Rep. Prog. Phys., Vol. 41, 1978. Printed in Great Britain

The quark model

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Abstract

The quark model is reviewed in a version which includes three colours and at least five quark flavours. Methods are discussed which are applicable to any number of flavours, provided the underlying symmetry is SU(n). The relation between quark bound states and hadron spectroscopy is discussed, and sum rules are given for baryon magnetic moments and hadron mass splittings. Quark–parton models of high-energy hadron–hadron and lepton–hadron scattering are treated. It is concluded that the quark model has been on the whole very successful in accounting for the properties and interactions of hadrons in a qualitative way.

This review was received in January 1978.

0034-4885/78/0011-1707 $05.00

Contents

1. Introduction

In the past few years, several exciting new discoveries have been made in experimental high-energy physics. New kinds of hadrons (strongly interacting elementary particles) have been discovered (Aubert *et al* 1974, Augustin *et al* 1974, Goldhaber *et al* 1976, Knapp *et al* 1976) which do not fit into the older accepted classification scheme of Gell-Mann (1961) and Ne'eman (1961) based on the symmetry group SU(3). Their masses are in the 2–4 GeV region, much greater than previously observed hadrons. Many of them are very long-lived, a surprising property in view of their large masses. A higher classification scheme, based on SU(4), is required.

As an alternative to pure symmetry schemes, one can consider hadrons as being made up of quarks. Originally, a three-quark model was proposed (Gell-Mann 1964, Zweig 1964) to correspond to SU(3). The new hadrons apparently can be accommodated very nicely within the framework of the quark model, provided a fourth quark (the charmed quark) is added to the original three. Several early papers suggesting a fourth quark had been based on arguments such as quark–lepton symmetry, but these were not compelling. Later, Glashow *et al* (1970) pointed out that the weak interactions could be better understood if a charmed quark were included in the model. The latest data essentially necessitate this fourth quark.

Even more recently, Herb *et al* (1977) and Innes *et al* (1977) have observed structure in the di-muon spectrum in the mass region 9–10 GeV. This possibly indicates the existence of yet another new family of hadrons, which in the quark picture would require an additional quark (carrying 'beauty') beyond the quark with 'charm'.

We shall here discuss the quark model in its up-to-date version with four (or more) quarks, while not overly slighting the earlier work, because the whole subject is only about 15 years old. Good older treatments of the quark model are contained in books by Kokkedee (1969) and Feld (1969). A more recent treatment is contained in the book by Lichtenberg (1978) on unitary symmetry. Many books on elementary particle physics contain briefer sections on the quark model. Good examples include, among others, Gasiorowicz (1966), Frauenfelder and Henley (1974) and Perl (1974). In addition there are a large number of review articles on aspects of the quark model. Among these, we call attention to Lipkin (1973), Dalitz (1967, 1976), Weinberg (1974), Greenberg and Nelson (1977) and Harari (1977). Gaillard *et al* (1975) have written a good review of charm, which contains material on the four-quark model. Recent articles on the quark model with charm have been written for the well-educated layman by Glashow (1975), Nambu (1976) and Schwitters (1977). We shall also refer to some of the original literature on the quark model, but there are so many papers that we cannot cite even a substantial fraction of them.

It will become apparent during the course of this review that, although we use the term 'the quark model', in fact, many different quark models have been proposed and discussed. These models have in common that quarks have half-integral spin and are a principal constituent of hadrons. More general composite models of hadrons with point-like constituents are called parton models, the constituents being called partons. At the very least, the size of quarks and partons should be small compared to 1 Fermi, the typical size of hadrons. Constituent models with quarks are sometimes referred to as quark–parton models.

We begin our review with a discussion of the physical quantum number attributes of quarks and hadrons. This leads to a study of the connection between quarks and the symmetry group SU(n), and how hadrons can be accommodated in such a scheme. Next we discuss various models for excited quark systems and their decay systematics. Lastly, we consider collision processes in which the hadrons are treated as quark–parton composites. As we shall see, the great achievement of the quark model is how it ties together and correlates so many diverse properties of hadrons. But at the same time, it raises for us many new questions about the inner workings of nature's array of particles.

2. Properties of quarks

2.1. Hadron quantum numbers

The quantum numbers of the hadrons include, in addition to *spin* and *parity*, a set of quantum numbers having to do with internal symmetry; that is, symmetry under transformations which do not change space–time points. The known quantum numbers which correspond to internal symmetry are electric charge number (or simply *charge*), *baryon number*, *isospin*, *strangeness*, *charm* and *charge-conjugation parity* (for some). There may well be other quantum numbers still to be discovered, for example *beauty*.

It is possible to regard all the known hadrons as composites of a much smaller number of simpler entities—the quarks. This comes about because of relations between the internal quantum numbers of hadrons and their spins and parities. For example, one of these relations is that all known *baryons* (hadrons with baryon number one) have half-integral spin, and all known *mesons* (hadrons with baryon number zero) have integral spin.

Not all the internal quantum numbers used to describe hadrons are independent. For example, the charge Q (in units of the proton charge), the z component of isospin I_z, the baryon number B, and the strangeness S are related by the formula of Gell-Mann (1953) and Nakano and Nishijima (1953). This formula, generalised to include hadrons with charm C and beauty b, is:

$$Q = I_z + \tfrac{1}{2}(B + S + C + b). \tag{2.1}$$

This equation holds for hadrons if it holds for quarks, because Q, I_z, B, S, C and b are all additive quantum numbers. Gell-Mann and Nishijima were unaware of the existence of charm and beauty (and of quarks) when they introduced their formula. (To obtain their original equation, set $C = b = 0$.) If hadrons with additional quantum numbers are discovered, this formula will probably have to be generalised once again. So far, no hadrons with b different from zero have as yet been observed. The case for the existence of b rests on the indirect evidence of Herb *et al* (1977) and Innes *et al* (1977), but we include beauty in the present review in anticipation of hadrons with non-zero b values being discovered in the future.

Sometimes another quantum number, the hypercharge Y, is used. It is related to the others by:

$$Y = B + S - C + b. \tag{2.2}$$

The definitions of the quantum numbers have a certain arbitrariness about them, and the conventions for C and b are not yet standard.

For hadrons, the internal quantum numbers Q, I_z, etc, take on values within a very limited range. As we shall see, these values are such that a normal baryon can be considered simply as a composite of three quarks qqq, and a meson as a composite of a quark and an antiquark $q\bar{q}$. (Generically we refer to a quark by the symbol q. As usual, a bar on the symbol for a particle denotes its antiparticle.) Normal hadrons may also contain in addition a *sea* of $q\bar{q}$ pairs whose net effect does not change the hadron quantum numbers. The quarks which determine the quantum numbers of a hadron are often said to be the *valence* quarks.

At present, there is evidence for the existence of more than a hundred of these normal hadrons. There is also some evidence for *exotic* hadrons, though it is not yet conclusive. Exotic baryons and mesons have quantum numbers such that they cannot be composites of qqq and $q\bar{q}$, respectively, but must contain one or more additional quark–antiquark pairs. The simplest configuration of an exotic baryon is $qqqq\bar{q}$, while for an exotic meson it is $q\bar{q}q\bar{q}$.

The status of the evidence for the existence of individual hadrons is reviewed extensively by the Particle Data Group (Trippe *et al* 1976, 1977).

2.2. *Flavour*

The different kinds of quarks are distinguished by a quantity now usually called flavour. Gell-Mann (1964) and Zweig (1964) originally proposed that three different kinds (flavours) of quarks exist, because at that time the approximate symmetry group of the strong interactions was thought to be SU(3). For a discussion of SU(3) and other unitary groups, see, for example, Lipkin (1965) or Lichtenberg (1978). A collection of original papers on SU(3) is given in a book by Gell-Mann and Ne'eman (1964).

The group SU(3) contains an SU(2) subgroup corresponding to isospin and a U(1) subgroup corresponding to strangeness. Any value of isospin I can be constructed from two building blocks of $I=\frac{1}{2}$, one with $I_z=\frac{1}{2}$ and the other with $I_z=-\frac{1}{2}$. Accordingly, two of the quark flavours were chosen to be the two degrees of freedom associated with $I=\frac{1}{2}$ ('up' and 'down' quarks). Similarly, to account for the strangeness degrees of freedom in hadrons, a third quark flavour was introduced (strange quark). Many calculations based on the group SU(3) were carried out during the 1960s. Generally speaking, the agreement with experiment is remarkable, thus substantiating SU(3) with its quarks of three flavours as an internal symmetry group of hadrons.

In the past few years, however, experimental evidence was found (Aubert *et al* 1974, Augustin *et al* 1974, Goldhaber *et al* 1976, Peruzzi *et al* 1976, Knapp *et al* 1976) for another degree of freedom in hadrons, known as *charm*. A natural generalisation therefore is to extend the internal symmetry group to SU(4), corresponding to the addition of a fourth quark (a quark with charm). Still more recently, Herb *et al* (1977) and Innes *et al* (1977) have observed resonance structure around 9·4 GeV which can be interpreted as providing indirect evidence for a fifth quark (a quark with *beauty*). However, this interpretation is not yet compelling.

The group SU(4) was postulated by a number of authors (Katayama *et al* 1962, Maki *et al* 1962, Tarjanne and Teplitz 1963) to be an approximate symmetry of nature prior to any apparent need for it. After the quark model was proposed, several authors (Maki 1964, Hara 1964, Amati *et al* 1964, Bjorken and Glashow 1964) added a fourth quark to the original three of Gell-Mann and Zweig. One reason for introducing a fourth quark was to achieve a quark–lepton symmetry because, at the time, only four leptons were known. Later, Glashow *et al* (1970) pointed out that the experimental absence of strangeness-changing weak neutral currents could be explained by the existence of a fourth quark.

At present, there is no known principle which unambiguously determines how many quark flavours are needed. Thus, future experiments may turn up still other hadrons which will require the introduction of additional quark flavours. If the argument that there is a quark–lepton symmetry has merit, there should be at least six quarks, because there is at present rather good evidence for two additional leptons (Perl *et al* 1975, Feldman and Perl 1977). Several theorists have already proposed models with five, six, seven, eight, or more quark flavours. Among the authors proposing models with more than four quarks are Kobayashi and Maskawa (1973), Barnett (1975), Fritsch *et al* (1975) and Eichten and Gottfried (1977). Additional authors are cited in Harari's (1977) review of models with more than four quarks. Most of the recent models contain an even number of quarks.

In order to account for the properties of hadrons, quarks must have properties not shared by any known hadrons. In particular, in the model of Gell-Mann and Zweig, quarks have fractional electric charge and fractional baryon number. We use the symbols u and d for the up and down quarks of isospin $\frac{1}{2}$, s for the strange quark, and c for the charmed quark, although other notations are also seen in the literature. We use the symbols b and t for the fifth and sixth quarks. The internal additive quantum numbers B, Q, I_z, S, Y, C and b are opposite in sign for the antiquarks.

The quantum numbers of the quarks are given in table 1. The s quark has negative strangeness because of the historical accident that strangeness was defined before the quark model was invented. We give the b quark negative beauty in analogy with the quantum numbers of the strange quark. Because no hadrons with $b \neq 0$ have been observed, the quantum numbers of the b quark, particularly the charge, are based on conjecture.

Table 1. Quantum numbers of the quarks in a five-quark model. All quarks have baryon number $B=\frac{1}{3}$ and spin and parity $J^P=\frac{1}{2}^+$. Not all the quark quantum numbers are independent, as they are related by equations (2.1) and (2.2) of the text. The quantum numbers listed for the b quark have not been verified by experiment, and so at present are speculative. Each of the five quarks comes in three colours.

Flavour	Charge Q	Isospin I	I_z	Strangeness S	Charm C	Beauty b	Hypercharge Y
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{3}$
d	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{3}$
s	$-\frac{1}{3}$	0	0	-1	0	0	$-\frac{2}{3}$
c	$\frac{2}{3}$	0	0	0	1	0	$-\frac{2}{3}$
b	$-\frac{1}{3}$	0	0	0	0	-1	$-\frac{2}{3}$

2.3. *Colour*

There is indirect evidence that quarks have, in addition to flavour, another internal degree of freedom called colour. The reason for introducing colour has to do with quark statistics. Quarks are supposed to be particles of spin $\frac{1}{2}$. According to the usual field theory (e.g. Streater and Wightman 1964), the wavefunction of a collection of identical particles of half-integral spin is antisymmetric under the interchange of any two of them. But the wavefunction of quarks inside a baryon (including just the usual quantum numbers) appears to be symmetric under this interchange.

A way out of the difficulty is to assume that quarks carry a colour degree of freedom, and that the wavefunction is antisymmetric in the colour variable. A baryon contains three quarks, so it is natural to let the colour degree of freedom take on three values. In other words, a quark of a given flavour comes in three colours, say red, green and blue. A baryon then is made up of three quarks, each one with a different colour in an antisymmetric combination. Such a combination is a colour singlet, and is said to be colourless. Thus, the present quark model, in its version with five flavours and three colours, contains in all 15 varieties of quarks (and an antiquark for each). Already some of the simplicity of the original three-quark model has been lost. Models with even more than five flavours only compound this difficulty. In fact, it has already been proposed that quarks are themselves composite particles (Chang 1972). In one scheme (Greenberg 1975), a quark is composed of two objects, one carrying flavour and the other colour. This idea reduces the number of fundamental entities from $3n$, where n is the number of flavours, to $3+n$. However, in this review we shall not concern ourselves with the possible composite nature of quarks.

Actually the first proposal (Greenberg 1964) to remove the statistics problem did not use colour. It was suggested that quarks were not ordinary fermions, but obeyed a generalised kind of statistics called *parastatistics*. In particular according to Greenberg, quarks are parafermions of order three. This implies that, unlike the case of ordinary fermions, up to three quarks can be put into symmetric states. Greenberg's parastatistics idea is similar to the colour model, provided the symmetry associated with the colour degree of freedom is exact. The colour description, however, is simpler to understand, fits in better with conventional ideas, and lends itself to the construction of a gauge field theory of quarks. Therefore, instead of working with parafermion quarks, we shall in later sections use only coloured quarks.

Shortly after Greenberg introduced parafermion quarks, Han and Nambu (1965) proposed that each quark flavour should come in three varieties (now called colour). They also pointed out that, once the number of quarks was enlarged by a factor of three, there was no need for quarks to have fractional charge or baryon number, and not all the members with the same flavour need have the same charge. An example of Han–Nambu quarks with integral charge is given in table 2, for the case of four flavours. The average charge of all three members of a given flavour is the same as in the quark model with fractional charge.

Table 2. A possible set of twelve integrally charged Han–Nambu quarks. The subscripts r, g, b stand for the different colours (red, green and blue). Note that the average charge $\langle Q\rangle$ is the same as the charge of the corresponding fractionally charged quark of table 1. Some of the other quantum numbers of integrally charged quarks are flexible. For example, the quarks need not all have baryon number $B=\frac{1}{3}$.

Flavour	Q_r	Q_g	Q_b	$\langle Q\rangle$
u	1	1	0	$\frac{2}{3}$
d	0	0	-1	$-\frac{1}{3}$
s	0	0	-1	$-\frac{1}{3}$
c	1	1	0	$\frac{2}{3}$

A colour symmetry with fractionally charged quarks is sometimes called Gell-Mann–Zweig colour, and a colour symmetry with integrally charged quarks is sometimes called Han–Nambu colour. It is usually assumed that Gell-Mann–Zweig colour symmetry is exact. However, Han–Nambu colour symmetry must be broken. This is because not all quarks with the same flavour and different colours have the same charge, and therefore they must interact differently with the electromagnetic field. In this review we principally use the Gell-Mann–Zweig model with fractionally charged quarks.

Although it is sufficient to use quarks of three colours to account for the symmetry properties of quarks in baryons, Pati and Salam (1973, 1974) have proposed to unify leptons and quarks by proposing leptons as a fourth colour. As a consequence of their theory (in a version in which quarks have integral charges), quarks can decay into leptons and baryon number is not strictly conserved. Georgi and Glashow (1974) have proposed another quark model, based on SU(5), in which baryon number is not conserved. Rosen (1974) generalised this model to any SU(n). For details of these and other models, see the original papers and a recent review of colour by Greenberg and Nelson (1977); see also Nambu (1976). Here we shall consider principally models with only three colours.

2.4. *Gluons*

If our present ideas on matter are to hold for quarks, then quarks cannot be the only constituents of hadrons. Just as the photon carries the electromagnetic interaction between charged particles, there should exist field quanta to carry the strong interaction between quarks. In analogy with the electromagnetic case, the carriers of the strong interaction can be taken to be vector gauge fields, and if the analogy is a good one, the quanta of such fields are, like photons, massless vector bosons. These bosons are referred to as *gluons*.

According to one version of gauge field theory, colour symmetry is an exact SU(3) symmetry, and the gluons form an SU(3) colour octet. An important difference between this theory, called *quantum chromodynamics* (QCD) (Fritsch *et al* 1973, Weinberg 1973, Gross and Wilczek 1973b), and ordinary quantum electrodynamics (QED), however, is that the eight gluon fields do not commute with one another. Thus the theory is a non-Abelian gauge field theory, the prototype of which is the Yang–Mills (1954) field. All states of the theory which are not colour SU(3) singlets, such as quarks, gluons, bound states of two quarks or di-quarks, etc, are called coloured, while colour SU(3) singlets are called colourless. (All observed hadrons are colourless.)

Non-Abelian gauge field theories like QCD have some unusual properties. One intriguing property (Gross and Wilczek 1973a, Politzer 1973, 1974) is that the effective interaction between the quarks decreases as the energy and momentum transfer increase. As these variables increase asymptotically to infinity, the theory approaches a free field theory. This property is called *asymptotic freedom.*

However, although the theory has nice asymptotic (or ultraviolet) properties, it has infrared divergences which may be even more serious than those of QED. It has been speculated that the infrared behaviour of the theory leads to the confinement of all coloured states, including quarks and gluons. Such confinement is called *infrared slavery*. Much work has been done on the problem of confinement, for example, by Wilson (1974) and Kogut and Susskind (1975). Nevertheless, it is not yet known whether the theory really does confine the quarks and gluons, and in many papers on the quark model this is merely assumed. For further discussion of gauge field theories, see Abers and Lee (1973), Weinberg (1974, 1977) and Iliopoulos (1976).

In other models, including models with quarks of integral charge, colour symmetry is not exact. In some of these models, the gluons acquire mass and charge. If their masses are large enough, or if they are sufficiently unstable, they might not have been seen. Alternatively, some or all of the known mesons, although themselves composites of quark–antiquark pairs, may provide the glue to bind the quarks.

2.5. *Bound quarks and possible free quarks*

It should be easy to identify free quarks of fractional charge by their electromagnetic interaction. In particular, high-energy quarks of charge $\frac{2}{3}$ travelling through matter would give rise to ionisation only $\frac{4}{9}$ as great as would high-energy protons. If quarks have integral charge, it will be more difficult to identify them, especially if they are unstable as in the theory of Pati and Salam (1973). These authors have suggested (Pati *et al* 1976) that unstable quarks of integral charge may have already been observed in the experiment of Perl *et al* (1975), but the usual interpretation of that experiment is that there is an additional charged lepton, the τ, of mass about 1·8 GeV.

Many physicists have searched for free quarks without success. Goldhaber and Smith (1975) and Jones (1977), among others, have reviewed the subject of quark searches. Recently, LaRue *et al* (1977) have claimed some evidence for charges close to $\frac{1}{3}$ and $-\frac{1}{3}$ of the electron charge (modulo 1). The result is not conclusive because of the possibility of systematic error. If the experiment of LaRue *et al* is confirmed, then either the conjecture that QCD leads to quark confinement is wrong, or QCD is wrong (or both). Also, theories with integrally charged quarks will then have only historical interest.

Independent of whether free quarks exist, the quark model is useful to explain the properties of hadrons in terms of the properties of bound quarks. The four well-established quarks form the basis, as we shall discuss in §3, for an SU(4) symmetry and, as we have noted, there is tentative evidence for a fifth quark, which would presumably make the symmetry SU(5). If the symmetry of n quarks is indeed SU(n), it is badly broken, or it would have been recognised much earlier. If the symmetry were exact, all quarks would have the same mass, but the effective masses of bound quarks differ from one another. (The effective masses of bound quarks may be quite different from the masses of free quarks, if indeed free quarks exist.)

There are a number of ways to estimate the quark effective masses. Two of these ways, which we shall discuss further in §3, are from the hadron masses and the baryon magnetic moments. From these methods we can deduce that the quark effective masses are approximately (for example, De Rújula *et al* 1975, Cheng and James 1975, Wu 1976):

$$\begin{aligned} &m_u \approx m_d \approx 350 \text{ MeV} \qquad &m_s \approx 500 \text{ MeV} \\ &m_c \approx 1500 \text{ MeV} \qquad &m_b \approx 4700 \text{ MeV}. \end{aligned} \tag{2.3}$$

The difference between the effective masses of the d and u quarks is around 2–6 MeV, with the d quark having the larger mass. Masses around the values given in equation (2.3) appear to lead to the best agreement with the experimental properties of hadrons. These are masses appropriate to what is known as a *constituent* quark model, and not to a model of *current* quarks on which Gell-Mann (1964) based his current algebra. Melosh (1974) has suggested a transformation between current and constituent quarks. In this review, we confine ourselves to the constituent quark picture, and when we refer to the mass of a quark, it will mean the effective mass of a bound quark as in equation (2.3).

3. Quark model and hadron multiplets

3.1. Quarks and SU(n)

At present, we have no guiding principle to tell us how many different flavours of quarks we should include in a model. In the absence of such a principle, let us consider a model with n distinct flavours. For definiteness, let the second quark be distinguished from the first by an (internal) additive quantum number (z component of isospin I_z), the third from the first two by a second additive quantum number (strangeness S), the fourth from the first three by a third additive quantum number (charm C), and so on. Then, if there are n quarks, $n-1$ additive quantum numbers serve to distinguish them. An nth additive quantum number, the baryon number B, is common to all quarks.

A model with n objects distinguished by $n-1$ additive quantum numbers is well-suited to be described by the special unitary group in n dimensions, SU(n). This is the group of n-by-n unitary matrices with determinants equal to one. However, even if a model contains n quarks, the relevant symmetry group need not be SU(n). For example, Gürsey and Sikivi (1976), in an attempt to unify strong, electromagnetic, and weak interactions, have proposed that the symmetry group of nature is the exceptional group E_7. In the Gürsey–Sikivi model, however, the quarks transform among themselves according to an SU(6) subgroup of E_7.

We see from the above that there are many possible ways to generalise a four-quark model to include additional quarks. We cannot discuss all the ways proposed thus far without making this review unduly long. We therefore restrict ourselves to SU(n) (see, for example, Baird and Biedenharn 1963). The group SU(n) has n^2-1 generators, of which $n-1$ can be simultaneously diagonalised. The eigenvalues of the $n-1$ diagonal generators are the additive quantum numbers of the quarks. In addition, all quarks have a common value of the baryon number B. The inclusion of B enlarges the group from SU(n) to U(n). However, it is usually convenient to consider SU(n) and to treat B separately.

Each quark has a definite value of each of the $n-1$ additive quantum numbers of SU(n). Therefore, each quark can be represented as a point on a diagram of $n-1$ dimensions, with each additive quantum number given on a mutually perpendicular axis. Such a diagram is called a *weight diagram.*

The group SU(n) has two representations of n dimensions. The eigenvectors of the first can be taken to be the state vectors of the n quarks. The second is conjugate to the first, and its eigenvectors represent the antiquarks. The quark representation (or the multiplet of its eigenvectors) is often simply denoted by $\boldsymbol{n}$, and the conjugate or antiquark representation by $\bar{\boldsymbol{n}}$.

Because a meson consists of a quark and antiquark, the dimensionalities of meson multiplets are given by the dimensionalities of the irreducible representations contained in the product $\boldsymbol{n} \otimes \bar{\boldsymbol{n}}$. This decomposition can be conveniently carried out by means of Young tableaux (see, for example, Lichtenberg 1978). The result is:

$$\boldsymbol{n} \otimes \bar{\boldsymbol{n}} = \boldsymbol{n^2-1} \oplus \boldsymbol{1}. \tag{3.1}$$

The interpretation of equation (3.1) is that the model predicts that mesons should exist in multiplets of n^2-1 and 1 particles. If SU(n) is a broken symmetry, mixing between these multiplets should occur. Thus, if SU(n) is an approximate symmetry of nature, we ought to be able to classify mesons in mixed multiplets of n^2 particles, all with the same spin and parity.

Because a baryon is composed of three quarks, the numbers of particles in baryon multiplets are given by the dimensionalities of the irreducible representations contained in $\boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n}$. The decomposition is:

$$\boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n} = \tfrac{1}{6}\, \boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}+\mathbf{2}) \oplus \tfrac{1}{3}\, \boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}-\mathbf{1}) \oplus \tfrac{1}{3}\, \boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}-\mathbf{1}) \oplus \tfrac{1}{6}\, \boldsymbol{n}(\boldsymbol{n}-\mathbf{1})(\boldsymbol{n}-\mathbf{2}). \tag{3.2}$$

For the group SU(3), equations (3.1) and (3.2) reduce to:

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \tag{3.3}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}. \tag{3.4}$$

Thus, the very successful SU(3) scheme predicted that mesons should occur in mixed octets and singlets, or nonets, and that (neglecting mixing) baryons should exist in decuplets, octets and singlets. The experimental evidence for the existence of these meson and baryon multiplets is reviewed in §§3.4 and 3.5.

As we remarked in the introduction, the recent discovery of additional hadrons which do not fit into the SU(3) scheme has led to its enlargement to SU(4) and beyond. For SU(4) equations (3.1) and (3.2) become:

$$\mathbf{4} \otimes \bar{\mathbf{4}} = \mathbf{15} \oplus \mathbf{1} \tag{3.5}$$

$$\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_{\mathrm{s}} \oplus \mathbf{20}_{\mathrm{m}} \oplus \mathbf{20}_{\mathrm{m}} \oplus \bar{\mathbf{4}} \tag{3.6}$$

where the subscript s refers to a multiplet which is symmetric under the interchange of the SU(4) indices of the quarks and m refers to a multiplet which has mixed symmetry. It is only for SU(4) that the symmetric representation and the representation of mixed symmetry happen to have the same number of dimensions.

3.2. Approximate dynamical symmetry

Thus far, we have restricted our discussion to SU(n) considered as an internal symmetry group. But quarks must also have space–time degrees of freedom as well. In particular, quarks have half-integral spin, because otherwise the model would not be able to account for the half-integral spins of baryons. In the usual quark model, quarks are assumed to have spin $\frac{1}{2}$.

If the interactions between quarks do not depend on their spin configuration, we may enlarge the symmetry group from SU(n) to SU($2n$). This enlarged symmetry group is called a dynamical group. It contains as a subgroup the direct product of the internal symmetry group SU(n) and the spin group SU(2), or SU(n) $\otimes$ SU(2).

