(\tau_i x) \\ \sinh(\tau_i x) \\ \cos(\tau_i x) \\ \sin(\tau_i x) \end{pmatrix}^T \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}_i$$

**important: provide 1 and only 1 damping coefficient for each mode**



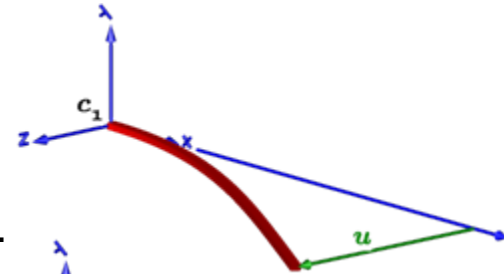
# Boundary Conditions I

$$\mathbf{r}(c, t) = \mathbf{r}_R(t) + \mathbf{c} + \mathbf{u}(c, t)$$



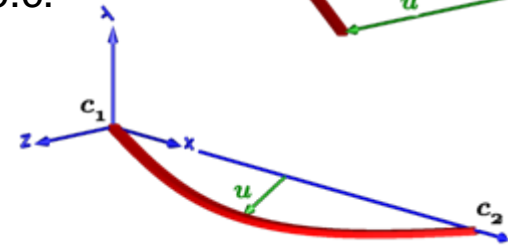
- 1st Option: tangent frame: clamped-free b.c. corresponds to cantilever beam

$$\mathbf{u}(c = 0, t) = \mathbf{0} \quad \frac{\partial \mathbf{u}}{\partial c}(c = 0, t) = \mathbf{0}$$



- 2nd Option: chord frame: supported-supported b.c.

$$\mathbf{u}(c_1) = \mathbf{0} \quad \mathbf{u}(c_2) \cdot \overline{c_1 c_2} = 0$$



- 3rd Option: Buckens frame: free-free b.c.  ${}^0C_r = {}^0C_t = {}^1d_C = \mathbf{0}$

$$\begin{pmatrix} mI_3 & & \text{sym.} \\ m\tilde{d}_C & J & \\ C_t & C_r & M_e \end{pmatrix} \begin{pmatrix} \mathbf{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\mathbf{q}} \end{pmatrix} = \mathbf{h}_\omega - \begin{pmatrix} 0 \\ 0 \\ K_e \mathbf{q} + D_e \dot{\mathbf{q}} \end{pmatrix} + \begin{pmatrix} \mathbf{f}_a \\ \mathbf{f}_\alpha \\ \mathbf{f}_e \end{pmatrix}$$

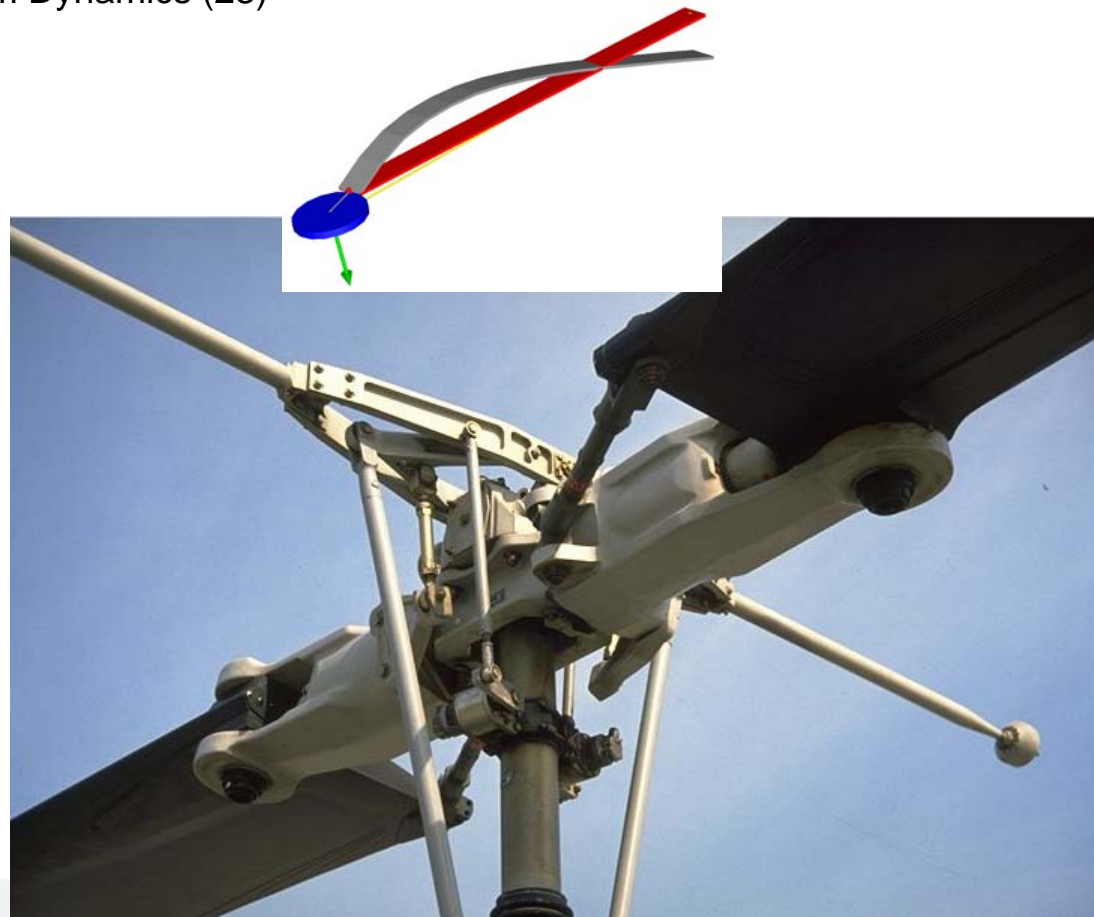
- Linearisation: choose reference frame in such a way that is as small as possible

