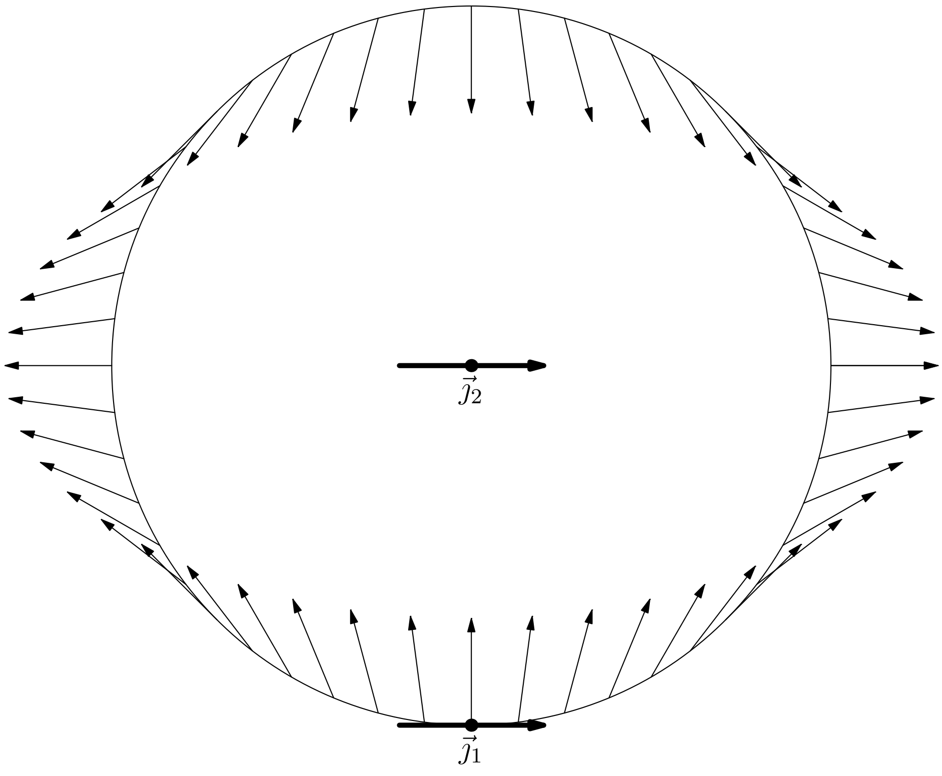
Quaternionic Electrodynamics

Saint Alban



November 6, 2006

Contents

Introduction

This treatise is about quaternionic electrodynamics, which is a description of the electromagnetic phenomena using quaternions. The history of quaternionic electrodynamics goes back to the time of Sir William Rowan Hamilton (1805-1865), Peter Guthrie Tait (1831-1901) and James Clerk Maxwell (1831-1879).

When I first began this project, my objective was to learn as much about the roots of modern electromagnetism as possible, to make a solid basis for possible further experimental studies on the subject.

I started out reading the first edition of Maxwell's TREATISE ON ELECTRICITY AND MAGNETISM, in which Maxwell shows how to write down the general equations of electrodynamics using scalars and quaternion vectors. Investigating this quaternion formulation, I found that using the idea of a scalar field instead of a gauge condition, you could write the electrodynamic equations in full quaternion notation.

While searching the literature for quaternionic models of electrodynamics I found that other models used biquaternions[1], and I did not find any new work, after Maxwell and Tait, which used normal quaternions.

To investigate physical reality, I looked at the differences between Maxwell's equation and the quaternion model, focusing on finding published experiments which could indicate if the predictions from the quaternion model are true.

[1] Biquaternions is quaternion where the four components are complex numbers.

Chapter 1

Hamilton's quaternions

1.1 The quaternion bridge

Our journey starts Monday morning on the 16. October 1843 near Broome Bridge[1] close to Dublin. Sir William Rowan Hamilton and Lady Hamilton is on the way to Dublin, where Sir Hamilton is to lead a meeting in the Royal Irish Academy. While walking along the royal canal Sir Hamilton is worrying about a mathematical problem that he has been working on for some time.

The problem occupying Sir Hamilton's mind is, if it is possible to find an algebra of triplets; back in 1835 he had been helping with the foundation for the algebra of couples now known as complex numbers.

How to find a algebra of triplets is an important question in the academic society at the time, and for Hamilton the pressure is not only professional but also private, as it can be seen in a letter from Hamilton to his son Archbald Henry Hamilton.[2]

> Every morning in the early part of the above-cited month [Oct. 1843] on my coming down to breakfast, your brother William Edwin and yourself used to ask me, 'Well, Papa, can you multiply triplets¿Whereto I was always obliged to reply, with a sad shake of the head, 'No, I can only add and subtract them.'[2]

The problem Sir Hamilton is considering while walking to Dublin, where that the terms ij and ji appear when multiplying two triplets in the form $x+iy+jz$, as he approaches Broome bridge he tries to let ij be equal to k, and he realizes that he has got an algebra of quartets. He quickly writes down the ideas in his pocket book and then he takes a knife and carves the relation

$$i^2 = j^2 = k^2 = ijk = -1 \tag{1.1}$$

into a stone on the bridge. Within an hour he has asked for leave from the Royal Irish Academy meeting, in exchange for a promise that he will prepare a presentation on this new algebra of quartets for the next meeting a month later, and the same evening he writes a basic analysis in his notebook.[3] The story

[1] Hamilton refers to this bridge as Brougham Bridge, but according to Graves the real name is Broome Bridge

[2] Thanks to `http://quaternions.com/` for making me aware of this letter.

about the discovery is later revealed in letters to his son Archbald [2] and to his friend Peter G. Tait [4].

The solution Hamilton has discovered is 4 dimensional. By letting the elements i, j, k follow the relation $i^2 = j^2 = k^2 = ijk = -1$ one can write a quaternion as

$$\mathbf{q} = q + iq_x + jq_y + kq_z \tag{1.2}$$

Where $\mathbf{q}$ are called a quaternion, q is named the scalar part and $iq_x+jq_y+kq_z$ is called the vector part and q, q_x, q_y, q_z are all real numbers.

In this treatise quaternions are written in bold, and the vector part is written $\vec{q}$ so that quaternions may be written like $\mathbf{q} = q + \vec{q}$ and we can define the conjugated to this quaternion as $\mathbf{q}^* = q - \vec{q}$.

Given another quaternion $\mathbf{p} = p + \vec{p}$ one can show that in this notation the product of two quaternions is:

$$\mathbf{qp} = (qp - \vec{q} \cdot \vec{p}) + (q\vec{p} + p\vec{q} + \vec{q} \times \vec{p}) \tag{1.3}$$

It should be noted that the quaternion product doesn't commutate because $\mathbf{pq} - \mathbf{qp} = 2\vec{q} \times \vec{p}$ but we do have the following product rule for conjugation: $(\mathbf{qp})^* = \mathbf{p}^*\mathbf{q}^*$

On the 17 October Hamilton wrote a letter to John T. Graves[5] about how he had discovered the quaternions. Hamilton where looking for the algebra of triplets so in this letter he discarded the scalar part of the quaternions, and call the resulting vector part for pure quaternion. While making a product of two pure quaternions he writes down the formulas for what we today call the dot and cross product of vectors and in a way laying the foundation for vector calculus. Differential operators for quaternions appeared for the first time in a note appended to an article on a quaternion formulation of Pascal's theorem[6], here he defines the differential operator

$$i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \tag{1.4}$$

and find the following product with a pure quaternion $(it + ju + kv)$.

$$-(\frac{\partial t}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z}) + i(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial z}) + j(\frac{\partial t}{\partial z} - \frac{\partial v}{\partial x}) + k(\frac{\partial u}{\partial x} - \frac{\partial t}{\partial y}) \tag{1.5}$$

which contain the divergent and curl differential operator which latter make it possible for Maxwell to formulate his theory of electrodynamics on a compact and elegant form.

1.2 Quaternions and physics

In the preface to the second edition of 'An elementary treatise on quaternions'[7] Tait writes:

> Sir W. Hamilton, when I saw him but a few days before his death, urged me to prepare my work as soon as possible, his being almost ready for publication. He then expressed, more strongly perhaps than he had ever done before, his profound conviction of the importance of quaternions to the progress of physical science; and his

> desire that a really elementary treatise on the subject should soon be published.[3]

To understand Hamilton's standpoint, we will take a look on how quaternion algebra can be used in physics.

In the first example, let us look at the electrostatic force from a charge n on another charge m.

$$\vec{F}_{mn} = -\frac{q_n q_m(\vec{r}_n - \vec{r}_m)}{|\vec{r}_n - \vec{r}_m|^3} \tag{1.6}$$

Let $\mathbf{F}_{mn}$ be a pure quaternion where the vector part is $\vec{F}_{mn}$,then we have that:

$$\mathbf{F}^*_{mn} = -\vec{F}_{mn} = +\frac{q_m q_n(\vec{r}_n - \vec{r}_m)}{|\vec{r}_n - \vec{r}_m|^3} = -\frac{q_n q_m(\vec{r}_m - \vec{r}_n)}{|\vec{r}_m - \vec{r}_n|^3} = \vec{F}_{nm} = \mathbf{F}_{nm} \tag{1.7}$$

If we know the force on m from n we can find the reaction forces from m on n by simple conjugating.

Another interesting thing is to look at the operator $\frac{d}{dt}$, if we expand it in partials we get:

$$\begin{aligned} \frac{d}{dt} &= \frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z} && (1.8)\\ &= \frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z} && (1.9)\end{aligned}$$

With the use of quaternions we can elegantly introduce space orientation into the expression $\frac{d}{dt}$, and express the four parts in one algebraic unit:

$$\frac{\mathbf{d}}{\mathbf{dt}} = \frac{\partial}{\partial t} + iv_x\frac{\partial}{\partial x} + jv_y\frac{\partial}{\partial y} + kv_z\frac{\partial}{\partial z} \tag{1.10}$$

If we take $v_x = v_y = v_z = c$ as it is normally assumed with electromagnetic fields, then we can define a quaternionic nabla:

$$\boldsymbol{\nabla} = \frac{1}{c}\frac{\mathbf{d}}{\mathbf{dt}} = \frac{1}{c}\frac{\partial}{\partial t} + i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} = \frac{1}{c}\frac{\partial}{\partial t} + \vec{\nabla} \tag{1.11}$$

Because quaternions do not commutate, there is a difference if we use $\boldsymbol{\nabla}$ on the right or the left side. So lets define the right and left operator.

$$\begin{aligned} \boldsymbol{\nabla}\mathbf{q} &= (\frac{1}{c}\frac{\partial q}{\partial t} - \vec{\nabla}\cdot\vec{q}) + (\frac{1}{c}\frac{\partial \vec{q}}{\partial t} + \vec{\nabla}q + \vec{\nabla}\times\vec{q}) && (1.12)\\ \mathbf{q}\boldsymbol{\nabla} &= (\frac{1}{c}\frac{\partial q}{\partial t} - \vec{\nabla}\cdot\vec{q}) + (\frac{1}{c}\frac{\partial \vec{q}}{\partial t} + \vec{\nabla}q - \vec{\nabla}\times\vec{q}) && (1.13)\end{aligned}$$

The symmetric and antisymmetric product is also very useful, when doing quaternion algebra they are defined by.

$$\begin{aligned} \{\mathbf{q},\mathbf{p}\} &= \frac{1}{2}(\mathbf{qp} + \mathbf{pq}) && (1.14)\\ [\mathbf{q},\mathbf{p}] &= \frac{1}{2}(\mathbf{qp} - \mathbf{pq}) && (1.15)\end{aligned}$$

[3] Tait was waiting for Hamilton to publish his textbook first

1.3 Hamilton and electromagnetism

Hamilton tried to formulate the law of electromagnetism as quaternions.

At a meeting in the British Association which took place in June 1845 in Cambridge he stated the following [8]:

> That he wished to have placed on the records the following conjecture as to a future application of Quaternions:-'Is there not an analogy between the fundamental pair of equations ij=k ji=-k, and the facts of opposite currents of electricity corresponding to opposite rotations? '

and in a letter from H. Lloyd to Hamilton we find[9]:

> I hope you will not lose sight of the point you mentioned to me last night of meeting. If you can show grounds for the existence of a second system of forces in electrical propagation (varying as the cosine of inclination while the former vary as the sine) you will have attained one of the most important of the desiderata of modern physics.

and in another place Graves writes some important letters to Dr. Lloyd in 1854[10]

> They set forth what he calls a conjecture suggested by Quaternions which might prove 'a physical discovery respecting the mutual action of two elements of the same, or two different (electromagnetic) currents, considered as exerting (in addition to Ampère's attractive or repulsive force) a certain directive force, or as producing a system of two contrary couples.' He afterwards saw reason to doubt of the physical applicability of what he had called provisionally his electro-magnetic Quaternions; but Lloyd continued to assign to it a high possible value in relation to the theory of Electro-magnetism. Dr. Lloyd's words are – 'May 31st. – I am greatly interested with your electro-dynamic Quaternion. It seems to me to promise (not a new physical discovery, but what is yet more interesting) a theoretical explanation of the fundamental facts of electro-magnetism.... The similarity (or agreement) to these [Biot's laws, representing the action of an infinitely small magnet upon a magnetic particle] of the laws which govern you vectors, give, I think, ground for hope that you will be able, through it, to explain the true physical relation between the electric current and the magnet. And if so, the discovery will indeed be a great one'

Graves does not print the letter to Dr. Lloyd though he prints a similar letter sent on 25 May 1854 to Augustus De Morgan[11], which is also included in Appendix A on page 54. In this letter Hamilton writes about his experiment with formulating Ampère's force law on quaternion form. At Hamilton's time electrodynamics where described by Ampère's force law.1.17

This law expresses the force from one infinitesimal conductor element ds upon another ds' and it is given on the form:

$$f \sim -ii'dsds'\frac{2}{\sqrt{r}}\frac{\partial^2\sqrt{r}}{\partial s\partial s'} \tag{1.16}$$

Where i and i' is the current through ds and ds' respectively and r is the length of the vector between the two elements.