Even if SU(n) were an exact symmetry, SU($2n$) would not be, because in general only the total angular momentum of a system is conserved, and not spin and orbital angular momentum separately. It is only at low energy, in a non-relativistic approximation, that spin and orbital angular momentum have the possibility of being separately conserved. For this reason, SU($2n$) ought to be most useful at low energy. In fact, dynamical SU(6), containing the internal symmetry group SU(3) and the spin group SU(2), has proved to be very useful in classifying the low-mass hadrons. If the charmed quark is included in the model, the dynamical group SU(8) may also be useful, but at present not enough experimental information is known to provide a good test of SU(8) predictions. The dynamical group SU($2n$) $\supset$ SU(n) $\times$ SU(2) was first considered by Wigner (1937) for $n=2$, by Gürsey and Radicati (1964) for $n=3$, and by Moffat (1965) and Iwao (1965) for $n=4$.

3.3. Colour SU(3)

As we have remarked, it is now generally accepted that, in addition to flavour, quarks must have a colour degree of freedom which can take on three values. It is not known what is the nature of this new degree of freedom, because the only known hadrons are colour singlets. Despite this lack of knowledge, it is usually assumed (Greenberg 1964) that the colour symmetry group is SU(3). For another possibility see, for example, Franklin (1968). In this review, we confine ourselves to models in which colour symmetry is SU(3). In the presently most popular version of this model, colour SU(3) is an exact symmetry and forms the basis for QCD. The weight diagrams for colour SU(3) multiplets look the same as for flavour SU(3) multiplets, except that the additive quantum numbers are abstract quantities unrelated to the I_z and S of ordinary SU(3). These abstract colour quantum numbers are zero for colour singlets.

3.4. Meson multiplets

We give here a discussion of the mesons expected in the quark model and some properties of the observed mesons. A more detailed treatment of meson spectroscopy has been given recently by Hey and Morgan (1977).

Once we allow a colour degree of freedom, we can assume that quarks are fermions and that their behaviour can be described by a local field theory. (A non-Abelian gauge theory is just a special case of a local field theory.) Then the *CPT* theorem holds (see, for example, Streater and Wightman 1964). This theorem says that a local field theory is invariant under the combined operation of charge conjugation C, parity P and time reversal T. It follows from the *CPT* theorem that a quark and an antiquark of the same flavour have the same mass. Furthermore, any bound state of quarks and/or antiquarks has the same mass as the corresponding bound state with each quark replaced by its antiquark and vice versa. In particular, every hadron must have an antiparticle of the same mass; and if the hadron is unstable, it must have the same lifetime as its antiparticle. No violation of the *CPT* theorem has been observed for any elementary particle.

If we neglect the weak interactions of quarks, then parity is a good quantum number. If a quark and an antiquark are in a bound state with orbital angular momentum L, then the state has parity:

$$P=(-1)^{L+1}. \tag{3.7}$$

If the quark and antiquark have the same flavour, the state is neutral and has charge-conjugation parity or C parity:

$$\mathrm{C}=(-1)^{L+S} \tag{3.8}$$

where S is the total spin ($S=S_1+S_2$). If a meson has isospin I but no strangeness, charm, or beauty, then it has a G parity:

$$G=(-1)^I\mathrm{C}$$

where C is the C parity of the neutral member of the multiplet.

In considering pseudoscalar mesons, we temporarily restrict ourselves to a model with four quark flavours. Then there should exist 16 different mesons belonging to a mixed **15** $\oplus$ **1** multiplet, each having spin and parity $J^P = 0^-$ and each being a bound $^1S_0$ state (the notation S means $L = 0$) of a quark and antiquark. Of these 16 states, four correspond to a quark and antiquark of the same flavour and are eigenstates of charge conjugation with $C = +$. In general, the wavefunctions of the four mesons with positive C parity are orthogonal linear combinations of the wavefunctions of these four quark–antiquark pairs $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ and $c\bar{c}$. We shall discuss these linear combinations in more detail later in this section.

If SU(4) were an exact symmetry, 15 of the 16 pseudoscalar mesons would belong to a fifteen-dimensional representation of SU(4) and would all have the same mass, while the sixteenth meson would be an SU(4) singlet and in general would have a different mass. However, even if the quark–antiquark interactions were independent of flavour, the mass differences of the quarks would still break SU(4) symmetry. Therefore, the fifteen-plet and the singlet will, in general, be mixed into a collection of 16 mesons, and there will be in general 10 different meson masses. Not all 16 masses will be different because six of the mesons have distinct antiparticles (differing from their particles in one or more internal quantum numbers) belonging to the same multiplet.

Because the 16 mesons are composites of only four quarks and their antiquarks, their internal quantum numbers will exhibit striking regularities. The pattern of the I_z, S and C quantum numbers can be exhibited in a three-dimensional weight diagram. This weight diagram can be constructed from the quark quantum numbers of table 1 using the fact that I_z, S and C are all additive quantum numbers and that quarks and antiquarks have opposite values of these quantum numbers.

The weight diagram and quark content of the 16 0^- mesons is shown in figure 1. All these mesons have been observed, although at present the experimental evidence is not conclusive for all of them. In table 3 we list the experimental values (where known) of the masses, widths, lifetimes, and principal decay modes of the 16 pseudoscalar mesons. These values are adapted primarily from the reviews of the Particle Data Group (Trippe *et al* 1976, 1977) and also from Goldhaber *et al* (1976), Peruzzi *et al* (1977), Feldman (1977) and De Boer (1977). These 16 mesons correspond to the ground states of quark–antiquark. Radially excited multiplets with $J^P = 0^-$ are also

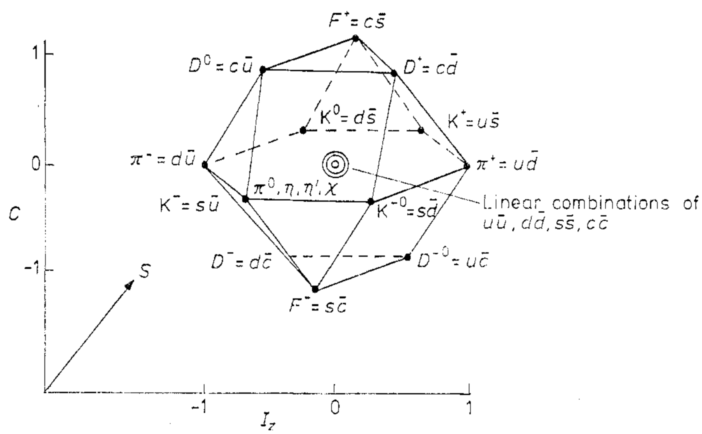

Figure 1. SU(4) weight diagram and quark content of the pseudoscalar mesons (after Gaillard *et al* 1975).

Table 3. Some properties of the ground-state pseudoscalar mesons ($J^P=0^-$). All these mesons are $^1S_0$ states of $q\bar{q}$. Experimental values from Trippe *et al* (1976, 1977), De Boer (1977) and Peruzzi *et al* (1977).

Symbol	I^G	Mass (MeV)	Mean lifetime (s) or width	Principal decay modes
$\pi^\pm$	1^-	$139{\cdot}57\pm0{\cdot}01$	$(2{\cdot}603\pm0{\cdot}003)\times10^{-8}$	$\mu\nu$
π^0	1^-	$134{\cdot}96\pm0{\cdot}01$	$(0{\cdot}83\pm0{\cdot}06)\times10^{-16}$	$\gamma\gamma$
$K^\pm$	$\frac{1}{2}$	$493{\cdot}71\pm0{\cdot}04$	$(1{\cdot}237\pm0{\cdot}003)\times10^{-8}$	$\mu\nu$ $(63{\cdot}6\pm0{\cdot}2)\%$
				$\pi^\pm\pi^0$ $(21{\cdot}0\pm0{\cdot}2)\%$
				$\pi^\pm\pi^+\pi^-$ $(5{\cdot}6\pm0{\cdot}1)\%$
				$\pi^\pm\pi^0\pi^0$ $(1{\cdot}7\pm0{\cdot}1)\%$
				$\mu\pi^0\nu$ $(3{\cdot}2\pm0{\cdot}1)\%$
				$e\pi^0\nu$ $(4{\cdot}8\pm0{\cdot}1)\%$
$K^0, \overline{K^0}$	$\frac{1}{2}$	$497{\cdot}7\pm0{\cdot}2$	50% K_L, 50% K_S	
(K_S)			$(0{\cdot}893\pm0{\cdot}003)\times10^{-10}$	$\pi^+\pi^-$ $(68{\cdot}7\pm0{\cdot}3)\%$
				$\pi^0\pi^0$ $(31{\cdot}3\pm0{\cdot}3)\%$
(K_L)			$(5{\cdot}18\pm0{\cdot}04)\times10^{-8}$	$3\pi^0$ $(21{\cdot}4\pm0{\cdot}7)\%$
				$\pi^+\pi^-\pi^0$ $(12{\cdot}2\pm0{\cdot}2)\%$
				$\pi\mu\nu$ $(27{\cdot}1\pm0{\cdot}5)\%$
				$\pi e\nu$ $(39{\cdot}0\pm0{\cdot}5)\%$
η	0^+	$548{\cdot}8\pm0{\cdot}6$	$(0{\cdot}8\pm0{\cdot}2)\times10^{-3}$ MeV	$\gamma\gamma$ $(38\pm1)\%$
				$\pi^0\gamma\gamma$ $(3\pm1)\%$
				$3\pi^0$ $(30\pm1)\%$
				$\pi^+\pi^-\pi^0$ $(24\pm1)\%$
				$\pi^+\pi^-\gamma$ $(4{\cdot}9\pm0{\cdot}2)\%$
η'	0^+	$957{\cdot}6\pm0{\cdot}3$	<1 MeV	$\eta\pi\pi$ $(68\pm2)\%$
				$\rho^0\gamma$ $(30\pm2)\%$
				$\gamma\gamma$ $(2{\cdot}0\pm0{\cdot}3)\%$
χ	0^+	2830	?	$\gamma\gamma$
$D^\pm$	$\frac{1}{2}$	$1868{\cdot}3\pm0{\cdot}9$	?	$K\pi\pi$
$D^0, \bar{D}^0$	$\frac{1}{2}$	$1863{\cdot}3\pm0{\cdot}9$	?	$K\pi$, $K\pi\pi\pi$
$F^\pm$	0	2030 ± 60	?	$\pi^\pm\eta$

expected to exist, but the evidence for them is rather sparse (see the reviews of the Particle Data Group (Trippe *et al* 1976, 1977) and Hey and Morgan (1977)).

We now turn to the vector mesons with $J^P=1^-$. These are primarily bound $^3S_1$ states of $q\bar{q}$, although they will in general have some admixture of $L=2$. The SU(4) weight diagram of the 16 vector mesons is just the same as that of the pseudoscalar mesons because the quark content is the same (except that the four vector mesons which are eigenstates of C are composed of different linear combinations of $q\bar{q}$ pairs). Rather than show the weight diagram for the vector mesons, we instead give in figure 2 slices or sectors of the weight diagram corresponding to definite values of charm; namely $C=1$, 0 and -1. In the $C=0$ slice of figure 2, there is a state Υ in parentheses. This state cannot be accommodated by SU(4), and we temporarily defer discussion of it.

Once the number of quarks increases beyond four, it is impractical to try to picture weight diagrams. However, we can still use the procedure of showing two-dimensional sectors of weight diagrams having fixed values of all other additive quantum numbers.

If we omit the c quark, three meson states with $I_z=S=0$ can be formed from the quarks u, d, s, and their antiparticles. Note, however, from figure 2 that (temporarily ignoring the Υ) four states, rather than three, appear at the centre of the sector of the SU(4) weight diagram with $C=0$. The fourth state, although it has $C=0$, is a bound

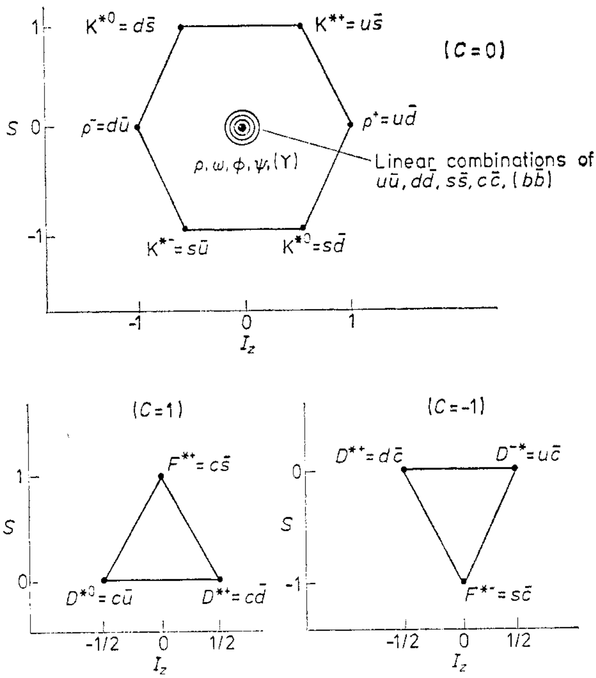

Figure 2. Slices of the SU(4) weight diagram of the vector mesons. The Υ is shown in parentheses because it cannot be accommodated within the SU(4) scheme.

state of a $c\bar{c}$ pair, and so can be said to have 'hidden' charm. Thus, when the $\psi(3098)$, often called the J/ψ, was discovered (Aubert *et al* 1974, Augustin *et al* 1974), many physicists, for example, Appelquist and Politzer (1975) and De Rújula and Glashow (1975), felt that it would not be long before a meson with manifest charm was seen. That expectation was realised in an experiment of Goldhaber *et al* (1976).

If the $\Upsilon(9400)$ is still another vector meson, then there is a fifth state with $I_z = S = 0$, and SU(4) is too small to accommodate it. Thus, we apparently need a fifth quark and the group SU(5). If this interpretation is correct, then the Υ is a $b\bar{b}$ bound state and carries hidden beauty. To observe manifest beauty will not be an easy task.

In table 4 we list some of the properties of the ground-state vector mesons. These are adapted primarily from Trippe *et al* (1976, 1977), but also from Goldhaber *et al* (1976), Peruzzi *et al* (1976, 1977), Feldman (1977), De Boer (1977) and Innes *et al* (1977). For a discussion of excited states, see Hey and Morgan (1977). It can be seen from table 4 that the ψ, which is composed of $c\bar{c}$, has a much larger mass than ρ^0, ω and ϕ, which are composed of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$. This is interpreted as telling us that the c quark has a much larger mass than the u, d and s quarks. The very large mass of the Υ tells us that the b quark has a still larger mass than the c quark.

But if the mass of the ψ is much larger than the masses of the ρ, ω and ϕ, the question arises as to why it should be classified in the same multiplet with them. An alternative possibility would be to classify the ψ as a highly excited state of the $q\bar{q}$ system. This latter interpretation is not plausible because the ψ has a very narrow width (about 70 keV), whereas highly excited states typically have widths of considerably more than 100 MeV. Thus, the ψ has a width three orders of magnitude smaller than would be expected if it were made out of an uncharmed quark and antiquark.

Table 4. Some properties of the observed ground-state vector mesons ($J^P=1^-$). All these mesons are $^3S_1$ states of $q\bar{q}$. Experimental values from Trippe *et al* (1976, 1977), De Boer (1977), Peruzzi *et al* (1977) and Innes *et al* (1977).

Symbol	I^G	Mass (MeV)	Width (MeV)	Principal decay modes
$\rho^{\pm}$	1^+	770 ± 5	150 ± 5	$\pi^{\pm}\pi^0$
ρ^0	1^+	770 ± 5	150 ± 5	$\pi^+\pi^-$
$K^{*\pm}$	$\frac{1}{2}$	892 ± 1	50 ± 2	$K\pi$
K^{*0}, $\bar{K}^{*0}$	$\frac{1}{2}$	896 ± 1	50 ± 1	$K\pi$
ϕ	0^-	$1019{\cdot}7\pm0{\cdot}3$	$4{\cdot}1\pm0{\cdot}2$	K^+K^- $(46\pm3)\%$ K_LK_S $(35\pm2)\%$ $\pi^+\pi^-\pi^0$ $(16\pm2)\%$
ω	0^-	$782{\cdot}7\pm0{\cdot}3$	$10{\cdot}0\pm0{\cdot}4$	$\pi^+\pi^-\pi^0$ $(90\pm1)\%$ $\pi^0\gamma$ $(9\pm1)\%$
ψ	0^-	3095 ± 1	$0{\cdot}067\pm0{\cdot}012$	Hadrons $(86\pm2)\%$ e^+e^- $(7\pm1)\%$ $\mu^+\mu^-$ $(7\pm1)\%$
$D^{*\pm}$	$\frac{1}{2}$	$2008{\cdot}6\pm1{\cdot}0$		$D\pi$, $D\gamma$
D^{*0}, $\bar{D}^{*0}$	$\frac{1}{2}$	$2006\pm1{\cdot}5$		$D\pi$, $D\gamma$
$F^{*\pm}$	0	2140 ± 60		$F^{\pm}\gamma$ (only mode seen so far)
Υ	0^-	9400 ± 13	<400	$\mu^+\mu^-$ (only mode seen so far)

An empirical rule, called Zweig's rule or the OZI rule (Okubo 1963, Zweig 1964, Iizuka 1966), which inhibits the decay of the ϕ meson compared to the decay of the ω, also inhibits the decay of the ψ if it is composed of $c\bar{c}$. Thus, the ψ is narrow because it has hidden charm. We shall discuss the OZI rule in §5.3.

Similar arguments apply to the $\Upsilon(9400)$, although so far with less force. This is because the experiments of Herb *et al* (1977) and Innes *et al* (1977) do not have very good energy resolution, so that it cannot be said definitely that the $\Upsilon(9400)$ is a very narrow resonance. However, the assumption that this resonance is narrow is not contradicted by the data. In fact, the evidence of Innes *et al* (1977) indicates that the structure around 9·4 GeV consists of at least two resonances and a possible third, with masses 9·40, 10·01 and 10·40 GeV. In view of the fact that the energies of these resonances are so much higher than the energy of any previously known resonance, it is plausible to regard them as a manifestation of a new phenomenon. Because the $\Upsilon(9400)$ cannot be accommodated by a four-quark model, we assume that it has hidden beauty. If so, mesons should exist in multiples of 25, including mesons with manifest beauty. Rather than introduce new symbols for mesons with beauty, we use the same symbols as for charmed mesons, with a subscript b to denote that a b quark has replaced a c quark. We do introduce a new symbol E^- for the meson containing $b\bar{c}$.

In table 5 we give the quark content and quantum numbers of the 25 pseudoscalar mesons expected on the basis of the existence of five quarks. The mesons π^0, η, η', χ and χ_b are expected to be linear combinations of the pairs $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$. Just what these linear combinations are depends on the details of SU(5) mixing. In constructing table 5, we have assumed for simplicity that χ and χ_b are 'ideally' mixed such that they contain only $c\bar{c}$ and $b\bar{b}$ respectively. Conservation of isospin assures that the π^0 is a linear combination of $u\bar{u}$ and $d\bar{d}$ only. We have not specified what linear combinations of pairs the η and η' contain except to ensure that these states are orthogonal to π^0, χ and χ_b.

The quark model predicts that other meson multiplets should exist besides the

Table 5. Quark content and quantum numbers of pseudoscalar mesons in a five-quark model. We have assumed that the b quark has $I=0$ and $Q=-\frac{1}{3}$, although there is as yet no definite convincing evidence for these assignments.

Symbol	Quark content	I	I_z	S	C	b
π^+	$u\bar{d}$	1	1	0	0	0
π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	1	0	0	0	0
π^-	$d\bar{u}$	1	-1	0	0	0
K^+	$u\bar{s}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0
K^0	$d\bar{s}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	0
$\bar{K}^0$	$s\bar{d}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	0
K^-	$s\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	0	0
η	$(u\bar{u}+d\bar{d})\sqrt{2}$, $s\bar{s}$	0	0	0	0	0
η'	$(u\bar{u}+d\bar{d})\sqrt{2}$, $s\bar{s}$	0	0	0	0	0
χ	$c\bar{c}$	0	0	0	0	0
D^+	$c\bar{d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	0
D^0	$c\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0
$\bar{D}^0$	$u\bar{c}$	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	0
D^-	$d\bar{c}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	0
F^+	$c\bar{s}$	0	0	1	1	0
F^-	$s\bar{c}$	0	0	-1	-1	0
χ_b	$b\bar{b}$	0	0	0	0	0
$D_b{}^+$	$u\bar{b}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
$D_b{}^0$	$d\bar{b}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1
$\bar{D}_b{}^0$	$b\bar{d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-1
$D_b{}^-$	$b\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$F_b{}^0$	$s\bar{b}$	0	0	-1	0	1
$\bar{F}_b{}^0$	$b\bar{s}$	0	0	1	0	-1
E^+	$c\bar{b}$	0	0	0	1	1
E^-	$b\bar{c}$	0	0	0	-1	-1

ground-state pseudoscalar and vector multiplets. In fact, if quarks are totally confined to the interior of hadrons, it is plausible that there should exist an infinite number of bound states of different energy, although only a finite number should exist below a given energy. The masses and angular momenta of the mesons will depend on the details of the $q\bar{q}$ interaction. For a large class of interactions, the number of states per unit energy will increase as the energy increases. Furthermore, most of the high-energy states will have large decay widths because of the large number of open decay channels and the large available phase space. Both because of the increasing energy level density and increasing decay widths, it becomes harder to detect individual states as the energy increases.

In several models, to be discussed in §4, the P-wave bound states of $q\bar{q}$ have masses not too far above the masses of S-wave bound states. These P-wave states can be classified in spectroscopic notation as $^3P_0$, $^3P_1$, $^3P_2$ and $^1P_1$. The $^3P_2$ state is not pure $L=1$, but has some admixture of $L=3$. Except for weak, parity non-conserving effects, the $^3P_0$, $^3P_1$ and $^1P_1$ states are pure $L=1$. The P-wave multiplets are not so well-established experimentally as the S-wave multiplets. Other excited states of quark–antiquark pairs include the D waves. These can be classified as $^3D_1$, $^3D_2$, $^3D_3$ and $^1D_2$. The $^3D_1$ state will have some admixture of S wave and the $^3D_3$ state will have some admixture of G wave. According to the quark model there will also exist radially excited states for each value of L. The details of this spectrum depend on

Table 6. Some properties of mesons containing $c\bar{c}$ or $b\bar{b}$ quarks excluding ground-state mesons already included in tables 3 and 4 (Trippe *et al* 1977, Feldman and Perl 1977, Innes *et al* 1977).

Symbol	J^{PC}	$^{2S+1}L_J$	Mass (MeV)	Width (MeV)	Principal decay modes
ψ'	1^{--}	$^3S_1$	3684 ± 1	$0{\cdot}23\pm0{\cdot}06$	Hadrons
ψ''	1^{--}	$^3D_1$	3772 ± 3	28 ± 5	$D\bar{D}$
ψ	1^{--}	$^3S_1$	~4028	?	Hadrons
ψ	1^{--}	$^3S_1$	4414 ± 5	33 ± 10	Hadrons
χ	0^{++}	$^3P_0$	~3415	?	Hadrons $\gamma\psi$
χ	0^{-+}	$^1S_0$	~3455	?	$\gamma\psi$
χ	1^{++}	$^3P_1$	3510 ± 4	?	Hadrons $\gamma\psi$
χ	2^{++}	$^3P_2$	3554 ± 5	?	Hadrons $\gamma\psi$
Υ'	1^{--}	$^3S_1$	10010 ± 40	?	$\mu^+\mu^-$ (only mode seen so far)
Υ''?	1^{--}	$^3S_1$	10400 ± 120	?	$\mu^+\mu^-$ (only mode seen so far)

the nature of the quark–antiquark interaction. In table 6 we give some of the properties of the excited mesons containing *c* or *b* quarks. For information about excited states containing only *u*, *d* or *s* quarks, see Trippe *et al* (1976).

3.5. Baryon multiplets

As we have previously noted, a baryon is assumed to be a bound state of three quarks. For an ordinary attractive interaction between quarks, the bound state of lowest energy has a configuration in which the quarks are all in relative S states ($L=0$). If this is the case, it follows that the wavefunction of a baryon is symmetric under the interchange of the spatial coordinates of any two quarks. Three quarks of spin $\frac{1}{2}$ can combine to give a total spin S of either $\frac{3}{2}$ or $\frac{1}{2}$. Then, with orbital angular momentum $L=0$, the baryons of lowest mass should have total angular momentum J of either $\frac{3}{2}$ or $\frac{1}{2}$.

In fact, the baryon of lowest mass is the proton, with $J=\frac{1}{2}$. In their original papers on SU(3), Gell-Mann and Ne'eman classified the proton as one member of an SU(3) octet (see Gell-Mann and Ne'eman 1964). Another low-mass baryon state is the pion–nucleon resonance Δ. This state has $J=\frac{3}{2}$ and is classified as one member of an SU(3) decuplet (also known as a decimet). At the time of this classification, only nine members of this decuplet were known. The subsequent discovery of the tenth member, the Ω^-, was considered a triumph for the SU(3) scheme.

With an additional charmed quark, the baryon octet becomes part of a $\mathbf{20}_{\mathrm{m}}$ multiplet of SU(4), and the baryon decuplet becomes part of a $\mathbf{20}_{\mathrm{s}}$ multiplet. Thus far, only one or two of the 12 charmed baryons belonging to the $\mathbf{20}_{\mathrm{m}}$ have been observed (Knapp *et al* 1976), and possibly one belonging to the $\mathbf{20}_{\mathrm{s}}$. This evidence is still preliminary. With the addition of a *b* quark, the decuplet becomes part of a **35**, and the octet becomes part of a **40**. There is as yet no direct evidence for the existence of any baryon containing a *b* quark.

There is no standard notation for baryons containing *c* or *b* quarks. The notation of Gaillard *et al* (1975) used a large number of new symbols to denote baryons containing charm, while the present authors (Hendry and Lichtenberg 1975) used the

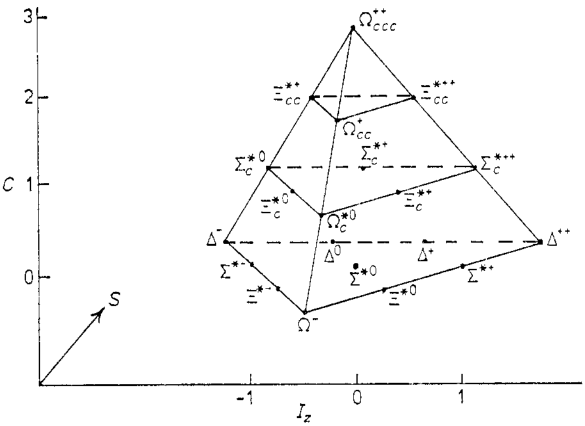

Figure 3. Weight diagram for the $\mathbf{20}_s$ of SU(4).

same symbols as for uncharmed baryons, with a subscript on the symbol for a baryon to denote how many charmed quarks have replaced strange quarks. In this review we use a symbol for a baryon which depends on its isospin and hypercharge. But, as we see from table 1, the *s*, *c* and *b* quarks have the same hypercharge. We remove this ambiguity by putting a subscript *c* (or *b*) on the symbol for each *c* (or *b*) quark the baryon contains.