$$\mathbf{u} \ll 1 \quad \Rightarrow \quad \text{prefer Buckenssystem}$$

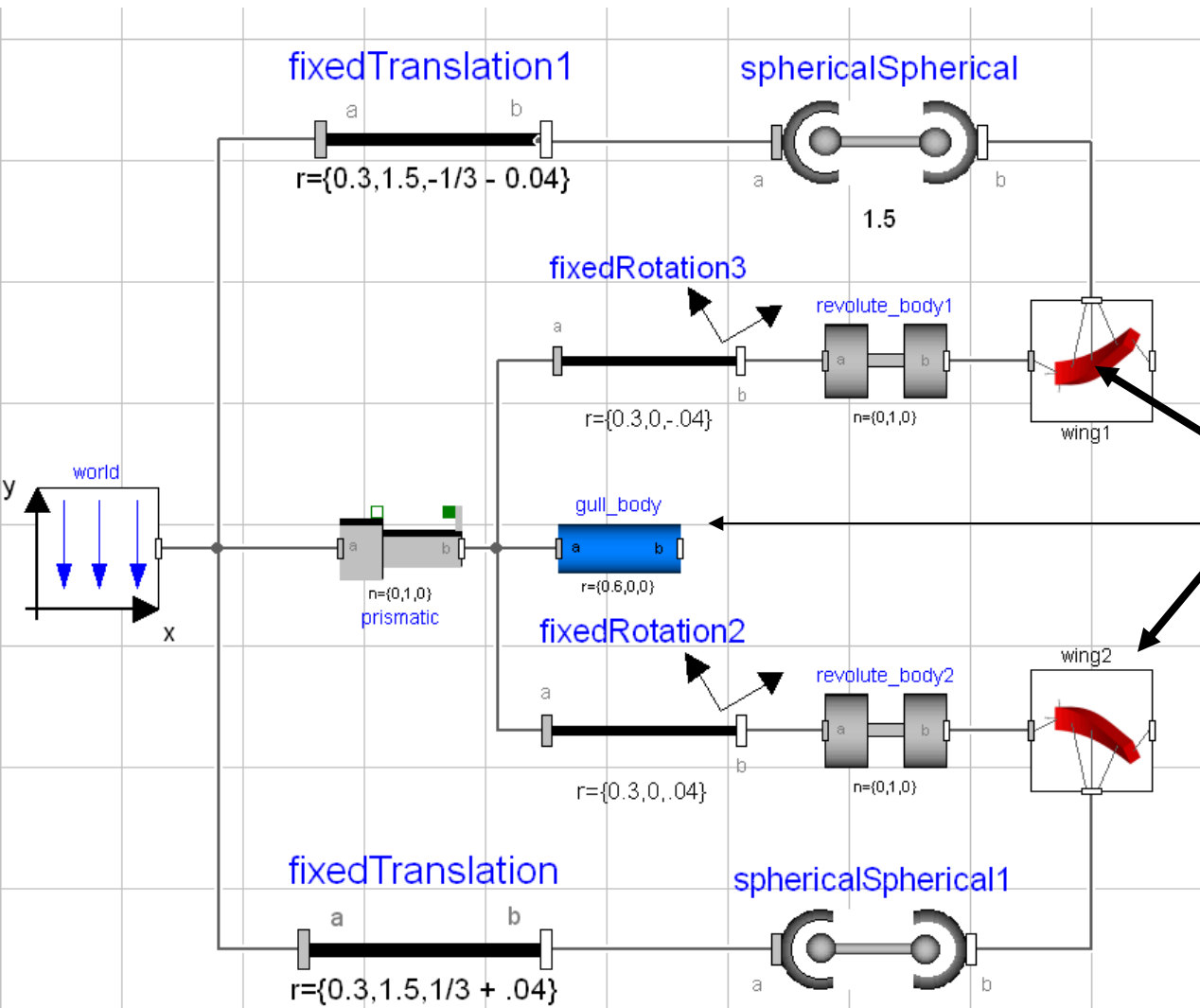
see Schwertassek/Wallrapp/Shabana99

# Boundary Conditions II

- Helikopter-Rotor (see Examples/Beam)
  - choose the boundary conditions according to the attachment joint
  - Heckmann2010: On the Choice of Boundary Conditions for Mode Shapes, Multibody System Dynamics (23)



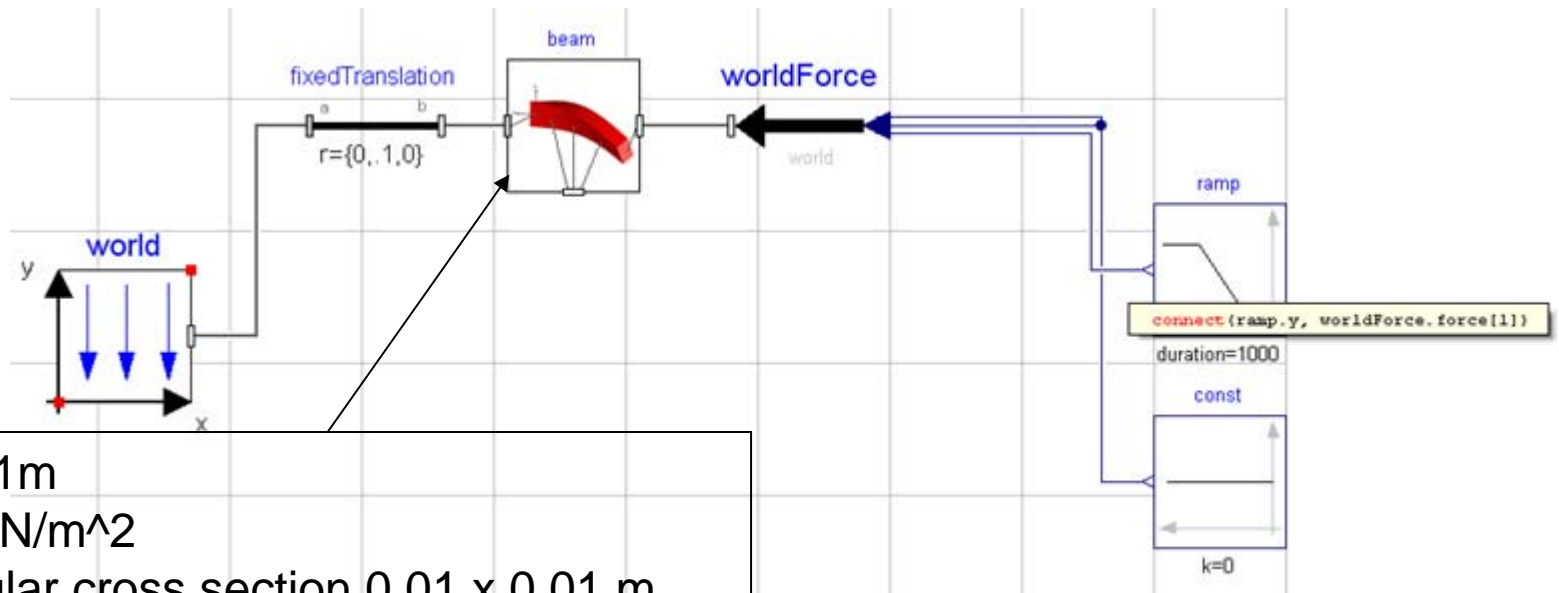
# Exercise 3: The Flying Gull II



rectangle crosssection  
 width=0.12 height=0.005  
 l= 1m  
 $\rho=1000 \text{ kg/m}^3$   
 $E=1e9 \text{ N/m}^2$   
 $\text{xsi}=\{1/3\}$   
 bending\_xz  
 supported/free, {1}, {0.02}  
 all other BeamModeData  
 fill(0,0), fill(0,0)

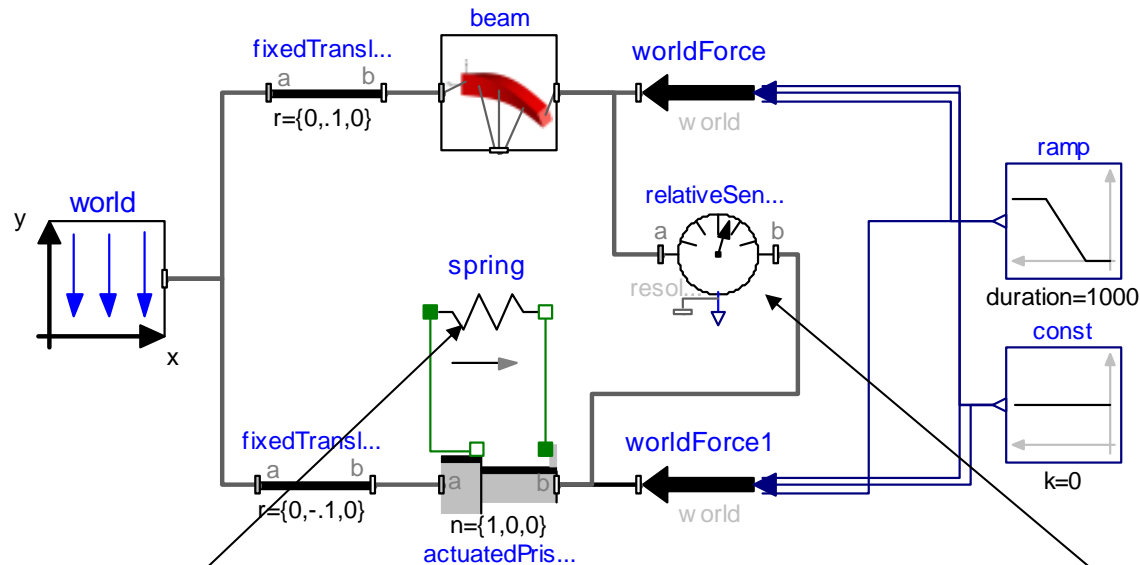
# Exercise 4: a classic Pitfall I

- Model the following system
  - (quasi-) static deformation: a thrust-force shortens the beam



# Exercise 4: a classic Pitfall II

➤ the system is now extended by an equivalent spring !