Hamilton tries to find a quaternion such that the scalar part would express Ampère's force law, and after some investigation he derives what he names the "ELECTRO-MAGNETIC QUATERNION".

$$Q = -\frac{1}{2}\left(\frac{d\rho d\rho'}{\Delta\rho^2} + 3V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}\right). \tag{1.17}$$

Here $d\rho$ and $d\rho'$ are vectors representing the two current elements and $\Delta\rho = \rho' - \rho$ where ρ' and ρ are vectors to the beginning of the elements.

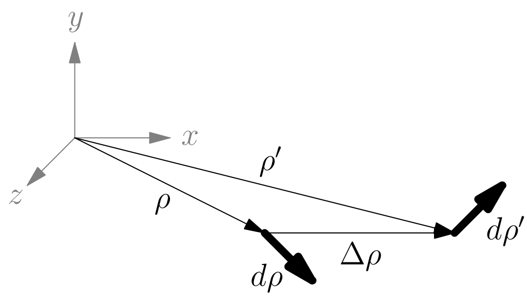


Figure 1.1: Hamilton's notation for Ampère's force law

Hamilton then considered what the vector part of Q would express and he ends the letter very optimistic about the electro-magnetic quaternion.

A few days later, on the 27 May[12] (See Appendix A) he writes another letter to A. De Morgan where he feels less optimistic about Q, not seeing that it should have any physical value. The argument he provides is that if one takes any vector and add it to the quaternion Q then the scalar part would still equal Ampère's force law.

That Hamilton didn't lose all hope for a quaternionic electrodynamic can be seen in the first edition of 'elements of quaternions' where his son William Edwin Hamilton writes:

> Shortly before my father's death, I had several conversations with him on the subject of the 'Elements.' In these he spoke of anticipated applications of Quaternions to Electricity, and to all questions in which the idea of Polarity is involved - applications which he never in his own lifetime expected to be able fully to develop...[4]

Hamilton planned to devote his last chapter in his book to the application of quaternions in physics, but unfortunately Hamilton died before he finished his book, so we will never know if the book which was published after his death, contain all Hamilton's ideas on quaternion application in electrodynamics.

Another place which mentions Hamilton's work on quaternionic electrodynamics are in the article 'On the nabla of Quaternions'[13] by Shunkichi Kimura:

[4]Thanks to `http://www.hypercomplex.com/education/intro_tutorial/nabla.html` for guiding me to this quotation.

> Thus it is seen that nablas in their extended form have direct physical application, not to mention the 'electrodynamic quaternion' of Hamilton.

The question posted here is, if this electrodynamic quaternion is the same as Hamilton's electromagnetic quaternion ? If the answer is yes, then why does Kimura thinks that it has physical meaning when Hamilton himself does not ? On the other hand if this where two different quaternions, then it should be possible to find some sources, unfortunately Kimura does not give a reference for the electrodynamic quaternions of Hamilton, and I have not had any luck in finding anything about it.

What Hamilton might have done to find this electrodynamic quaternion, is instead of expressing the Ampère's force law in the scalar part and then examine the vector part, he might have expressed Ampère's force law as the vector part and then examined the scalar part, because Ampère's force law is already in agreement with Newton's law of action and reaction, which Hamilton knew where equivalent with quaternionic conjugation.

I think that it is possible that Hamilton developed his ideas for quaternionic electrodynamic between 1854 and his death in 1865 which the citations from his son and Shunkichi Kimura also seems to hint at, but I have not had any luck in finding any sources on quaternionic electrodynamics from Hamilton's hand after the 1854 letters.

Bibliography

[1] Robert P. Graves. *Life of Sir William Rowan Hamilton.* Arno Press, New York, 1975. ISBN 0-405-06594-9.

[2] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter XXVIII, pages 434–435. Volume II of III [1], 1975.

[3] William Rowan Hamilton. Note book entry for 16 october 1843. In Halberstam and Ingram [15], pages 103–104.

[4] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter XXVIII, pages 435–436. Volume II of III [1], 1975.

[5] William Rowan Hamilton. Letter to graves on quaternions. In Halberstam and Ingram [15], pages 106–110.

[6] William Rowan Hamilton. On a proof of pascal's theorem by means of quaternions. In Halberstam and Ingram [15], pages 367–377.

[7] Peter Guthrie Tait. *An elementary treatise on quaternions.* Cambridge university press, 3. edition, 1890.

[8] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter XXX, page 489. Volume II of III [1], 1975.

[9] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter XXXII, page 565. Volume II of III [1], 1975.

[10] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter XXVIII, page 5. Volume III of III [1], 1975.

[11] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter LII, pages 480–485. Volume III of III [1], 1975.

[12] Robert P. Graves. *Life of Sir William Rowan Hamilton*, chapter LII, pages 485–487. Volume III of III [1], 1975.

[13] Shunkichi Kimura. On the nabla of quaternions. *Annals of Mathematics*, pages 127–155, 1895.

[14] Thomas L. Hankins. *Sir William Rowan Hamilton.* The Johns Hopkins University Press, Baltimore, 1980. ISBN 0-8018-2203-3.

[15] H. Halberstam and R. E. Ingram, editors. *The mathematical papers of sir William Rowan Hamilton, Algebra*, volume III. Cambridge University Press, Cambridge, 1967.

Chapter 2

Maxwell's equations

This chapter is about Maxwell and his contribution to electromagnetism and his relationship to quaternions. Extensive biographical works have been made on Maxwell and his works on electromagnetism, so this treatise will only contain a summary of this, it will contain more details on his relationship to quaternions.

2.1 Maxwell's education

James Clerk Maxwell was born in Edinburgh, Scotland 1831. His mother was in charge of his early education until she died of illness in 1839, when he where eight years old. After an unsuccessful experiment with a tutor at home, He was sent to school at Edinburgh academy in November 1841. It was here that Maxwell formed a friendship with Peter Guthrie Tait, and it was also in this period where he made his first scientific paper on a method for drawing oval curves[1].

In 1847 he began three years of study at the university of Edinburgh. Maxwell's friend Peter Guthrie Bait describes the time in the following passage[2].

> The winter of 1847 found us together in the classes of Forbes and Welland, where he highly distinguished himself. With the former he was a particular favorite, lingered here behind most of his former associates, having spent three years at the University of Edinburgh, working (without any assistance or supervision) with physical and chemical apparatus, and devouring all sorts of scientific works in the library*. During this period he wrote two valuable papers, which are published in our Transactions, on "The Theory of Rolling Curves" and "On the Equilibrium of Elastic Solids"

The footnote says.

> * From the University Library lists for this period it appears that Maxwell perused at home Fourier's ThéBrie be la Chaleur, Monge's Géométrie Descriptive, Newton's optics, Willis's Principles of Mechanism, Cauchy's Calcul Différentiel, Taylor's Scientific Memoirs, and many other works of a high order. Unfortunately no record is kept of books consulted in the reading-room.

The footnote in Tait's writing tells us that Maxwell at an early age privately had studied the works of the scientific giants before him.

In 1850 Maxwell went to Cambridge university first staying at Peterhouse and later moving on to Trinity College, graduating with a degree of second wrangler in January 1854.

2.2 Papers on electromagnetism

Maxwell's primary work on electromagnetism was published in 3 articles and 2 books. Unfortunately, Maxwell died in the early age of 48 years, just before finishing hes work on the second edition of his book, named 'a treatise on electricity and magnetism'.

2.2.1 On Faraday's lines of force

Maxwell first publication on electrodynamics was the article 'On Faraday's Lines of Force' [3] In this article he studies the electromagnetic phenomena with the help of physical analogies, as a reason for this choice of research he writes:

> The first process therefore in the effectual study of the science, must be one of simplification and reduction of the results of previous investigation to a form in which the mind can grasp them. The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis. In the first case we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexion's of the subject. If, on the other hand, we adopt a physical hypothesis, we see the phenomena only though a medium, and are liable to that blindness to facts and rashness in assumption which a partial explanation encourages. We must therefore discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on the physical science from which that conception is borrowed, so that it is neither drawn aside from the subject in pursuit of analytical subtleties, nor carried beyond the truth by a favorite hypothesis.
>
> In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies.

He then explores the analogy between the electromagnetic phenomena and the motion of an incompressible fluid.

2.2.2 On physical lines of force

On the motivation and object for the paper 'On physical lines of force' first published in 1861 and 1862, Maxwell writes:[4]

> We are dissatisfied with the explanation founded on the hypothesis of attractive and repellent forces directed towards the magnetic

> poles, even though we may have satisfied ourselves that the phenomenon is in strict accordance with that hypothesis, and we cannot help thinking that in every place where we find these lines of force, some physical state or action must exist in sufficient energy to produce the actual phenomena.
>
> My object in this paper is to clear the way for speculation in this direction, by investigating the mechanical results of certain states of tension and motion in a medium, and comparing these with the observed phenomena of magnetism and electricity. By pointing out the mechanical consequences of such hypotheses, I hope to be of some use to those who consider the phenomena as due to the action of a medium, but are in doubt as to the relation of this hypothesis to the experimental laws already established, which have generally been expressed in the language of other hypotheses.

The medium that Maxwell investigates can be summarized with the following characteristics:

1. Electromagnetic phenomena are due to motion or pressure in a medium.
2. The magnetic field is due to unequal pressure in the medium and the line of force represent the direction of the least pressure.
3. The difference in pressure is generated by vortices or eddies, which have their axes of rotation aligned with the lines of force.
4. The vortices are separated from each other by a layer of round particles.
5. These particles are in rolling contact with the vortices they separate and motion of the particles represented electric current.
6. Electric current through the medium makes the vortices around the current move in the same direction, while vortices further from the current will move in opposite direction.
7. When an electric current or a magnet is moved, the velocity of rotation of the vortices are changed by the motion, this creates an electromotive force if a conductor is present.
8. When a conductor is moved in a magnetic field, the vortices in and around it are moved and change form, this also creates a electromotive force in the conductor.

Maxwell was well aware that this model was not necessarily realistic, but he suggested it as a model which was mechanically conceivable and easy to investigate.

2.2.3 A dynamical theory of the electromagnetic field

The dynamical theory known as Maxwell's equation's was published in a paper in 1864 [5]. In this paper, Maxwell still tries to move away from a action as a distance theory and over to a motion through a medium theory, but without trying to build a model, like in the previous paper. Instead he presents a set of dynamical equation to describe the motion through the aether.

In an abstract of the paper, read before the Royal Society, Maxwell describes these equations in the following way[6]:

> The next part of the paper is devoted to the mathematical expression of the electromagnetic quantities referred to each point in the field, and to the establishment of the general equations of the electromagnetic field, which express the relations among these quantities.
>
> The quantities which enter into these equations are - Electric currents by conduction, electric displacements, and Total Currents; Magnetic forces, Electromotive forces, and Electromagnetic Momenta. Each of these quantities being a directed quantity, has three components; and besides these we have two others, the Free Electricity and the Electric Potential, making twenty quantities in all.
>
> There are twenty equations between these quantities, namely Equations of Total Currents, of Magnetic Force, of Electric Currents, of Electromotive Force, of Electric Elasticity, and of Electric Resistance, making six sets of three equations, together with one equation of Free Electricity, and another of Electric Continuity.

These twenty equations, are summarized in the following table, with Maxwell's original notation to the left, modern coordinate notation in the middle and vectors to the right. [1]

$$\left.\begin{aligned} p' &= p + \frac{df}{dt} \\ q' &= q + \frac{dg}{dt} \\ r' &= r + \frac{dh}{dt} \end{aligned}\right\} \rightarrow \left.\begin{aligned} J_x &= j_x + \frac{\partial D_x}{\partial t} \\ J_y &= j_y + \frac{\partial D_y}{\partial t} \\ J_z &= j_z + \frac{\partial D_z}{\partial t} \end{aligned}\right\} \rightarrow \vec{J} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \tag{2.1}$$

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx} \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned}\right\} \rightarrow \left.\begin{aligned} \mu H_x &= \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \mu H_y &= \frac{\partial A_x}{\partial z} - \frac{\partial A_y}{\partial x} \\ \mu H_z &= \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{aligned}\right\} \rightarrow \mu\vec{H} = \vec{\nabla} \times \vec{A} \tag{2.2}$$

$$\left.\begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p' \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q' \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r' \end{aligned}\right\} \rightarrow \left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= 4\pi J_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_y}{\partial x} &= 4\pi J_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= 4\pi J_z \end{aligned}\right\} \rightarrow \vec{\nabla} \times \vec{H} = 4\pi\vec{J} \tag{2.3}$$

[1] The idea for the table is borrowed from [7]

$$\left.\begin{aligned} P &= \mu\left(\gamma\frac{dy}{dt} - \beta\frac{dz}{dt}\right) - \frac{dF}{dt} - \frac{d\Psi}{dx} \\ Q &= \mu\left(\alpha\frac{dz}{dt} - \gamma\frac{dx}{dt}\right) - \frac{dG}{dt} - \frac{d\Psi}{dy} \\ R &= \mu\left(\beta\frac{dx}{dt} - \alpha\frac{dy}{dt}\right) - \frac{dH}{dt} - \frac{d\Psi}{dz} \end{aligned}\right\} \rightarrow \left.\begin{aligned} E_x &= \mu(H_z v_y - H_y v_z) - \frac{\partial A_x}{\partial t} - \frac{\partial\phi}{\partial x} \\ E_y &= \mu(H_x v_z - H_z v_x) - \frac{\partial A_y}{\partial t} - \frac{\partial\phi}{\partial y} \\ E_z &= \mu(H_y v_x - H_x v_y) - \frac{\partial A_z}{\partial t} - \frac{\partial\phi}{\partial z} \end{aligned}\right\}$$

$$\rightarrow \quad \vec{E} = \mu(\vec{v}\times\vec{H}) - \frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\left.\begin{aligned} P &= kf \\ Q &= kg \\ R &= kh \end{aligned}\right\} \rightarrow \left.\begin{aligned} \varepsilon E_x &= D_x \\ \varepsilon E_y &= D_y \\ \varepsilon E_z &= D_z \end{aligned}\right\} \rightarrow \varepsilon\vec{E} = \vec{D} \tag{2.4}$$

$$\left.\begin{aligned} P &= -\zeta p \\ Q &= -\zeta q \\ R &= -\zeta r \end{aligned}\right\} \rightarrow \left.\begin{aligned} \sigma E_x &= j_x \\ \sigma E_y &= j_y \\ \sigma E_z &= j_z \end{aligned}\right\} \rightarrow \sigma\vec{E} = \vec{\jmath} \tag{2.5}$$

$$e + \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0 \rightarrow \rho + \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 \rightarrow \rho + \vec{\nabla}\cdot\vec{D} = 0 \tag{2.6}$$

$$\frac{de}{dt} + \frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} = 0 \rightarrow \frac{\partial\rho}{\partial t} + \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0 \rightarrow \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{\jmath} = 0 \tag{2.7}$$

Here we should note that there is an error in the equations in this paper and the privies paper. By using that the divergent on the rotation is zero on (2.3) we can write;

$$0 = \vec{\nabla}\cdot(\frac{1}{4\pi}\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} = \vec{\nabla}\cdot(\vec{\jmath}+\frac{\partial\vec{D}}{\partial t}) = \vec{\nabla}\cdot\jmath + \frac{\partial(\vec{\nabla}\cdot\vec{D})}{\partial t} = \vec{\nabla}\cdot\jmath - \frac{\partial\rho}{\partial t} \tag{2.8}$$

where the last terms are in conflict with (2.7). This error where corrected by changing sign on the charge density ρ in equation (2.6).