We first restrict ourselves to four quarks. From the additive quantum numbers of the quarks given in table 1, we can construct the three-dimensional weight diagrams for the baryons in the $\mathbf{20}_s$ and $\mathbf{20}_m$ of SU(4). In figure 3 we show the weight diagram for the $\mathbf{20}_s$, and in figure 4 for the $\mathbf{20}_m$. In tables 7 and 8 we give the masses, widths and principal decay modes of the known baryons of the $\mathbf{20}_m$ and $\mathbf{20}_s$ respectively (Trippe *et al* 1976, 1977). For information about other excited baryon states, see Trippe *et al* (1976).

If five quarks exist, baryons will belong to multiplets of 35 and 40 particles. In table 9 we give the quark content and quantum numbers of the baryons belonging to these SU(5) multiplets.

We next consider the dynamical group SU($2n$), restricting ourselves for simplicity to $n \leqslant 4$. The baryon $\mathbf{20}_s$ and $\mathbf{20}_m$ multiplets of SU(4) can be classified together in a

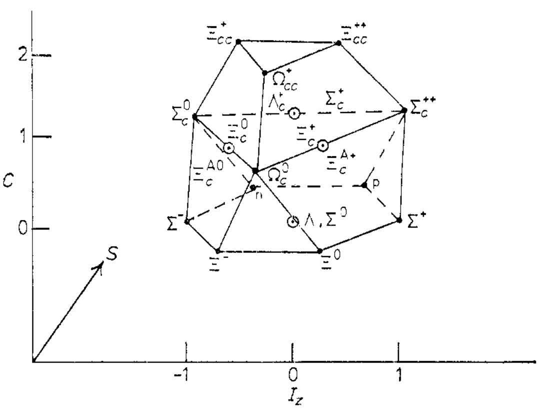

Figure 4. Weight diagram for the $\mathbf{20}_m$ of SU(4).

Table 7. Some properties of the known baryons belonging to the $\mathbf{20_m}$ of SU(4). Experimental values adapted from Trippe *et al* (1976, 1977).

Symbol	Mass (MeV)	Mean lifetime (s) or width	Principal decay modes
p	$938{\cdot}280 \pm 0{\cdot}003$	Stable	
n	$939{\cdot}573 \pm 0{\cdot}003$	918 ± 14	$pe^-\nu$ 100%
Λ	$1115{\cdot}60 \pm 0{\cdot}05$	$(2{\cdot}58 \pm 0{\cdot}02) \times 10^{-10}$	$p\pi^-$ $(64{\cdot}2 \pm 0{\cdot}5)$% $n\pi^0$ $(35{\cdot}8 \pm 0{\cdot}5)$%
Σ^+	$1189{\cdot}37 \pm 0{\cdot}06$	$(0{\cdot}800 \pm 0{\cdot}006) \times 10^{-10}$	$p\pi^0$ $(51{\cdot}6 \pm 0{\cdot}7)$% $n\pi^+$ $(48{\cdot}4 \pm 0{\cdot}7)$%
Σ^0	$1192{\cdot}47 \pm 0{\cdot}08$	$< 10^{-14}$	$\Lambda\gamma$ 100%
Σ^-	$1197{\cdot}35 \pm 0{\cdot}06$	$(1{\cdot}48 \pm 0{\cdot}02) \times 10^{-10}$	$n\pi^-$ 100%
Ξ^0	$1314{\cdot}9 \pm 0{\cdot}6$	$(2{\cdot}96 \pm 0{\cdot}12) \times 10^{-10}$	$\Lambda\pi^0$ 100%
Ξ^-	$1321{\cdot}3 \pm 0{\cdot}2$	$(1{\cdot}65 \pm 0{\cdot}02) \times 10^{-10}$	$\Lambda\pi^-$ 100%
$\Lambda_c{}^+$	2260 ± 10	< 75 MeV	$\Lambda\pi^+\pi^+\pi^-$ (only mode seen so far)
$\Sigma_c{}^{++}$	2426 ± 12	?	$\Lambda_c{}^+\pi^+$ (only mode seen so far)
$\Sigma_c{}^0$	2426 ± 12	?	$\Lambda_c{}^+\pi^-$ (only mode seen so far)

single multiplet of the dynamical group SU(8). The relevant multiplet is the **120** which has just the right SU(4) and spin content, as seen from the relation $\mathbf{120} \supset (\mathbf{20_s}, 4) \oplus (\mathbf{20_m}, 2)$ where in the symbol (N_1, N_2), N_1 stands for the SU(n) multiplicity and N_2 for the spin multiplicity.

The **120** is symmetric in the combined spin and unitary spin indices of the quarks. Furthermore, as we have remarked earlier, it is most plausible that the spatial wavefunction is also symmetric. The observed shape of the proton form factor adds weight to this conjecture (Mitra and Majumdar 1966). Thus, the wavefunctions of the baryons belonging to the **120** of SU(8) are apparently symmetric under the combined interchange of SU(4), spin and space coordinates of any two quarks. But, according to the spin-statistics theorem, the wavefunction of a composite system of identical fermions is antisymmetric under the interchange of all the coordinates of any pair.

Table 8. Some properties of the known baryons belonging to the $\mathbf{20_s}$ of SU(4). Experimental values adapted from Trippe *et al* (1976, 1977).

Symbol	Mass (MeV)	Width (MeV) or mean lifetime	Principal decay modes
Δ^{++}	1232 ± 2	115 ± 5	$p\pi^+ \sim 100$%
Δ^+	?	?	$p\pi^0$, $n\pi^+ \sim 99{\cdot}4$%
Δ^0	1232 ± 2	120 ± 5	$p\pi^-$, $n\pi^0 \sim 99{\cdot}4$%
Δ^-	1237 ± 5		$n\pi^- \sim 100$%
Σ^{*+}	$1382{\cdot}5 \pm 0{\cdot}5$	35 ± 2	$\Lambda\pi^+$ (88 ± 2)% $\Sigma^+\pi^0$, $\Sigma^0\pi^+$ (12 ± 2)%
Σ^{*0}	~ 1385	~ 40	$\Lambda\pi^0 \sim 88$% $\Sigma^+\pi^-$, $\Sigma^-\pi^+ \sim 12$%
Σ^{*-}	$1386{\cdot}6 \pm 1{\cdot}2$	42 ± 4	$\Lambda\pi^- \sim 88$% $\Sigma^0\pi^-$, $\Sigma^-\pi^0 \sim 12$%
Ξ^{*0}	$1531{\cdot}8 \pm 0{\cdot}3$	$9{\cdot}1 \pm 0{\cdot}5$	$\Xi^0\pi^0$, $\Xi^-\pi^+ \sim 100$%
Ξ^{*-}	$1535{\cdot}1 \pm 0{\cdot}6$	10 ± 2	$\Xi^-\pi^0$, $\Xi^0\pi^- \sim 100$%
Ω^-	$1672{\cdot}2 \pm 0{\cdot}4$	$(1{\cdot}3 \pm 0{\cdot}3) \times 10^{-10}$ s	$\Xi^0\pi^-$, $\Xi^-\pi^0$, ΔK^-
$\Sigma_c{}^{*++}$?	2500	?	$\Lambda_c{}^+\pi^+$
$\Sigma_c{}^{*0}$?	2500	?	$\Lambda_c{}^+\pi^-$

Table 9. Quantum numbers and quark content of baryons in a five-quark model. We have assumed the b quark is an isospin singlet with charge $-\frac{1}{3}$, although at present there is no convincing evidence for that assignment.

Symbol 35	40	Quark content	35 I	40 I	I_z	S	C	b
Δ^{++}		uuu	$\frac{3}{2}$		$\frac{3}{2}$	0	0	0
Δ^{+}	p	uud	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
Δ^{0}	n	udd	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
Δ^{-}		ddd	$\frac{3}{2}$		$-\frac{3}{2}$	0	0	0
Σ^{*+}	Σ^{+}	uus	1	1	1	-1	0	0
Σ^{*0}	Σ^{0}	uds	1	1	0	-1	0	0
Σ^{*-}	Σ^{-}	dds	1	1	-1	-1	0	0
	Λ^{0}	uds		0	0	-1	0	0
Ξ^{*0}	Ξ^{0}	uss	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2	0	0
Ξ^{*-}	Ξ^{-}	dss	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-2	0	0
Ω^{-}		sss	0		0	-3	0	0
Σ_c^{*++}	Σ_c^{++}	uuc	1	1	1	0	1	0
Σ_c^{*+}	Σ_c^{+}	udc	1	1	0	0	1	0
Σ_c^{*0}	Σ^{0}	ddc	1	1	-1	0	1	0
	Λ_c^{+}	udc		0	0	0	1	0
Ξ_c^{*+}	Ξ_c^{+}, Ξ_c^{A+}	usc	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	1	0
Ξ_c^{*0}	Ξ_c^{0}, Ξ_c^{A0}	dsc	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	1	0
Ω_c^{*0}	Ω_c^{0}	ssc	0	0	0	-2	1	0
Ξ_{cc}^{*++}	Ξ_{cc}^{++}	ucc	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	2	0
Ξ_{cc}^{*+}	Ξ_{cc}^{+}	dcc	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	2	0
Ω_{cc}^{*+}	Ω_{cc}^{+}	scc	0	0	0	-1	2	0
Ω_{ccc}^{++}		ccc	0	0	0	0	3	0
Σ_b^{*+}	Σ_b^{+}	uub	1	1	1	0	0	-1
Σ_b^{*0}	Σ_b^{0}	udb	1	1	0	0	0	-1
Σ_b^{*-}	Σ_b^{-}	ddb	1	1	-1	0	0	-1
	Λ_b^{0}	udb		0	0	0	0	-1
Ξ_b^{*0}	Ξ_b^{0}, Ξ_b^{A0}	usb	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	-1
Ξ_b^{*-}	Ξ_b^{-}, Ξ_b^{A-}	dsb	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	0	-1
Ω_b^{*-}	Ω_b^{-}	ssb	0	0	0	-2	0	-1
Ξ_{cb}^{*+}	Ξ_{cb}^{+}, Ξ_{cb}^{A+}	ucb	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	-1
Ξ_{cb}^{*0}	Ξ_{cb}^{0}, Ξ_{cb}^{A0}	dcb	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	-1
Ω_{cb}^{*0}	Ω_{cb}^{0}, Ω_{cb}^{A0}	scb	0	0	0	-1	1	-1
Ω_{ccb}^{*+}	Ω_{ccb}^{+}	ccb	0	0	0	0	2	-1
Ξ_{bb}^{*0}	Ξ_{bb}^{0}	ubb	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-2
Ξ_{bb}^{*-}	Ξ_{bb}^{-}	dbb	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-2
Ω_{bb}^{*-}	Ω_{bb}^{-}	sbb	0	0	0	-1	0	-2
Ω_{cbb}^{*0}	Ω_{cbb}^{0}	cbb	0	0	0	0	1	-2
Ω_{bbb}^{-}		bbb	0		0	0	0	-3

This was the paradox that led to the introduction of the colour. If the wavefunction is antisymmetric under interchange of colour indices of two quarks, then it must be symmetric under the interchange of the remaining indices.

This argument can be put more simply without making use of SU(8), which is, after all, a broken symmetry. The Δ^{++} baryon is made up of three u quarks in a spin-$\frac{3}{2}$ state. Now a spin-$\frac{3}{2}$ wavefunction constructed from three spin-$\frac{1}{2}$ particles is symmetric. Furthermore, because the Δ^{++} is the lowest-mass state with spin $\frac{3}{2}$, its space wavefunction should also be symmetric. Therefore, the wavefunction of the Δ^{++} is most plausibly symmetric under the interchange of the combined space–spin coordinates of any two u quarks. If this is so, then the spin-statistics theorem tells us that the three u quarks in the Δ^{++} cannot be really identical. In the absence of any real understanding of what constitutes the difference between the three u quarks, we label them red, green and blue.

It might be argued that we should give up the idea the Δ^{++} space wavefunction is symmetric rather than introduce a new degree of freedom. There are two reasons why this alternative possibility is much less popular. First, the quark model is relatively simple and has had much success in the version in which the spatial wavefunctions of the low-mass baryons are symmetric, but the model is contrived and has had little success in the version in which the baryon space wavefunctions are antisymmetric. The second, and perhaps more compelling, reason is that there exist other indirect experimental evidence for the existence of the colour degree of freedom in quarks. We discuss this evidence in §6.7.

In a model with ordinary attractive quark–quark interactions, the mixed symmetry **168** multiplet of SU(8) will lie higher in energy than the symmetric **120**, and the antisymmetric **56** will lie higher still. Also, radially excited states of these multiplets ought to occur. The details of the spectrum depend on the particular form of the quark–quark interaction, discussed in §4.

3.6. Hadron sum rules and quark masses

The magnetic moments of baryons may be readily calculated in terms of the quark moments μ_q. We write down the results only for the baryons whose moments have been measured. More complete results are contained in papers of Franklin (1968), Lichtenberg (1977) and Johnson and Shah-Jahan (1977). The expressions for moments are:

$$\mu_{\mathrm{p}}=\tfrac{1}{3}(4\mu_u-\mu_d) \qquad \mu_{\mathrm{n}}=\tfrac{1}{3}(4\mu_d-\mu_u) \qquad \mu_\Lambda=\mu_s \tag{3.9}$$

$$\mu_{\Sigma^+}=\tfrac{1}{3}(4\mu_u-\mu_s) \qquad \mu_{\Sigma^-}=\tfrac{1}{3}(4\mu_d-\mu_s) \qquad \mu_{\Xi^-}=\tfrac{1}{3}(4\mu_s-\mu_d). \tag{3.10}$$

From equations (3.9) we can obtain the quark moments in terms of the moments of the proton, neutron and Λ. We can then substitute these expressions in equations (3.10) to obtain sum rules for the moments of Σ^+, Σ^- and Ξ^- in terms of the moments of p, n and Λ. The resulting expressions have been given by Franklin (1968). If we assume that the quarks have Dirac moments, then we can use the experimental values of μ_{p}, μ_{n} and μ_Λ in equations (3.9) to obtain the masses of the u, d and s quarks. These are:

$$m_u=338\ \mathrm{MeV} \qquad m_d=322\ \mathrm{MeV} \qquad m_s=467\pm42\ \mathrm{MeV} \tag{3.11}$$

where the indicated error in m_s comes from the error in the measured Λ moment.

According to equation (3.11), the u quark is heavier than the d quark. However, there is contrary evidence from hadron electromagnetic mass splittings, to be discussed later in this section. The evidence from hadron masses is considered to be stronger than the evidence from the baryon magnetic moments. Possible sources of error in the magnetic moment calculation are neglect of exchange currents and other relativistic effects, and neglect of orbital angular momentum in the wavefunctions.

If the masses of the u and d quarks are set equal to one another, we obtain a sum rule which follows from SU(6) without the quark model (Pais 1966). The good agreement of the predicted ratio $\mu_{\mathrm{p}}/\mu_{\mathrm{n}}=-1{\cdot}5$ with the experimental value $\mu_{\mathrm{p}}/\mu_{\mathrm{n}}=-1{\cdot}46$ provides evidence that the proton and neutron wavefunctions are symmetric under the interchange of quark spin and unitary spin indices. With any other choice of symmetry, there is a larger discrepancy between theory and experiment.

If SU(4) symmetry were exact, all members of an SU(4) multiplet would have the same mass. But SU(4) is actually a badly broken symmetry, with the charmed members of a multiplet having considerably larger masses than the uncharmed members. The SU(3) and SU(2) symmetries contained in SU(4) are also not exact, but the symmetry breaking is smaller.

Gell-Mann (1962) and Okubo (1962) obtained sum rules for the masses of hadrons belonging to a given SU(3) multiplet. Their formula was based on the assumption that the mass operator transforms as a component of an SU(3) octet. Okubo (1975) generalised this formula to SU(4), assuming that the mass operator transforms like a component of a fifteen-dimensional representation of SU(4). Additional mass formulae have been obtained by Moffat (1975), Okubo (1975), and others using SU(8) symmetry, assuming definite transformation properties for the mass operator. It is also possible to obtain SU(3), SU(4), SU(6) and SU(8) mass formulae within the framework of the quark model. This has been done by many authors, including Federman *et al* (1966) and Hendry (1967) for broken SU(6), and Hendry and Lichtenberg (1975) for broken SU(8). Sum rules for the masses of charmed hadrons have also been given by Gaillard *et al* (1975), De Rújula *et al* (1975) and others. Most of the sum rules for baryons, to the extent they have been tested, are in qualitative, but not quantitative, agreement with the data. The sum rules for mesons, on the other hand, do not agree with experiment unless the meson wavefunctions break SU(n) invariance with mixing angles. The SU(3) mixing angle was defined by Glashow and Socolow (1966). If the mixing is ideal for vector mesons, the ρ and ω contain only $u\bar{u}$ and $d\bar{d}$, the ϕ only $s\bar{s}$, the ψ only $c\bar{c}$, and the Υ only $b\bar{b}$.

There are some indications from high-energy hadron scattering (see §6) that the binding energy of quarks in hadrons does not play a major role in the collision process. We can get a crude estimate of the effective quark masses from the masses of hadrons by neglecting the quark binding energies. In obtaining this estimate, we shall also neglect the mass difference between the u and d quarks. The N and Δ each contain three quarks with only u and d flavours. Therefore, our crude estimate says (the symbol for a hadron denotes its mass):

$$m_u \simeq m_d \simeq \tfrac{1}{6}(\mathrm{N}+\Delta) = 360 \text{ MeV}. \tag{3.12}$$

Since the Ω contains only strange quarks, we obtain:

$$m_s \simeq \tfrac{1}{3}\,\Omega = 560 \text{ MeV}. \tag{3.13}$$

Using these values, we can now calculate the average mass of the Λ, Σ, Σ^*, each of which contains two ordinary quarks and one strange quark. We get:

$$\Lambda \simeq \Sigma \simeq \Sigma^* \simeq 2\,m_u + m_s \simeq 1280 \text{ MeV}.$$

Likewise, the average mass of the Ξ and Ξ^* is:

$$\Xi \simeq \Xi^* \simeq m_u + 2m_s \simeq 1480 \text{ MeV}.$$

In fact, the average mass of the Λ, Σ and Σ^* is 1230 MeV, and the average mass of the Ξ and Ξ^* is 1425 MeV, not too far from these estimates. One reason that these quark masses are plausible is that they are not too different from the quark masses estimated from the baryon magnetic moments. Going on to the charmed baryons Λ_c and Σ_c, we obtain:

$$m_c \simeq \tfrac{1}{2}(\Lambda_c + \Sigma_c) - 2m_u \simeq 1625 \text{ MeV}. \tag{3.14}$$

We can get similar estimates from the mesons. In this case, because the vector mesons are nearly ideally mixed, we confine ourselves to these mesons. (An additional difficulty with the pseudoscalar mesons is that the pion is anomalously light.) Since the ρ and ω contain only ordinary quarks, we obtain:

$$m_u \simeq m_d \simeq \tfrac{1}{4}(\rho + \omega) \simeq 390 \text{ MeV} \tag{3.15}$$

from the ϕ and ψ, which contain strange and charmed quarks respectively, we get:

$$m_s \simeq \tfrac{1}{2}\phi \simeq 510 \qquad m_c \simeq \tfrac{1}{2} \simeq 1550 \text{ MeV}. \tag{3.16}$$

Again, these estimates of quark masses are similar to those obtained from the baryon masses and magnetic moments. The masses of the K^*, D^* and F^* then are calculated to be:

$$K^* \simeq m_u + m_s \simeq 900 \qquad D^* \simeq m_u + m_c \simeq 1940 \qquad F^* \simeq m_s + m_c \simeq 2060 \text{ MeV}$$

not too far from their actual values of 894, 2007 and 2140 MeV respectively.

We now consider the splitting of isospin multiplets. In the case of baryons, sum rules have been obtained by Rubinstein (1966), Rubinstein *et al* (1967) and Franklin (1975) by assuming that baryon mass differences are caused by the quark mass differences plus two-body quark–quark interactions acting in an additive way. Among these sum rules is the well-known Coleman–Glashow (1961) relation, obtained without the quark model. Some of the others, not yet tested, involve the masses of charmed baryons. Using these methods and assuming SU(3) invariance of the unperturbed wavefunctions, we obtain for the quark mass difference $m_d - m_u$ (Franklin 1968):

$$m_d - m_u = n - p + \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0) = 1{\cdot}9 \pm 0{\cdot}1 \text{ MeV}. \tag{3.17}$$

In the meson case, the assumption that the quark–antiquark interactions are two-body interactions does not lead to any mass relations, because mesons are composed only of two particles. To get mass relations for mesons, we assume that the interaction which breaks isospin symmetry is a Coulomb plus a contact magnetic moment interaction between $q\bar{q}$ of the following form (Kuo and Yao 1965, Miyamoto 1966):

$$V_{12} = Q_1 Q_2 / r - \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 \delta(\boldsymbol{r}). \tag{3.18}$$

We then obtain the inequalities (Gal and Scheck 1967, Lichtenberg 1975):

$$\pi^+ - \pi^0 > 0 \qquad D^+ - D^0 > \mathrm{K}^0 - \mathrm{K}^+. \tag{3.19}$$

These inequalities depend on the positivity of the expectation values $\langle 1/r \rangle$ and $\langle \delta(\boldsymbol{r}) \rangle$. They are both consistent with experiment.

Recently a number of authors (e.g. De Rújula *et al* 1975, Itoh *et al* 1975, Lane and Weinberg 1976, Celmaster 1976, Chan 1977, Ono 1976, 1977, Deshpande *et al* 1977, Peaslee 1977) have made additional or different assumptions about quark interactions and calculated charmed hadron electromagnetic mass differences.

Using an interaction like that of equation (3.18) for qq, and assuming that the effective mass difference between the d and u quark is the same in baryons and mesons, we can obtain two inequalities relating meson and baryon masses, both in agreement with experiment (Lichtenberg 1976). However, the same assumptions which lead to the inequalities (3.19) lead to a condition on $m_d - m_u$:

$$m_d - m_u > \mathrm{K}^0 - \mathrm{K}^+ = 4 \text{ MeV}. \tag{3.20}$$

Comparing the inequality (3.20) with equation (3.17), we see that, although in both cases the d quark is heavier than the u quark, the numerical estimates from baryons and mesons do not agree. A similar result was noted by Itoh *et al* (1975).

There are a number of possible ways to circumvent this difficulty, of which we shall mention only two. The first is to give up SU(3) invariance of the unperturbed wavefunctions. The second is to note that the mass difference between the d and u quark has the indirect effect of leading to breaking of isospin symmetry in the strong interaction. This effect can be calculated if a specific model of the strong interaction is adopted. Celmaster (1976) and Chan (1977) have made such calculations.

4. Quark bound-state dynamics

4.1. *Hadronic excited states*

As we have seen in §3, a large number of hadrons are known to exist. We now discuss further details of the simplification which follows when hadrons are considered as bound systems of quarks. We shall consider the question: can the observed hadrons be accommodated into the ground and excited states of these bound quark systems?

The ground state of a system of quarks with n flavours consists of a badly-broken multiplet of SU($2n$). The symmetry breaking is caused by the different masses of the different flavoured quarks, by flavour-dependent quark interactions, by spin dependence of the quark interactions, and by mixing of spin and orbital angular momentum. Despite all these symmetry-breaking effects, the symmetry SU($2n$) can still be recognised by means of the multiplicities of the states. For example, in a model with three flavours, the ground-state baryons belong to a broken 56-dimensional multiplet of SU(6). All the states of this multiplet have been observed. In addition to the ground state, excited hadron states should also exist. These excitations can be of at least three kinds: (i) orbital excitations, (ii) radial excitations, and (iii) excitations involving the creation of quark–antiquark pairs.

In the approximation that spin and orbital angular momentum are separately conserved, the orbital angular momentum is a good quantum number. Therefore, in this approximation, states of definite orbital angular momentum can be classified according to the three-dimensional orthogonal group O(3), which includes rotations and reflections. The multiplets can then be classified according to the group SU($2n$)$\otimes$ O(3). We denote a multiplet of this group by (N, L^P), where N is the SU($2n$) multiplicity, L is the orbital angular momentum, and P is the parity.

To compare with experiment, we shall confine ourselves mostly to the case with $n=3$ flavours. For baryons, the ground-state multiplet in the SU(6)$\otimes$O(3) scheme is (56, 0^+). Spin-dependent forces cause this multiplet to split up into the SU(3) octet of baryons of spin $\frac{1}{2}$ and decuplet of spin $\frac{3}{2}$. Flavour-breaking forces split individual isospin multiplets within the octet and decuplet. In spite of all this splitting, the (56, 0^+) multiplet is recognisable from the total number of states, their isospins, strangeness, spins and parity. A similar association is done for the excited baryonic states. The harmonic oscillator model (to be described in the next section) has played a particularly useful role in the description of these states.

In the case of mesons, the ground state corresponds to a set of pseudoscalar and vector mesons belonging to mixed multiplet 35 $\oplus$ 1 of SU(6). This has already been discussed to some extent in §3. In §4.3, we therefore concentrate more on the presently popular one-gluon-exchange model for mesons. In later subsections, we describe briefly other important models for hadrons and the possibility of exotic hadrons.

4.2. *Harmonic oscillator model*

In order to be able to calculate the energy spectrum of excited states, one must assume a specific form for the qq and $q\bar{q}$ interactions. As a first approximation, we treat the quarks as moving non-relativistically (Morpurgo 1965), since then one can carry over ideas from atomic and nuclear physics. In this case, a reasonable choice for the potential between quarks (at least for the low-lying states) is the harmonic oscillator potential, which also forms the basis for the shell model in nuclear physics. The harmonic oscillator model was first suggested by Greenberg (1964) to describe the baryon spectrum, and has since been developed by Dalitz (1967), Faiman and Hendry (1968, 1969a, b), Feynman *et al* (1971) and Horgan (1976). As we shall show, the SU(6) ⊗ O(3) harmonic oscillator model provides a convenient classification scheme for all the well-established baryons. We shall also indicate how this scheme has been used to study the various mass splittings among the baryons when the pure symmetry is broken.