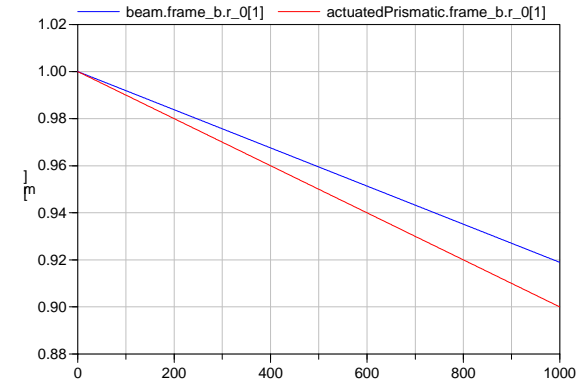
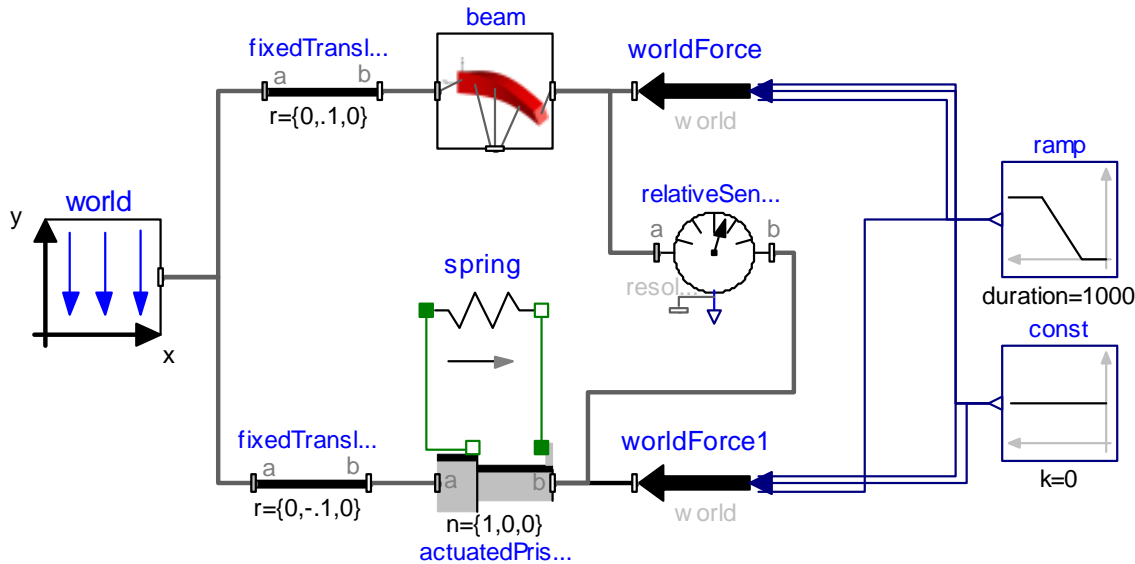
$c = 0.01\text{m} \cdot 0.01\text{m} \cdot 1\text{e}10 \text{ N/m}^2 / (1 \text{ m})$   
unstretched length = 1m

compare the deformations  
by measurements !

Plot the `relativeSensor.r_rel[1]` !  
Gradually increase the number of modes !

# Exercise 4: a classic Pitfall III

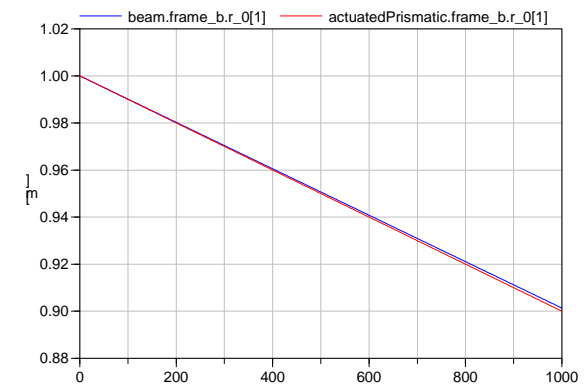
➤ static deflection: thrust force shortens beam and equivalent spring



1 eigenmode

	spring	beam	error
1 eigenmode	-10 cm	-8.1 cm	19 %
5 eigenmodes	-10 cm	-9.6 cm	4 %
10 eigenmodes	-10 cm	-9.8 cm	2 %
15 eigenmodes	-10 cm	-9.9 cm	1 %