2.3 Maxwell's treatise on electricity and magnetism

Maxwell's most prominent work on electromagnetism is his book "A treatise on electricity and Magnetism"[2] published in 1873. This work is in two volumes, and

[2] In the following I referrer to it as 'the treatise'

it discusses a lot of the mathematical and physical aspects of electromagnetism known at the time. While most of the calculation in the treatise is done in ordinary Cartesian coordinates, Maxwell shows how to write his general equation for electrodynamics using Hamilton's quaternions. The following is Maxwell's quaternion equation adopted to the notation used in this treatise:[8]

$$\begin{array}{cc}
\vec{E} = V\vec{v}\vec{B} - \frac{\partial \vec{A}_e}{\partial t} - \vec{\nabla}\phi_e & \vec{B} = V\vec{\nabla}\vec{A}_e \\
\vec{F} = V\vec{J}_e\vec{B} - \rho_e\vec{\nabla}\phi_e - \rho_m\vec{\nabla}\phi_m & \vec{B} = \mu\vec{H} \\
4\pi\vec{J}_e = V\vec{\nabla}\vec{H} & \vec{j}_e = \sigma\vec{E} \\
\vec{D} = \frac{1}{4\pi}k\vec{E} & \vec{J}_e = \vec{j}_e + \frac{\partial \vec{D}}{\partial t} \\
\vec{B} = \vec{H} + 4\pi\vec{M} & \rho_e = S\vec{\nabla}\vec{D} \\
\rho_m = S\vec{\nabla}\vec{M} & \vec{H} = -\vec{\nabla}\phi_m
\end{array}$$

As can be seen, the quaternions that Maxwell uses are not full quaternions, only pure quaternions and scalars, as this is how quaternions were normally used at his time.

Maxwell died in 1879 while he was still working on the second edition of the treatise. The second edition was published in 1881 with W. D. Niven as editor, the first nine chapters were replaced with Maxwell's rewriting, and the last part was a reprint of the first edition. Unfortunately, only a few of the first nine chapters, are chapters particularly suited for quaternion treatment, so we only know little about what Maxwell had in mind concerning quaternion treatment in the second edition.

2.4 Scientific letters

A good source for Maxwell's scientific letters are P. M. Herman 3 volume collection[9][10][11], in which he also describes the history of the letters: After Maxwell's death in 1879, Professor George Gabriel Stokes and Professor George Edward Paget were chosen by Maxwell's will, to go through his personal papers and decide what should be destroyed, and what should be published, a job which Stokes was asked to do. Stokes, who was very busy at the time, delegated the task to William Garnett, who had worked with Maxwell at Cavendish Laboratory. Then Garnett teamed up with Maxwell's life long friend, Professor Lewis Campbell, and in 1882 they published the book '*Life of Maxwell*'. The biography focused on Maxwell as a person and his philosophy, so it is not a good source for his scientific correspondences. After Garnett and Campbell had finished their work, the letters were returned to the owners and Maxwell's papers were returned to Mrs. Maxwell in Cambridge, after which they were presumably moved to Maxwell's house in Glenlair, where they were lost when Glenlair later burnt to the ground. This may be the reason that there is a gab in Maxwell's scientific correspondence with Tait, Stokes and Lord Kelvin. Unfortunately, it is with them, that Maxwell would have discussed his ideas for further quaternion formulation for his electrodynamic theory.

2.4.1 Quaternions

The first available letter, where Maxwell write about quaternions, is a letter to Tait from 1865[12] where he asks:

> Does any one write quaternions but Sir. W. Hamilton & you?

Tait had had an active correspondence with Hamilton regarding quaternions since 1858, and Tait was the one Maxwell turned to, when he needed advice on quaternions.

The next question about quaternion is in 1867 were he asks, if Tait's book on quaternions had been published [13], in the same letter he also asks.

> Is there any virtue in turning Δ round $30°$?

Here Maxwell is referring to the fact that Hamilton used $\triangleleft$ as the Nabla operator while Tait used ∇. It is not known, if Maxwell had already discovered the right side form of nabla and is trying to give Tait a hint, or if he is just wondering about Tait's change of notation.

2.4.2 First edition

When Maxwell start hes work on the first edition of the treatise, the frequency of questions concerning quaternions began to increase.

In a letter from 1870[14], Maxwell asks Tait about the ∇ operator:

> Dear Tait
>
> $$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}.$$
>
> What do you call this? Atled?[3] I want to get a name or names for the result of it on scalar or vector functions of the vector of a point.

Then he suggests the name slope for $\vec{\nabla}$ on a scalar function, for the scalar part of $\vec{\nabla}$ on a vector function he suggests the name Convergence and for the vector part he discusses several options; twist, turn, version, twirl and curl. For the result of $\vec{\nabla}^2$ he suggests the name concentration. He asks Tait if these names are inconsistent with any terms in the domain of quaternions.

A week later, Maxwell writes to Tait again[15], returning a borrowed letter from Tait's friend William Robertson Smith. In this letter, Smith suggests the that symbol ∇ is called Nabla, as the Assyrian harp with the same shape. The name Nabla becomes a source of amusement for Maxwell and Tait, in their further correspondence they use words like Nablody, Nabladist and nabble.

Apparently Tait had forgotten to comment on the names for the results for ∇ because Maxwell writes:

> The names which I sent you were not for ∇ but the results of ∇. I shall send you presently what I have written, which though it is in the form of a chapter of my book is not to be put in but to assist in leavening the rest.

[3] Atled is delta spelled backwards

The reprint[10, p.570] of the chapter Maxwell talks about, is titled '*Manuscript on the application of quaternions to electromagnetism*'. In this manuscript Maxwell shows that he has finished most of the quaternion notions that he used in the first edition of the treatise.

In October 1872 Maxwell write to Lewis Campbell:

> I am getting converted to Quaternions, and have put some in my book, in a heretical form, however, for as the Greek alphabet was used up, I have used German capitals from $\mathfrak{A}$ to $\mathfrak{J}$ to stand for Vectors, and, of course, ∇ occurs continually. This letter is called 'Nabla', and the investigation a Nablody.

2.4.3 Second edition

When Maxwell began writing on the second edition, again we see an increase in the frequency of letter to Tait with questions on quaternions.

In June 1878 Maxwell ask [16]:

> What is the correct statement as to the right handed system of unit vector adopted in Hamilton's & in Tait's Quaternions?.
>
> Also any other remarks on Electricity & Magnetism which is being revised for 2^{nd} edition.

In September 1878 he asks [17].

> May one plough with an ox & an ass together? The like of you may write everything and prove everything in pure 4^{nions} but in the transition period the bilingual method may help to introduce and explain the more perfect system.
>
> But even when when that which is perfect is come, that which builds on 3 axes will be useful for purposes of calculations by Cassios of the future.
>
> Now in a bilingual treatise it is troublesome, to say the least, to find that the square of AB is always positive in Cartesian and always negative in 4^{nions} and that when the thing is mentioned incidentally you do not know which language is being spoken.

Here Maxwell raises a critical question, for quaternion expression is not alway the easies to comprehend, or the most simple to write down, and it might be best to make a bilingual notation even though this isn't without problems either.

Later in the same month he writes on a postcard to Tait[18].

> What is the best expression in 4^{nions} for a Stress?

Here Maxwell poses question about a quaternion expression for stress, a subject much later, in the treatise than the nine chapters which were replaced in the second edition, indicating that he might have been working on rewrites of those chapters as well.

Finally, in Maxwell's last known letter to Tait, there are hints that Maxwell knew about both the left from and the right form of the nabla operator even though there is no evidence that he didn't use the latter in written form.

In this cryptical letter[19][4], dated 28 august 1879 and titled 'Headstone in search of a new sensation'[5], Maxwell pretends to be Tait writing a diary entry.

The section which seems to indicate that Maxwell knows of the right side form of Nabla, is the following:

> Might not I, too, under the invocation of the holy ALBAN [6] become inspired with some germinating idea, some age-making notion by which I might burst the shell of circumstance and hatch myself something for which we have not even a name, freed for ever from the sickening round of possible activities and exulting in life every action of which would be a practical refutation of the arithmetic of this present world.
>
> Hastily turning the page on which I had recorded these meditations, I noticed just opposite the name of the saint another name which I did not recollect having written. Here it is - ИAᗺ⅃A.
>
> Here then was the indication, impressed by the saint himself, of the way out of all my troubles. But what could the symbol mean?

In the next chapter, we will see that both forms of Nabla is necessary to write the quaternionic form of Maxwell's equations. So, in that sense the right side form of Nable is the solution to all the trouble. This is another indication that Maxwell was working on parts of the books far beyond the nine chapters replaced in the second edition, because that form is most interesting when working with the general equations for electromagnetism in the last part of the treatise.

[4] The letter is included in appendix B on page 60

[5] Here Maxwell referrer to Tait's pseudonym 'Guthrie Headstone'

[6] Alban is Nabla (∇) spelled backwards

Bibliography

[1] James Clerk Maxwell. *Paper on the description of oval curves*, pages 35–42. Volume I of Harman [9], 1990.

[2] Peter Gutherie Tait. Obituary notice of James Clerk Maxwell. In *Scientific papers by Peter Gutherie Tait*, volume I, pages 396–401. C. J. Clay and sons, Cambridge, 1898.

[3] James Clerk Maxwell. On Faraday's lines of force. In Niven et al. [21], pages 155–229.

[4] James Clerk Maxwell. On physical lines of force. In Niven et al. [21], pages 452–513.

[5] James Clerk Maxwell. A dynamical theory of the electromagnetic field. In Niven et al. [21], pages 527–597.

[6] James Clerk Maxwell. *Abstract of paper 'A dynamical theory of the electromagnetic field'*, pages 189–196. Volume II of Harman [10], 1995.

[7] André Waser. On the notation of Maxwell's field equation. AWVerlag, `www.aw-verlag.ch`, June 2000.

[8] James Clerk Maxwell. *A Treatise on electricity and magnetism*, volume II. Clarendon press, Oxford, second edition, 1881.

[9] P. M. Harman, editor. *The scientific letters and papers of James Clerk Maxwell, 1846-1862*, volume I. Cambridge University Press, Cambridge, 1990.

[10] P. M. Harman, editor. *The scientific letters and papers of James Clerk Maxwell, 1862-1873*, volume II. Cambridge University Press, Cambridge, 1995.

[11] P. M. Harman, editor. *The scientific letters and papers of James Clerk Maxwell, 1874-1879*, volume III. Cambridge University Press, Cambridge, 2002.

[12] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 7 March 1865*, pages 214–215. Volume II of Harman [10], 1995.

[13] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 11 December 1867*, pages 328–333. Volume II of Harman [10], 1995.

[14] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 7 November 1870*, pages 568–569. Volume II of Harman [10], 1995.

[15] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 14 November 1870*, pages 577–579. Volume II of Harman [10], 1995.

[16] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 11 June 1878*, pages 676–677. Volume III of Harman [11], 2002.

[17] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 7 September 1878*, pages 709–710. Volume III of Harman [11], 2002.

[18] James Clerk Maxwell. *Postcard to Peter Guthrie Tait, 21 September 1878*, page 712. Volume III of Harman [11], 2002.

[19] James Clerk Maxwell. *Letter to Peter Guthrie Tait, 28 August 1879*, pages 840–841. Volume III of Harman [11], 2002.

[20] Thomas K. Simpson. *Maxwell on the Electromagnetic Field*. Masterwork of discovery series. Rutgers University Press, New Brunswick, 1997.

[21] W.D. Niven, M.A., and F.R.S., editors. *The scientific papers of James Clerk Maxwell*, New York, 1965. Dover publication inc.

[22] James Clerk Maxwell. *A Treatise on electricity and magnetism*. Clarendon press, Oxford, second edition, 1881.

Chapter 3

Quaternion equations

This chapter is about extending Maxwell's equations to their full quaternion form.

3.1 Quaternions in Maxwell's model

When you looks at Maxwell's description of electromagnetism. you notice that the electric scalar potential ϕ_e together with the electric vector potential $\vec{A}_e$, can form a single quaternion $\mathbf{A}_e = \phi_e + \vec{A}_e$ and that the electric charge density ρ_e together with the electric current density $\vec{\jmath}_e$, also forms a quaternion $\mathbf{j}_e = \rho_e + \vec{\jmath}_e$ and the relation between these two quaternions can be written as

$$\mathbf{A}_e(t, \vec{r}) = \frac{1}{4\pi} \int_V \frac{\mathbf{j}_e(t - |\vec{r}_s - \vec{r}|/c, \vec{r}_s)}{|\vec{r}_s - \vec{r}|} d\vec{r}_s \tag{3.1}$$

3.2 Quaternion fields

In Maxwell's model the field is pure vector parts $\vec{E}$ and $\vec{B}$, but in the quaternion model we would like to make them full quaternions.