As an indication of the regularity that is exhibited among the baryons, we show in figure 5 the observed spectrum of even- and odd-parity baryon states with strangeness 0. These correspond to the nucleon as the ground state and the first few excited levels. These excited states have been obtained from analyses of pion–nucleon scattering by

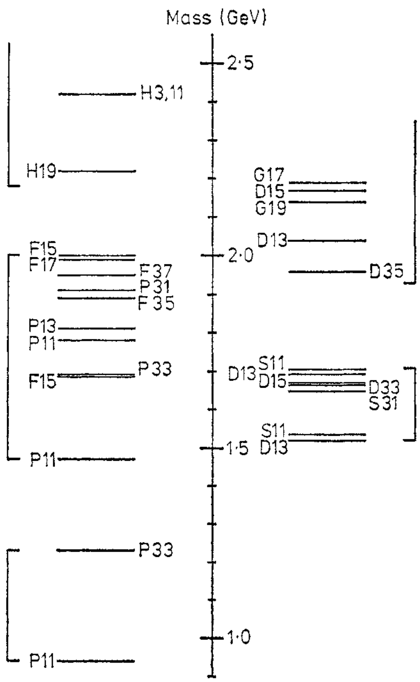

Figure 5. Observed spectrum of N* and Δ* states, with even-parity states on the left and odd-parity states on the right. The notation for a state is $l_{2I,2J}$ where I is the isospin, J is the total spin and l is the orbital angular momentum of the pion–nucleon system to which the resonance can couple.

phase-shift techniques (see Donnachie 1973). The notation used for these resonances is $l_{2I,\,2J}$ where I is the isospin of the resonance, J is its total spin and l is the orbital angular momentum of the pion–nucleon system to which it couples. Since the pion has odd intrinsic parity, the parity of the resonance is $(-1)^{l+1}$. It is clear from figure 5 that, at least for these low-lying levels, the resonances seem to fall into bands of alternating parity.

Besides the strangeness 0 baryons, there are many others of type Λ^*, Σ^* and Ξ^* with strangeness -1 and -2 which accompany the particles shown in figure 5, and which fill out the corresponding SU(3) octets, decuplets and singlets. A report on the status of baryon resonances has been given recently by Lanius (1976). A summary of the present state of the art in resonance hunting is provided in Ross and Saxon (1976).

We consider now the energy levels of a three-quark system interacting via harmonic oscillator forces. We restrict ourselves for simplicity to states with zero strangeness and neglect the mass differences between the d and u quarks. The non-relativistic Hamiltonian is:

$$H=\frac{1}{2m}\sum_{j}\boldsymbol{p}_j^2+\tfrac{1}{2}\,m\omega^2\sum_{i<j}(\boldsymbol{r}_i-\boldsymbol{r}_j)^2 \tag{4.1}$$

where $\boldsymbol{r}_j$, $\boldsymbol{p}_j$ ($j=1, 2, 3$) are the locations and momenta of the three quarks. In terms of the CM (centre-of-mass) coordinate $\boldsymbol{R}$ and two relative coordinates $\boldsymbol{\lambda}$, $\boldsymbol{\rho}$ defined by:

$$\boldsymbol{R}=\tfrac{1}{3}\,(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3) \qquad \boldsymbol{\lambda}=(\boldsymbol{r}_1+\boldsymbol{r}_2-2\boldsymbol{r}_3)/\sqrt{6} \qquad \boldsymbol{\rho}=(\boldsymbol{r}_1-\boldsymbol{r}_2)/\sqrt{2} \tag{4.2}$$

the Hamiltonian becomes:

$$H=\frac{\boldsymbol{P}^2}{2(3m)}+\frac{1}{2m}\,(\boldsymbol{p}_\lambda{}^2+\boldsymbol{p}_\rho{}^2)+\tfrac{1}{2}\,m(\sqrt{3}\,\omega)^2\,(\boldsymbol{\lambda}^2+\boldsymbol{\rho}^2) \tag{4.3}$$

where $\boldsymbol{P}$, $\boldsymbol{p}_\lambda$ and $\boldsymbol{p}_\rho$ are the momenta canonically conjugate to $\boldsymbol{R}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ respectively. It follows from equation (4.3) that, apart from the CM motion, the eigenfunctions are products of one-body harmonic oscillator wavefunctions in the coordinates $\boldsymbol{\lambda}$, $\boldsymbol{\rho}$. In order to obtain the allowed complete wavefunctions, these spatial parts have to be combined with SU(6) wavefunctions belonging to the symmetric 56, mixed 70 or antisymmetric 20 representations, so that the final wavefunction is symmetric. (Colour, which we ignore, makes the overall wavefunction antisymmetric.)

The first three levels of the spectrum of states generated in this scheme are listed in table 10. This table gives in the right-hand columns the expected nucleonic states N*, Δ^*. By comparing with figure 5, we can see that the states observed in the lowest even- and odd-parity bands are precisely those expected in the ground state ($56, 0^+$) and first excited level ($70, 1^-$) of the harmonic oscillator scheme. Moreover, the states occurring in the next band of even-parity states can all be accommodated in the second excited level, although there are still some states missing. These latter states may yet be uncovered in future phase-shift analyses, although their widths may be so large and their couplings to the pion–nucleon channel so small that they may never be found.

The observed strangeness -1 and -2 states Λ^*, Σ^* and Ξ^* can also be accommodated and form the remaining members of the SU(3) octets, decuplets and singlets in table 10. The ($56, 0^+$) and ($70, 1^-$) levels are almost complete and the other known baryons of this kind can fit into the next excited level. (See Litchfield (1974, 1976) and Litchfield *et al* (1975a, b, 1976) for a detailed discussion.)

Table 10. Lowest levels of a three-quark harmonic oscillator. The right-hand column gives the corresponding SU(3) decomposition with octets, decuplets and singlets having total quark spin $\frac{1}{2}$ or $\frac{3}{2}$. The states in parentheses are the N*, Δ* states belonging to these multiplets; these should show up as resonances in pion–nucleon scattering processes.

	Spatial	SU(6), L^P	SU(3) content		
Ground state ($n=0$)	$(1s)^2$	56, 0^+	$8^{1/2}(P_{11})$	$10^{3/2}(P_{33})$	
First excited level ($n=1$)	(1s) (1p)	70, 1^-	$8^{1/2}(D_{13}, S_{11})$ $8^{3/2}(D_{15}, D_{13}, S_{11})$	$10^{1/2}(D_{33}, S_{31})$	$1^{1/2}$
Second excited level ($n=2$)	(1s) (2s), (1s) (1d), $(1p)^2$	56, 0^+	$8^{1/2}(P_{11})$	$10^{3/2}(P_{33})$	
		70, 0^+	$8^{1/2}(P_{11})$ $8^{3/2}(P_{13})$	$10^{1/2}(P_{31})$	$1^{1/2}$
		20, 1^+	$8^{1/2}(P_{13}, P_{11})$		$1^{3/2}$
		56, 2^+	$8^{1/2}(F_{15}, P_{13})$	$10^{3/2}(F_{37}, F_{35}, P_{33}, P_{31})$	
		70, 2^+	$8^{1/2}(F_{15}, P_{13})$ $8^{3/2}(F_{17}, F_{15}, P_{13}, P_{31})$	$10^{1/2}(F_{35}, P_{33})$	$1^{1/2}$

The major achievement of the harmonic oscillator model is that it provides a classification of this large array of resonant states which have been discovered during the last twenty years. They can be understood, at least in principle, in terms of simple excitations of a three-quark system. We say 'in principle' because in reality the resonance spectrum is much more complicated than we have described so far. In pure SU(6) ⊗ O(3), the particles belonging to each excited level all have the same mass, but from figure 5, the observed resonances fall into fairly broad bands. In nature, therefore, the exact symmetry is badly broken.

Symmetry breaking leads to an enormous complication of the simple picture, but nonetheless deserves serious consideration. One interesting problem is to try to determine the nature of the main symmetry-breaking pieces which bring about the splitting among the various members in each level. This has been examined in detail by Greenberg and Resnikoff (1967), Horgan and Dalitz (1973, 1974), Horgan (1974) and Jones *et al* (1974). The mass operator is expressed as a term which is invariant under the symmetry, plus terms which break the symmetry in all possible ways, such as spin–orbital and SU(3)-breaking contributions. Matrix elements are then calculated using harmonic oscillator wavefunctions. The situation is complicated further by possible mixing between wavefunctions (for example, each of the two physical S_{11} states in the (70, 1^-) level are linear combinations of pure harmonic oscillator states). Jones *et al* (1977) compare the various methods used by the different groups and present a final joint analysis. The pattern of symmetry-breaking terms turns out to be very complicated, with symmetry-breaking pieces frequently being as large as the symmetry-invariant pieces.

From figure 5, we see that there is evidence for resonances with spins $J=\frac{9}{2}$ and $\frac{11}{2}$. Other high-spin resonances with $J=\frac{15}{2}, \frac{17}{2}, \ldots$ have been suggested by Hendry (1976a, b). These can be accommodated in levels that are even more highly excited than the ones we have discussed so far. However, in the harmonic oscillator model, the number of expected states increases rather rapidly as the three-quark system becomes more excited (Horgan 1976), so this model may not be so useful for these high-spin states. Perhaps the more economical picture of treating the three-quark system as a quark–di-quark system (Lichtenberg 1969) may be more appropriate, since this generates only the 'minimal' spectrum (56, L even) and (70, L odd). (A further discussion of di-quarks is given in §4.5.) Alternatively one should also consider other models such as the linear string model of Cutkosky and Hendrick (1977a,b) or bag models (see §4.4), but as yet less quantitative work has been done with these models for baryons than with the harmonic oscillator model.

4.3. One-gluon-exchange model with quark confinement

Quantum chromodynamics is a theory based on analogy with quantum electrodynamics. Therefore, if the coupling constant governing the strength of the quark–gluon interaction is sufficiently small, the qq interaction ought to bear a certain resemblance to the electron–electron interaction. If the electron–electron interaction arising from one-photon exchange is reduced to non-relativistic form, the resulting potential is known as the Fermi–Breit potential (see, for example, Bethe and Salpeter 1957). This potential contains, in addition to a Coulomb term $1/r$, where r is the separation between quarks, a spin-dependent term, a tensor term, and still other terms.

As expected, the one-gluon-exchange potential is very similar to the one-photon-exchange potential between two electrons. The main difference, aside from the difference in coupling strength, is a numerical factor arising from the non-Abelian nature of the gluon field. Let α_s be the strong-interaction coupling constant analogous to the fine-structure constant α. Then the strength of the quark–antiquark interaction in a colour singlet state is $-4\alpha_s/3$, and the strength of the quark–quark interaction in an antisymmetric colour state is $-2\alpha_s/3$. (The analogous strengths in the electron–positron and electron–electron cases are $-\alpha$ and $+\alpha$ respectively.) The one-gluon-exchange potential (or part of it) has been considered by Appelquist and Politzer (1975), De Rújula *et al* (1975), Barbieri *et al* (1976), Lichtenberg and Wills (1975), Wills *et al* (1977) and Celmaster (1976, 1977), among others.

Using the renormalisation group (Gell-Mann and Low 1954, Callan 1970, Symanzik 1970), Politzer (1973, 1974) and Gross and Wilczek (1973a) have shown that non-Abelian gauge field theories like QCD are *asymptotically free*. This means that the high-energy, short-distance behaviour of such theories logarithmically approaches the behaviour of a field theory without interactions. Because of this behaviour, the strong-interaction coupling strength α_s is often taken to be a function of momentum, or of the quark masses. Thus, α_s is taken to be largest for the interaction between u and d quarks, somewhat smaller for the interaction between two s quarks, and still smaller for the interaction between two c quarks. Because the dependence of α_s on quark mass is logarithmic, with a scale which appears to be around 1 GeV, α_s will decrease only slightly for quarks heavier than the c quark.

The one-gluon-exchange potential model has been used primarily in discussing the meson spectrum. This potential by itself leads to a meson energy spectrum which in first approximation is similar to the hydrogen atom spectrum. As the energy increases the energy levels rapidly crowd closer together, and there is a definite ionisation energy. The observed meson spectrum does not look like this. As the energy increases the level spacing decreases, but not nearly so fast as would be expected from a $1/r$ potential. Furthermore, an ionisation marked by the appearance of free quarks has not yet been seen (or at least not yet been recognised).

The most popular proposal to get around this problem has been to assume that the one-gluon-exchange potential applies at small distances but that a confining interaction takes over at large distances. For a number of reasons, various authors (e.g. Tryon 1972, 1976, Gunion and Willey 1975, Harrington *et al* 1975, Eichten *et al* 1975, Kang and Schnitzer 1975) have assumed that the quark confining interaction is a linear potential. (This implies that the force between two quarks is a constant, independent of distance once their separation increases beyond a certain value.) One motivation for a linear potential comes from consideration of gauge field theories on a lattice (Wilson 1974, Kogut and Susskind 1975). More recently, Machacek and Tomozawa (1976) and Quigg and Rosner (1977) have suggested on phenomenological grounds that the confining potential may grow only logarithmically.

De Rújula *et al* (1975) argue that all the spin dependence of the interaction is in the one-gluon-exchange potential. On the other hand, Schnitzer (1975), Pumplin *et al* (1975), Wills *et al* (1977) and Jackson (1977) include spin-dependent terms which include derivatives of the confining potential. In the treatment of Celmaster *et al* (1977), α_s is taken to be independent of momentum, but instead the one-gluon-exchange potential is modified by a factor which depends on the logarithm of position. Despite attempts to justify the one-gluon-exchange plus confining potential on theoretical grounds, at present such a potential should be regarded as phenomenological.

Another problem concerns what wave equation to use to describe the motion of the quarks. Most authors have used the non-relativistic Schrödinger equation for ease of calculation, but the Klein–Gordon equation (Gunion and Li 1975), Dirac equation (Goldman and Yankielowicz 1975), Bethe–Salpeter equation (Cung *et al* 1976), and others have also been used.

Calculations with these models have led to predicted meson spectra which are in qualitative, but not quantitative, agreement with the experimental meson masses. The non-relativistic approximation ought to improve as the quark masses increase. Therefore, there has been considerable effort put into calculating the $c\bar{c}$ spectrum (called charmonium). The experimental spectrum is shown in figure 6. The charmonium models have had qualitative success in fitting this spectrum, but have not been able to account for the large singlet–triplet splitting in a convincing way. For a recent review of charmonium spectroscopy, see Jackson (1977).

4.4. *Bag, lattice and string models*

Approaches to hadron spectroscopy which are somewhat different from the potential method are the so-called bag and string models of hadrons. We discuss these models only briefly.

In the MIT bag model (Chodos *et al* 1974a,b, DeGrand *et al* 1975, Jaffe and Kiskis 1976, Deshpande *et al* 1977), quarks bound in a hadron satisfy the free Dirac equation within a region having a boundary, called the bag surface. The boundary conditions are: first, that all vector currents vanish at the bag surface, and second, that no energy or momentum is carried across the surface.

The model contains a universal constant B, which is a positive potential energy per unit volume inside the bag. This constant is introduced to confine the quarks and gluons, as with it, an infinite amount of energy is necessary to expand the volume to

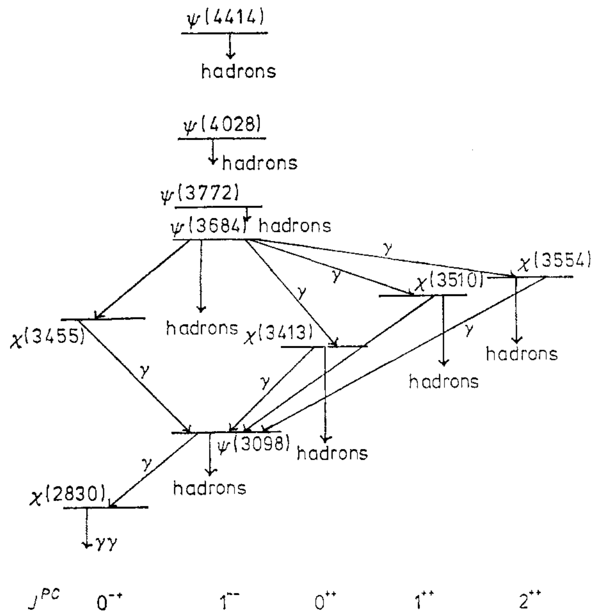

Figure 6. Observed charmonium spectrum.

infinity—in other words, to free quarks or gluons. The constant B is regarded as an adjustable parameter. A number of calculations have been carried out in the approximation in which the bag itself is treated classically. The decay of a hadron is described by a fissioning of a colour-singlet bag into two or more bags, each of which is also a colour singlet. If a bag were to fission into bags which were not coloured singlets, coloured gluon flux lines would be cut in violation of the boundary conditions.

In the SLAC bag model (Bardeen *et al* 1975), the authors start from a canonical field theory, with fields to describe coloured quarks and gluons. The ground state of the system is approximated by a variational method. The classical field equations with scalar coupling lead to quarks which have much lower effective masses when bound than their original large bare masses. One feature of the solutions is that quarks are confined to a thin shell at the surface of what is known as the SLAC bag. Unlike the MIT bag, however, the SLAC bag does not require an extra energy-density term, but apparently emerges from classical field theory with strong coupling.

Thus far, no one has succeeded in working out the basic consequences of non-Abelian gauge field theory. For this reason, a number of authors (Wilson 1974, 1976, Balian *et al* 1974, Kogut and Susskind 1975) have considered the more tractable problem of gauge theory on a lattice. In this theory, quarks are confined to points on a lattice, and the lattice spacing is a parameter of the theory. Time is sometimes treated as a discrete variable and sometimes as a continuous one.

The hope in lattice gauge theory is that it will be possible to obtain a well-defined limit as the lattice spacing goes to zero and, furthermore, that such a limit will give the same results as a continuum gauge field theory. Even if such a hope is not realised, the lattice theory may be a good approximation to the continuum theory. The lattice theory has the defect of not being Lorentz-invariant. But the potential and bag models also have their shortcomings, the former not allowing for pair creation, and the latter thus far being solvable only semiclassically.

In a lattice model reviewed by Wilson (1976), the theory contains both coloured quarks and coloured strings (the gauge field). The strings are necessary because colour is conserved at each lattice site. In the absence of strings an isolated coloured quark would be unable to move from one site to another, as the colour of the site would thereby change. In the model, the colour gauge field behaves like a string with colour at one end and anticolour at the other. A meson, which is a combination of a quark and antiquark attached to a string, can move without any change of colour at a site. Likewise a baryon, which is a combination of three quarks and three strings, is free to move.

It has been shown that such a theory is asymptotically free in the continuum limit. However, although for sufficiently large coupling, quarks are confined in a lattice gauge theory (Challifour and Weingarten 1978), it has not yet been possible to discover whether confinement holds when the lattice spacing goes to zero.

Up to six quarks and six antiquarks of each flavour may be on a single lattice site (three colour degrees of freedom, two spin degrees of freedom). A meson in its ground state contains a quark and antiquark on the same site in a singlet colour state. Likewise, a baryon in its ground state contains three quarks on the same site in a colour singlet state. Excited states contain strings of length equal to one or more lattice spacings. In a static approximation, the mass of a hadron in its ground state is just the sum of the masses of the quarks it contains. However, the dynamics gives rise to corrections which depend on the lattice spacing. A quark–di-quark structure for excited baryons results from a configuration in which two quarks are on one site and a third is on a neighbouring site.

4.5. Di-quarks and exotic hadrons

In Gell-Mann's (1964) original paper on quarks, he discussed possible ways of observing these particles. In a footnote to this discussion, he cautioned that it was conceivable that the state of lowest mass with fractional charge might not be a quark but a bound multiquark state, for example, a di-quark.

Subsequently, the possibility of bound di-quarks existing in hadrons has been considered by a number of authors. Ida and Kobayashi (1966) and Lichtenberg and Tassie (1967) considered models in which a baryon is composed of a quark and di-quark. Later, mass sum rules and other properties of baryons were obtained in the quark–di-quark model (Lichtenberg *et al* 1968, Carroll *et al* 1968, Ono 1973), and baryon multiplets of SU(6) $\otimes$ O(3) were considered (Lichtenberg 1969). In a quark–di-quark model with strong exchange forces, the **56** multiplets have even L and positive parity, while the **70** multiplets have odd L and negative parity. These multiplets are the ones for which the best experimental evidence exists. Capps (1974) has suggested that colour-symmetry breaking by massive coloured gluons could account for a quark–di-quark structure of baryons.

Recently, Rosenzweig (1976) has suggested that di-quark–antidi-quark bound states exist. A meson consisting of a di-quark–antidi-quark pair may be exotic, i.e. it may have quantum numbers which forbid it from existing (part of the time) as a bound $q\bar{q}$ state. Such a meson may not necessarily belong to $\boldsymbol{n} \otimes \bar{\boldsymbol{n}}$, but to a larger multiplet. Of course not all members of the larger multiplet will have exotic quantum numbers. Also, such a meson may have odd spin, negative parity, and positive C parity: such a state cannot be made from $q\bar{q}$, but can be made from a di-quark–antidi-quark pair.

Other models for exotic states exist. Bander *et al* (1976) and De Rújula *et al* (1977) have suggested that an exotic meson is a loosely bound state of two more tightly bound systems, each of the latter being a quark–antiquark pair (see also Kenny *et al* 1976). At present there is no firm evidence for the existence of exotic mesons.

Exotic baryons, containing an extra quark–antiquark pair, may also exist. Again, we can speculate about the configuration of such a state. One possibility is that the configuration of an exotic baryon is primarily that of a bound or resonant state of a baryon and meson. Another possibility is that the state consists principally of two di-quarks and an antiquark. The evidence for exotic baryons is summarised by Trippe *et al* (1976) and Lanius (1976). Of particular interest here is the possible existence of a strangeness +1 baryon. Such a state cannot be made up from only three ordinary quarks, but could be produced in K^+p and K^+n collisions. However, although there were early claims for strangeness +1 baryons $Z_0{}^*$, $Z_1{}^*$ with isospin 0 and 1 respectively, their existence is still in doubt (see Martin 1976).

Although the experimental evidence for exotics is at present inconclusive, there is no reason in principle why they should not exist. If they do not, there must be something in the dynamics which forbids them. Exotics, if they exist, may, however, be hard to find, especially if they have relatively high mass and small production cross sections.

5. Hadron decays

5.1. Leptonic decays of vector mesons

Quark models can also be used to describe the decays of hadrons. We begin the consideration of this subject by examining the leptonic decays of vector mesons. Van Royen and Weisskopf (1967) calculated the partial decay width of neutral vector mesons into lepton pairs under the assumption that the mesons are bound states of quark–antiquark pairs. Including an additional factor of three for colour, the Van Royen–Weisskopf formula is:

$$\Gamma_{l^+l^-} = 16\,\pi\alpha^2\,|\Psi(0)|^2\,|\sum_i C_i Q_i|^2/m^2 \tag{5.1}$$

where $\Psi(0)$ is the wavefunction at the origin, C_i is the Clebsch–Gordan coefficient of quark i in the meson, Q_i is the charge of the quark, and m is the meson mass.

Using this formula, we can calculate the leptonic decay widths of the ρ, ω, ϕ and ψ mesons into lepton pairs, provided we can estimate $\Psi(0)$. Alternatively, we can use the measured values of $\Gamma_{e^+e^-}$ and $\Gamma_{\mu^+\mu^-}$ to obtain the values of $\Psi(0)$. In table 11 we give the values of $|\Sigma_i C_i Q_i|^2$, the experimental partial decay widths from Trippe *et al* (1976, 1977) and Rapidis *et al* (1977), and the values of $\Psi(0)$ calculated from equation (5.1). Although quark model calculations have been done which are in qualitative agreement with the values of $|\Psi(0)|$, no one has yet achieved quantitative agreement.

5.2. Strong decays of hadrons

When possible, excited mesons and baryons usually prefer to decay into lower states by means of the emission of mesons, for example $\Delta^*(1232)\to N\pi$ and $K^*(892)\to K\pi$ (exceptions are discussed in the next subsection). In previous sections, we discussed how different hadrons can be assigned to various multiplets. As a result, many

Table 11. Partial decay widths of vector mesons into lepton pairs.

Process	$\lvert\Sigma_i C_i Q_i\rvert^2$	$\Gamma_{l^+l^-}$ experiment (keV)	$\lvert\Psi(0)\rvert$ (GeV)$^{3/2}$
$\rho^0\to e^+e^-$	1/2	$6{\cdot}6\pm1$	0·054
$\rho^0\to\mu^+\mu^-$	1/2	10 ± 2	0·066
$\omega\to e^+e^-$	1/18	$0{\cdot}76\pm0{\cdot}17$	0·056
$\phi\to e^+e^-$	1/9	$1{\cdot}3\pm0{\cdot}1$	0·067
$\phi\to\mu^+\mu^-$	1/9	$1{\cdot}0\pm0{\cdot}1$	0·059
$\psi(3095)\to e^+e^-$	4/9	$4{\cdot}8\pm0{\cdot}6$	0·20
$\psi(3095)\to\mu^+\mu^-$	4/9	$4{\cdot}8\pm0{\cdot}6$	0·20
$\psi(3684)\to e^+e^-$	4/9	$2{\cdot}1\pm0{\cdot}3$	0·15
$\psi(3684)\to\mu^+\mu^-$	4/9	$1{\cdot}8\pm0{\cdot}3$	0·14
$\psi(3772)\to e^+e^-$	4/9	$0{\cdot}37\pm0{\cdot}10$	0·066
$\psi(4414)\to e^+e^-$	4/9	$0{\cdot}44\pm0{\cdot}14$	0·085

of the decay rates of excited hadrons ought to be correlated, and indeed one way of checking the possibility of an underlying symmetry scheme is to compare the decay rates of the various members of a suspected multiplet.

As an example, let us consider the decays of the spin-$\frac{3}{2}^+$ SU(3) decuplet of baryons into the spin-$\frac{1}{2}^+$ octet B(N, Σ, Λ, Ξ) and the pseudoscalar 0^- nonet of mesons P(π, K, $\bar{\text{K}}$, η, η'). Because of the mass differences, only four decays are possible: $\Delta(1232)\to\text{N}\pi$, $\Sigma(1385)\to\Sigma\pi$ and $\Lambda\pi$, and $\Xi(1530)\to\Xi\pi$. The decay rate can be written in the form (Samios *et al* 1974, Trippe *et al* 1976 appendix II):

$$\Gamma=C^2g^2k^{2l+1}/M^* \tag{5.2}$$

where C is an SU(3) coefficient (different for each decay), M^* is the mass of the decaying decuplet resonance, k is the CM momentum of the particles in the decay channel, and l is the orbital angular momentum in the final state. For the decays $\frac{3}{2}^+\to\frac{1}{2}^++0^-$, we have $l=1$. The group SU(3) correlates the four decays and, as seen in equation (5.2), there is only one unknown coupling constant g for all four decays. (The form of the expression (5.2) is not unique, and some authors prefer to incorporate a barrier penetration factor.) Inserting the physical values for M^* and k for each decay allows for some symmetry breaking.