comparison: deflections at the end



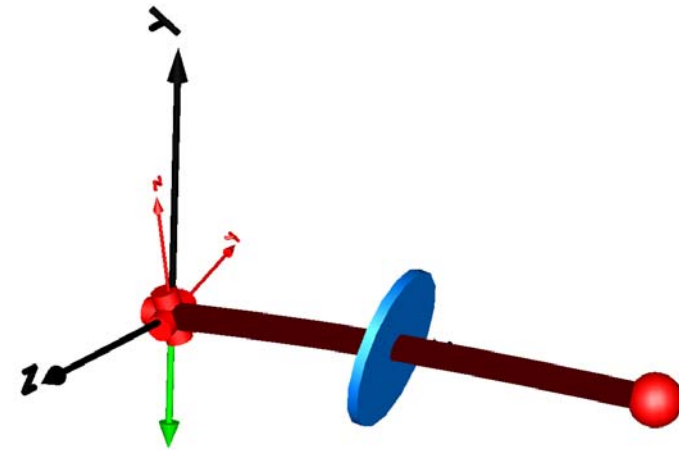
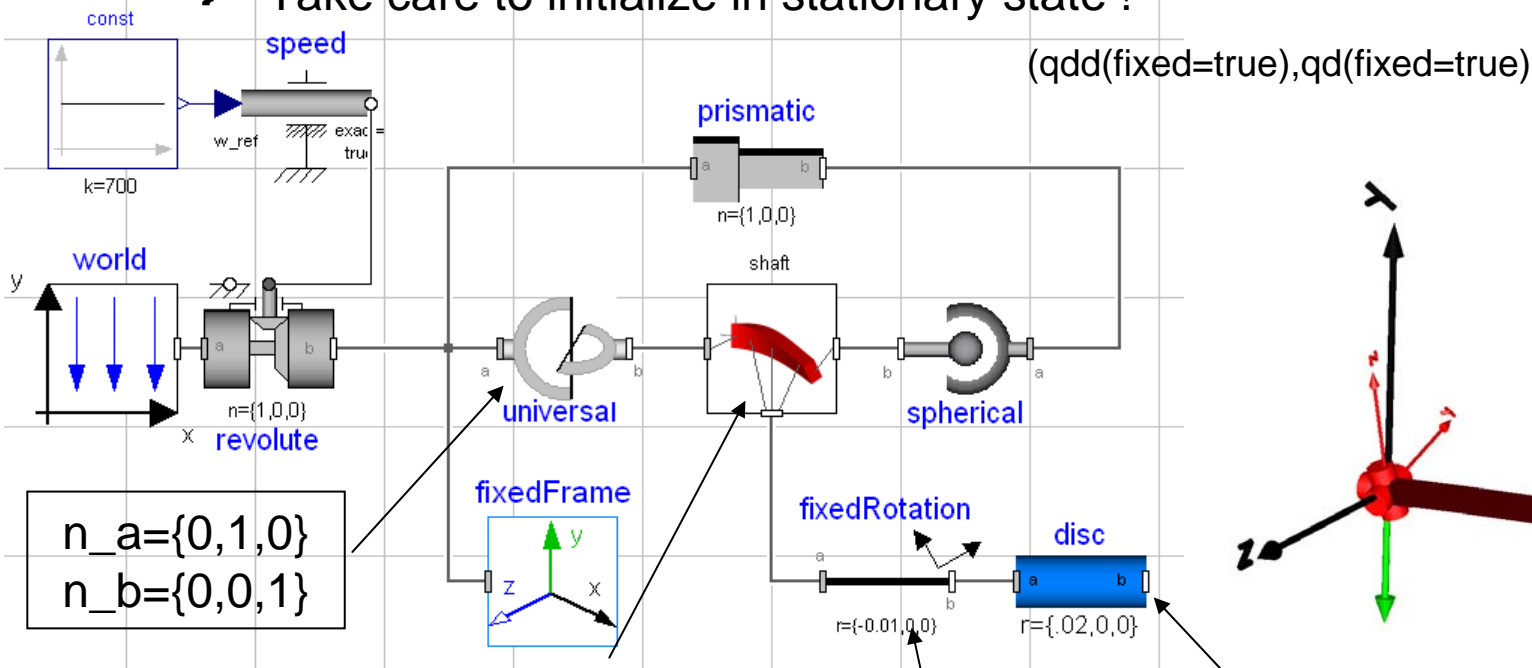
15 eigenmodes

# Exercise 4: a classic Pitfall IV

- Mechanical background
  - static deflections rely on elastic properties only
  - eigenmodes consider elastic and inertia properties
    - that's why they are well suited for dynamic problems
- Geometrical background
  - analytically:  $u = c \cdot x$
  - expansion with eigenmodes:  $u = \sin\left(\frac{2x}{\pi l}\right) + \sin\left(\frac{2x}{3\pi l}\right) + \dots$
- It is proven that Raleigh-Ritz approach converges against true value
  - but how fast ?
  - this is an extreme example, e.g. bending is less sensitive
- Check whether a higher number of modes changes results !

# Erexercice 5: unbalanced Shaft

- Instability at which rotational velocity ?
- Take care to initialize in stationary state !



circle  $\varnothing$  0.05m  
 $l=1$  m  
 1 xy- + 1xz- Bending mode  
 supported-supported

15° around y-axis

$\varnothing$  0.3m



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# Recall Theory: the equations of motion

➤ principle of virtual power

$$\int \delta \mathbf{v} (\mathbf{f} - \mathbf{a}) \, dm = 0$$

➤ equations of motion: here

$$\boldsymbol{\omega} := \boldsymbol{\omega}_R, \quad \tilde{\boldsymbol{\omega}} := \boldsymbol{\omega} \times$$

$$\begin{pmatrix} m\mathbf{I}_3 & & \text{sym.} \\ m\tilde{\mathbf{d}} & \mathbf{J} & \\ \mathbf{C}_t & \mathbf{C}_r & \mathbf{M}_e \end{pmatrix} \begin{pmatrix} \mathbf{a}_R \\ \boldsymbol{\alpha}_R \\ \ddot{\mathbf{q}} \end{pmatrix} + \begin{pmatrix} 2\tilde{\boldsymbol{\omega}}\mathbf{C}_t^T\dot{\mathbf{q}} + \tilde{\boldsymbol{\omega}}\tilde{\boldsymbol{\omega}}\mathbf{d} \\ \mathbf{G}_r\dot{\mathbf{q}}\boldsymbol{\omega} + \tilde{\boldsymbol{\omega}}\mathbf{J}\boldsymbol{\omega} \\ \mathbf{G}_e\dot{\mathbf{q}}\boldsymbol{\omega} + \mathbf{O}_e\boldsymbol{\Omega} \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{K}_e\mathbf{q} + \mathbf{D}_e\dot{\mathbf{q}} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_a \\ \mathbf{f}_\alpha \\ \mathbf{f}_q \end{pmatrix}$$

the generalized Newton-Euler-equations of motion of an unconstrained deformable body

➤ SID structure: definition of file format to file volume integrals

$$\mathbf{C}_r, \mathbf{C}_t, \mathbf{J}, \mathbf{M}_e, \mathbf{K}_e, \mathbf{D}_e, \mathbf{G}_r \dots$$