We will start our extension with Maxwell's expression for the fields:

$$\begin{aligned} \vec{E} &= -\vec{\nabla}\phi_e - \frac{1}{c}\frac{\partial \vec{A}_e}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A}_e \end{aligned} \tag{3.2}$$

Then we add a simple and very used extension for magnetic monopoles. This looks like:

$$\begin{aligned} \vec{E} &= -\vec{\nabla} \times \vec{A}_m - \vec{\nabla}\phi_e - \frac{1}{c}\frac{\partial \vec{A}_e}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A}_e - \vec{\nabla}\phi_m - \frac{1}{c}\frac{\partial \vec{A}_m}{\partial t} \end{aligned} \tag{3.3}$$

Where ϕ_m is the scalar potential and $\vec{A}_m$ is the vector potential due to magnetic currents. Just like their electric counterparts they might be expressed in a quaternion $\mathbf{A}_m = \phi_m + \vec{A}_m$. To get to Maxwell's equation from the field expression, you have to chose a gauge condition. One of the most used gauge conditions is the Lorenz's gauge:

$$\frac{1}{c}\frac{\partial \phi_e}{\partial t} + \vec{\nabla} \cdot \vec{A}_e = 0 \tag{3.4}$$

But using this gauge condition would effectively prevent us from using the equations to investigate systems where the condition is not true. Instead of using a gauge condition we will use an idea from an article by Koen van Vlaenderen [1] and define scalar field components as an extension of Lorenz's gauge:

$$\begin{aligned} E &= -\frac{1}{c}\frac{\partial \phi_e}{\partial t} - \vec{\nabla} \cdot \vec{A}_e \\ B &= -\frac{1}{c}\frac{\partial \phi_m}{\partial t} - \vec{\nabla} \cdot \vec{A}_m \end{aligned} \tag{3.5}$$

Defining the scalar field allow for simpler symbolic manipulation compared to not choosing a gauge condition, while it does not impose the limitations that choosing a gauge condition does.

Having defined a scalar field, it is now possible to express (3.3) and (3.5) using only quaternions:

$$\begin{aligned} \mathbf{E} = \frac{1}{2}(&\{\boldsymbol{\nabla}^*, \mathbf{A}_e^*\} - \{\boldsymbol{\nabla}, \mathbf{A}_e\} - \{\boldsymbol{\nabla}^*, \mathbf{A}_e\} - \{\boldsymbol{\nabla}, \mathbf{A}_e^*\} \\ &- [\boldsymbol{\nabla}^*, \mathbf{A}_m^*] + [\boldsymbol{\nabla}, \mathbf{A}_m] + [\boldsymbol{\nabla}^*, \mathbf{A}_m] + [\boldsymbol{\nabla}, \mathbf{A}_m^*]) \end{aligned} \tag{3.6}$$

$$\begin{aligned} \mathbf{B} = \frac{1}{2}(&\{\boldsymbol{\nabla}^*, \mathbf{A}_m^*\} - \{\boldsymbol{\nabla}, \mathbf{A}_m\} - \{\boldsymbol{\nabla}^*, \mathbf{A}_m\} - \{\boldsymbol{\nabla}, \mathbf{A}_m^*\} \\ &+ [\boldsymbol{\nabla}^*, \mathbf{A}_e^*] - [\boldsymbol{\nabla}, \mathbf{A}_e] - [\boldsymbol{\nabla}^*, \mathbf{A}_e] - [\boldsymbol{\nabla}, \mathbf{A}_e^*]) \end{aligned} \tag{3.7}$$

It is worth noticing that to express the fields as a quaternion we need to use every combination of $\boldsymbol{\nabla}$, $\mathbf{A}_m$, $\mathbf{A}_e$ and conjugation.

In similar way, you can write the quaternion expression for the currents, which is the quaternion equivalent to Maxwell's equations:

$$\begin{aligned} \mathbf{j}_e = \frac{1}{2}(&-\{\boldsymbol{\nabla}^*, \mathbf{E}^*\} - \{\boldsymbol{\nabla}, \mathbf{E}\} - \{\boldsymbol{\nabla}^*, \mathbf{E}\} + \{\boldsymbol{\nabla}, \mathbf{E}^*\} \\ &+ [\boldsymbol{\nabla}^*, \mathbf{B}^*] + [\boldsymbol{\nabla}, \mathbf{B}] - [\boldsymbol{\nabla}^*, \mathbf{B}] + [\boldsymbol{\nabla}, \mathbf{B}^*]) \end{aligned} \tag{3.8}$$

$$\begin{aligned} \mathbf{j}_m = \frac{1}{2}(&\{\boldsymbol{\nabla}^*, \mathbf{B}^*\} + \{\boldsymbol{\nabla}, \mathbf{B}\} - \{\boldsymbol{\nabla}^*, \mathbf{B}\} + \{\boldsymbol{\nabla}, \mathbf{B}^*\} \\ &+ [\boldsymbol{\nabla}^*, \mathbf{E}^*] + [\boldsymbol{\nabla}, \mathbf{E}] + [\boldsymbol{\nabla}^*, \mathbf{E}] - [\boldsymbol{\nabla}, \mathbf{E}^*]) \end{aligned} \tag{3.9}$$

Reduced to scalar and vector parts it can be written in a simpler form:

$$\begin{aligned}
\rho_e &= \vec{\nabla}\cdot\vec{E} - \frac{1}{c}\frac{\partial E}{\partial t} && (3.10)\\
\vec{\jmath}_e &= \vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \vec{\nabla}E && (3.11)\\
\rho_m &= -\vec{\nabla}\cdot\vec{B} + \frac{1}{c}\frac{\partial B}{\partial t} && (3.12)\\
\vec{\jmath}_m &= -\vec{\nabla}\times\vec{E} - \frac{1}{c}\frac{\partial \vec{B}}{\partial t} + \vec{\nabla}B && (3.13)
\end{aligned}$$

Notice that in the expression for charge, the extended Gauss's laws, now include a time derived of the scalar fields. And that the generalized Faraday's and Ampere's law, now include a gradient of the scalar fields.

While the physical understanding of magnetic monopoles is quite common, it is not the case with the understanding of the scalar fields. If we compare the scaler field to the electric vector field which express the amount of work done on a unit charge moved a distance $\delta w = q\vec{E}\delta x$ or force per unit charge, then it becomes apparent that the electric scalar field express the amount of work done on a unit charge doing a interval of time $\delta w = qE\delta t$ or effect per unit charge.

This means that the presence of a scaler field indicates energy flowing in or out of the electromagnetic system. The scalar field has been suggested as a model for electrodynamic interaction with the gravitation field [2] and as a model for interaction with heat[3], while those interactions properly can contribute to the scaler field, I don't consider it wise to consider it due to only one of the models, but would rather view them as contributing to the scalar field, like the Hamilton operator in quantum mechanics in which the actual terms depend on the physical systems being modeled. The question about which type of outside systems contribute to this energy flow will not be treated in this treatise, but it is a subject for further research.

3.3 Wave equations

As a result of the scalar fields we get terms for longitudinal weaves in the vector wave equations;

$$\begin{aligned}
\vec{\nabla}\vec{\nabla}\cdot\vec{E} - \vec{\nabla}\times\vec{\nabla}\times\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} &= \frac{1}{c}\frac{\partial\vec{\jmath}_e}{\partial t} + \vec{\nabla}\rho_e + \vec{\nabla}\times\vec{\jmath}_m\\
\vec{\nabla}\vec{\nabla}\cdot\vec{B} - \vec{\nabla}\times\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} &= \frac{1}{c}\frac{\partial\vec{\jmath}_m}{\partial t} - \vec{\nabla}\rho_m - \vec{\nabla}\times\vec{\jmath}_e
\end{aligned} \tag{3.14}$$

and we get scalar wave equations:

$$\begin{aligned}
\vec{\nabla}^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} &= \vec{\nabla}\cdot\vec{\jmath}_e + \frac{1}{c}\frac{\partial\rho_e}{\partial t}\\
\vec{\nabla}^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} &= \vec{\nabla}\cdot\vec{\jmath}_m - \frac{1}{c}\frac{\partial\rho_m}{\partial t}
\end{aligned} \tag{3.15}$$

Compared to Maxwell's equations the electric vector wave equation includes the extra terms $\vec{\nabla}\vec{\nabla}\cdot\vec{E}$, $\vec{\nabla}\rho_e$ and $\vec{\nabla}\times\vec{\jmath}_m$. The first term $\vec{\nabla}\vec{\nabla}\cdot\vec{E}$ makes it possible for this equation to describe longitudinal waves. The second term $\vec{\nabla}\rho_e$ is very interesting because it tells us that an electro static system also is a source of electric radiation and the 3th term shows that the same is a curved magnetic monopole current. The new terms in the magnetic wave equation are $\vec{\nabla}\vec{\nabla}\cdot\vec{B}$, $\frac{1}{c}\frac{\partial\vec{\jmath}_m}{\partial t}$ and $\vec{\nabla}\rho_m$ which is a longitudinal wave term and two terms which are magnetic counterparts to the electric terms in the electric case.

On the right of the scalar wave equations (3.15) we have the electric and the magnetic charge continuity equation which functions as the source term for the scalar wave terms on the left side. This tell us that a scalar wave may interfere with charge continuity by generating or annihilating charge $\frac{1}{c}\frac{\partial\rho}{\partial t}$ or generate divergence or convergence of currents $\vec{\nabla}\cdot\vec{\jmath}$.

With this wave equations, an interesting interpretation for electromagnetism is possible, if one adopts Maxwell's analogy that electromagnetism is due to motion in a medium. In the late 19th century it was discovered that vibration in a medium can cause attraction and repulsion [4][5]. If the field intensity is inversely proportional to the density of the medium, then the force between charge might be explained by such vibration. [1]

3.4 Quaternion power-force

In quaternionic electrodynamics, the power and the force becomes one quaternion, called the quaternion force $\mathbf{F}$.

You can write the electric part as the quaternion expression;

$$\mathbf{F}_e = \frac{1}{2}(\{\mathbf{j}_e,\mathbf{E}\} - \{\mathbf{j}_e^*,\mathbf{E}\} - \{\mathbf{j}_e,\mathbf{E}^*\} - \{\mathbf{j}_e^*,\mathbf{E}^*\} + [\mathbf{j}_e,\mathbf{B}] - [\mathbf{j}_e^*,\mathbf{B}] - [\mathbf{j}_e,\mathbf{B}^*] - [\mathbf{j}_e^*,\mathbf{B}^*]) \quad (3.16)$$

and the magnetic part as:

$$\mathbf{F}_m = \frac{1}{2}(\{\mathbf{j}_m,\mathbf{B}\} - \{\mathbf{j}_m^*,\mathbf{B}\} + \{\mathbf{j}_m,\mathbf{B}^*\} + \{\mathbf{j}_m^*,\mathbf{B}^*\} - [\mathbf{j}_m,\mathbf{E}] + [\mathbf{j}_m^*,\mathbf{E}] - [\mathbf{j}_m,\mathbf{E}^*] - [\mathbf{j}_m^*,\mathbf{E}^*]) \quad (3.17)$$

Or expressed as scalar and vector components:

$$\mathbf{F}_e = -\rho_e E - \vec{\jmath}_e\cdot\vec{E} + \rho_e\vec{E} + \vec{\jmath}_e E + \vec{\jmath}_e\times\vec{B} \quad (3.18)$$

$$\mathbf{F}_m = \rho_m B - \vec{\jmath}_m\cdot\vec{B} - \rho_m\vec{B} + \vec{\jmath}_m B - \vec{\jmath}_m\times\vec{E} \quad (3.19)$$

The scalar part of the total quaternion force becomes:

$$\begin{aligned} F = F_e + F_m &= -\rho_e E - \vec{\jmath}_e\cdot\vec{E} + \rho_m B - \vec{\jmath}_m\cdot\vec{B} \\ &= \frac{1}{2}\frac{1}{c}\frac{\partial(E^2+\vec{E}^2+B^2+\vec{B}^2)}{\partial t} - \vec{\nabla}\cdot(B\vec{B}+\vec{B}\times\vec{E}+E\vec{E}) \end{aligned} \quad (3.20)$$

Here we get new term in the time derived of the scalar E^2 and B^2 and we also notice that the Poynting energy flow $\vec{B}\times\vec{E}$ now got company by extra

[1] Such views of electromagnetism have before been held by researches such as John Keely[6], and Mel Winfield[7].

scalar-electric $E\vec{E}$ and scalar-magnetic $B\vec{B}$ energy flows in the direction of the electrical and magnetic fields.

Deriving the vector part of the quaternion force we get:

$$\begin{aligned}\vec{F} = \vec{F}_e + \vec{F}_m &= \rho_e\vec{E} + \vec{\jmath}_e E + \vec{\jmath}_e \times \vec{B} - \rho_m\vec{B} + \vec{\jmath}_m B - \vec{\jmath}_m \times \vec{E} \\ &= -\frac{1}{c}\frac{\partial(E\vec{E} + \vec{E}\times\vec{B} + B\vec{B})}{\partial t} + \vec{E}(\vec{\nabla}\cdot\vec{E}) + \vec{B}(\vec{\nabla}\cdot\vec{B}) + \frac{1}{2}\vec{\nabla}(E^2 + B^2) \\ &\quad + \vec{\nabla}\times(E\vec{B} - B\vec{E}) + (\vec{\nabla}\times\vec{E})\times\vec{E} + (\vec{\nabla}\times\vec{B})\times\vec{B} \qquad (3.21)\end{aligned}$$

Here we get lot of new interesting terms due to scalar fields and the magnetic monopoles.

Bibliography

[1] Koen van Vlaenderen. Generalised classical electrodynamics for the prediction of scalar field effects. In Andrew Chubykalo, Vladimir Onoochin, Agusto Espinoza, and Roman Smirnov-Rueda, editors, *Has the Last word been said on Classical Electrodynamics*, chapter 15. Rinton press, Paramus, Febuary 2004. ISBN 1-58949-036-3.

[2] Doug Sweetser. Einstein's vision 1: Classical unification of gravity and electromagnetism using riemannian quaternions. Online `http://www.theworld.com/~sweetser/quaternions/gravity/unified_field/unified_field.html`.

[3] Peter Michael Jack. Physical space as a quaternion structure. arXiv:math-ph/0307038, July 2003.

[4] Frederick Guthrie. On approach caused by vibration. *Proceedings of the Royal Society of London*, 19:35–41, 1870.

[5] Sir William Thomson. On attractions and repulsions due to vibration. In *Mathematical and physical papers*, volume IV, pages 98–100. Cambridge university press, Cambridge, 1910.

[6] Clara Bloomfield-Moore. *Keely and His Discoveries*. Kegan Paul and co., London, 1893.