A comparison of the experimental decay rates for the four possible decays and theoretical values obtained by adjusting g^2 to give a best overall fit is given in table 12. The agreement is reasonable, but not perfect. Similar results are found in the case of excited meson decays (Samios *et al* 1974).

One can examine higher symmetry schemes. For example, we found that baryons could be classified in the harmonic oscillator scheme SU(6) $\otimes$ O(3). The various

Table 12. Comparison of experimental decay rates (Trippe *et al* 1976) for $\frac{3}{2}^+\to\frac{1}{2}^++0^-$ with SU(3) (Samios *et al* 1974).

Decay	Γ (MeV) (theory)	Γ (MeV) (experiment)
$\Delta(1232)\to\text{N}\pi$	107	115 ± 5
$\Sigma(1385)\to\Sigma\pi$	5	$4{\cdot}2\pm0{\cdot}9$
$\to\Lambda\pi$	35	$30{\cdot}8\pm2{\cdot}5$
$\Xi(1530)\to\Xi\pi$	12	$9{\cdot}1\pm0{\cdot}5$

excited levels generally include many SU(3) multiplets (the 70, 1^- representation alone contains a total of nine different SU(3) multiplets when one allows for quark spin and orbital angular momentum coupling). The larger symmetry, however, correlates the decays of all these multiplets. Let us consider an excited baryon in this scheme decaying into the ground-state baryon octet B, emitting a pseudoscalar meson P. If the decay takes place by means of single quark de-excitation, and we use as a first approximation the non-relativistic form of this interaction, the decay rate takes the form (Faiman and Hendry 1968):

$$\Gamma = C\frac{f_q^2}{4\pi}\frac{E}{\mu^2 M^*}\frac{k^{2n+3}}{\alpha^{2n}}\exp\left(-\frac{k^2}{3\alpha^2}\right). \tag{5.3}$$

Here C is a numerical coefficient which depends upon the particles involved, f_q is the quark–meson coupling constant, μ is the mass of the emitted meson, E is the energy of the final-state baryon, and $\alpha^2 = m\omega$ is the harmonic oscillator constant (see equation (4.1)). The quantity n takes the values $n = 0, 1, 2, \ldots,$ for baryons B^* belonging to the successive excited levels (see table 10). Again, the physical masses are used in (5.3) to allow for some symmetry breaking. There is also the complication, mentioned in §4.2, of physical states with the same set of experimental quantum numbers (isospin, total spin J and parity) being mixtures of pure symmetry states. For these states, the formula for the decay rates is more complicated than (5.3) to allow for the mixing.

Table 13. Predicted $N\pi$ partial widths in the harmonic oscillator model with $f_q^2/4\pi = 0{\cdot}053$ and $\alpha^2 = 0{\cdot}10$ (GeV/c)^2.

Resonance	(N, L^P)	$\Gamma_{N\pi}$ (MeV) (theory)	$\Gamma_{N\pi}$ (MeV) (experiment)
$P_{33}(1232)$	$(56, 0^+)$	115	115
$D_{15}(1670)$	$(70, 1^-)$	32	70 ± 15
$D_{33}(1670)$	$(70, 1^-)$	27	30 ± 15
$S_{31}(1650)$	$(70, 1^-)$	23	49 ± 15
$F_{37}(1950)$	$(56, 2^+)$	82	88 ± 10
$F_{17}(1990)$	$(70, 2^+)$	11	10 ± 5

We show in table 13 the decay rates for the unmixed states of strangeness 0 in the $n = 0$, 1, 2 levels. The quark coupling constant $f_q^2/4\pi$ and the harmonic oscillator constant α^2 were chosen to give the width for $\Delta(1232) \to N\pi$ correctly, and simultaneously to yield a best fit to the other widths. The agreement is encouraging, and calculations have been extended to include the full SU(3) multiplet substructure (Faiman 1971), as well as decays by photon emission (Faiman and Hendry 1969a, Copley *et al* 1969a,b).

Several improvements, however, need to be made. First, many of the decays involve relativistically moving particles, so that the recoil of the decaying resonance is important. A model which incorporates recoil was suggested some time ago by Mitra and Ross (1967). A more recent model with recoil has been developed by Feynman *et al* (1971). This has been particularly successful in describing the amplitudes in pion photoproduction processes (Moorhouse and Oberlack 1973, Moorhouse *et al* 1974, Knies *et al* 1974, Metcalf and Walker 1974).

A second difficulty can be seen in expression (5.3). Normally we would expect the decay rate to be proportional to k^{2l+1}, where l is the orbital angular momentum of the two particles in the final state. However, the above decay rate is proportional to k^{2n+3}, where n is the degree of excitation of the initial resonance (see table 10). The effect of this k^{2n+3} factor is two-fold. First, if a resonance with sizeable n decays into a final state where the CM momentum k is small, the corresponding calculated decay rate is much too small. This happens, for example, in the case of the $n=2$ (56, 0^+) resonance $P_{11}(1470)$ decaying into $N\pi$; the calculated decay rate is 37 MeV compared to the estimated experimental value of about 140 MeV. This problem is alleviated to some extent by models which incorporate recoil terms. Secondly, a more serious situation arises when a resonance can decay into a particular final state by two different orbital angular momenta. For example, the $n=1$ (70, 1^-) $D_{33}(1670)$ can decay into $\Delta(1232)\pi$ with both $l=0$ and $l=2$ in the final state. We would expect these decay rates to be proportional to k and k^5 respectively, but the harmonic oscillator model gives k^5 for both of them.

There is further evidence that the harmonic oscillator model is too restrictive when two partial waves are possible in the final state. Not only does it give the magnitudes of the contributions from the two waves, it also gives their relative phases. These phases can be determined experimentally (Herndon *et al* 1975, Barnham 1976). Although the analyses are not completely reliable, there are indications that the relative phases are not always the same as those given by the harmonic oscillator model.

Several authors (Colglazier and Rosner 1971, Faiman and Plane 1972a,b, Peterson and Rosner 1972, 1973) have suggested a less stringent group structure to describe resonance decays, namely l-broken $SU(6)_W \otimes O(2)_{L_z}$. The group $SU(6)_W$ (Lipkin and Meshkov 1966) is suitable for describing collinear processes, while $O(2)_{L_z}$ allows for orbital excitations. The combined vertex symmetry group $SU(6)_W \otimes O(2)_{L_z}$ gives essentially the same answers as the harmonic oscillator model, but without the characteristic momentum and Gaussian factors in equation (5.3). It still relates different l values in the final state where two values are possible. The l-broken symmetry implies that the couplings for the two l values are to be treated separately. This separation has also received support from a completely algebraic approach due to Melosh (1974); see also Gilman (1973) and Gilman *et al* (1973, 1974).

Much effort has gone into fitting a large number of decay rates, allowing for the possibility of l-broken symmetry and mixing between states with the same quantum numbers (see Dalitz (1976) and Hey (1976) for a discussion of the latest fits). The results are somewhat perplexing since some decays seem to prefer the relative phases of the harmonic oscillator or $SU(6)_W \otimes O(2)_{L_z}$ model, while other decays seem to prefer opposite phases. Also there is some disagreement about the mixing of states as determined from the decays and from fits to the mass spectrum (Jones *et al* 1977). With more accurate data, some of these puzzles may be resolved.

5.3. *Okubo–Zweig–Iizuka rule*

The collisions and decays of hadrons can be pictured in terms of interactions and decays of their constituent quarks. Diagrams showing these processes explicitly in terms of quarks are known as quark line diagrams. They were introduced in their present form by Harari (1969) and Rosner (1969), although Zweig (1964, 1965) used essentially equivalent diagrams. They are now frequently used to pictorialise the underlying quark dynamics. In figure 7 we give a quark line diagram for pion–nucleon elastic scattering showing the contribution from one-meson exchange.

Quark line diagrams have been useful in suggesting quark mechanisms to explain

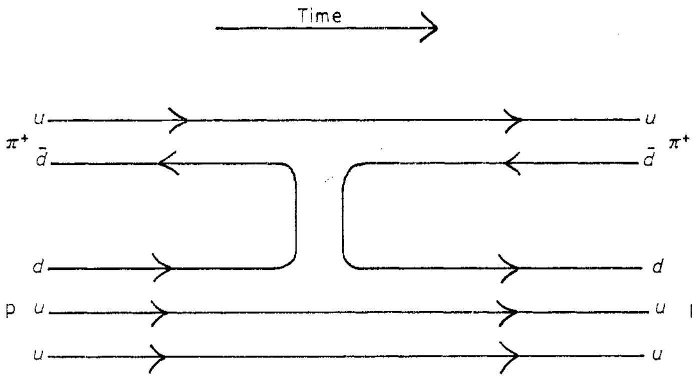

Figure 7. Quark line diagram for elastic π^+p scattering.

properties of hadron interactions. They have also led to a puzzle which may be an important clue about interactions between the quarks themselves. Consider for example the decays of the ω and ϕ mesons. These both have the same quantum numbers $J^{PC}=1^{--}$. One would therefore expect that the ϕ, which has a greater mass than the ω, would also have a greater decay width. In fact, as can be seen from table 4, the decay width of the ω is more than twice as great as that of the ϕ. The puzzle seems even greater when we look at the partial decay widths of the ω and ϕ into three pions. From table 4 we see that the partial decay width of the ω into 3π is about fifteen times that of the ϕ into the 3π. We conclude that there is a dynamical mechanism which inhibits the decay of the ϕ meson into three pions. This mechanism is not well understood, but it can be expressed in terms of an empirical rule known as the Okubo (1963), Zweig (1964), Iizuka (1966) rule or OZI rule.

To explain this rule, let us consider the decays:

$$\omega \to 3\pi \qquad \phi \to 3\pi \qquad \phi \to \mathrm{K}\bar{\mathrm{K}}$$

in terms of the quark line diagrams of figure 8. There we have used the information from §3 that the ω consists predominantly of $u\bar{u}$ and $d\bar{d}$ quarks while the ϕ consists predominantly of $s\bar{s}$ quarks. We see that each quark line in figure 8(a) and (c) is part of two hadrons, but that the s quark line in figure 8(b) is confined to the ϕ. In other words, the vertex is disconnected. This is an illustration of the OZI rule which states that any process which incorporates a disconnected quark line diagram is forbidden. A disconnected diagram can be defined as one in which one or more hadrons can be isolated by a line which does not cut any quark lines. Okubo (1963) obtained this rule from algebraic considerations without using the quark model explicitly.

The OZI rule inhibits the decay $\phi \to 3\pi$ while allowing the decays $\omega \to 3\pi$ and $\phi \to \mathrm{K}\bar{\mathrm{K}}$. The overall narrow width of the ϕ is explained because there is not much available phase space for the ϕ decay into $\mathrm{K}\bar{\mathrm{K}}$. The small decay of the ϕ into 3π can be accounted for either as a result of a small violation of the OZI rule or as a result of a deviation of the ϕ wavefunction from pure $s\bar{s}$.

The OZI rule also accounts for the extremely narrow widths of the $\psi(3095)$ and $\psi'(3684)$ mesons, and the much larger width of the $\psi''(3772)$. The ψ'' has enough mass to decay into a $D\bar{D}$ pair, and this decay is allowed by the OZI rule (see figure 9). On the other hand, the ψ and ψ' have masses less than twice the D mass, and so must decay via OZI-forbidden processes.

It is suspected (Feldman 1977) that the ψ decays into an η or η' meson (plus other

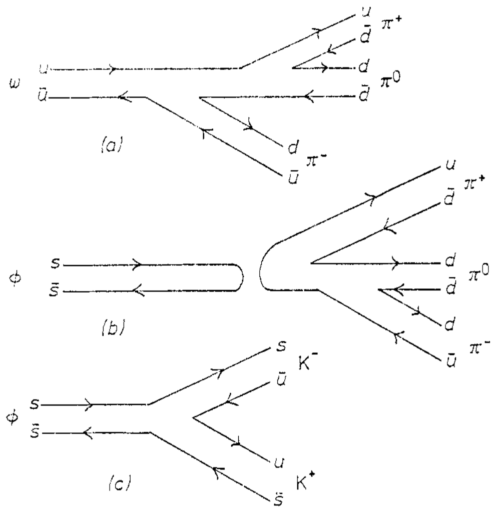

Figure 8. Quark line diagrams showing (*a*) the allowed decay of the ω into three pions, (*b*) the Zweig-forbidden decay of the ϕ into three pions, and (*c*) the allowed decay of the ϕ into K^+K^-.

things) with an unusually large branching ratio. If this interpretation of the data is correct, it indicates that the η and η' have an admixture of $c\bar{c}$ quarks in their wave-functions. Thus the pseudoscalar mesons do not seem to be ideally mixed.

While the OZI rule remains a puzzle, a possible way of explaining it may come from QCD. In QED an electron–positron pair bound into positronium can virtually annihilate into a single photon. In contrast, in QCD a quark–antiquark pair bound into a meson cannot annihilate into a single gluon, because a gluon has colour while a meson is colourless. Thus, the annihilation must take place into at least two gluons. If the coupling between quarks and gluons is sufficiently small, then the fact that the process

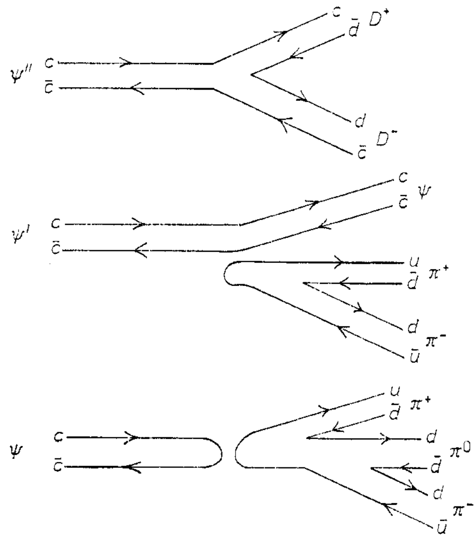

Figure 9. Quark line diagrams illustrating the allowed decay of the ψ'' into D^+D^- and examples of Zweig-forbidden decays of the ψ' and ψ.

must be of higher order can inhibit it. As mentioned in §2.4, according to the asymptotic freedom hypothesis, the effective quark–gluon coupling is indeed expected to be small for quarks that are close together. Thus, if the quark–antiquark annihilation takes place at short distances, the process might be severely inhibited.

5.4. *Weak decays of hadrons*

The so-called standard model of the weak interactions of hadrons is that of Weinberg (1967) and Salam (1968), generalised by Glashow *et al* (1970) to include the charmed quark. In this model, both weak and electromagnetic interactions are described by a gauge field theory. The bosons $W^{\pm}$, Z^0, which are coupled to the weak vector and axial vector currents, are gauge fields. But unlike the photon, which couples to the electromagnetic current, the weak bosons are thought to be massive (70–100 GeV) because the symmetry is spontaneously broken.

In the model the gauge fields interact with hadronic and leptonic currents. The charged hadronic current couples the quarks in only certain ways; namely, the members within the pairs (u, d') and (c, s') are coupled together. Here u and c are fields representing the u and c quarks respectively, while d' and s' are the following linear combinations of quark fields:

$$d' = d \cos \theta_C + s \sin \theta_C \qquad s' = -d \sin \theta_C + s \cos \theta_C$$

where θ_C is the Cabibbo (1963) angle. Once the charged currents are given, the gauge theory predicts that neutral currents should also exist. These have in fact been observed.

Many interesting results follow from this theory because the Cabibbo angle is experimentally observed to be rather small ($\sin^2 \theta_C \simeq 0{\cdot}06$). One consequence is that strangeness-changing decays of light (uncharmed) hadrons are inhibited, whereas charmed hadrons preferentially decay into strange particles. This can be seen most readily by means of quark line diagrams describing hadron decays. We illustrate in figure 10 the strangeness-changing decay $\Lambda \to p + \pi^-$. As one can see from the figure, an s quark emits a W^- boson, becoming a u quark. But the u quark is coupled to d',

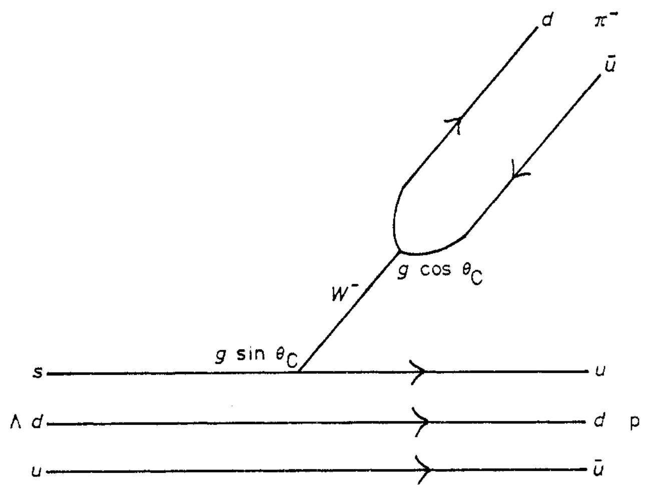

Figure 10. Quark line diagram illustrating the Cabibbo-suppressed decay of the Λ into $p\pi^-$.

which contains $s \sin \theta_C$, and so this decay is inhibited. The decay $D^0 \to K^-\pi^+$ is favoured, but $D^0 \to K^+\pi^-$ is forbidden. Its antiparticle D^{-0} does, however, decay into $K^+\pi^-$. So far, we have discussed non-leptonic decays of hadrons. However, charged lepton currents (ν_e, e), (ν_μ, μ) also couple to the W, and therefore, semileptonic weak decays of hadrons also occur, for example, $K^+ \to \pi^0 e^+ \nu_e$.

Although the standard model with four quarks and four leptons successfully explains a large body of data on weak decays, it probably needs to be enlarged. This is because there is some evidence that at least two more leptons (τ, ν_τ) and at least one more quark (the b quark contained in the Υ) exist. The weak interaction of this quark is not yet known. Models of the weak interactions which are simple extensions of the Weinberg–Salam model require that the quarks interact as left-handed doublets. According to such models, the number of quarks is even. Thus, these models predict that if the b exists then a sixth quark (sometimes called the t quark) should also exist. Still additional pairs of quarks and leptons are not excluded.

Another possible difficulty with the standard model is that it leads to the prediction of parity violation in atomic physics but, so far, parity violation has not been observed. More theoretical and experimental work is needed to discover whether indeed a discrepancy exists (see, for example, Wolfenstein (1977) for additional discussion).

6. High-energy collisions

6.1. *Additivity and total cross sections*

If hadrons are made up of quarks, their internal composition should show up when one hadron collides with another. In analogy with two atoms colliding, we may visualise two clouds of quarks approaching and penetrating each other; the hadrons scatter as a result of the interactions between their constituent quarks.

As a first guess, one might write the scattering of two hadrons simply as the sum of the scatters from the individual constituents (Levin and Frankfurt 1965, Lipkin and Scheck 1966). Thus, for example, for proton–proton scattering, the scattering amplitude would be:

$$\begin{aligned}\langle pp \mid pp \rangle &= \langle (uud)(uud) \mid (uud)(uud) \rangle \\ &= 4\langle uu \mid uu \rangle + \langle dd \mid dd \rangle + 2\langle ud \mid ud \rangle + 2\langle du \mid du \rangle.\end{aligned}$$

This additivity of the individual quark contributions is similar to the impulse approximation in nuclear physics. It is expected to be a reasonable approximation near the forward scattering direction—at larger angles, multiple quark scattering and spin effects might become important. Also, the energy should be high so that incident particle velocities are much greater than the velocities associated with the internal motion of the quarks.

The optical theorem relates the total collision cross section to the imaginary part of the forward scattering amplitude. Thus if all the forward qq and $q\bar{q}$ amplitudes were equal at high energies, we would expect the total cross sections for meson–baryon (MB) processes to be equal to one another, and likewise for all baryon–baryon (BB) processes. Moreover, we would expect that they are related by:

$$\sigma_{tot}(\text{MB})/\sigma_{tot}(\text{BB}) = \tfrac{2}{3}. \tag{6.1}$$

This amazingly simple result, which relates MB and BB processes (generally difficult to do in non-constituent models), follows essentially from 'quark counting'—since a meson has two constituents while a baryon contains three, there are six quark collisions in MB scattering and nine in BB scattering, yielding a ratio of $\frac{2}{3}$. Constituent models with different numbers of constituents will, of course, usually give different answers for this ratio. For example, the Fermi–Yang model (Fermi and Yang 1949), in which baryons are treated as elementary and mesons as baryon–antibaryon bound states, gives the answer 2.

The experimental total cross sections for several MB and BB processes are shown in figure 11. Clearly, the total cross sections for all MB processes are not equal to one another, nor are the BB cross sections equal. However, the MB and BB processes do tend to fall into separate groups, their averages at 200 GeV/c being $\sigma_{\text{tot}}(\text{MB}) \approx 21{\cdot}5$ mb and $\sigma_{\text{tot}}(\text{BB}) \approx 40{\cdot}1$ mb. These yield a ratio of 0·54, encouragingly near the value 0·67 of the above equation.

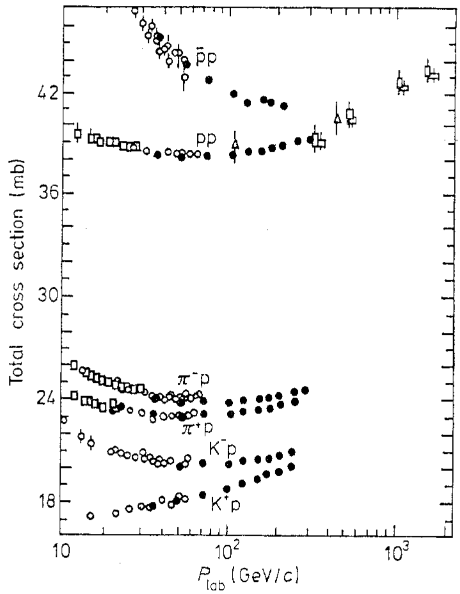

Figure 11. Total cross sections for several meson–baryon and baryon–baryon processes. Data from Carroll *et al* (1976) and earlier data cited therein.

Figure 11 shows that there must be differences between the individual quark amplitudes. By relaxing the constraint that these amplitudes are all equal, one can obtain relations among the MB and BB processes that are slightly better satisfied than (6.1). For example, if we use only isospin invariance for the quark amplitudes in addition to the additivity assumption, the following relations (Lipkin 1966, 1973, Feld 1969) can be derived:

$$\frac{2\ \Sigma(\pi\text{p})}{\Sigma(\text{pp})+\Sigma(\text{pn})} = \frac{2}{3} = \frac{\Sigma(\pi\text{p})+[\Sigma(\text{Kp})-\Sigma(\text{Kn})]/3}{\Sigma(\text{pp})} \tag{6.2(a)}$$

$$\Delta(\text{Kp})-\Delta(\text{Kn}) = \Delta(\pi\text{p}) = \Delta(\text{pp})-\Delta(\text{pn}) \tag{6.2(b)}$$

where $\Sigma(\pi\text{p}) = \sigma_{\text{tot}}(\pi^-\text{p})+\sigma_{\text{tot}}(\pi^+\text{p})$, $\Delta(\pi\text{p}) = \sigma_{\text{tot}}(\pi^-\text{p})-\sigma_{\text{tot}}(\pi^+\text{p})$, etc, stand for the sums and differences of total cross sections respectively. These relations are compared with experiment in figure 12. The relations (6.2(b)) involving the cross-section differ-

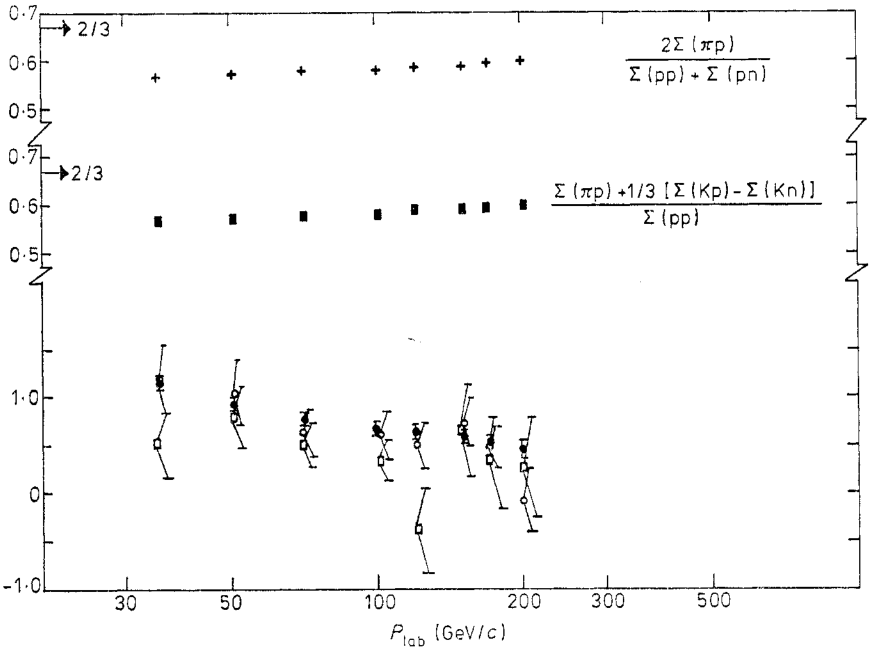

Figure 12. Comparison of the relations (8.2) involving sums Σ and differences Δ of total cross sections. Data from Carroll *et al* (1976). ●, $\Delta(\pi p)$; ○, $\Delta(Kp)-\Delta(Kn)$; □, $\Delta(pp)-\Delta(pn)$, (values in mb).

ences are quite well satisfied. However, the ratios of the cross-section sums in (6.2(*a*)) fall somewhat below $\frac{2}{3}$, though agreement seems to be improving gradually as the energy increases.

One can easily think of many possible sources for these discrepancies. Spin effects have been neglected. Moreover, at these high energies, relativistic corrections might be important. Annihilation processes probably play an important role (Lipkin 1966), since $\bar{p}p$ and $\bar{p}n$ total cross sections (see figure 11) seem unusually large and may be one of the main reasons why the ratios in (6.2(*a*)) come out less than $\frac{2}{3}$. It is also possible that there is some as yet unknown component (Lipkin 1974, 1975) that has not been taken into account. Additivity itself, a basic ingredient in the above discussions, needs further examination. Just like the impulse approximation, it is based on the idea that collision processes involve two quarks at a time, the other quarks present in the projectile and target acting as spectators. Multiple scattering of quarks may not be negligible.

Before considering multiple scattering effects, however, we should point out that all of the relations that have been obtained above from the simple quark model picture can, with some effort, be derived from other models (Barger and Cline 1969, Lipkin 1973, Barger 1974) of particle scattering, such as models involving meson or baryon exchanges between the projectile and target, or from symmetry SU(3) and SU(6) arguments. The advantage of the quark model is its great simplicity and its ability to tie together a large amount of experimental data within a simple framework.