# SID-Data from FE: Where do they come from ?

- Consider the linear FE-equation

$$M\ddot{u}_{fe} + Ku_{fe} = f_{fe}$$

- the related eigenvalue problem

$$[M\omega_i^2 + K]v_i = 0$$

- a set of eigenvectors  $v_1, v_2, \dots$

- a selection of nodes  $c_1, c_2, \dots$

- for each node mode shapes are collected from set of eigenvectors

$$\Phi(c_1), \Phi(c_2), \dots$$

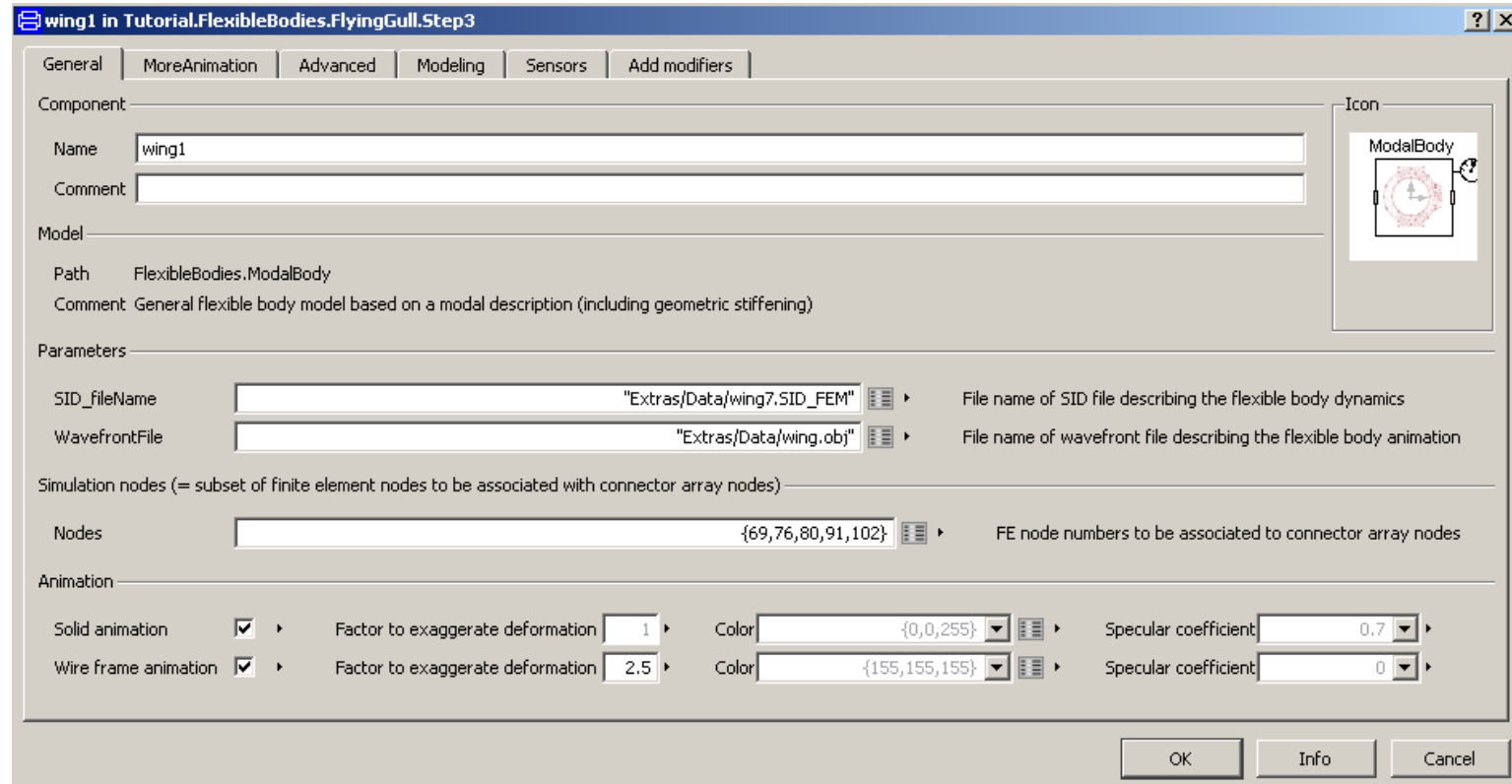
- the related rotational terms (non-volume-elements only)

$$\Psi(c_1), \Psi(c_2), \dots$$

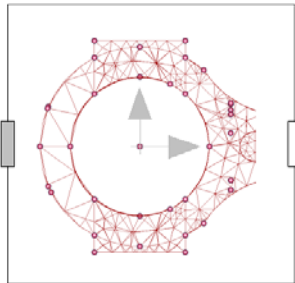
- the volume integrals are reassembled from (substructure) element inertia and stiffness data

$$C_r, C_t, J, M_e, K_e, D_e, G_r \dots$$

# FlexibleBodies Library: ModalBody Menu I



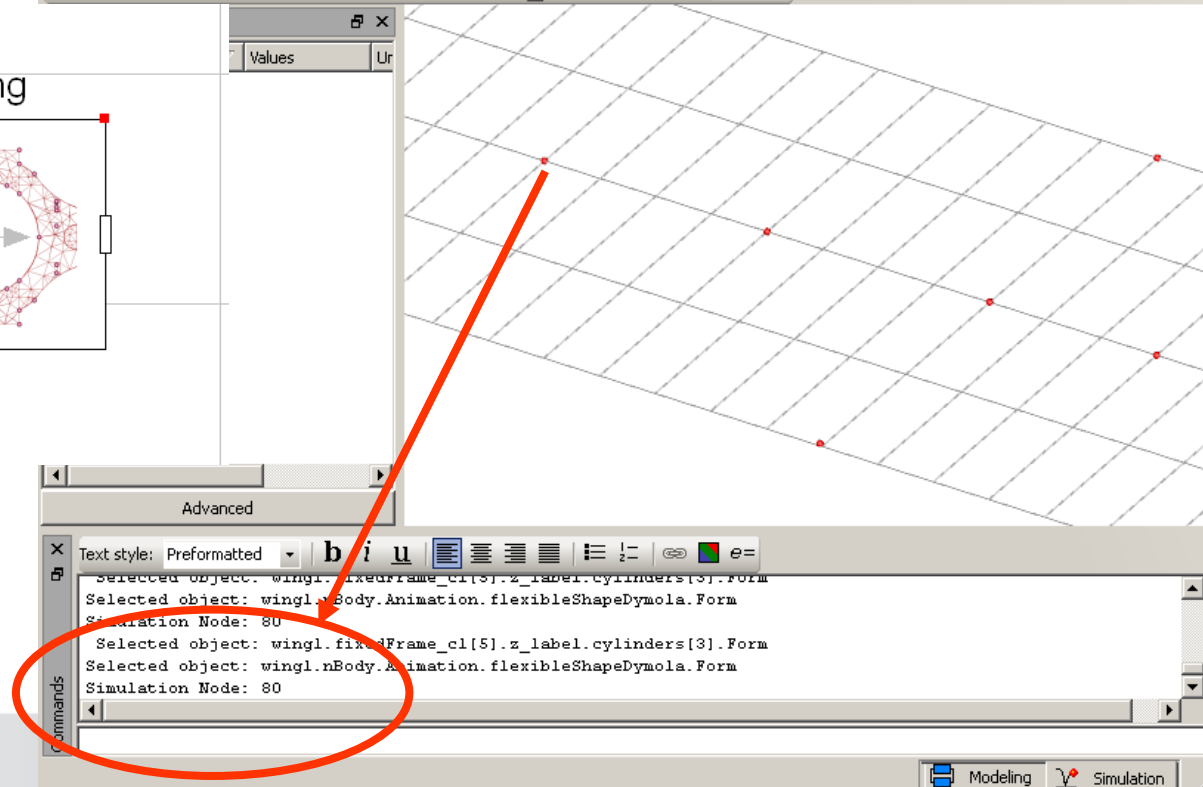
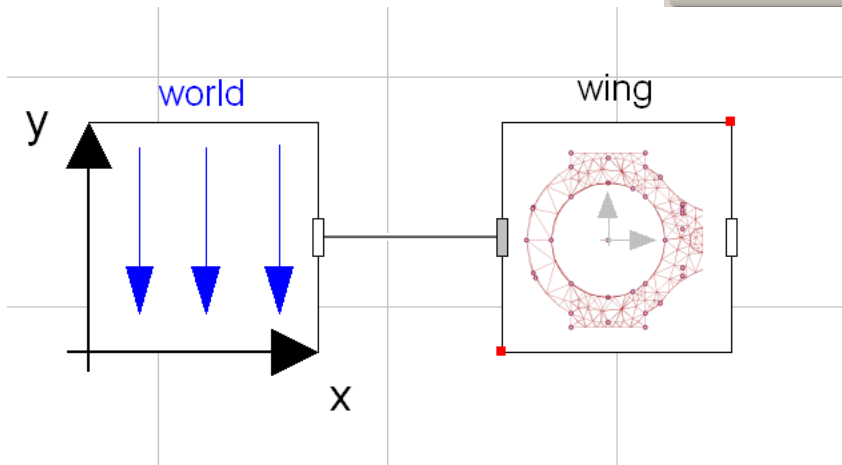
modalBody