[7] Mel E. Winfield. *The Science of Actuality*. Universology Press, 2004. ISBN 0-9739347-0-0.

[8] Artice M. Davis. A generalized helmholtz theorem for time-varying vector fields. *American journal of physics*, 74(1):72–76, January 2006.

Chapter 4

Longitudinal force

In this chapter we will study the forces which act between current elements in a circuit. To compare the different models we will consider two currents elements with the electrical current densities $\vec{\jmath}_1$ and $\vec{\jmath}_2$ and let $\vec{r}$ be the vector from the location of $\vec{\jmath}_1$ to $\vec{\jmath}_2$.

4.1 Ampère's force law

Before Maxwell's equations, electrodynamics were formulated by Ampère's force law which has already been mentioned in (1.17). When Ampère heard about H.C. Ørsted's discovery of the action a current exercises on a magnet he investigated whether two electric currents also exercise an action upon each other. After performing experiments with steady current and closed circuits. Ampère summarized his experimental observations in four laws.

1. The effect of a current is reversed when the direction of the current is reversed.
2. The effect of a current flowing in a circuit twisted into small sinuousities are the same as if the circuit were smoothed out.
3. The force exerted by a closed circuit on an element of another circuit is at right angles to the latter.
4. The force between two elements of circuits are unaffected when all linear dimensions are increased proportionally, the current-strengths remaining unaltered.

Together with the assumptions that the force should act along the line connecting the two conductor elements and that the force should follow Newton's 3. law that the reaction force is equal and opposite the action force. Ampère's formulated a force law, which in addition to describing a force between conductor elements running beside each other, also included longitudinal forces between conductor elements on the same line. Ampère force law can be written as

$$d^2\vec{f}_{12} \sim -\frac{\vec{r}}{|\vec{r}|^3}\left(\vec{\jmath}_1 \cdot \vec{\jmath}_2 - \frac{3}{2|\vec{r}|^2}(\vec{r}\cdot\vec{\jmath}_1)(\vec{r}\cdot\vec{\jmath}_2)\right). \tag{4.1}$$

Here it is easy to see that the force acts along $\vec{r}$, and that if one makes the transformation $\vec{\jmath}_1 \to \vec{\jmath}_2$, $\vec{\jmath}_2 \to \vec{\jmath}_1$ and $\vec{r} \to -\vec{r}$ we get a force which is opposite and equal in size such that Newton's 3. law is satisfied.

A weakness in Ampère's argument is that his 3. law was based on experiments with closed circuits and therefore the current elements could have a force component breaking his law, if it disappears when integrating all the elements in the circuit. Another weakness is naturally his two assumptions which he uses without experimental validation.

4.2 Grassmann's force

In Maxwell's equations we find that the force on the current elements comes from the term $\vec{\jmath}_1 \times \vec{B}_2$ which can be written as

$$d^2\vec{f}_{12} \sim -\frac{1}{|\vec{r}|^3}\vec{\jmath}_1 \times (\vec{r} \times \vec{\jmath}_2) = -\frac{1}{|\vec{r}|^3}(\vec{r}(\vec{\jmath}_1 \cdot \vec{\jmath}_2) - \vec{\jmath}_2(\vec{r} \cdot \vec{\jmath}_1)) \tag{4.2}$$

this term was first suggested by Grassmann in 1845. This force doesn't satisfy Newton's 3. Law because $\vec{\jmath}_2(\vec{r} \cdot \vec{\jmath}_1) \neq \vec{\jmath}_1(\vec{r} \cdot \vec{\jmath}_2)$, furthermore $\vec{\jmath}_2(\vec{r} \cdot \vec{\jmath}_1)$ doesn't follow $\vec{r}$ and therefore Maxwell's equations break with both of Ampère's original assumptions.

It is interesting to note that in his treatise Maxwell makes a detailed analysis of the force between two current elements[1, p.147] in which he concludes that Ampère's force law properly is the correct one, but then when he lists his general electromagnetic equations[1, p.239] they are in agreement with Grassmann's formula.

4.3 Tait's quaternion forces

In 1860 Tait had published[2] a quaternion investigation of Ampère's law in which he follows Ampere's assumptions that the force between two current elements follows the line between them, in this paper he promises to make another one in which he investigates Ampère's experimental data, without the assumptions. When Maxwell in 1873 published his treatise on electricity and magnetism, where he investigated the same question, Tait remember his old promise and publishes[3] hes own investigation, which is somewhat different from Maxwell's. Tait obviously makes his investigation using quaternions, and by studying which type of terms disappears when you integrate over a close circuit, he finds that the force between the two conductor elements can be expressed simply as the vector part of the quaternion product $\vec{\jmath}_1\vec{r}\vec{\jmath}_2$[1]. It is interesting is that the vector part of this quaternion product between the 3 vectors is exactly the same as we get from the quaternion model.

4.4 The quaternion formula

In the quaternion model we have an additional term $\vec{\jmath}_1 E_2$ compared to Grassmann's formula and it expands to $-\vec{\jmath}_1\vec{\nabla} \cdot \vec{A}_2$ in this situation. Then the force

[1] This force is often called the Whittaker's formula because he gives a non-quaternionic expression for it in [4]

becomes:

$$d^2\vec{f}_{12} \sim -\frac{1}{|\vec{r}|^3}(\vec{\jmath}_1 \times (\vec{r} \times \vec{\jmath}_2) - \vec{\jmath}_1(\vec{r} \cdot \vec{\jmath}_2))$$
$$= -\frac{1}{|\vec{r}|^3}(\vec{r}(\vec{\jmath}_1 \cdot \vec{\jmath}_2) - \vec{\jmath}_2(\vec{r} \cdot \vec{\jmath}_1) - \vec{\jmath}_1(\vec{r} \cdot \vec{\jmath}_2)) \quad (4.3)$$

Here it is easy to see that the law of action and reaction is restored. We also have that the last two parts of the force which are not along $\vec{r}$ disappear when integrated over in a close circuit.

If Tait had extended the vectors to be full quaternions, he would have been on his way to discover the full quaternion model.

If you set $\mathbf{j_1} = \rho_1 + \vec{\jmath}_1$ and $\mathbf{j_2} = \rho_2 + \vec{\jmath}_2$ you get

$$\mathbf{j_1}\vec{r}\mathbf{j_2} = -\rho_1(\vec{r} \cdot \vec{\jmath}_2) - (\vec{\jmath}_1 \cdot \vec{r})\rho_2 - j_1 \cdot (\vec{r} \times \vec{\jmath}_2)$$
$$+ \rho_1\vec{r}\rho_2 + \vec{\jmath}_1 \times (\vec{r} \times \vec{\jmath}_2) - \vec{\jmath}_1(\vec{r} \cdot \vec{\jmath}_2) + \rho_1(\vec{r} \times \vec{\jmath}_2) + (\vec{\jmath}_1 \times \vec{r})\rho_2 \quad (4.4)$$

The last line is the vector part and proportional to the force, here can we see that $\rho_1\vec{r}\rho_2$ is proportional to the electrostatic force and $\vec{\jmath}_1 \times (\vec{r} \times \vec{\jmath}_2) - \vec{\jmath}_1(\vec{r} \cdot \vec{\jmath}_2)$ is the electrodynamic force, the last two terms are the asymmetric part of the quaternion. This is the part we remove by using the symmetric product in the quaternion model.

If Tait had thought of extending his expression, he could have found a quaternion expression which would express both electrostatic and electrodynamic forces.

Then the next step towards the quaternion model, would be to notice that $\vec{\nabla}\frac{1}{|\vec{r}|} = -\frac{\vec{r}}{|\vec{r}|^3}$ and then extend $\vec{\nabla}$ to $\mathbf{\nabla}$.

The problem which seems to stop Tait from expanding his formula beyond current elements, was primary, that they worked mainly with scalar and vector parts, at the time, and rarely with full quaternions, and secondary, that without being aware of the difference on right and left nabla, he would not have been able to write a full quaternion expression which would give the correct expression of the force.

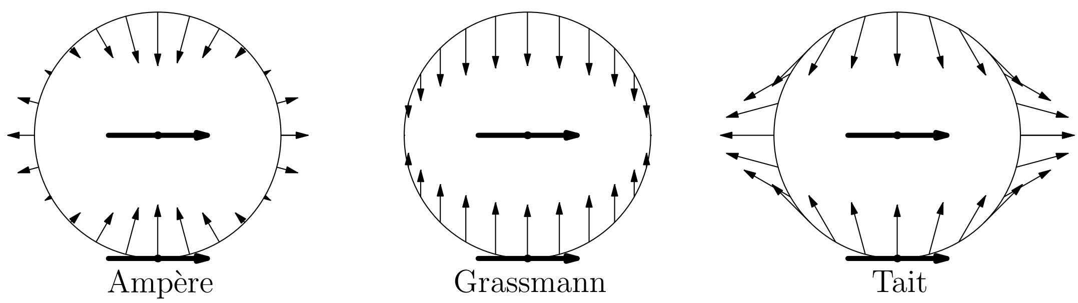


Figure 4.1: Force between parallel conductor elements

4.5 Auguste-Arthur de la Rive

In 1822 the Swiss scientist Auguste-Arthur de la Rive invited Ampère to come to Geneva and do an experiment, which De la Rive had designed to explicitly demonstrate the longitudinal force. The experiment is known as the hairpin experiment and consists of two pools of mercury arranged next to each other, with an isolating barrier in between. Then an insulated copper wire in the shape of an hairpin is placed with a leg in each pool. When current is supplied to the pool from a battery, then the wire would move along the barrier. See figure 4.2.

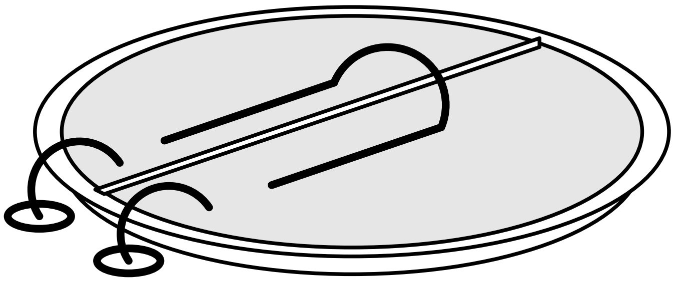


Figure 4.2: Hairpin experiment

4.6 Peter Guthrie Tait

In 1861 the hairpin experiment was repeated by Maxwell's friend Peter Guthrie Tait[5]. First he confirmed Ampere's and De la Rive's results, but then he replaced the copper wire with a mercury filled glass tube to get rid of the possibility that the effect could be due to thermo-electric or other effects between the two different conductors. After this modification to the experiment he was still able to observe the same effect.

4.7 Peter Graneau

In 1981 a modern version of the hairpin experiment was performed by Peter Graneau at MIT[6]. Here the hairpin conductor floated on two channels of liquid mercury and would move to the end of the channels when a current of 200 A was applied. In his paper a new effect was noticed and reported for the first time. When the conductor was blocked from moving, jets were seen emerging at the ends of the conductor, at 500 A the effect became unmistakable, and at 1000 A the were a danger of the mercury spilling out of the channel. [2]

A demonstration specially designed to show this jet effect, is the Liquid Mercury Fountain experiment [7], where one electrode is inserted through the bottom of a cup filled with liquid mercury, and a ring electrode is partly submerged in the mercury at the top. When a current between 500 and 1000 A is

[2] A good book for further information on Ampère's force law, are Peter and Neal Graneau's 'Newtonian Electrodynamics' [7].

made to flow through the arrangement, a fountain is observed rising from the cup. See figure 4.3.

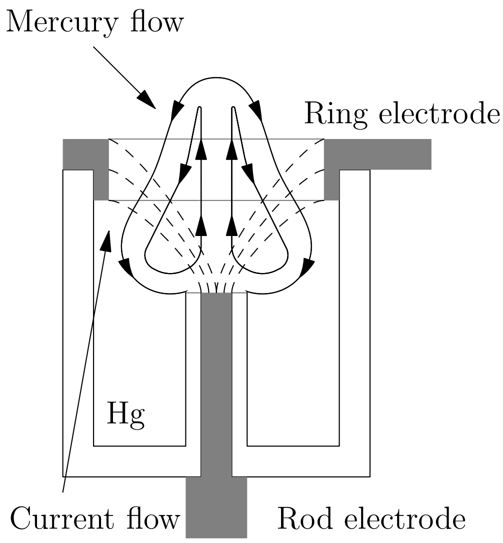


Figure 4.3: Mercury fountain

One of the barriers to reproduce the experiments about are their use of very high currents, but new experiments remove this barrier and bring a reproduction of the experiments within the reach of every serious student.

4.8 Remi Saumont

In 1992 Remi Saumont published an experiment [8] where the conductor from Ampère's and De la Rive's experiment is turned vertically, and placed on a high precision weight scale, so that the longitudinal force on the conductor could be precisely measured. Different setups made it possible to measure the force both as a push and a pull. The currents used in the experiments were 4 A to 12 A for a duration of around 1 s. The conclusion of this experiment was also a verification of a longitudinal force. See figure 4.4.

4.9 Thomas Phipps jr.

In 1995 Thomas Phipps jr. published an article [9] where he shows how you can verify the existence of a longitudinal force with simple equipment.

In his experiment, he used a turning fork and connected a pair of conductors in such a way that an oscillating longitudinal force would result in a driving force on the turning fork. The two conductors were attached to each leg of the turning fork with a gab of around 1 mm. This gab was then connected with a piece of plastic straw holding a drop of mercury. The oscillation of the turning fork was detected using a laser, a photo diode, an amplifier, and a razor-blade attached to the top leg of the fork. See figure 4.5.