6.2. *Collisions with small momentum transfer; multiple scattering*

The extension of quark ideas away from the exact forward direction requires some knowledge about the quark distributions inside the interacting particles, and thus probes in slightly more detail the internal structure of hadrons. As a first step, we consider processes where only a small amount of momentum is transferred from the projectile to the target, but where we retain the assumption of additivity of the amplitudes and neglect multiple quark scattering.

For a reaction of the type $A+B\rightarrow C+D$, two convenient kinematic variables are:

$$s=(p_A+p_B)^2 \qquad t=(p_A-p_C)^2$$

where $p_A, \ldots, p_D$ are the four-momenta of the particles $A, \ldots, D$, s is the square of the total CM energy, and t is the square of the four-momentum transfer. Thus, for the elastic scattering of two particles A and B, additivity gives (Kokkedee and Van Hove 1966, Kokkedee 1969) for the scattering amplitude:

$$\langle AB\,|\,AB\rangle = T_{AB}(s,t) = \sum_{i,j} f_i{}^A(t) f_j{}^B(t) T_{ij}(s_{ij}, t) \tag{6.3}$$

where on the right-hand side T_{ij} is the amplitude for the scattering of quark i on quark j, and $f_i{}^A$, $f_j{}^B$ are t-dependent form factors associated with the bound-state structure of quarks inside A, B, respectively. If we assume a simple picture of hard, small (so that multiple scattering can be assumed negligible) quarks colliding, with all the quark amplitudes T_{ij} taken equal to ig at high energies, the expansion for T_{AB} may be rewritten as:

$$T_{AB}(s,t)\rightarrow ig\left[\sum_i f_i{}^A(t)\right]\left[\sum_j f_j{}^B(t)\right] = igF^A(t)F^B(t)$$

where $F^{A,B}(t)$ are form factors associated with the distribution of matter inside the scattering objects A, B. Though $F^{A,B}(t)$ are unknown, a reasonable guess might be that they are similar to the electric charge distributions $G_E{}^{A,B}(t)$ inside A, B. This latter quantity is well known, at least for protons, from the study of elastic scattering of electrons off protons. Thus if A and B are protons, we obtain for the differential cross section $\mathrm{d}\sigma/\mathrm{d}t$ the estimate:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\mathrm{pp}) = \left[\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\mathrm{pp})\right]_{t=0}[G_E{}^{\mathrm{p}}(t)]^4. \tag{6.4}$$

We have used the fact that $\mathrm{d}\sigma/\mathrm{d}t = |T_{AB}|^2$.

In figure 13, the experimental data for pp differential cross sections for beam momenta $p_{\mathrm{lab}} = 10$, 19·2, 100 and 200 GeV/c are compared with $[G_E{}^{\mathrm{p}}(t)]^4$. For convenience the proton charge form factor is represented by:

$$G_E{}^{\mathrm{p}}(t) = (1-t/0{\cdot}71)^{-2} \tag{6.5}$$

which is a very good fit to the form factor data (Coward *et al* 1968). For small values of t, there is reasonable agreement, indicating that near the forward direction multiple scattering is not too large an effect. This last observation can be corroborated from other experimental data, namely the cross sections for processes which manifestly require at least two quark scatters to take place (such as $\pi^-\mathrm{p}\rightarrow\pi^+\Delta^-$, which involves a charge exchange of two units). These cross sections always seem to be small (Dauber *et al* 1969). Likewise, since the quark content of π^- is $u\bar{d}$ and that of ϕ is $s\bar{s}$ (both quarks being different from those in π^-), we would expect (Alexander *et al* 1966) the cross section for $\pi^-\mathrm{p}\rightarrow\phi\mathrm{n}$ to be much smaller than, say, the cross section for ρ^0 or ω production. This is again the case experimentally, ϕ production being typically two orders of magnitude smaller than ρ or ω production (Ayres *et al* 1974, Blobel *et al* 1975).

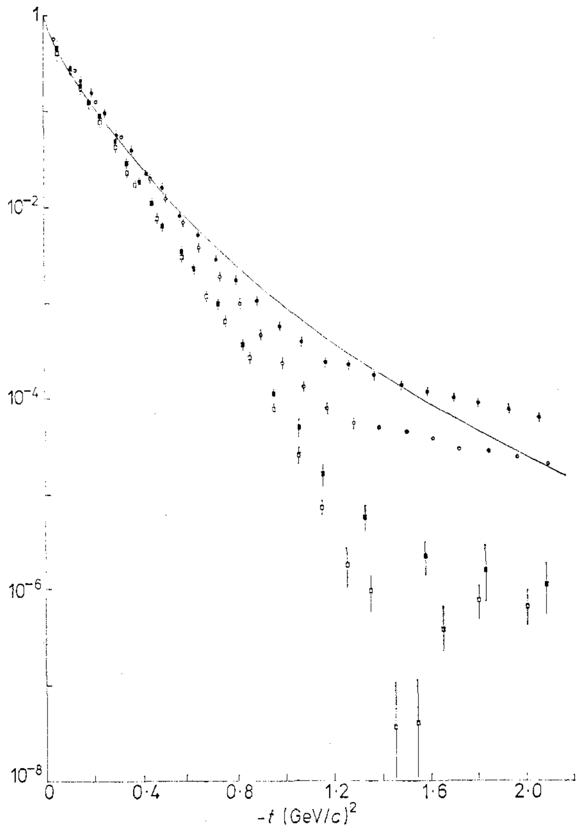

Figure 13. Comparison of $(d\sigma/dt)/(d\sigma/dt)_{t=0}$ for proton–proton elastic scattering at laboratory beam momenta 10·0 (●), 19·2 (○), 100 (■), 200 (□) GeV/c, with $[G_E{}^p(t)]^4$(—). Proton data from Allaby *et al* (1968, 1973) and Akerlof *et al* (1976).

However, as we see in figure 13, at larger values of the momentum transfer there is disagreement between the cross-section data and equation (6.5), indicating that, for those values of t, multiple scattering effects are no longer negligible. We therefore turn to considering corrections due to multiple scattering between the quarks. Fortunately, a formalism which incorporates multiple scattering effects was developed by Glauber (1959). It was used originally in nuclear collisions, but it has been taken over directly to provide a description for hadron–hadron scattering.

Consider the collision of two particles A and B, each composed of several constituents: in figure 14, we have illustrated this with three constituents each. Viewed in the CM frame, A and B approach each other with their centres separated by a distance $\boldsymbol{b}$ (the impact parameter) in the plane perpendicular to the CM momentum. The constituents are momentarily at distances $\boldsymbol{a}_i$, $\boldsymbol{a}_j'$ ($i, j = 1, 2, 3$) from the lines of direction of their respective centres of mass. Then the elastic scattering amplitude can be written as:

$$F(\boldsymbol{\Delta}) = \frac{ip}{2\pi} \int d^2b \exp(-i\boldsymbol{\Delta}.\boldsymbol{b}) \langle AB | \Gamma | AB \rangle \tag{6.6}$$

where $\boldsymbol{\Delta} = \boldsymbol{p}' - \boldsymbol{p}$ is the momentum transferred and $\boldsymbol{p}$, $\boldsymbol{p}'$ are the initial, final momenta in the CM system. The quantity Γ is called the profile function for the scattering and

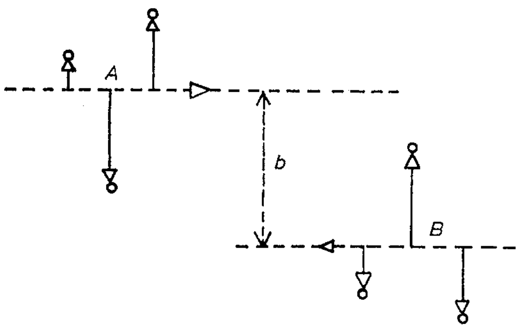

Figure 14. Collision of two composite hadrons A and B, each taken to have three constituents. b is the impact parameter.

can be expressed in the form $\Gamma = 1 - \exp(\mathrm{i}\chi)$. The eikonal χ can be thought of as the total phase shift experienced when the two particles scatter.

In Glauber's approximation, χ is taken as a sum of all the individual phase shifts arising from the successive scatters between pairs of constituents:

$$\chi = \sum_{i,j} \chi_{ij}.$$

This gives:

$$\begin{aligned}\Gamma &= 1 - \prod_{i,j} \exp[\mathrm{i}\,\chi_{ij}(\boldsymbol{b} - \boldsymbol{a}_i + \boldsymbol{a}_j')] \\ &= \sum_{i,j} \Gamma_{ij} - \sum_{i,\ldots,n} \Gamma_{ij}\Gamma_{mn} + \ldots \qquad (6.7)\end{aligned}$$

where $\Gamma_{ij} = [1 - \exp(\mathrm{i}\,\chi_{ij})]$ is the Fourier transform of the amplitude $f_{ij}(\boldsymbol{\delta})$ for the scattering of constituent i in A off constituent j in B with momentum transfer $\boldsymbol{\delta}$. Combining these equations we see that the full scattering amplitude $F(\boldsymbol{\Delta})$ can be expressed as a series, the first term of which comes from scatters between a single pair of constituents, the second from double scatters, and so on. As in (6.7), successive terms alternate in sign.

Examination (Trefil 1967) of this series shows that the contributions from successive terms decrease in magnitude. Also, as functions of the momentum transfer $\Delta = \sqrt{-t}$, successive terms have flatter Δ dependence. This means that near the forward direction single scattering dominates, but as Δ increases, the single-scattering term falls off faster than the double-scattering term. Thus for sufficiently large Δ, the double-scattering contribution dominates. Physically, it has become easier to have the overall scattering take place via two small-angle scatters than through one large angle. At even larger Δ the triple-scattering term will dominate, and so on. Because of the alternating signs in the Glauber series, a characteristic feature is the sequence of minima in the differential cross section, corresponding to the places where successive terms become comparable and destructively interfere. In optics, these are just the successive diffraction minima.

Some years ago, strong diffraction-like minima were observed to occur in nuclear collisions, such as in proton–deuterium and proton–helium scattering. Glauber theory was applied (Franco and Glauber 1966, Saudinos and Wilkin 1974) with considerable success. In high-energy physics, one again finds dips in the data. Figure 13, for example, shows structure developing at $-t \approx 1{\cdot}5\ (\mathrm{GeV}/c)^2$ in the pp elastic differential cross section as the beam energy is increased. It is tempting to try to explain these dips in terms of a multiple scattering approach. However, before one can proceed, at least two pieces of information have to be provided. First, the wavefunctions Ψ_A, Ψ_B of the colliding particles in terms of their constituents must be known. These are usually taken as Gaussians, corresponding to a ground state with harmonic oscillator forces. In addition, one needs to know something about the constituent–constituent scattering amplitude $f(\delta)$. In nuclear physics, this corresponds to nucleon–nucleon scattering, about which a great deal is known. In hadron physics, however, almost nothing is known about quark–quark scattering. Therefore, the theory becomes much more speculative and one has to resort to somewhat arbitrary parametrisations of qq scattering.

In most attempts (Franco 1967, Deloff 1967, Harrington and Pagnamenta 1967, 1968, Schrauner *et al* 1969, Klenk and Kanofsky 1973) to fit the data, a Gaussian form has been used for qq scattering. Other attempts (Dean 1969, Dorren 1974, Licht *et al* 1976) have incorporated Regge exchanges between quarks. Elastic scattering of hadrons has received the most attention, although a few inelastic processes such as $\pi + \mathrm{N} \to \pi + \mathrm{N}^*$ (Hendry and Trefil 1969, Le Yaouanc *et al* 1971, 1972, Ravndal 1971) and processes involving double charge or double strangeness exchange (Dean 1968) have also been examined. We shall not go into the details of these fits here. Suffice it to say that reasonable fits to the experimental data can be obtained. However, beyond the actual fits, it is not completely clear exactly what has been accomplished. The parametrisations used for qq scattering presumably do not correctly represent the underlying quark dynamics but only form a convenient ansatz. Moreover, relativistic effects, negligible in nuclear physics applications, are probably important (Lipkin 1969, 1970, Krzywicki and Le Yaouanc 1969) at high energies. Some improvements along these lines, such as using Lorentz-boosted wavefunctions (Licht and Pagnamenta 1970a,b), have been made recently, so there is some hope that a more satisfactory theory for near-forward scattering will be forthcoming.

6.3. Collisions at large momentum transfer

In the last few years, most of the interest about particle collisions has moved from small to large momentum transfers. As we have seen, the scattering of particles near the forward direction is a coherent effect with many types of collisions of the subparticles contributing to the overall scattering. Thus, it looks like a painstaking task to extract properties of the underlying quark interactions from near-forward scattering data. Therefore, following Rutherford, we turn our attention to larger scattering angles where possibly a much cleaner picture of hadron scattering in terms of quark interactions may hold.

As the scattering angle increases, differential cross sections fall off very rapidly—typically exponentially in t for $|t|$ not too large. This means that events which take place with large scattering angles are comparatively rare. What is the mechanism which produces these rare events? While it is certainly possible for coherent effects to contribute, a more tempting hypothesis is that they come about from hard, incoherent, collisions between the constituents of the colliding particles. The hope of studying large momentum transfer processes therefore is that one might learn directly something about the properties of the constituents themselves.

There are three principal ways of doing this, each way differing from the other by the type of probe used as a projectile. The target in each case consists of either protons or deuterons. The simplest kind of projectile is an object that itself has no internal structure, but which is essentially a point particle. There are two convenient kinds of projectiles of this type—electrons or muons (which interact via electromagnetic forces), and neutrinos or antineutrinos (which interact via the weak force). Another possibility, which we do not consider in detail, is to use real photons (coming from electron bremsstrahlung) as projectiles. Alternatively, we can give up the advantage of point-like projectiles and use hadrons beams, although this is a more difficult way for extracting the desired information.

Historically, experiments with electrons beams were done first, in the late sixties. Almost immediately, surprising results were obtained when the electrons were observed to scatter inelastically off nucleons with large momentum transfer. The simplest interpretation of the data seemed to be that the electron was scattering from hard, point objects, rather from the extended nucleon as a whole! This observation led to an upsurge of interest in quark–parton models in the early seventies, and pointed the way to further detailed experimental studies of large momentum transfer events.

In the next three subsections, we discuss some of the work that has been done with these different beams. Generally speaking, high-energy physicists nowadays believe that all the main features of the large momentum transfer data are reasonably well understood (at least qualitatively) in terms of constituent collisions, although there is some disagreement about the actual underlying processes. However, as in the small momentum transfer models, the qq interactions again have to be parametrised in a somewhat arbitrary way. The details of quark–parton dynamics are still unknown and lie tantalisingly to be discovered in the future.

6.4. *Deep inelastic electron scattering*

In the early days of electron accelerators, much experimental effort was directed towards the study of elastic electron–nucleon scattering (Hofstadter 1963). From this one can extract the electric and magnetic form factors $G_{E,M}(t)$ of the proton and neutron. As we have noted in equation (6.5), these form factors fall off roughly as the fourth power of the momentum transfer $\sqrt{-t}$, indicating the general softness and fuzziness of nucleons. In the late 1960s, however, interest swung over to inelastic electron scattering, especially at large momentum transfers where one might expect to probe short distances. For recent reviews of deep inelastic electron scattering, we refer the reader to Drees (1971), Feynman (1972), Friedman and Kendall (1972), Llewellyn-Smith (1972), Roy (1975) and Yan (1976).

The reactions studied in greatest detail are of the type:

$$\mathrm{e} + \mathrm{N} \rightarrow \mathrm{e}' + X$$

where X stands for all the other particles in the final state besides the scattered electron. In these reactions, only the scattered electron was detected. (A reaction in which only one of the final-state particles is detected is usually called an *inclusive process* (Feynman 1969).) To lowest order in the electromagnetic coupling, the electron interacts with the nucleon through the exchange of a single virtual photon, as indicated in figure 15. In general, this inclusive process can be described in terms of three independent variables, two of which are usually taken as q^2 and ν, defined by:

$$t = q^2 = (p' - p)^2 \qquad \nu = P.q/M.$$

Here p and p' are the four-momenta of the incoming and scattered electron respectively, P is the four-momentum of the target nucleon, q is the four-momentum of the

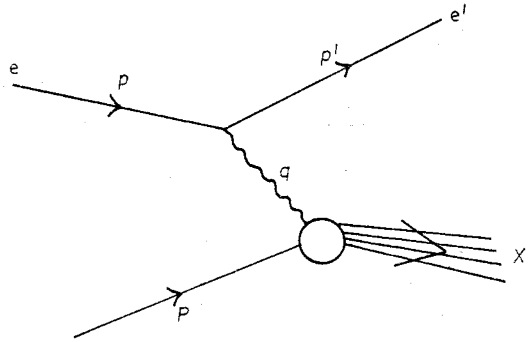

Figure 15. Electron–nucleon inelastic scattering with single-photon exchange. X represents the (undetected) final hadronic state.

exchanged photon, and M is the mass of the target nucleon. In the laboratory frame (the nucleon rest frame), these quantities reduce to:

$$q^2 = -Q^2 = -EE' \sin^2 \tfrac{1}{2}\,\theta \qquad \nu = E - E'$$

where E, E' are the energies of the incoming and scattered electron, and θ is the scattering angle. In the formula for q^2, we have neglected the electron mass. The produced hadronic part X has total mass M_X given by:

$$M_X{}^2 = (P+q)^2 = M^2 - Q^2 + 2M\nu.$$

We shall also frequently make use of the variable x defined by:

$$x = Q^2/2M\nu.$$

Since $M_X \geqslant M$, we have that $0 \leqslant x \leqslant 1$, the upper limit of x corresponding to elastic electron–nucleon scattering.

By writing down the Feynman amplitudes for the process in figure 15, one can show (Drell and Walecka 1964) that the differential cross section may be expressed in the following form:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{1}{2}\theta} \left[2\,W_1(\nu, Q^2) \sin^2 \tfrac{1}{2}\theta + W_2(\nu, Q^2) \cos^2 \tfrac{1}{2}\theta\right] \qquad (6.8)$$

where α is the fine-structure constant and W_1, W_2 are called the ***structure functions***. Since the electron is a point particle whose electromagnetic interaction is known, these structure functions reflect properties at the nucleon vertex only. They are analogous to the nucleon form factors in elastic scattering; however, in general, they depend on two variables (such as ν and Q^2) rather than just one. At small angles θ, W_2 gives the major contribution to the cross section; the contribution from W_1 becomes more important as the scattering angle increases.

There is another convenient way (Hand 1963) of writing this differential cross section. At the nucleon vertex, the process that takes place is just the absorption of the virtual photon by the nucleon. It is therefore possible to express the above differential cross section in terms of σ_T and σ_L, the total photon–nucleon absorption cross sections for the virtual photon in its transverse and longitudinal polarisation states:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{\alpha}{4\pi^2} \frac{E'}{EQ^2} \left(\nu - \frac{Q^2}{2M}\right)\left(\frac{2}{1-\epsilon}\right)(\sigma_\mathrm{T} + \epsilon\sigma_\mathrm{L}) \qquad (6.9)$$

where $\epsilon = [1 + 2(1 + \nu^2/Q^2) \tan^2 \frac{1}{2}\theta]^{-1}$ Comparison of these two expressions (6.8) and (6.9) for the differential cross section gives:

$$
\begin{aligned}
W_1(\nu, Q^2) &= \frac{1}{4\pi^2\alpha}\left(\nu - \frac{Q^2}{2M}\right)\sigma_\mathrm{T} \\
W_2(\nu, Q^2) &= \frac{1}{4\pi^2\alpha}\left(\nu - \frac{Q^2}{2M}\right)\left(\frac{Q^2}{Q^2 + \nu^2}\right)(\sigma_\mathrm{T} + \sigma_\mathrm{L})
\end{aligned}
\tag{6.10}
$$

or equivalently one can deduce that the ratio R of the absorption cross sections is related to W_1 and W_2 by:

$$R = \frac{\sigma_\mathrm{L}}{\sigma_\mathrm{T}} = \left[\left(1 + \frac{\nu^2}{Q^2}\right)\frac{W_2}{W_1} - 1\right]. \tag{6.11}$$

Data have been taken at several values of θ (6–34°) for a range of incident energies E (4·5–18 GeV) (Bloom *et al* 1969a,b, 1972). To obtain a separation of σ_T and σ_L, radiatively corrected cross sections at constant values of Q^2 and $M_X{}^2$ are plotted as functions of ϵ (which corresponds to different values of θ). Once σ_T and σ_L are known, one can calculate $R = \sigma_\mathrm{L}/\sigma_\mathrm{T}$ and the structure functions W_1 and W_2 from (6.10).

What might one expect to find? Certainly if $M_X{}^2$ is such that it corresponds to one of the more distinct low-mass nucleon resonances, for example the $\Delta(1232)$, N*(1520) or N*(1690), one would expect to see resonance bumps. This is indeed the case, as illustrated in figure 16 where the unseparated combination $(\sigma_\mathrm{T} + \epsilon\sigma_\mathrm{L})$ is plotted against M_X. However, for fixed M_X, these resonance bumps quickly disappear as Q^2 increases. The resonance contribution falls off roughly as the power Q^{-8} as Q^2 becomes larger, in a similar manner to the square of the elastic nucleon form factors. This strong

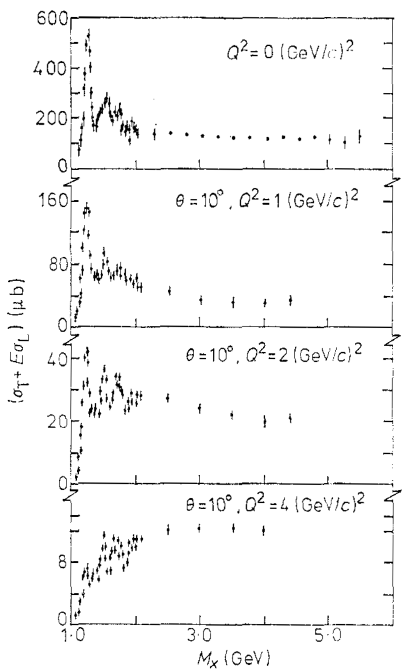

Figure 16. Electron–nucleon cross-section combination $(\sigma_\mathrm{T} + \epsilon\sigma_\mathrm{L})$ in μb.

momentum-transfer dependence is again a reflection of the extended nature of these objects.

At larger Q^2, there is no trace of resonances. In fact, as we can see from figure 16 for fixed M_X above the resonance region, the cross section remains quite large and is only slightly dependent on the momentum transfer (it varies at most as Q^{-2}). This is strongly indicative of scattering from a point-like object, rather than from an extended fuzzy object! The temptation is to think that the electrons are probing inside the nucleon, and that they are scattering from point-like constituents. Feynman (1972) gave these objects the name *partons*. (These partons may be the quarks that were used in building up resonance spectra for mesons and baryons, but there is no need to assume that at the outset.) Observation of these almost constant cross sections at large momentum transfers was one of the most crucial results in high-energy physics in the late 1960s. It provided a great stimulus for constituent theories of the hadrons (regarded with some scepticism until then), and led the way to a large effort, both theoretical and experimental, to try to elicit more detailed properties of the constituents.

Before turning to a simple theoretical model with partons, we draw attention to two other important aspects of the data. One concerns the value of $R=\sigma_L/\sigma_T$; the other is the property known as 'scaling'. The ratio R can be obtained where there are sufficient data to separate out both σ_L and σ_T. With present data, this cannot be done very accurately, and values of R turn out to vary from 0 to 0·5 with sizable errors. However, on the assumption that R is constant throughout the kinematical region investigated, the average value of R is $0{\cdot}18\pm0{\cdot}10$ for a proton target. A similar value is obtained for deuterons. It would appear therefore that the transverse photo-absorption cross section dominates over the longitudinal cross section in these reactions.

Experimentalists have also attempted to explore a region which is of special interest to theorists. This is the asymptotic region where both ν and Q^2 become very large, $x=Q^2/2M\nu$ remaining finite. In general, W_1 and W_2 should everywhere depend separately on the two variables ν and Q^2. However, before the data were available, Bjorken (1969) suggested on the basis of various field theoretical calculations that the structure functions in this asymptotic region should have the property that:

$$\begin{aligned} MW_1(\nu, Q^2)&\rightarrow F_1(x) \\ \nu W_2(\nu, Q^2)&\rightarrow F_2(x) \end{aligned} \tag{6.12}$$

where the functions on the right-hand side depend solely on the *one* variable x. This important result (6.12) is called *Bjorken scaling*. A crucial ingredient of his derivation is the idea that, in this deep inelastic region, there is no size available (such as the overall size of the proton) to set the scale. With this assumption, the structure functions turn out to be functions only of the dimensionless ratio x.

Although the experimental range of the variables is far from asymptotic ($Q^2\lesssim 11$ $(\mathrm{GeV}/c)^2$ and $\nu\lesssim 13$ GeV/c), it is interesting in view of Bjorken's suggestion to plot the structure functions to see whether they do in fact show this scaling property. The extracted values of $2MW_1$ and νW_2 are shown in figure 17, where R was taken to be 0·18 and $M_X>2{\cdot}6$ GeV; the variable $\omega=1/x$. Apparently all the data tend to fall within reasonable bands in each case, showing that $2MW_1$ and νW_2 do indeed seem to scale at least approximately in this kinematical range. (The setting-in of approximate scaling so far from the asymptotic region of the variables is usually referred to as 'precocious scaling' in the literature.)

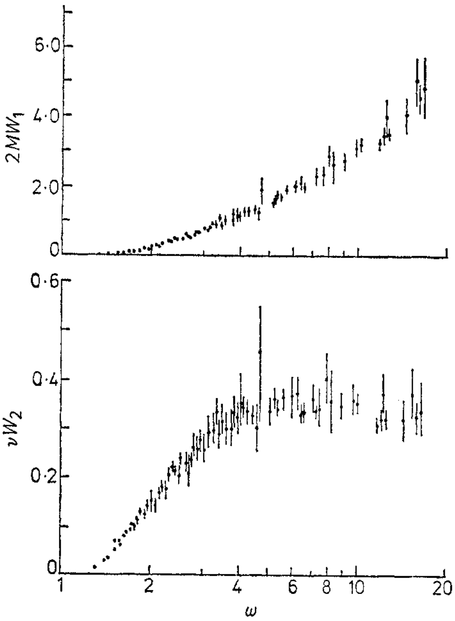

Figure 17. Proton structure functions $2MW_1$ and νW_2, extracted with $R=0{\cdot}18$ and $M_X>2{\cdot}6$ GeV (Bloom *et al* 1972).

Several experiments have been done recently using muons as projectiles (Anderson *et al* 1975, 1976, Chang *et al* 1974, 1975a,b). One would expect similar results, because of μ–e universality. The data for a deuterium target show no great departures from scaling; however, with an iron target, definite violations are observed. This might be an effect due to the nuclear complexity of the target and requires further investigation.