# Exercise 6: The Flying Gull III

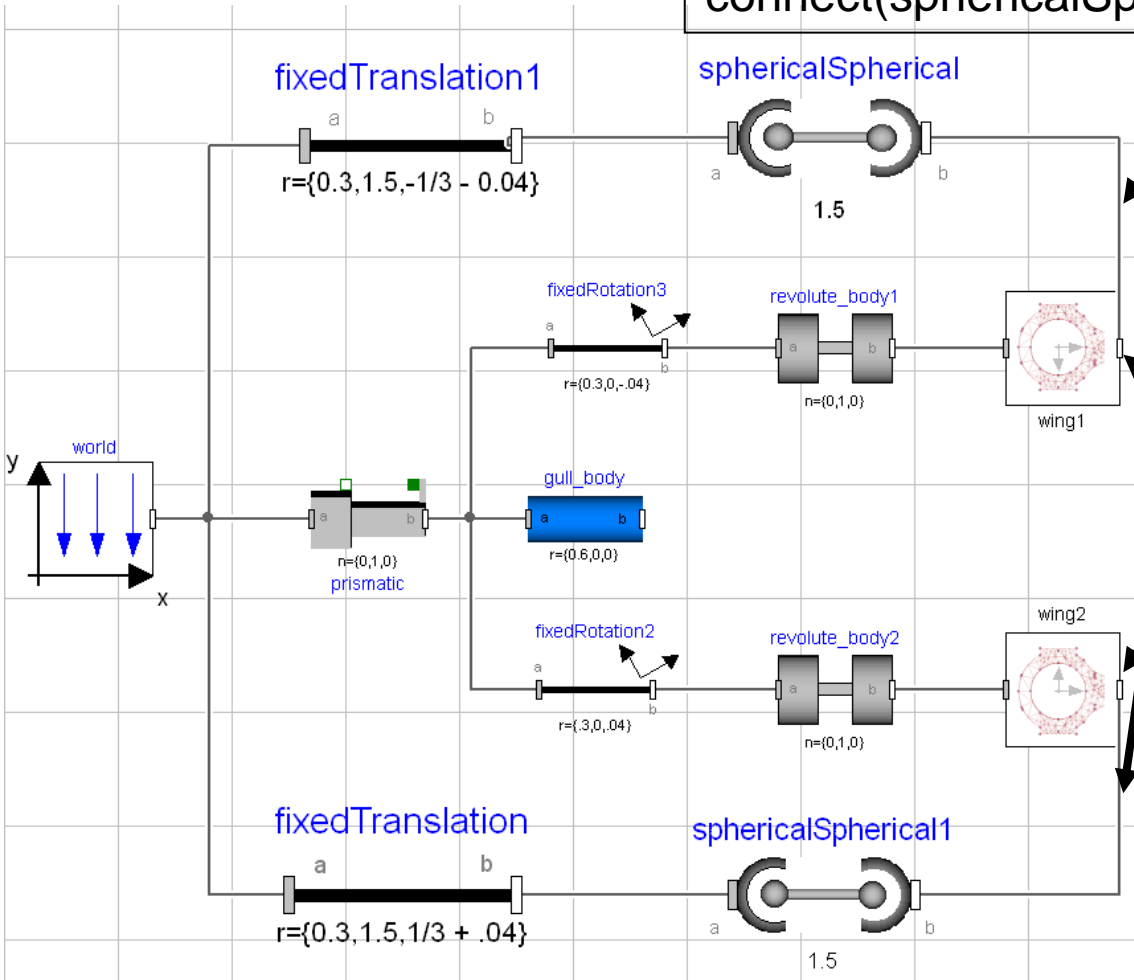
## ➤ 1st step:

- introduce world and ModalBody- model
- assign SID-file .../Extras/Data/wing7.SID\_FEM
- assign OBJ-file .../Extras/Data/wing.obj



# Exercise 6: The Flying Gull III

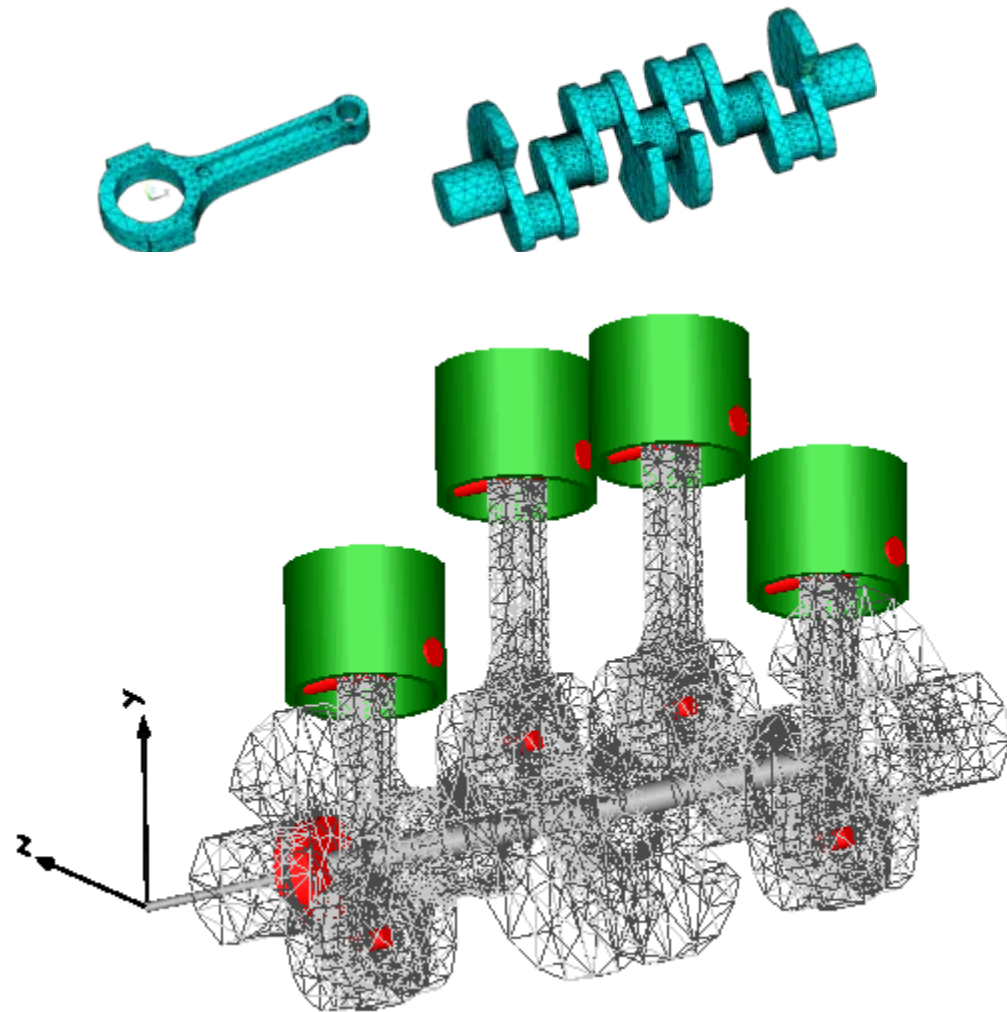
```
connect(sphericalSpherical.frame_b, wing1.nodes[3])
```



Extras/Data/wing7.SID\_FEM  
 Extras/Data/wing.obj  
 Nodes={69,76,80,91,102}

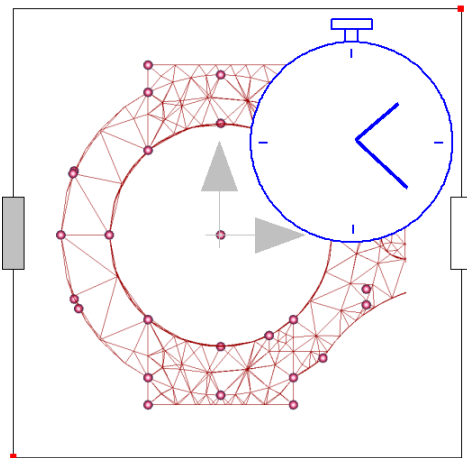
# ModalBody example: 4-Cylinder-Engine

- FEM-models
  - Crankshaft : 106.789 nodes
  - Rod: 22777 nodes
- Multibody representation
  - < 1900 Hz
  - Crankshaft:
    - 2 torsional eigenmodes
    - 305 simulation nodes
  - Rod
    - 4 eigenmodes each
    - 148 simulation nodes each
- Time-integration with gas forces  
38 states, ~6 cpu-s for 1 s