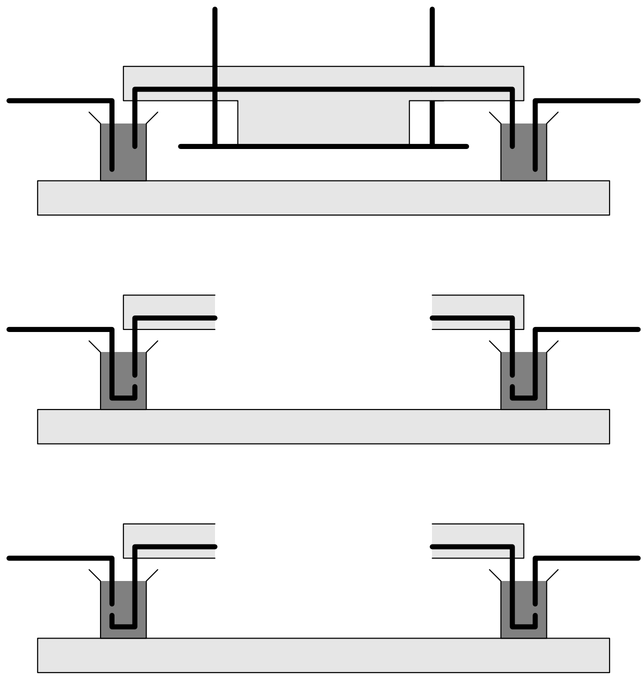


Figure 4.4: Saumont's experiment

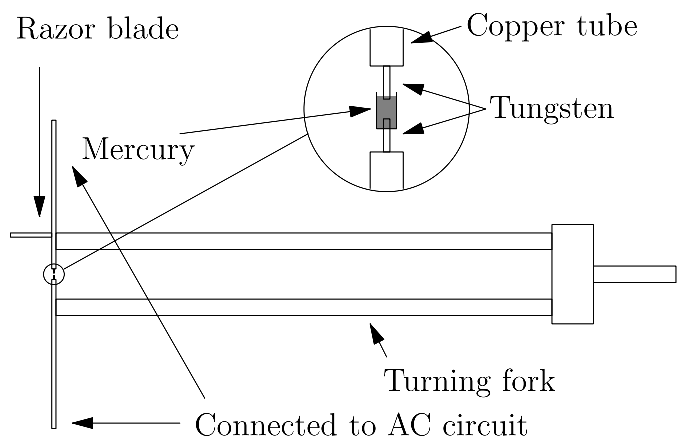


Figure 4.5: Phipps experiment

4.10 Roundup

As has been seen, fundamental uncertainties in Ampère's original experiments lead to certain freedoms when formulating the force law between two current elements. Later experiments explicitly show the existence of a longitudinal force, this shows that Grassmann's formula and Maxwell's equation can't provide a complete description of the forces. A more detailed theoretical investigation can be found in [1],[4] and [10] and a further experimental overview can be found in [7] and [11].

Bibliography

[1] James Clerk Maxwell. *A Treatise on electricity and magnetism*, volume II. Clarendon press, Oxford, second edition, 1881.

[2] Peter Gutherie Tait. Quaternion investigation connected with electrodynamics and magnetism. In *Scientific papers by Peter Gutherie Tait* [13], pages 22–32.

[3] Peter Gutherie Tait. Note on the various possible expressions for the force exerted by an element of one linear conductor on an element of another. In *Scientific papers by Peter Gutherie Tait* [13], pages 237–244.

[4] Sir Edmund Whittaker. *A History of the Theories of Aether and Electricity.* Thomas Nelson and Sons ltd, Edinburgh, 1951.

[5] Peter Gutherie Tait. Note on a modification of the apparatus employed for one of ampère's fundamental experiments in electrodynamics. In *Scientific papers by Peter Gutherie Tait* [13], pages 35–36.

[6] Peter Graneau. Electromagnetic jet propulsion in the direction of current flow. *Nature*, 295:311–312, 1982.

[7] Peter Graneau and Neal Graneau. *Newtonian Electrodynamics.* World Scientific, Singapore, 1996. ISBN 981-02-2284-X.

[8] Remi Saumont. Mechanical effects of an electrical current in conductive media. experimental investigation of the longitudinal ampère force. *Physics Letters A*, 165:307–313, 1992.

[9] Thomas E. Phipps Jr. A do-it-yourself refutation of modern physics. *Galilean electrodynamics*, 6(5):92–97, 1995.

[10] Patrick Cornille. *Advanced Electromagnetism and Vacuum physics*, volume 21 of *World Scientific Series in Contemporary Chemical Physics.* World Scientific Publishing, Singapore, 2003. ISBN 981-238-367-0.

[11] Stefan Marinov. *Divine electromagnetism.* East-West, Graz, 1993.

[12] Olivier Darrigol. *Electrodynamics From Ampère to Einstein.* Oxford university press, Oxford, 2000. ISBN 0-19-850594-9.

[13] Peter Gutherie Tait. *Scientific papers by Peter Gutherie Tait.* C. J. Clay and sons, Cambridge, 1898.

Chapter 5

Magnetic monopoles

When making the quaternion model for electromagnetism, magnetic monopoles were included in the equations. In this chapter we are going to study experimental research on magnetic monopoles and magnetic currents.

With Ampère's hypothesis from 1820 that magnets owe their properties to closed currents in the mass [1, p.7], the concept of magnetic charges started to take a secondary place in the models for electromagnetism, even though Maxwell included the magnetic potential and charge density in his original equations, the subject began to receive less attention in mainstream physics. That might be why most research on magnetic currents and monopoles are from researchers who are considered controversial in mainstream physics.

5.1 Magnetosphotophoresis

Magnetosphotophoresis was first observed by the Austrian physicist Felix Ehrenhaft around 1910. In an article[2] he published in 1941 together with Leo Barrnet, they describe this phenomenon in the following way.

> One of us (F.E.) However has shown that, submicroscopic particles irradiated by light do move in a homogeneous magnetic field, some toward the North and some toward the South Magnetrode, that they reverse their direction if the direction of the field is reversed, etc. These particles thus actually behave like single magnetic poles (magnetic charges, magnetic ions). On them are a preponderance of North or South magnetism and their movement in a homogeneous magnetic field constitutes a magnetic current.

In 1944 V.D. Hopper[3] reports that he had failed to reproduce Ehrenhaft's experiment using a permanent magnet and he also suggests, without verification that Ehrenhaft's result may be due to some stray electrical fields.

The stray fields hypothesis was rejected in 1947 by B. G. Kane [4], he writes;

> It cannot be due to the presence of electrostatic fields since the experiments have been performed with the pole pieces shielded by a fine wire cage and the whole apparatus grounded.

and then he continues with an important observation:

> The explanation may be found by examining the conditions under which the particles jump in the dark. Some fine metal powder such as iron, nickel, cobalt, tungsten, chromium, etc, is placed on the lower pole piece B of a strong electromagnet. When the field is suddenly reversed, a number of these particles jump to the upper pole piece A.

From this observation he comes to the following conclusion:

> The obvious explanation seems to be that a current is induced in a particle by the rapidly changing magnetic field. If this induced current is more than 90 degrees out of phase with the current in the electromagnet, the particle will be repelled by the lower pole.

and then he verifies his conclusion in the following way:

> This interpretation of the phenomenon was checked in two ways. First, when a weak field is used, the particles jump a short distance and fall back to the lower pole face; if they were true single poles, they should continue to rise to the upper pole face. Secondly, the magnetic field was gradually reduced to zero before reversing it and gradually increased after reversal. As a result, no particles were observes to jump.

This might at first seem like a good explanation, but one could ask why good conductors like gold, silver and copper are missing from his list of materials, which is mostly ferromagnetic? One would think that good conductors should be primary candidates if the effect is caused by an induced current while ferromagnetic materials would indicate a magnetic phenomenon.

Another objection is that Ehrenhaft used a very homogeneous magnetic field and he reported that the free floating particles got a north or south magnetic charge with equal possibility as far as he could tell. But if the effect should be due to induced current from the magnetic switching, then one would expect that all the particles in an area would get similar polarization.

There could naturally be a problem with the last argument if the particle was not perfectly spherical, but an investigation of this problem was provided in 1988 by V.F. Mikhailov [5]. In an experiment, where he provide the ferromagnetic particles with a liquid shell which would not be able to contain any stable non-uniformity. He is still able to reproduce the magnetic charge effect and also determine the distribution of the magnetic charge.

The magnetic charge distribution has a peek at $2.5^{+1.6}_{-1.3} \times 10^{-8}$gauss cm^2 which is close to the 3.29×10^{-8}gauss cm^2 which is the value calculated by Dirac for the magnetic elementary charge. He then finishes the article with the conclusion:

> Further work is necessary to exclude the possibility of systematic errors but from the above numerical result we conclude that the observed effect would be consistent with the presence of Dirac monopoles within the droplet, possibly held as bound states with the magnetic moment of the ferromagnetic particle.

In 1995 V. F. Mikhailov published another article in which he measured the magnitude of the magnetic elementary charge. By observing how particles that had both an electric and a magnetic charge moved in space having a magnetic and a electric field orthogonal to each other, he was able to determine the magnitude of the magnetic elementary charge to $3.27 \pm 0.16 \times 10^{-8}$gauss cm^2 and in this paper he concludes.

> The magnitude of the magnetic charge by Dirac theoretically is obtained experimentally.

As fare as I know Mikhailov result still need to be verified by other researches.

A possible explanation that seems to fit all the experiments on magnetosphotophoresis, is that a rapid shifting magnetic field splits up the flow of magnetic monopoles and allow single monopoles to get trapped in the ferromagnetic particles where the light beam is necessary to keep them from escaping. This would explain why the effect don't show up when using a permanent or in a slow varying magnetic field and why it don't show up without a light beam.

5.2 Magnetic current

If magnetic monopoles really exist, then one might consider if the electromagnetic phenomenon is a consequence of interaction between magnetic monopoles. A researcher who advocated this view, was Edward Leedskalnin (1887-1951).

Edward was not educated as a scientist, and many of his views and explanations are based upon hes own experiments, were far from accepted scientific ones. What make Edward Leedskalnin's story interesting, is that while his views, were not those of our times scientist, he did seem to show superior understanding of the nature of gravity.

In the years 1920-1940 he single handedly built what are called the coral castle from 1,100 tons of coral rock in Homestead, Florida. The entrance is through a gate made of a 9 tons block of rock, so perfectly balanced that a child can open it with a finger.

Hes own words about how he was able to build his castle is:

> I have discovered the secrets of the pyramids, and have found out how the Egyptians and the ancient builders in Peru, Yucatan, and Asia, with only primitive tools, raised and set in place blocks of stone weighing many tons!

Edward never did reveal his secret directly, and most of hes work on the castle, he did at night by the light of a lantern. It has been told that, some teenagers once sneaked up on him one night and returned home with stories, about rocks floating in the air, like helium balloons.

Edward's view on electromagnetism is known from a little booklet he published in 1945, called 'Magnetic current' [12]. The primary content of this booklet is a description of experiments with magnets and simple circuits.

According to Edward, electric current is an effect generated by magnetic monopoles flowing against each other in a spinning fashion. North pole monopoles from electric positive to electric negative, and south pole monopoles the other way, always flowing against each other and never alone.

Edward argued that magnetic monopoles are much smaller than the particles of sunlight, by the simple fact that sunlight cannot penetrate things like wood, rock and iron, but the magnetic force can penetrate everything.

Edward's view may be summarized by the following quote:

> The real magnet is the substance that is circulating in the metal. Each particle in the substance is an individual magnet by itself, and both North and South Pole individual magnets. They are so small that they can pass through anything. In fact they can pass through metal easier than through the air. They are in constant motion, they are running one kind of magnets against the other kind, and if guided in the right channels they possess perpetual power.

One might say that Edward's view is the reverse version of Ampère's hypothesis.

Bibliography

[1] Olivier Darrigol. *Electrodynamics From Ampère to Einstein*. Oxford university press, Oxford, 2000. ISBN 0-19-850594-9.

[2] Felix Ehranhaft and Leo Barnet. Is there a "true magnetism´´ or not? *Phil. Sci.*, 8:458–462, 1941.

[3] V.D. Hopper. Magnetic ions. *Physical Review*, 66:93–94, 1944. Letter to the editor.

[4] Brother Gabriel Kane. Magnetic ions and magnetic currents. *Physical Review*, 71:458–459, 1947.

[5] V. F. Mikhailov. Observation of apparent magnetic charges carried by ferromagnetic particles in water droplets. *Journal of Physics A*, 24:53–57, 1991.

[6] Felix Ehranhaft. Photophoresis of small particles in magnetic field. *Physical review*, 57:659, 1940.

[7] Felix Ehranhaft. Magnetolysis and electric field around the magnetic current. *Physical Review*, 63:461, 1943. Proceedings of the American Physical Society.

[8] Felix Ehranhaft. The decommposition of water by the so-called permanent magnet and the measurement of the intensity of the magnetic current. *Physical Review*, 65:287–289, 1944. Letter to the editor.

[9] Felix Ehranhaft. The measurement of single magnetic charge and the electroststic field around the permanent magnet. *Physical Review*, 67:201–202, 1945. Proceedings of the American Physical Society.

[10] Felix Ehranhaft. Magnetism as a polar vector. *Physical Review*, 68:105, 1945. Proceedings of the American Physical Society.

[11] V F Mikhailov. Magnetic charge and optical levitation of ferromagnetic particles in magnetic field. *Journal of Physics D*, 27:2241–2245, 1994.

[12] Edward Leedskalnin. Magnetic current. Rock Gate, Homestead, Florida USA, 1945.

Chapter 6

Scalar effects

This chapter is about effects caused by the scalar electric field E and scalar magnetic field B. The study of scalar fields have in the past been difficult for a number of reasons.

- There is no commonly known way to measure the scalar fields directly.
- The effects are not described in the normally teached model of electromagnetism.
- The scalar effect seems to contradict some generally teached laws of physics, this has made it very difficult for researchers on this subject to get their findings published in the top peer reviewed journals.

6.1 Tesla's discovery

In 1892 Nikola Tesla published an article [1] where he describes the discovery of what he called "electrical sound waves". The event leading up to the discovery, is described in detail in the book *Lost Science* by Gerry Vassilatos[2], here it should be noted that he is basing some of his material on interviews with persons who knew Tesla, and that there is no written documentation:

Tesla was studying the phenomena that instantaneous application of current from his polyphase system often caused exploding effects. To be able to study this effect, he built a test system consisting of a high voltage dynamo and a capacitor bank. Then he configured it in such way that all the possible current alternations were eliminated, so that this test system would supply a single pulse of high current and high voltage, which would instantaneously vaporize a short and thin piece of wire.

When he made the first wire explosion, he felt a powerful sensation of a stinging pressure waves. At first he thought that the irritation was caused by small particles from the explosion but he could find no wounds or other evidence of that. Then he placed a large glass plate between himself and the exploding wire and he could still feel the stinging sensation. This made him exclude the possibility that the sensation where caused by a mechanical pressure waves and directed him on to believe that the effect was electrical.