We now turn to a theoretical model for understanding these experimental observations. As we have said, the fact that the deep inelastic cross section is large and only slightly Q^2-dependent suggests scattering from hard point-like partons within the target nucleon. The model we shall describe here is the parton model, introduced originally by Feynman (1969, 1972). It is assumed that the exchanged photon interacts not with the nucleon as a whole but with a single point parton, the other partons remaining unaffected. The time for this interaction is assumed to be small compared to the lifetime of any excited state of the nucleon so that the partons may be considered as quasi-free objects within the nucleon. This picture of the electron–nucleon collision is probably most valid at high energies and large momentum transfers.

Let us first write down the differential cross section for elastic scattering of an electron off a spin-$\frac{1}{2}$ point object of mass m and charge e:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}=\frac{\alpha^2}{4E^2\sin^4\frac{1}{2}\theta}\,\delta\left(\nu-\frac{Q^2}{2m}\right)\left(\frac{Q^2}{2m^2}\sin^2\tfrac{1}{2}\theta+\cos^2\tfrac{1}{2}\theta\right)$$

where the δ function reflects the fact that the scattering is elastic. Comparing with equation (6.8) we see that for scattering from a point-like particle of spin $\frac{1}{2}$, the equivalent structure functions are:

$$w_1=\frac{Q^2}{4m^2}\,\delta\left(\nu-\frac{Q^2}{2m}\right)\qquad w_2=\delta\left(\nu-\frac{Q^2}{2m}\right). \tag{6.13}$$

Suppose (Feynman 1972, Bjorken and Paschos 1969) in the high-energy regime a nucleon consists of N partons. The four-momentum of the ith parton is $P_i = x_i P$, i.e. the ith parton carries a fraction x_i of the nucleon's four-momentum. Let $f(x_i)$ be the probability distribution of x_i within a nucleon: the number of partons with momentum fraction between x_i and $x_i + \mathrm{d}x_i$ is $f(x_i)\,\mathrm{d}x_i$. The contribution of the ith parton to the nucleon structure function W_2, say, is therefore:

$$W_2{}^i = \int_0^1 e_i{}^2 f(x_i)\,\mathrm{d}x_i \delta\left(\nu - \frac{Q^2}{2x_i M}\right)$$

where we have allowed for the possibility of the parton carrying a fraction e_i of the unit charge e. The total structure function W_2 for a nucleon made up of N partons is:

$$\begin{aligned}\nu W_2(\nu, Q^2) &= \sum_{i=1}^{N} \nu W_2{}^i = \sum_{i=1}^{N} e_i{}^2 \int_0^1 \mathrm{d}x_i\, x_i f(x_i) \delta\left(x_i - \frac{Q^2}{2M\nu}\right) \\ &= \left(\sum_{i=1}^{N} e_i{}^2\right) x f(x)\end{aligned}$$

where $x = Q^2/2M\nu$. Likewise we may evaluate the corresponding expression for W_1. The simple parton model gives for the structure functions:

$$2MW_1(\nu, Q^2) = (\textstyle\sum e_i{}^2) f(x) \qquad \nu W_2(\nu, Q^2) = (\textstyle\sum e_i{}^2) x f(x). \tag{6.14}$$

The right-hand sides of the equations in (6.14) depend on ν, Q^2 only through the variable x. Thus Bjorken scaling turns out to be a consequence of this simple parton picture. In addition, the structure functions are related:

$$2MxW_1 = \nu W_2 \qquad 2xF_1(x) = F_2(x) \tag{6.15}$$

and so

$$R = \sigma_{\mathrm{L}}/\sigma_{\mathrm{T}} = 2Mx/\nu. \tag{6.16}$$

Since $x \leqslant 1$, R should be small for large ν, and as we have seen this seems to be the case experimentally. If the partons had spin 0, then $\sigma_{\mathrm{T}} = 0$ identically (Yan 1976) and R would be large in contradiction with experiment. The observed small value of R is therefore consistent with the exchanged photon interacting with partons of spin $\frac{1}{2}$ inside the nucleon.

Some further insight can be gained by considering sum rules which follow from the normalisation conditions on the probability distribution $f(x)$. We have that:

$$\textstyle\int_0^1 \mathrm{d}x\, f(x) = 1$$

and if the charged partons carry a fraction a of the total four-momentum, then:

$$\textstyle\int_0^1 \mathrm{d}x\, x f(x) = a/N.$$

If we define the function $F(x) = \nu W_2(\nu, Q^2)$, we obtain the following sum rules for $F(x)$:

$$I_1 = \int_0^1 \mathrm{d}x\, F(x) = \frac{a}{N} (\textstyle\sum e_i{}^2) \tag{6.17}$$

$$I_2 = \int_0^1 \mathrm{d}x\, \frac{F(x)}{x} = (\textstyle\sum e_i{}^2). \tag{6.18}$$

The right-hand sides of the last two equations can be evaluated for any specific parton model. For example, if the only partons present were the three charged valence quarks, the right-hand sides would be:

$$\sum e_i^2=\frac{4}{9}+\frac{4}{9}+\frac{1}{9}=1 \qquad \frac{1}{N}\sum e_i^2=\frac{1}{3} \qquad \text{for the proton } (uud)$$

and

$$\sum e_i^2=\frac{4}{9}+\frac{1}{9}+\frac{1}{9}=\frac{2}{3} \qquad \frac{1}{N}\sum e_i^2=\frac{2}{9} \qquad \text{for the neutron } (udd).$$

The integrals on the left-hand sides of (6.17) and (6.18) may be evaluated (Miller *et al* 1972, Friedman and Kendall 1972) from the experimental data on νW_2. Unfortunately, the data do not extend over the complete x range, but start at $x\approx 0{\cdot}05$. (The I_1 integral is less sensitive to this than the I_2 integral.) Taking this as the lower limit of the integrations, we obtain:

$$I_1^{\mathrm{p}}=0{\cdot}17 \qquad I_2^{\mathrm{p}}=0{\cdot}78 \qquad \text{for the proton}$$

and

$$I_1^{\mathrm{n}}=0{\cdot}11 \qquad I_2^{\mathrm{n}}=0{\cdot}59 \qquad \text{for the neutron.}$$

The value of I_1 is smaller than the value of I_2 in each case, while the values of I_1 and I_2 are smaller for the neutron than for the proton. This is qualitatively what one would expect from the three-valence quark model, but there is no quantitative agreement, the I_1 integrals being as much as a factor of two too small. Physically, this means that in a fast-moving nucleon pictured as being made up of three valence quarks and neutral gluons, as much as 50% of the energy momentum is carried by the neutral gluons.

More general parton models have been formulated (Bjorken and Paschos 1969, Feynman 1972). One can include, as well as gluons, a background 'sea' of $q\bar{q}$ pairs (with $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs in equal proportions). One can also allow for different distributions for the various kinds of quarks $u(x)$, $d(x)$, $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$ and $\bar{s}(x)$ in the nucleon. Better agreement with experiment can be achieved, but as yet no really adequate model has been developed.

6.5. *Deep inelastic neutrino and antineutrino scattering*

Neutrinos and antineutrinos are also very convenient particles for probing the nucleon since, like electrons, they have no known structure. However, instead of coupling to particles by means of the electromagnetic interaction, neutrinos and antineutrinos couple through the weak interaction. They themselves are produced in the weak decays of hadrons by initially having a beam of pions and kaons which are allowed to decay in flight ($\pi\to\mu\nu$, $\mathrm{K}\to\mu\nu$), the muons being subsequently removed by shielding. Here we describe some results that have an important bearing on quark–parton ideas. See also reviews by Perkins (1972, 1976), Llewellyn-Smith (1972), Roy (1975) and Yan (1976).

The process we are interested in is of the inclusive type:

$$\nu+\mathrm{N}\to\mu^-+X \qquad \bar{\nu}+\mathrm{N}\to\mu^++X$$

where in the final state only the muon is detected, and X stands for all the other products. In the lowest-order scattering diagram, ν and $\bar{\nu}$ exchange a vector boson of the weak interactions with the nucleon target, just as the electron in figure 15 exchanges a photon. In the laboratory frame, the differential cross section is of the form:

$$\frac{\mathrm{d}^2\,\sigma^{\nu,\bar{\nu}}}{\mathrm{d}\Omega\,\mathrm{d}E'}=\frac{G^2E'^2}{2\pi^2}\left[2\sin^2\tfrac{1}{2}\theta\, W_1{}^{\nu,\bar{\nu}}(\nu,Q^2)+\cos^2\tfrac{1}{2}\theta\, W_2{}^{\nu,\bar{\nu}}(\nu,Q^2)\mp\frac{E+E'}{M}\sin^2\tfrac{1}{2}\theta\, W_3{}^{\nu,\bar{\nu}}(\nu,Q^2)\right] \quad (6.19)$$

where the upper and lower signs refer to ν and $\bar{\nu}$ scattering, and the general notation is similar to that for the electron scattering case. The third structure function comes about from an interference between vector and axial vector coupling terms of weak interactions, and so contributes with opposite signs to ν and $\bar{\nu}$ scattering.

Again we are particularly interested in the scaling behaviour of the structure functions. According to Bjorken scaling (Bjorken 1969), if there is no size involved in the basic scattering process, we would expect that as ν and Q^2 become large with $x=Q^2/2M\nu$ remaining finite, then:

$$MW_1(\nu,Q^2)\to F_1(x) \qquad \nu W_2(\nu,Q^2)\to F_2(x) \qquad \nu W_3(\nu,Q^2)\to F_3(x) \quad (6.20)$$

where the functions F_1, F_2 and F_3 depend solely on x. Unfortunately, the experimental data are as yet too sparse to allow a determination of the three structure functions. However, if we assume that they do scale, the differential cross section may be re-expressed at high energies in the form:

$$\frac{\mathrm{d}^2\sigma^{\nu,\bar{\nu}}}{\mathrm{d}x\,\mathrm{d}y}=\frac{G^2ME}{\pi}\left[xy^2\,F_1{}^{\nu,\bar{\nu}}(x)+(1-y)\,F_2{}^{\nu,\bar{\nu}}(x)\mp(y-\tfrac{1}{2}y^2)xF_3{}^{\nu,\bar{\nu}}(x)\right] \quad (6.21)$$

where $y=(E-E')/E$.

Before deriving some predictions, we note that most ν and $\bar{\nu}$ experiments are done with complex targets to increase the reaction rate. It is convenient therefore to obtain expressions for the average scattering off nucleons, rather than off individual protons and neutrons. For strangeness-conserving processes, charge symmetry gives (Llewellyn-Smith 1972):

$$F_i{}^{\nu\mathrm{n}}=F_i{}^{\bar{\nu}\mathrm{p}} \qquad F_i{}^{\nu\mathrm{p}}=F_i{}^{\bar{\nu}\mathrm{n}}$$

so that

$$F_i{}^{\nu\mathrm{N}}=\tfrac{1}{2}\,(F_i{}^{\nu\mathrm{p}}+F_i{}^{\nu\mathrm{n}})=\tfrac{1}{2}\,(F_i{}^{\bar{\nu}\mathrm{p}}+F_i{}^{\bar{\nu}\mathrm{n}})=F_i{}^{\bar{\nu}\mathrm{N}}.$$

Hence we may write:

$$\frac{\mathrm{d}^2\sigma^{\nu\mathrm{N},\bar{\nu}\mathrm{N}}}{\mathrm{d}x\,\mathrm{d}y}=\frac{G^2ME}{\pi}\left[xy^2\,F_1{}^{\nu\mathrm{N}}(x)+(1-y)F_2{}^{\nu\mathrm{N}}(x)\mp(y-\tfrac{1}{2}y^2)xF_3{}^{\nu\mathrm{N}}(x)\right]. \quad (6.22)$$

If we integrate both sides of equation (6.22) with respect to x and y, we find that the total cross sections for ν and $\bar{\nu}$ should rise linearly with the beam energy E. The CERN data (Eichten *et al* 1973, Deden *et al* 1975) for these total cross sections shown in figure 18 are consistent with this prediction. For $2<E<14$ GeV, the best fits are:

$$\sigma^{\nu\mathrm{N}}=0{\cdot}74\times10^{-38}E \qquad \mathrm{cm^2/nucleon}$$

$$\sigma^{\bar{\nu}\mathrm{N}}=0{\cdot}28\times10^{-38}E \qquad \mathrm{cm^2/nucleon}\ (E\ \text{in GeV}).$$

While this does not prove that the structure functions scale, at least it shows that

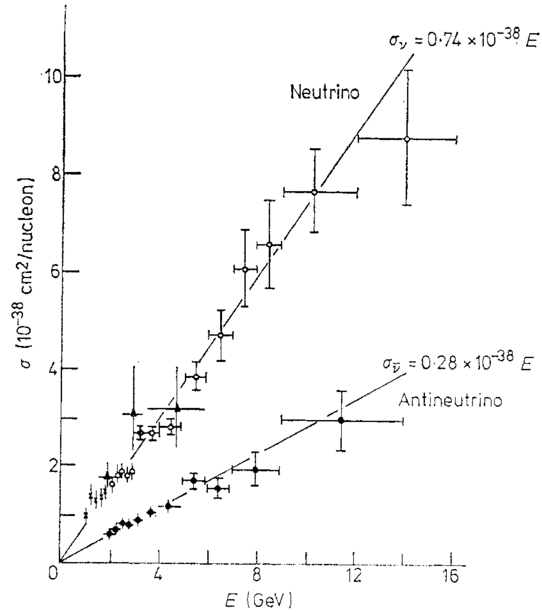

Figure 18. Neutrino and antineutrino total cross sections as functions of the beam energy. Data from Eichten *et al* (1973) and Perkins (1976).

scaling is consistent with the data. (In general, ν and $\bar{\nu}$ scattering from a point spin-$\frac{1}{2}$ particle give a total cross section proportional to E.)

Let us now see what extra predictions follow if we assume the three-valence standard quark model for nucleons: the quarks have spin $\frac{1}{2}$ and there are no antiquarks present. Just as in the electron case (6.15), the structure functions here are related to one another:

$$xF_1{}^{\nu,\bar{\nu}}(x) = F_2{}^{\nu,\bar{\nu}}(x) = -xF_3{}^{\nu,\bar{\nu}}(x) \tag{6.23}$$

and $F_2(x)$ is again simply related to the momentum distribution of the quarks. With these relations, the ν and $\bar{\nu}$ differential cross sections become:

$$\frac{\mathrm{d}^2\sigma^{\nu\mathrm{N}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2ME}{\pi}\,F_2{}^{\nu\mathrm{N}}(x) \qquad \frac{\mathrm{d}^2\sigma^{\bar{\nu}\mathrm{N}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2ME}{\pi}\,F_2{}^{\nu\mathrm{N}}(x)(1-y)^2. \tag{6.24}$$

These distributions are strikingly different in shape as functions of y. The most recent CERN data (Holder *et al* 1977) are highly consistent with a flat neutrino distribution in y, while the antineutrino distribution falls off to zero as y approaches 1, in approximate agreement with the predictions of (6.24). Further, on integrating with respect to both x and y, we obtain for the total cross sections:

$$\sigma^{\nu\mathrm{N}} = \frac{G^2ME}{\pi}\int_0^1 \mathrm{d}x\,F_2{}^{\nu\mathrm{N}}(x) \qquad \sigma^{\bar{\nu}\mathrm{N}} = \frac{G^2ME}{\pi}\,\frac{1}{3}\int_0^1 \mathrm{d}x\,F_2{}^{\nu\mathrm{N}}(x)$$

which yields for their ratio:

$$\sigma^{\bar{\nu}\mathrm{N}}/\sigma^{\nu\mathrm{N}} = \tfrac{1}{3}. \tag{6.25}$$

Figure 19 shows the latest experimental measurements (Holder *et al* 1977) for this ratio. It indeed seems to be constant over a wide energy range, $30 < E < 200$ GeV, though the experimental value is about 0·43 rather than $\frac{1}{3}$.

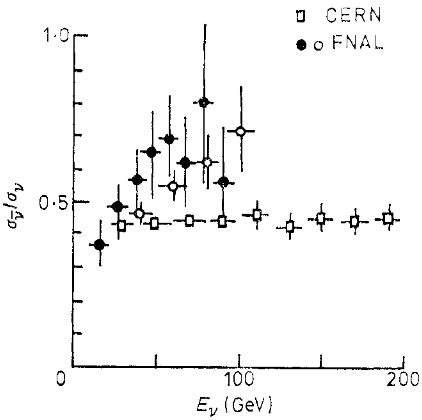

Figure 19. Ratio of antineutrino to neutrino total cross sections. Data from Holder *et al* (1977) and Benevenuti *et al* (1976).

This last result shows us again that, with the three-valence quark model, one can obtain answers which are of the correct order of magnitude, but clearly a better model is needed. This can be illustrated further by considering the sum:

$$\sigma^{\nu N} + \sigma^{\bar{\nu} N} = \frac{G^2 ME}{\pi} \frac{4}{3} \int_0^1 \mathrm{d}x\, F_2{}^{\nu N}(x). \tag{6.26}$$

From the experimental values of the total cross sections, we obtain the estimate:

$$\textstyle\int_0^1 \mathrm{d}x\, F_2{}^{\nu N}(x) \approx 0{\cdot}47 \tag{6.27}$$

showing again that only about 50% of the nucleon's momentum is carried by the valence quarks.

The ideas described here have been developed further to allow for different distributions for u, d and s quarks, as well as for antiquarks in a background sea of gluons (see Roy (1975) and Yan (1976) for a detailed discussion of these topics). While some clues can be obtained about these distributions (for example, the antipartons contribute primarily to small $x \lesssim 0{\cdot}2$), more data are still required to extract all the distributions.

Having seen how quark–parton ideas help to explain and qualitatively correlate much of the lepton–hadron scattering data, we turn now to hadron–hadron collisions.

6.6. Hadron–hadron collisions at large momentum transfer

As yet a third possibility of studying collisions at large momentum transfer, we can examine the collisions of hadrons with other hadrons. This is a more complicated situation than in lepton–hadron collisions since both projectile and target have internal constituents. However, at high enough energies and large enough momentum transfers, the process may again be dominated by a single collision between a constituent in one hadron and a constituent in the other.

Many theoretical models have been developed to provide a description of hadron–hadron collisions. We refer the reader to the following papers for the different details of the models: Berman *et al* (1971), Ellis and Kislinger (1974), Gunion *et al* (1972, 1973, 1975), Blankenbecler and Brodsky (1974), Brodsky and Farrar (1973, 1975), Matveev *et al* (1972, 1973), Landshoff and Polkinghorne (1973a,b, 1974) and Landshoff (1974). The article by Sivers *et al* (1976) provides a comprehensive review.

We begin by considering processes of the kind $A+B \to C+D$. Figure 20 depicts two of the more likely subprocesses that could contribute: (*a*) where partons from different hadrons collide and exchange a vector gluon between them; and (*b*) where partons from the projectile and target get interchanged. Of the many constituent models that have been developed, perhaps the most successful one is that due to Brodsky and Farrar (1973, 1975), which is based on dimensional counting arguments. Their method involves a renormalisable field theoretical treatment of the collisions process in which the effects of the binding of the quarks within a hadron are neglected, and certain assumptions are made about the tails of the hadronic form factors. Extraction of the high-energy limit leads to a surprisingly simple formula for the differential cross section at large (fixed) centre-of-mass scattering angle θ:

$$\mathrm{d}\sigma/\mathrm{d}t = s^{-N} f(\theta) \tag{6.28}$$

where $N = n_A + n_B + n_C + n_D - 2$, and $n_A, \ldots, n_D$ are the number of active fields in the

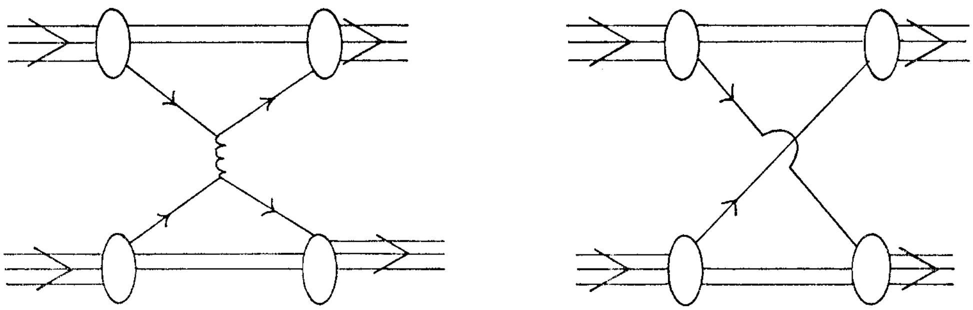

Figure 20. Hadron–hadron scattering with two possible subprocesses: (*a*) parton–parton scattering with vector gluon exchange, and (*b*) constituent interchange.

particles $A, \ldots, D$. For a nucleon, $n=3$ if we follow the simple three-valence quark picture. For a meson, we have $n=2$, corresponding to the quark–antiquark pair, while $n=1$ for a photon. Thus simple constituent models predict simple power law dependencies: for example

$$\mathrm{pp} \to \mathrm{pp}: \mathrm{d}\sigma/\mathrm{d}t \sim s^{-10} \qquad \pi\mathrm{p} \to \pi\mathrm{p}: \mathrm{d}\sigma/\mathrm{d}t \sim s^{-8} \qquad \gamma\mathrm{p} \to \pi\mathrm{p}: \mathrm{d}\sigma/\mathrm{d}t \sim s^{-7} \tag{6.29}$$

for fixed large angle θ.

How well are these predictions satisfied? For comparison, we show in figure 21 the data (Benary *et al* 1970) for pp elastic scattering at $\theta = 90°$. The s^{-10} line in the diagram gives a reasonable fit to the data. The experimental data for the other processes are less accurate, but are consistent with the anticipated power dependencies. The overall agreement with these power law dependencies is encouraging for the constituent picture.

It may be noticed in figure 21 that the data seem to oscillate around the s^{-10} line in a fairly regular fashion. From the periodicity of these oscillations, it has been suggested (Hendry 1974, Schrempp and Schrempp 1975) that they are due to the presence of a purely diffractive or peripheral component in the scattering (characteristic of a size of 1 Fermi), even at these large angles. Thus it would appear that

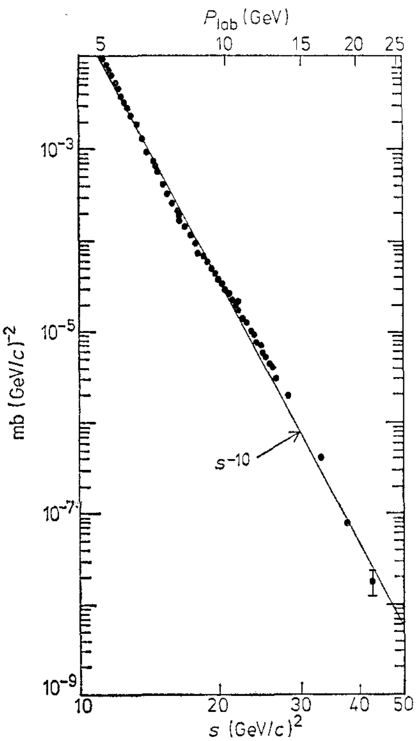

Figure 21. Comparison of differential cross section for proton–proton elastic scattering at CM angle 90° and the power dependence s^{-10}. Data from Benary *et al* (1970).

coherent parton effects are still observable at these energies ($p_{lab} \lesssim 24$ GeV/c) out at 90°; to isolate the incoherent scattering of constituents cleanly would seem to require somewhat higher energies.

As the energy is increased, it quickly becomes very difficult to measure elastic scattering at 90°—there are just not enough events. Physically, this means that with increasing energy it becomes more difficult for hadrons to collide and scatter at 90°, and still to remain the same particles. They much prefer to scatter inelastically, producing many mesons. It is more profitable therefore to study inelastic scatters at the higher energies where the number of events (even at large scattering angle) is large.

This leads us again to consider the simplest of these inelastic processes, namely the single-particle inclusive process $A+B \rightarrow C+X$, as illustrated in figure 22. The particle C is the only one detected in the final state, and X represents all the other produced

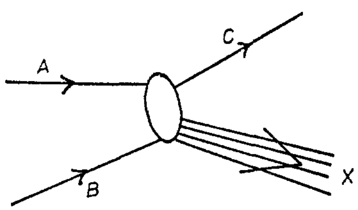

Figure 22. Inclusive process $A+B \rightarrow C+X$, where X goes undetected.

particles. The inclusive cross section for this process may be written as $E\,\mathrm{d}\sigma/\mathrm{d}^3p$, where E and p are the energy and momentum of the outgoing final-state particle C. This cross section is a Lorentz invariant, and in general it is a function of three independent variables. These are usually taken as p_T, the momentum of C perpendicular to the beam direction p_T, $x_\mathrm{T}=2p_\mathrm{T}/\sqrt{s}$, and the scattering angle of C in the CM system θ.

For p_T less than about 2 GeV/c (Banner *et al* 1972) the inclusive cross section falls exponentially with p_T, typically as $\exp(-6p_\mathrm{T})$, corresponding to an average transverse momentum $\langle p_\mathrm{T}\rangle$ of about 400 MeV/c. The smallness of $\langle p_\mathrm{T}\rangle$ implies that the reaction is rather well collimated (like elastic scattering), the bulk of the collision products produced in directions close to the beam direction. This is what one would expect from 'soft' exchanges between the colliding hadrons.

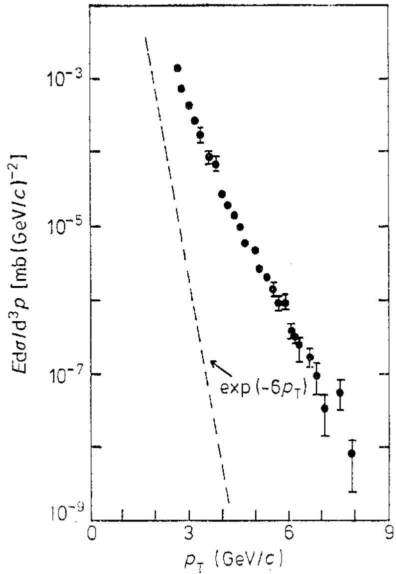

Figure 23. Inclusive cross section for the process $\mathrm{p}+\mathrm{p}\rightarrow\pi^0+X$ at total CM energy 53 GeV and scattering angle 90°, as a function of the transverse momentum p_T. Data from Büsser *et al* (1973).

At larger p_T, however, the inclusive cross section falls off much more slowly than an exponential. This result is illustrated in figure 23 for the case of $\mathrm{p}+\mathrm{p}\rightarrow\pi^0+X$ at fixed $\theta=90°$. It means that the likelihood of particles emerging at large p_T is much greater than would be anticipated from a smooth extrapolation of the low p_T data which follow the $\exp(-6p_\mathrm{T})$ curve. At $p_\mathrm{T}\approx 4$ GeV/c, for example, the particle yield is more than three orders of magnitude greater than originally expected! There must be an important mechanism for producing these large p_T events, the most likely candidate being constituent collisions.