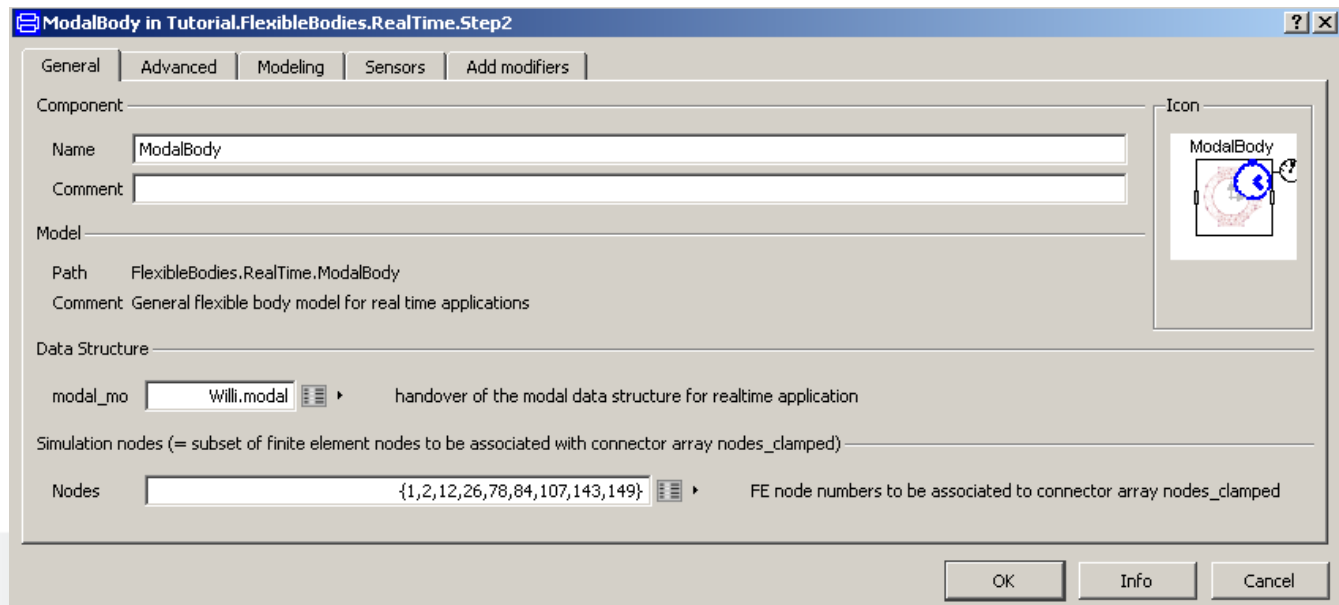


# RealTime Modal Body

## modalBody



- no external C-Code
  - 2. implementation( = parameter native=true)
  - con's: not suitable for large models
- no file access
  - SID-data filed as Modelica-record
  - ⇒ dsmodel.c contains all code and all data
  - no animation





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# FE-preprocessing: in summary

1. FE-modelling
  2. generate wavefront-file (export mesh-information)
  3. prepare and select nodes to retain
  4. solve FE-eigenvalue problem
    - care for boundary conditions and frequency range
  5. generate FE- substructure
  6. generate SID-file FE-from substructure
- 
7. introduce SID- and wavefront-file in Modelica

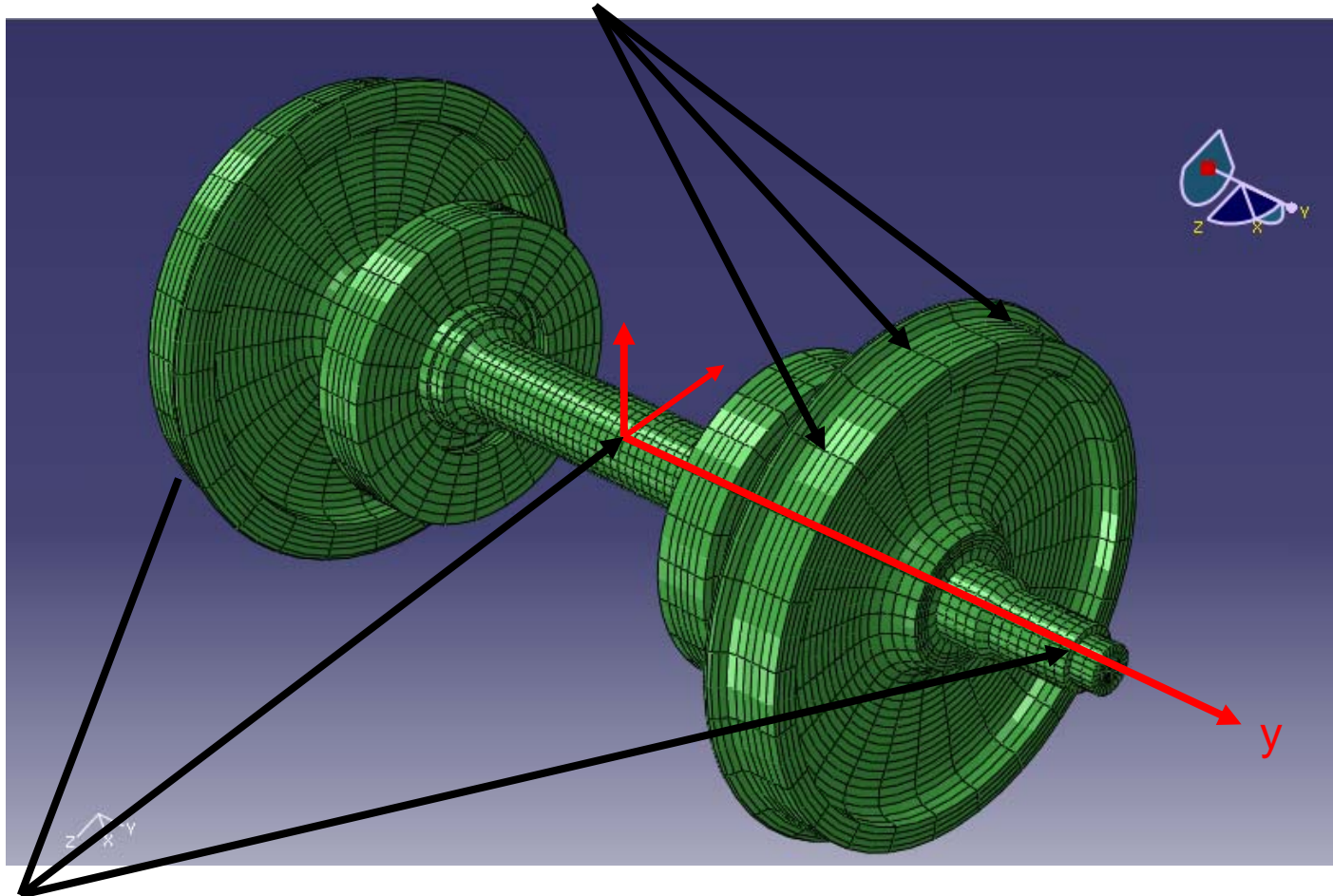
# FE-preprocessing Step 2: wavefront-file