He then exchanged the short and thin wire a for longer one, so that the wires did not explode, but still the powerful stinging sensation remained.

He then changed his test system again, and made it automatic, so that he could walk around in his lab while he made his experiments. He observed that the stinging effect could be sensed 10 feet away.

He realized that this effect was a rare electrical effect caused by very short current impulses. Painfully studying the discharge on very close range, he noted that the bright blue-white color of the discharge spark which stood strait out from the wire, seemed to indicate a voltage of 250,000 volt, while the test system only supplied 50,000 volt.

Tesla tried to change the number of impulses per second and he found that the manifestation of the effect changed with frequency. With some frequencies there was heat, with some there was light, and with some, it felt like a cold breeze and the painful stinging effect seemed to go away over a sudden threshold.

The phenomena that Tesla found are incredible and it is no wonder that almost no one could understand his inventions and discoveries at his time, lacking a mathematical model to guide them in this new territory of phenomena.

6.2 Tesla's article

We will take Tesla's own description from his 1892 article, and step by step show how this can be explained with quaternion electrodynamics.

> These waves are propagated at right angles from the charged surfaces when their charges are alternated, and dissipation occurs, even if the surfaces are covered with thick and excellent insulation.

This can be explained with the $E\vec{E}$ term in the flow of energy (3.20). While the $\vec{E}$ field is always at a right angle from the charged surface and charge alternation $\frac{1}{c}\frac{\partial \rho_e}{\partial t}$ is the source terms for the electric scalar wave (3.15). The $E\vec{E}$ term also explains the voltage scaling effect that Tesla observed, such that a 50 kV discharge could look like a 250 kV discharge.

In the article, Tesla has the following description.

> These waves are especially conspicuous when the discharges of a powerful battery are directed through a short and thick metal bar, the number of discharges per second being very small.

A powerful battery and with discharge through a short and thick metal bar, this a good way to optimize the term $\frac{1}{c}\frac{\partial \rho_e}{\partial t}$ and thereby the oscillations of the scalar field and the flow of energy $E\vec{E}$.

> The experimenter may feel the impact of the air at distances of six feet or more from the bar, especially if he takes the precaution to sprinkle the face or hands with ether.

and later in the article:

> These waves cannot be entirely stopped by the interposition of an insulated metal plate.

A normal insulation is meant to stop the electric current and not the electric field, and it would not be able to stop this scalar-electric radiation. The best insulator for this scalar-electric radiation would properly be a strong dielectric material.

> Most of the striking phenomena of mechanical displacement, sound, heat and light which have been observed.

Tesla only gives a few examples on the types of mechanical displacement he has been observing, but in a video[3] on the reproduction of Tesla's experiments some of the effects can be seen. The most remarkably experiment in this video, is when they show how a strip of copper is clearly attracted to a light blob, lighted with current impulses.

6.3 Current impulses and divergences

When you look at the experiments which show evidence of scalar effects, then two characteristics seem to emerge. Those experiments seem to include either impulse currents, divergences of current density, or both. Impulse currents are very short pulses of unidirectional current. It seems to be very important for obtaining a scalar effect, that the pulses are not alternating, but is only unidirectional.

A possible reason why a scalar effect is observed when impulse currents are used, might be that the decay time for an excitation of the scaler field might be very sort. By using short pulses more of the energy might be channeled through the terms governed by the scalar field as illustrated in figure (6.1).

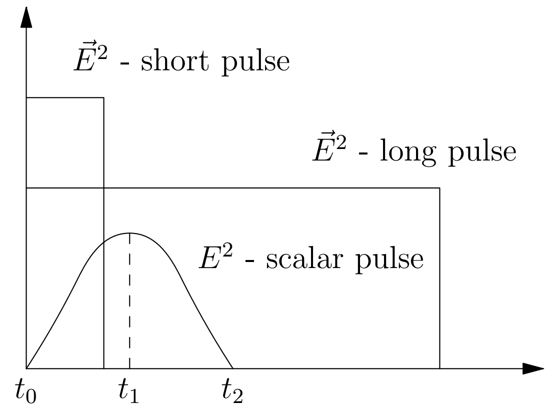


Figure 6.1: Scalar effect from impulse current

Here the first part of the pulse from t_0 to t_1 is where E^2 is increasing and energy is flowing into the system, the interval from t_1 to t_2 is where E^2 is decreasing and energy flows out of the system. The scaler effect is most dominant when the current impulse is shorter than t_1.

While there are divergences in the current density in current impulses, they can also be created when the current runs through conductors which expand or contract. This is another characteristic which is repeatedly observed in experiments with scaler fields.

6.4 Transmission through a single wire

In the energy power theorem (3.20) there is a flow of energy in the form of the $E\vec{E}$ term. Which indicates that it might be possible to set up a flow of energy along an electric field $\vec{E}$ if you can generate a scalar field E. This would make it possible to transmit energy through a single wire.

6.4.1 Tesla's transmission line

In [4] Tesla writes:

> In the course of development of my induction motors it became desirable to operate them at high speeds and for this purpose I constructed alternators of relatively high frequencies. The striking behavior of the currents soon captivated my attention and in 1889 I started a systematic investigation of their properties and possibilities of practical application. The first gratifying result of my efforts in this direction was the transmission of electrical energy thru one wire without return, of which I gave demonstrations in my lectures and addresses before several scientific bodies here and abroad in 1891 and 1892.

In order to understand Tesla's system, take a look at figure (6.2).

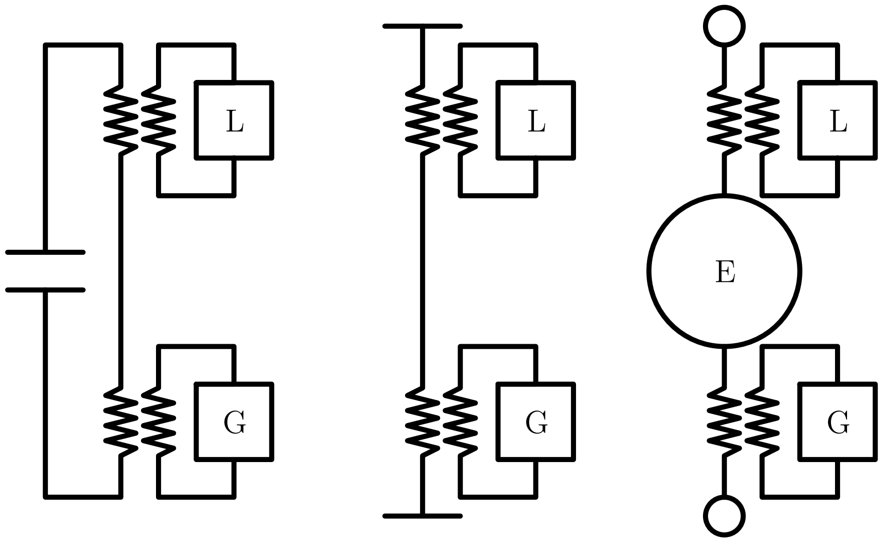


Figure 6.2: Tesla's transmission line

The first part shows a resonance circuit with a capacitor and two coils. The first coil is connected to a generator through a secondary coil and the other is connected to a load in the same way. When the generator is running the load draws energy, this is ordinary transformer theory. In the middle part, the capacitor has been split up and the energy is transmitted along a single line. In the last part, the terminals have been replaced with metal spheres and the line has been replaced with the earth. This is essentially Tesla's plan for transmitting energy to the entire earth. The use of impulse current, and the divergences at the terminal, would ensure a scalar scaling effect and a flow of energy along the line, and due to the scaling effect the generator was called a magnifying transmitter. Tesla seems to describe this effect in the following quote where he compares his system to pumping a bag of rubber[4]:

> This is a crude but correct representation of my wireless system in which, however, I resort to various refinements. Thus, for instance, the pump is made part of a resonant system of great inertia, enormously magnifying the force of the imprest impulses. The receiving devices are similarly conditioned and in this manner the amount of energy collected in them vastly increased.

6.4.2 Goubau's transmission line

In 1950 Georg Goubau published an article[5] where he describes another type of single wire transmission line. He describes a conductor with the surface modified in a saw tooth shaped pattern, which gives a divergence in the current density along the line. To launch and receive the wave Goubau is using a horn shaped conductor in each end. To keep the wave within the wave guide, it is coated with a dielectric material. Here the horn is a current density divergence which helps create the scalar field and a flow of energy along the conductor while the dielectric coating helps prevent the flow from escaping.

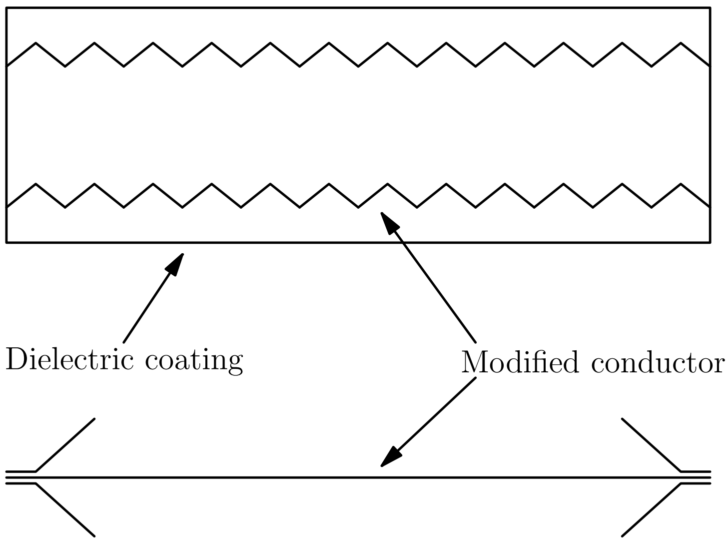


Figure 6.3: Goubau's transmission line

6.5 Scalar devices

6.5.1 Bedini's device

This scalar field device is attributed to researcher John Bedini.

It consists of 2 bar magnets which are glued with the north poles against each other and with 50 turns of wire around it. When impulse current is supplied to the coil, narrow beams are reported to extend a few inches from the magnets. (see figure 6.4) To verify the effect of the beam it is suggested that you buy two identical music cds, and after verifying that they sound the same, place one in the radiation for 2 minutes, and then compare them again.

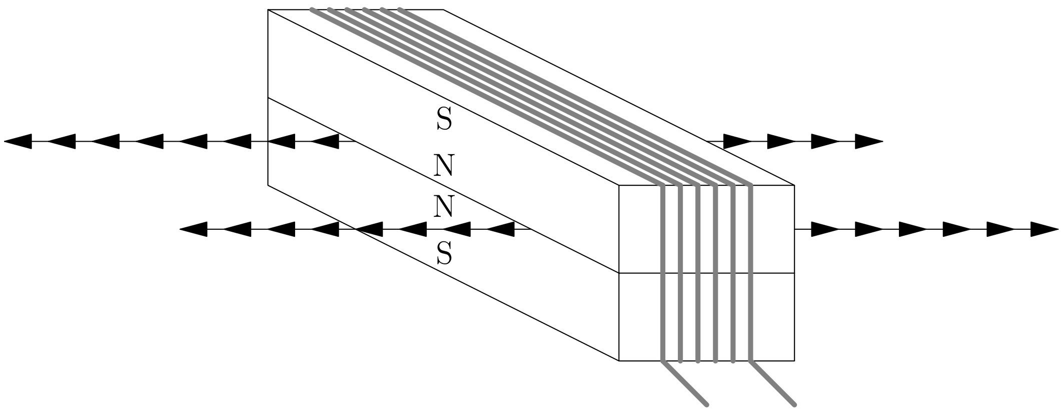


Figure 6.4: Bedini's scalar device

6.5.2 Fluxite device

An example of how it might be possible to make a scalar magnetic field B, might seen in [6]. Described in this document is a Tesla coil coiled around a cone with an iron tip (figure 6.5) and then a pulsing current is applied to the Tesla coil. The idea here is that this would give reason to a pulsing magnetic field with a divergence which would create a magnetic scalar field at the iron tip. [1]

In the fluxite non-patent, eight of these Tesla cones are mounted on a bar magnet, and used to magnify its magnetic field. A donut shaped copper coil is then used to gather electricity from the arrangement, and when the pulsing of the Tesla coils to tuned to the resonance frequency of the bar magnet, then more electricity is generated by the gathering coil than is used to drive the Tesla coils. See figure 6.6.

6.6 Quantum effects

While much research is still needed on scalar fields, I will use a little space to mention the possibility that the quantum effect might be due to scalar fields.

[1] It should be noted that the explanation from the non-patent differers from the explanation presented here.

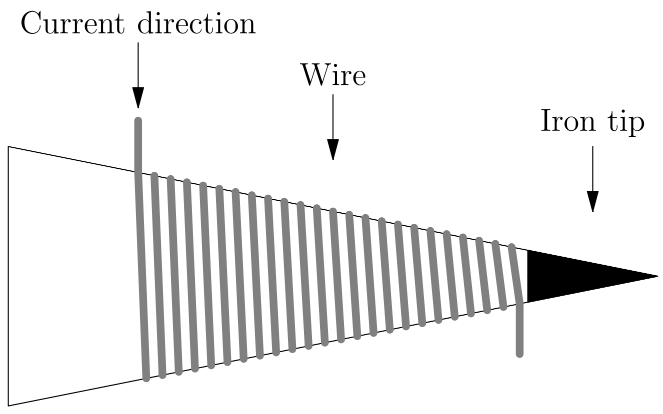


Figure 6.5: Tesla coil cone

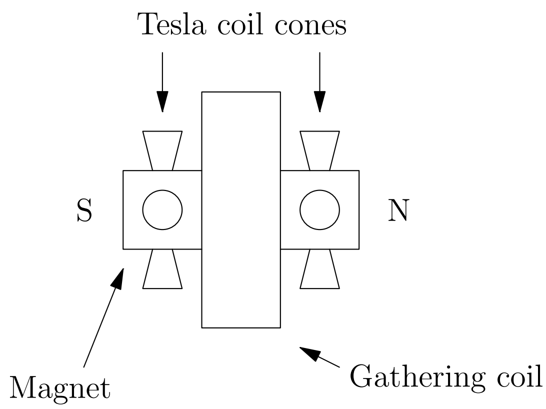


Figure 6.6: Fluxite sketch

Let us take the Heisenberg uncertainty principle, which in one form states the uncertainty in energy times the uncertainty in time is grater or equal than $\frac{\hbar}{2}$. Here it is important to remember that most quantum mechanical interaction observed in the laboratory has been almost exclusively in electromagnetic systems, so therefore it might be the case that the uncertainty in energy is due to an excitation of a scalar oscillation, and that the time for the energy in this scalar oscillation to return to the non-scaler system is grater than $\frac{\hbar}{2\Delta E}$. If this is what happens, it could help restore energy conservation back in the description of quantum effects. This would also give a better understanding of the quantum mechanical tunnel effect and give researchers a way to control it.