Figure 24 indicates the type of subprocess that might yield a high p_T event. Constituents a, b of particles A, B, respectively, are envisaged as having a 'hard' collision; they scatter, and among the debris is the particle C which is ultimately detected. An extension of the Brodsky–Farrar approach to inclusive processes (Brodsky and Farrar 1975, Blankenbeckler and Brodsky 1974, Gunion *et al* 1975) leads again to a very

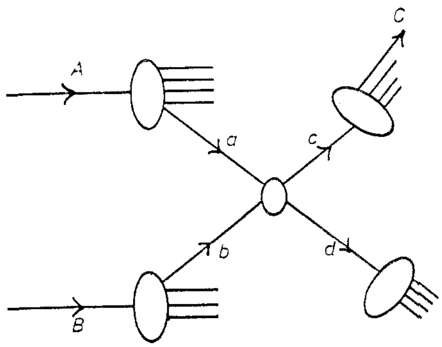

Figure 24. Inclusive process $A+B \rightarrow C+X$, with possible parton subprocess $a+b \rightarrow c+d$.

simple power law prediction. In this case, we find:

$$E\frac{d\sigma}{d^3p} = p_T^{-2N} f(x_T, \theta) \tag{6.30}$$

where $N = n_a + n_b + n_c + n_d - 2$, and $n_a, \ldots, n_d$ are the number of 'active' elementary fields in $a, \ldots, d$ which participate in the subprocess $ab \rightarrow cd$. Thus if the basic subprocess is pure qq scattering, each $n_i = 1$ so that $N = 2$ and we would expect the inclusive cross section to have a p_T^{-4} power law dependence. However, if the basic subprocess is of the constituent interchange type such as $Mq \rightarrow Mq$, $q\bar{q} \rightarrow MM$ or $qq \rightarrow B\bar{q}$ (where M stands for any meson state, B for any baryon state), then $N = 4$, yielding a p_T^{-8} power dependence. Moreover, for fixed large scattering angle θ and x_T close to 1, the function f is expected to have the form $(1-x_T)^{2n_p-1}$, where n_p is the total number of spectator or 'passive' fields in A, B and C.

At present energies, the data favour a p_T^{-8} dependence. Figure 25 shows some recent data taken at Fermilab for $pp \rightarrow \pi^{\pm}X$ with beam momenta 200, 300 and 400 GeV/c. For $x_T \gtrsim 0{\cdot}35$, the best fit to the π^+ production data gives $N = 8{\cdot}3 \pm 0{\cdot}5$, $(2n_p - 1) = 9{\cdot}0 \pm 0{\cdot}5$ and for π^- production $N = 8{\cdot}5 \pm 0{\cdot}5$, $(2n_p - 1) = 9{\cdot}9 \pm 0{\cdot}5$.

A value of $N = 8$ and $(2n_p - 1) = 9$ is exactly what is expected in the constituent interchange picture with the subprocess $Mq \rightarrow Mq$ dominating. These results contradict quark–quark scattering in its simplest form. However, several authors (Field and Feynmann 1977, Fischbach and Look 1977) have suggested that these results only

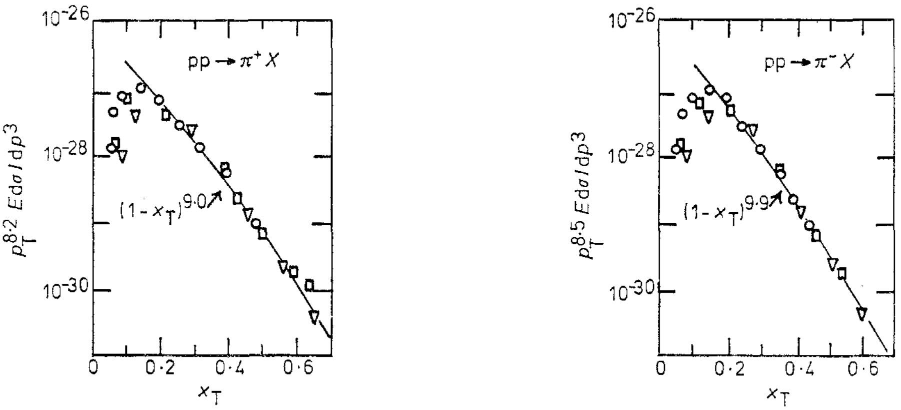

Figure 25. Inclusive data for $p+p \rightarrow \pi^{\pm} + X$, taken from Antreasyan *et al* (1977). ▽, 200 GeV; □, 300 GeV; ○, 400 GeV.

indicate that the dynamics of qq scattering are more complicated than originally anticipated, and have postulated various phenomenological formulae for qq scattering to obtain better agreement with experiment. Impressive fits to a wide range of data have been achieved by these models on the basis of only a few parameters (Field and Feynman 1977, Feynman *et al* 1977, Fischbach and Look 1977).

There is the possibility that at even higher transverse momenta ($p_T \gtrsim 8$ GeV/c), the p_T dependence will eventually change to p_T^{-4} to reveal the pure quark–quark collisions (Cutler and Sivers 1977). This will be one of the exciting things to examine at the proposed higher energy accelerators of the future.

6.7. *Electron–positron annihilation*

We now discuss some aspects of still another kind of collision process which provides valuable information about hadrons and quarks, namely electron–positron annihilation. In this process the electron and positron can annihilate into a virtual photon which can then couple via the electromagnetic interaction to charged particles, including quarks. We therefore have the possibility of the virtual photon converting into a $q\bar{q}$ pair which either remains in its bound state as a single vector meson (figure 26), or else converts into a cascade of hadrons (figure 27). Electron–positron annihilation is therefore a direct way of studying meson spectroscopy, limited only by the beam energy that can be provided.

Many experiments have been performed to measure the cross section for hadron production in e^+e^- collisions. This cross section shows much structure as a function of energy. Good reviews of the experimental situation, with some theoretical interpretation, are given by Feldman and Perl (1975, 1977).

Of special interest for the quark model is the value of the ratio of the cross section

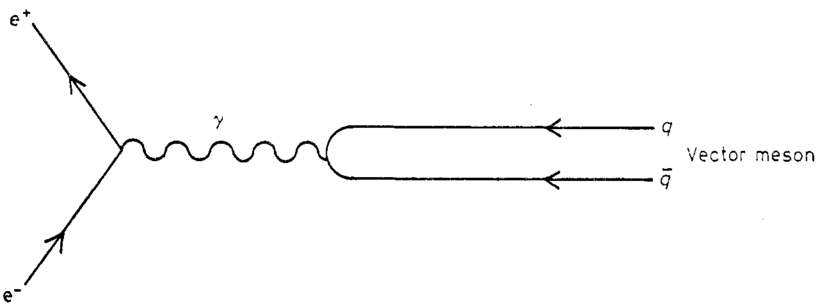

Figure 26. Electron–positron annihilation into a bound quark–antiquark pair forming a vector meson.

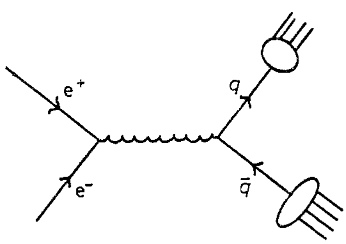

Figure 27. Electron–positron annihilation into a quark–antiquark pair which subsequently converts into many hadrons.

for the production of hadrons to the cross section for the production of $\mu^+\mu^-$ pairs. Like many other ratios, this ratio is called R. The $\mu^+\mu^-$ pair production cross section can be calculated using QED in the one-photon approximation. For values of the energy much greater than the muon mass, this cross section has the simple form:

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/(3s) \tag{6.31}$$

where α is the fine-structure constant and s is the square of the CM energy.

The experimental values for R are shown in figure 28. There are two different types of features in this figure which are very noticeable and which we shall discuss briefly. The first is the series of peaks in R, some narrow and some broad, and the second is the general trend of the average value of R as it goes from one plateau to another.

The peaks in R clearly indicate the presence of resonances. Since they are formed as shown in figure 26 through the conversion of an intermediate photon, these resonances correspond to vector mesons with spin and parity $J^P = 1^-$. The low-mass

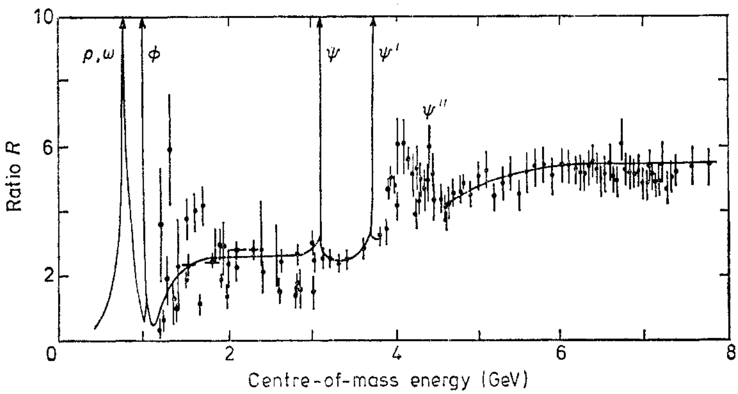

Figure 28. The experimental ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ as a function of the CM energy, taken from Schwitters (1977).

ρ, ω, ϕ mesons have, of course, been known for a long time, discovered originally in ordinary hadronic collisions. At higher energies, the extremely narrow $\psi(3095)$ and $\psi'(3684)$ mesons produce very sharp spikes, while a much broader state occurs at 4·4 GeV, denoted by ψ'' in the figure. There is also a peak at 3·772 GeV, now usually called ψ'', which decays prominently into $D\bar{D}$. The structure in the vicinity of 4 GeV is associated with the threshold for the production of pairs of charmed mesons D and D^*. For the known ψ spectrum see figure 6. All the information about charmed mesons has so far come solely from e^+e^- colliding beam experiments. The properties of the ψ family and the charmed mesons have already been discussed in §3.5, and we refer the reader back to this subsection for further information about these new particles, as well as to reviews by Goldhaber (1977) and Schwitters (1977).

Besides the various resonance and threshold structures in R, the average value of R changes from a little over 2 at the lower energies to about 5 at higher energies. Let us compare these values with the predictions of a simple quark model. One might expect that a formula analogous to (6.21) holds for the production of any charged particle of spin $\frac{1}{2}$ and its antiparticle, including presumably a $q\bar{q}$ pair. If so, we can write:

$$\sigma(e^+e^- \rightarrow q\bar{q}) = 4\pi\alpha^2 e_q{}^2/(3s) \tag{6.32}$$

where e_q is the fractional charge of the quark q. Since quarks have not been seen in electron–positron collisions, we envision the quark pair converting into hadrons after their initial production, as illustrated in figure 27. According to this model therefore, we obtain from (6.31) and (6.32):

$$R=\sum_i e_i^2 \tag{6.33}$$

where, at any energy, the sum is over all the kinds of quarks with masses well below that energy.

Below a CM energy of 3 GeV, only the u, d and s quarks contribute to the sum in (6.33). Because these quarks have charges $\frac{2}{3}$, $-\frac{1}{3}$ and $-\frac{1}{3}$ respectively, we obtain $R=\frac{4}{9}+\frac{1}{9}+\frac{1}{9}=\frac{2}{3}$. Because each quark comes in three colours, we must multiply this value by 3 to obtain $R=2$. This is in approximate agreement with experiment. The fact that experimentally R is much closer to 2 than to $\frac{2}{3}$ is considered an important success for the colour hypothesis.

Above a CM energy of 4 GeV, however, R rises and levels off again at a value of about $R=5$. In this region, the charmed quark (whose charge is $\frac{2}{3}$) should contribute. Including colour, we would therefore expect R to rise from 2 to a value of 10/3. But this lies well below the experimental value of 5. At least part of the discrepancy may be attributed to the production of the recently discovered heavy lepton τ whose mass is about 1·8 GeV (Perl *et al* 1975, 1976, Feldman *et al* 1976). Then the expected value of R would be 13/3, but this is still somewhat too low. Even poorer agreement is obtained using a model with integrally charged quarks. From table 2, R is calculated to be 4 below the $c\bar{c}$ threshold, and 6 above it. This latter value increases to 7 if we include the heavy lepton τ.

6.8. *Quark jets*

Whatever may be the specific form of constituent interactions, a clear type of signal should be observed if the energy is high enough and if constituent collisions take place. As can be seen from figure 24, the scattered constituents after collision evolve into a cascade of hadrons as they emerge from the interaction region. Large p_T events should therefore be characterised by the presence of two 'jets' of particles, and these should be coplanar with the beam direction. A similar situation will also hold in energetic e^+e^- annihilations if the intermediate photon changes into a quark–antiquark pair, as shown in figure 27. In this case the two jets should emerge in exactly opposite directions, as illustrated in figure 29. (In hadronic collisions, the jets need not be exactly 'back to back' in the CM system of the beam and target particles since this is not necessarily the CM system for the constituents that scatter.) The jet directions correspond to the initial directions of the partons.

The first definite indications of jet structure were found (Hanson *et al* 1975, Hanson 1976, Feldman and Perl 1977) in e^+e^- annihilations at the SPEAR storage rings for beam energies above the now-famous resonance region where ψ spectroscopy was discovered (see §6.7). To search for jets, the tensor:

$$T^{ij}=\sum_n (\delta^{ij}p_n^2-p_n^i p_n^j) \tag{6.34}$$

was first calculated for each event, where i and j refer to the spatial components of each particle momentum p_n, and the summation is over all the detected particles ($n=1$, $2, \ldots$) in the event. T^{ij} is analogous to a moment of inertial tensor, and can be diagonalised to find the principal moment eigenvalues λ_1, λ_2, λ_3 in momentum space;

they correspond to the sums of squares of transverse momenta with respect to the three eigenvector directions. If λ_3 is the smallest eigenvalue, its eigenvector represents the direction of the reconstructed jet axis (see figure 29). Defining the sphericity S by

$$S=3\lambda_3(\lambda_1+\lambda_2+\lambda_3)^{-1} \tag{6.35}$$

we see that S should be small (near 0) for well-collimated hadronic jets, and near 1 for events with large multiplicity and particles isotropically distributed in momentum phase space.

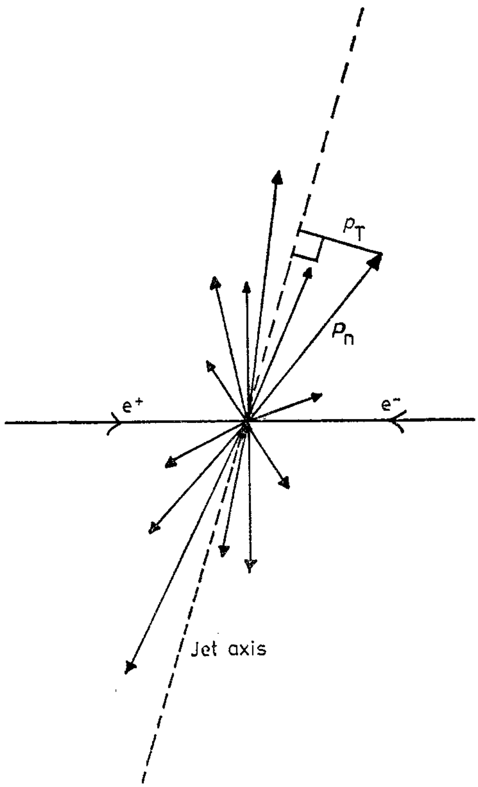

Figure 29. Example of jets in electron–positron annihilation.

The observed sphericity distributions at the three CM energies $E_{CM}=3{\cdot}0$, $6{\cdot}2$ and $7{\cdot}4$ GeV are displayed in figure 30(*a*). The distributions become more skewed towards low sphericity as the energy increases, the trend of the average sphericity with energy being shown in figure 30(*b*). These results are just the opposite of what one would expect if there were no distinct underlying dynamics involved (S should approach 1); instead, they strongly favour jet production, which seems to be becoming more evident as the beam energy increases.

Another result to come from the study of e^+e^- annihilations concerns the angular distribution of the jet axis at the highest energy $E_{CM}=7{\cdot}4$ GeV. The analysis is aided by the fact that at this energy the beam positrons and electrons are partially polarised (due to a synchrotron radiation effect). If the annihilation takes place through a single intermediate photon, the angular distribution for particle production is of the form:

$$d\sigma/d\Omega \propto 1+a\cos^2\theta+P^2 a\sin^2\theta\cos 2\phi$$

where θ is the polar angle measured from the positron direction, ϕ is the azimuthal angle measured from the plane of the storage rings, P is the known amount of polarisation in each beam, and $a=(\sigma_T-\sigma_L)(\sigma_T+\sigma_L)^{-1}$ with σ_T, σ_L here denoting the transverse and longitudinal production cross sections. For the production of two point spin-$\frac{1}{2}$ particles (such as in $e^+e^-\to\mu^+\mu^-$), the coefficient $a=1$.

In the SPEAR experiment, a was determined primarily from the azimuthal distribution, since the detector had an angular acceptance for only $|\cos\theta|\leqslant 0{\cdot}6$ but a full azimuthal acceptance. After corrections for this limitation, the value of a for jets was found to be $a=0{\cdot}97\pm0{\cdot}14$, which is consistent with the picture of the jets originating from a pair of spin-$\frac{1}{2}$ quarks in the quark–parton model.

Jet structure has also been observed in hadron–hadron collisions, both at CERN (Della Negra *et al* 1977) and at Fermilab (Bromberg *et al* 1977). Again, at large p_T, outgoing hadrons tend to emerge in two cones, with limited momentum transverse

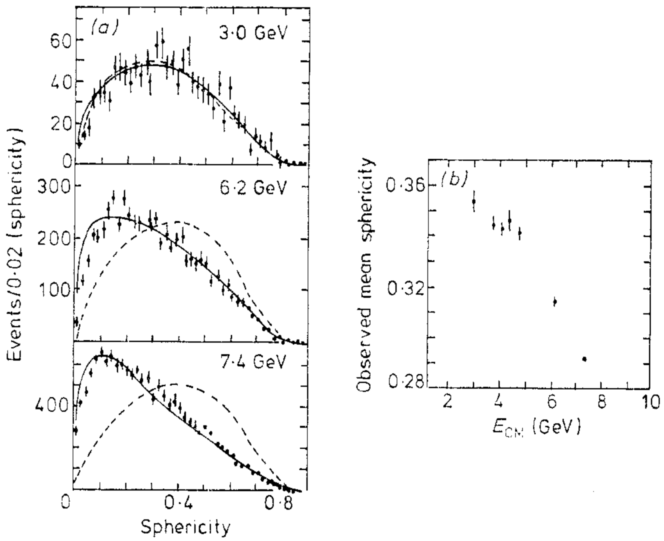

Figure 30. (*a*) Sphericity distributions for CM energies of 3·0, 6·2 and 7·4 GeV. (*b*) Variation of the mean sphericity with CM energy. Data from Hanson *et al* (1975) and Hanson (1976).

to the cones' axes. The jets are consistent with being coplanar with the beam, and generally their composition is similar to that of jets seen in e^+e^- annihilations. This tends to provide rather convincing support for the idea of jets originating from hard parton collisions.

A very recent result (Bromberg *et al* 1977, 1978) has been the comparison of the inclusive cross section for producing a jet ($A+B\to\text{jet}+X$), and single-particle production at large p_T. The experiment consisted of firing a beam containing π^- mesons and protons at a beryllium target. The detection spectrometer could be triggered either on single-particle production or on multiple-particle jets with net transverse momenta greater than 3, 4 or 4·5 GeV/c. The cross sections are shown in figure 31. It may be seen that, while the jet and single-particle production cross sections are similar in shape for $p_T\gtrsim 3$ GeV/c, the jet cross section is about two orders of magnitude greater than the single-particle production. This is what one would expect if the underlying subprocess involved a quark–quark collision with the scattered quarks subsequently fragmenting into hadrons, rather than a meson–quark collision where jet and single-particle production would be expected to be comparable. However, the picture still remains somewhat cloudy since the p_T dependence for jet production seems to be consistent with a p_T^{-8} power law (just like the single-particle inclusive case), and not the much sought-after p_T^{-4} fall-off that comes from simple quark scattering mediated by vector gluon exchange.

Many experiments are now in progress or are being planned to study properties of jets. How do quarks dissociate into bursts of hadrons? What is the distribution of different kinds of hadrons (pions, kaons, etc) in the decay products? Are jets the same, no matter whether they are produced in e^+e^- annihilation, deep inelastic lepton–hadron collisions or hadron–hadron collisions at large p_T? Is it possible to pin down

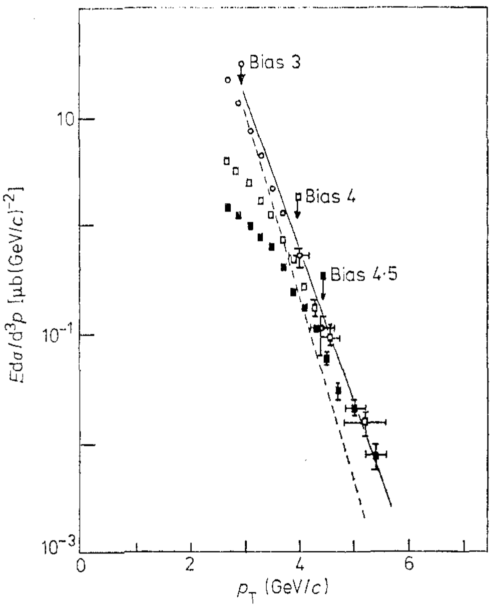

Figure 31. Inclusive cross section for jets in hadron collisions. The jets are triggered to have net transverse momenta greater than 3 (○), 4·0 (□) or 4·5 (■) GeV/*c*. Data from Bromberg *et al* (1977, 1978). ———, estimate of jet cross section; — — — 100 × single-particle cross section.

the nature of the quark which initiates a jet? There seems little doubt that jet physics is one of the more promising areas where one might be able to extract vital information about quark properties and how quarks interact with one another.

7. Concluding remarks

We have seen that the quark model has been very successful in enabling us to understand many qualitative and some quantitative features of hadron physics.

First, the model has proved to be a useful tool to enable us to classify the many baryons and mesons in terms of a far fewer number of quarks. It is true that since the quark model was introduced the number of quarks necessary to describe the hadrons has grown, but not nearly so fast as the number of hadrons. We seem to need a new flavour of quark only when we discover a new additive internal quantum number. But every time we find such a quantum number we discover a rich spectroscopy of many hadrons carrying that quantum number. The model enables us to control the problem of the classification of the exploding number of discovered hadrons.

However, we have seen that classification of hadron states is by no means the sole success of the model. There have also been qualitative successes in obtaining sum rules for the masses and magnetic moments of hadrons and also in obtaining the spectra of hadron states. Of course, it has proved necessary to assume something about the properties and interactions of quarks to obtain these more detailed results.

When we turn to scattering experiments, we find that there is evidence that hadrons are indeed made of smaller constituents, and that these constituents could well be quarks. Furthermore, colour, introduced as an *ad hoc* mechanism to explain the spin-statistics problem of quarks, really seems to show up in electron–positron colliding beam experiments, as otherwise the large ratio R of hadron production to muon pair production would be harder to understand.

On the theoretical side, the existence of quark fields are an important feature of the beautiful Weinberg–Salam model as modified by Glashow *et al.* Without quarks, it would be much more difficult to make a connection between the weak interactions of hadrons and leptons. Furthermore, turning to the strong interactions, we have the hope that quantum chromodynamics will prove to be a successful theory of quark interactions. Already this theory has apparently explained some of the scaling properties of high-energy collisions.

But a number of difficult questions remain. One of the most important of these is: how many quark flavours exist? A related question is: can a principle be discovered which can fix this number? And if there are n quarks, is the symmetry SU(n) or something different? From a practical point of view, we definitely need at least four flavours, and there is already some evidence for a fifth. Most of the presently aesthetically appealing models require an even number of quarks, so possibly we may look forward in the not-too-distant future to seeing evidence for a sixth quark. But, again as a practical matter, because (so far) the mass of each newly discovered quark has been larger than the mass of any of the preceding ones by an increasing amount, it may become increasingly difficult to find experimental evidence for many more quarks.

The introduction of colour gauge theory to describe the quarks, while pretty, leads to several unsolved questions. One of the most challenging is the question of whether coloured objects (quarks, gluons, di-quarks, etc) are really confined. But even if they are, we have the problem of understanding whether exotic hadrons exist, and if so, what are their distinguishing properties. Furthermore, although colour helps to explain the large value of the ratio R, the experimental value still exceeds the theoretical value. We still do not understand the reason for this.

Finally, what is the mystical nature of the number 3? Why are there three colours, and why do quarks have charges $\frac{2}{3}$ and $-\frac{1}{3}$? Or maybe we are deceived by thinking 3 is relevant. Perhaps Pati and Salam are right that the leptons constitute a fourth colour and that quarks have integral charge.

Judging from the past, we may expect more surprises from nature before our questions are answered—if, in fact, we are asking the right questions. But at least in the interim, the quark model will continue to bring a good measure of order into the complex subject of hadron physics.

Acknowledgment

This work was supported in part by the US Department of Energy.

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Note added in proof

After this review was submitted for publication, the 1978 review of particle properties by the Particle Data Group appeared (Bricman *et al* 1978). Their review contains some very recent data which occasionally differ in small respects from the data given in tables 3, 4, 6, 7 and 8 and in certain other parts of the text. These minor changes in measured masses, lifetimes, etc, are not important for the purposes of our review, but the reader who is interested in the best current values of particle properties should consult Bricman *et al* (1978) and future reviews of the Particle Data Group when they appear.

The Υ and Υ' have now been seen as narrow resonances in e^+e^- collision experiments at DESY (Berger *et al* 1978, Darden *et al* 1978, Flügge 1978). The masses of these states have been observed to be:

$$m(\Upsilon) = 9{\cdot}46 \pm 0{\cdot}01 \qquad m(\Upsilon) = 10{\cdot}01 \pm 0{\cdot}01 \text{ GeV}.$$

Their leptonic decay widths have been measured to be:

$$\Gamma(\Upsilon \rightarrow e^+e^-) = 1{\cdot}3 \pm 0{\cdot}2 \qquad \Gamma(\Upsilon' \rightarrow e^+e^-) = 0{\cdot}4 \pm 0{\cdot}1 \text{ keV}.$$

These widths are approximately what one would expect if the Υ and Υ' are bound states of a b quark of charge $\frac{1}{3}$ and its antiquark.

In a recent experiment at SLAC (Prescott *et al* 1978, Taylor 1978), parity violation has been observed in inelastic electron scattering from hydrogen and deuterium. The amount of parity violation in this experiment is in agreement with the prediction of the Weinberg–Salam model. Thus, at present, the evidence in favour of this model is quite good.

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