- the animation file in wavefront format \*.obj
  - an open (very) low level geometry format
  - freely available tools exist
  - represents geometrical shape of the body
  - interpolation for animation is completely independent from MBS-simulation
  - due to limited animation performance,
    - the „outside“ geometry is sufficient, e.g. the mesh of the surface

# FE-preprocessing Step 3: retained nodes

- retained nodes
  - prepare the body-model for interconnections of the MBS
    - select nodes where MBS-elements are supposed to be attached to
      - define of such nodes and associated MPCs
      - consider rotational degrees of freedom if needed
  - select an additional set of nodes necessary to support a „nice animation“
    - roughly equally distributed over surface of the body
  - in most cases all together 200, 250 retained nodes
  - you may use the specific Abaqus comand line
    - \*Nset SID\_SELECTED\_NODES

12 AttachmentPoints at radius 460 y 750 equally distributed at the circumference of each wheel (to introduce wheel/rail forces and torques )



3 AttachmentPoints # 90000, 90003, 90006 on the axis line of the wheelset (to attach suspension and measurements devices)

# FE-preprocessing Step 5: substructuring

- standard FE-capability
  - Gyuan-, Craig-Bampton-.....method
- Abaqus comand line

\*SUBSTRUCTURE GENERATE, FLEXIBLE BODY=S

SID assumes SI units	-slength	: scaling factor for the length unit (default: 1.0)
	-smass	: scaling factor for the mass unit (default: 1.0)
	-stime	: scaling factor for the time unit (default: 1.0)
alternative: number of modes	-fmin	: lower boundary of the frequency range (default: 0.001Hz)
	-fmax	: higher boundary of the frequency range (default: 1.E16Hz)
	-tol	: zero cutoff tolerance (default - 1E-12)
	-help	: this usage info

# FE-preprocessing Step 6: SID-file-generation

## ➤ abqtoSid

➤ additionally provided with Abaqus executable control of SID-generation by “substructureName.inp”

➤ ASCII-file with keywords e.g.

\*NSET

\*GENERATE

\*BOUNDARY

\*SELECT EIGENMODES

Set DEFINITION=MODE NUMBERS / FREQUENCY RANGE

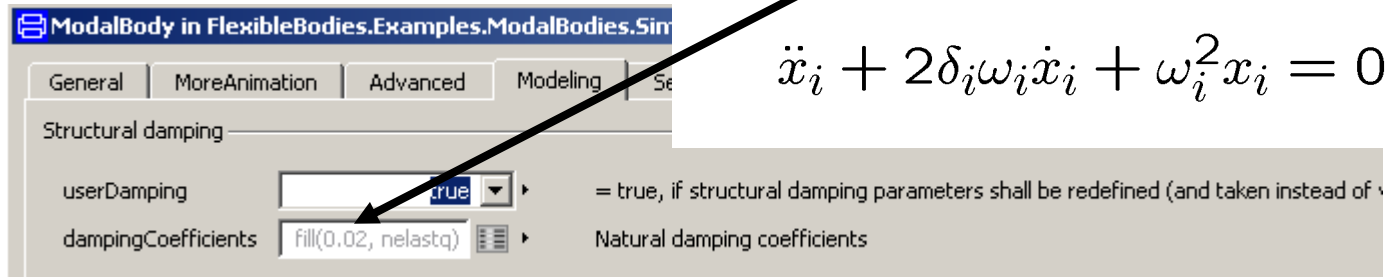
\*DAMPING CONTROLS , VISCOUS=FACTOR

\*DAMPING, ALPHA=0.0, BETA=0.02

Rayleigh-Damping:  
 $D = \alpha M + \beta K$

Alternative: natural damping:  $D_{ii} = 2\delta_i \sqrt{K_{ii} \cdot M_{ii}} = 2\delta_i \omega_i$

$$\ddot{x}_i + 2\delta_i \omega_i \dot{x}_i + \omega_i^2 x_i = 0$$



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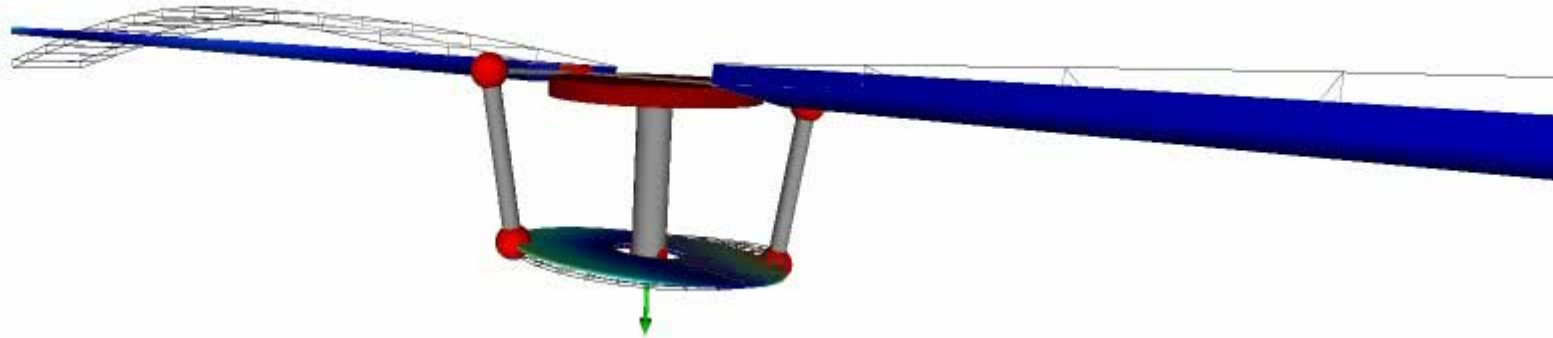




# FlexibleBodies Library extensions at this conference

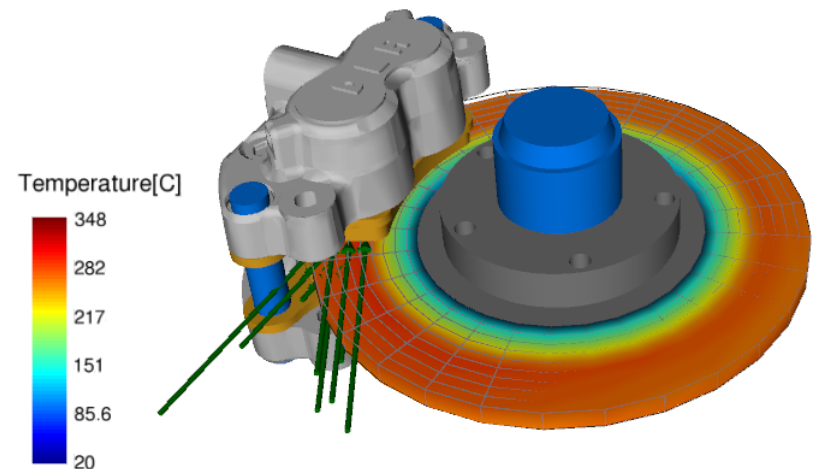
S. Hartweg, Monday HS3 12:00:

- An Annular Plate Model in Arbitrary Lagrangian-Eulerian Description for the DLR FlexibleBodies Library



L. Reyes Perez, Monday HS2 15:35

- A thermoelastic annular plate model for the modeling of brake systems



Thank you very much for your attention !