Bibliography

[1] Nikola Tesla. On the dissipation of the electrical energy of the hertz resonater. In Ratzlaff [8], pages 22–23.

[2] Gerry Vassilatos. *Lost Science*. Adventures Unlimited Press, Kempton, 1999. ISBN 0-932813-75-5.

[3] Eric Dollard and Peter Lindemann. Tesla's longitudinal electricity. Borderland Sciences video, 1988. ISBN 0-945685-89-0.

[4] Nikola Tesla. Famous scientific illusions. In Ratzlaff [8], pages 192–199.

[5] Georg Goubau. Surface waves and their application to transmission lines. *Journal of applied physics*, 21:1119–1128, November 1950.

[6] Anonymous author. A method and apparatus that produces excess alternating electricity in the form of a tunable round wave output for use with electricity requirements. Online `http://www.fluxite.com/`, June 2006.

[7] Charles Proteus Steinmetz. *Theory and calculation of transient electric phenomena and oscillations*, volume 8 of *Steinmetz electrical engineering library*. McGraw-Hill book company. Inc., New York, 3. edition, 1920.

[8] John T. Ratzlaff, editor. *Tesla said*. Tesla book company, Ventura, 1984. ISBN 0-914119-00-1.

Chapter 7

Closing

7.1 Considerations

In the following, I will write about considerations and open questions that I have encountered while researching this treatise.

A man named George Box once said ‘All models are wrong but some models are useful’ just as Maxwell’s equations, quaternionic electrodynamic is just a model, and while it might be useful, it also has limitations.

Every model has advantages and limitations, and in every discipline one of the things which separates the experts from the rest, is a deep knowledge about the advantages and limitations of the model in use. In the following section, we will take a closer look at some of the limitations of the quaternion model.

7.1.1 Faraday’s law

Lets take a closer look at the electromotive force in Maxwell’s system. From integrating

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{7.1}$$

over a surface we get

$$\vec{\epsilon} = \oint_S \vec{E} \cdot d\vec{l} = -\frac{1}{c}\int_S \frac{\partial \vec{B}(t)}{\partial t} \cdot d\vec{s} \tag{7.2}$$

But the law of electromotive force, discovered experimentally by Faraday where

$$\vec{\epsilon} = -\frac{1}{c}\frac{d}{dt}\int_{S(t)} \vec{B}(t) \cdot d\vec{s} \tag{7.3}$$

When we integrate over the surface which changes with time, then we have

$$\frac{d}{dt}\int_{S(t)} \vec{B}(t) \cdot d\vec{s} \neq \int_{S(t)} \frac{\partial \vec{B}(t)}{\partial t} \cdot d\vec{s}. \tag{7.4}$$

So, one limitation of Maxwell’s equations are that we should be very careful when using them on problems where we have paths, surfaces and volumes changing with time.

7.1.2 Galileo invariance

A way to solve some of the problem between Maxwell's equations and Faraday's law was suggested by Heinrich Rudolf Hertz. Hertz suggestion was to replace the partial time derivatives with the total time derivatives:

$$\begin{aligned}
\rho_e &= \vec{\nabla} \cdot \vec{E} && (7.5)\\
\vec{J}_e &= \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{d\vec{E}}{dt} && (7.6)\\
0 &= -\vec{\nabla} \cdot \vec{B} && (7.7)\\
0 &= -\vec{\nabla} \times \vec{E} - \frac{1}{c}\frac{d\vec{B}}{dt} && (7.8)
\end{aligned}$$

By the expansion $\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v} \cdot \vec{\nabla})$ we get

$$\begin{aligned}
\rho_e &= \vec{\nabla} \cdot \vec{E} && (7.9)\\
\vec{J}_e &= \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{1}{c}(\vec{v} \cdot \vec{\nabla})\vec{E} && (7.10)\\
0 &= -\vec{\nabla} \cdot \vec{B} && (7.11)\\
0 &= -\vec{\nabla} \times \vec{E} - -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \frac{1}{c}(\vec{v} \cdot \vec{\nabla})\vec{B} && (7.12)
\end{aligned}$$

This not only solves some of the problems with Faraday's law but also make the equations Galileo invariant.

Hertz interpreted $\vec{v}$ as the absolute velocity of aether elements.

Using Hertz suggestion on the full quaternion model, gives us the following equations:

$$\begin{aligned}
\rho_e &= \vec{\nabla} \cdot \vec{E} - \frac{1}{c}\frac{\partial E}{\partial t} - \frac{1}{c}(\vec{v} \cdot \vec{\nabla})E && (7.13)\\
\vec{J}_e &= \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{1}{c}(\vec{v} \cdot \vec{\nabla})\vec{E} + \vec{\nabla}E && (7.14)\\
\rho_m &= -\vec{\nabla} \cdot \vec{B} + \frac{1}{c}\frac{\partial B}{\partial t} + \frac{1}{c}(\vec{v} \cdot \vec{\nabla})B && (7.15)\\
\vec{J}_m &= -\vec{\nabla} \times \vec{E} - \frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \frac{1}{c}(\vec{v} \cdot \vec{\nabla})\vec{B} + \vec{\nabla}B && (7.16)
\end{aligned}$$

If this equation has any relevance to actual physics, is open for further research.

7.1.3 Speed of light

When deriving Maxwell's equations we assume that the speed of light c for the vacuum, does not change with position $\vec{r}$ and time t. Both Maxwell's equations and the quaternion model can be derived directly from the expression for the retarded potential[2]

$$\mathbf{A}_e(t, \vec{r}) = \frac{1}{4\pi}\int_V \frac{\mathbf{j}_e(t - |\vec{r}_s - \vec{r}|/c, \vec{r}_s)}{|\vec{r}_s - \vec{r}|} d\vec{r}_s \tag{7.17}$$

But if c is not constant over the space between $\vec{r}_s$ and $\vec{r}$, then the expression is not valid and the model might have to be changed.

In 1929 Tesla said[3]:

> The velocity of any sound wave depends on a certain ratio between elasticity and density, and for this ether or universal gas the ratio is 800,000,000,000 times greater than for air.

Tesla held the opinion that electromagnetic radiation were longitudinal (sound) waves through the aether. What is interesting in this connection, is that if you combine Tesla's opinion with the hypothesis that electromagnetic fields lower the density of the medium, then it might be possible to measure changes in the speed of light in regions with strong electric or magnetic fields.

7.1.4 Materials properties

In this treatise the electromagnetic models have been studied without considering material dependent factors like permittivity and permeability. There might be similar factors related to the electric and magnetic scaler fields, if this is so, then determining those factors for all types of materials will be a big task for future researchers.

7.2 Conclusion

In this treatise the primary conclusions are:

- Maxwell's equations can be extended to full quaternion fields.
- These extended equations can be written as two quaternion equations.
- That the quaternion model predicts a longitudinal force between current elements, which might explain the longitudinal force observed in many experiments since the time of Ampère.

While taking into account that independent experimental verification is needed, the secondary conclusions are:

- Experimental research seems to indicate that magnetic monopoles do exist, and that the magnetic elementary charge is close to Dirac's prediction.
- Effects indicating scalar fields have been observed since Tesla's discovery around 1890.

Bibliography

[1] André Waser. On the notation of Maxwell's field equation. AWVerlag, `www.aw-verlag.ch`, June 2000.

[2] Artice M. Davis. A generalized helmholtz theorem for time-varying vector fields. *American journal of physics*, 74(1):72–76, January 2006.

[3] Nikola Tesla. Nikola tesla tells of new radio theories. In John T. Ratzlaff, editor, *Tesla said*, pages 225–226. Tesla book company, Ventura, 1984.

Appendix A

Hamilton letters

The following is letters from W.R. Hamilton to A. De Morgan on the electromagnetic quaternion. Taken from the 1975 reprint of R. P. Graves, *Life of Sir William Rowan Hamilton.*

A.1 Letter to Augustus De Morgan, 25 May 1854

'OBSERVATORY, May 25, 1854.

'Your note has been received, and some things in it shall perhaps be noticed more fully hereafter. Meanwhile, I am in mood to tell you of something new, before my first interest in it shall have evaporated, and my mind turned to other matters. I MUST bring in quaternions (Hamlet *won't* let himself be left out), but shall assume little more than a quaternion

$$Q = \omega + \rho = SQ + VQ = \text{scalar plus vector},$$

where the scalar ω, or SQ, is a positive or negative *number*, while the vector ρ, or VQ, is by me usually constructed as a *directed right line* in tridimensional space. You will also allow me to assume that, in a certain definite sense, the product or quotient of two vectors is generally a quaternion, and will allow me to difference and differentiate.

'Well, I sought this morning to translate into my own notation the law of attractive or repulsive action (say f), of one element ds of a current on another element ds' of the same or another current, which was discovered by Ampère, and by him under the form (*Théorie des Phénomènes Électrodynamiques*, p. 217, &c.).

$$(f =) - \frac{2ii'dd'\sqrt{r}}{\sqrt{r}}, \text{ or } - 2ii'r^{-\frac{1}{2}}dd'.r^{\frac{1}{2}}; \tag{A.1}$$

the respective intensities of the elements being i, i', and their rectilinear distance r.

'I took ρ, ρ' for the *vectors* of the beginnings of the elements (drawn from an arbitrary origin), and therefore naturally denoted the directed elements themselves by $d\rho$, $d\rho'$; while $\Delta\rho = \rho' - \rho$ expressed the directed line from first to second; so that, on my principles,

$$d\rho^2 = -ds^2, d\rho'^2 = -ds'^2, \Delta\rho^2 = -r^2.$$

I was to pick out the part involving $d\rho$ and $d\rho'$ each in the 1st dimension, from the development of $\{-(\Delta\rho + d\Delta\rho)^2\}^{\frac{1}{4}}$; and then (treating the intensities of the currents as each =1) was to multiply the part so found by $-2(-\Delta\rho^2)^{-\frac{1}{4}}$, in order to get the required translation of Ampère's law. Or I was to select, by the condition mentioned, the proper part of the expansion of

$$-2\left(1 + 2S\frac{d\Delta\rho}{\Delta\rho} + \frac{d\Delta\rho^2}{\Delta\rho^2}\right)^{+\frac{1}{4}};$$

or the proper portion of this part thereof,

$$-\frac{d\Delta\rho^2}{2\Delta\rho^2} + \frac{3}{4}\left(S\frac{d\Delta\rho}{\Delta\rho}\right)^2;$$

or of

$$-\frac{(d\rho' - d\rho)^2}{2\Delta\rho^2} + \frac{3}{4}\left(S\frac{d\rho'}{\Delta\rho} - S\frac{d\rho}{\Delta\rho}\right)^2.$$

This part, or selection portion, is

$$f = \frac{S.d\rho d\rho'}{\Delta\rho^2} - \frac{3}{2}S\frac{d\rho}{\Delta\rho}S\frac{d\rho'}{\Delta\rho}; \tag{A.2}$$

which accordingly I found to agree, through spherical trigonometry, with a perhaps better known formula of Ampère (quoted by De la Rive, and investigated by Murphy, &c.),

$$r^{-2}dsds'(\sin\theta\sin\theta'\cos\omega - \frac{1}{2}\cos\theta\cos\theta'),$$

when θ, θ' are the angles made by the elements with their connecting line r, and w is the dihedral angle with that line for edge. So far, all is mere *practice* in my *calculus*; and you may say the same of these transformations of the expression (A.2),

$$f = -\frac{1}{2}S\left(\frac{d\rho}{\Delta\rho}\frac{d\rho'}{\Delta\rho} + V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}\right), \tag{A.3}$$

$$f = -\frac{1}{2}S\left(\frac{d\rho d\rho'}{\Delta\rho^2} + 3V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}\right), \tag{A.4}$$

But you must know that I have been for more than ten years haunted with visions, or amused by notions, of some future application of the *Calculus of Quaternions* to *Nature*, as furnishing a *Calculus of Polarities*. (See my printed letter of Oct. 17th, 1843, to J. T. G., *Philosophical Magazine*, Supplement to December number for 1844, p.490). More definitely, I have often stated, to Lloyd and others in conversation, my expectation that it would be found possible to *express two connected but diverse physical laws* by means of *one common quaternion*; and Faraday may possibly remember my chat with him at Cambridge, in 1845, upon the subject of the analogy of the products of ijk, to the laws of electrical currents ($ij = +k$, $ji = -k$, corresponding to phenomena of electricity (see British Association *Report*, for 1845).

'Knowing this little bit of my own history, you will not be surprised that I sought to realize my old expectation to-day, after having translated Ampère's law into the notations of the Calculus of Quaternions. Having expressed his *attractive force* between two elements, as the *scalar part* of a quaternion,

$$f = SQ, \tag{A.5}$$

where Q may have either of the two forms given by (A.3) and (A.4) – and indeed others also, which I thought less proper for my purpose – my old conjecture led me to surmise that there might be *some directive force*, ϕ, perhaps an axis of a couple, perhaps a magnetic element, or what else deponent sayeth not, which should be expressed as the *vector part* of the *same quaternion*, Q; in such a way that.

$$\phi = VQ. \tag{A.6}$$

The form which (A.3) would give for Q was at first tried; but I found that it had, what appeared to me to be inconsistent with the law of action and re-action, the property of *not* charging to its own *conjugate* quaternion, KQ, when ρ and ρ' were interchanged. From this fault the quaternion suggested by the formula (A.4) is free; and accordingly I assumed

$$Q = -\frac{1}{2}\left(\frac{d\rho d\rho'}{\Delta\rho^2} + 3V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}\right); \tag{A.7}$$

and proceeded to try, as a mathematical experiment, whether my old conjecture, expressed by the recent equation (A.6), might not lead me to the re-discovery of some known law of nature, or at least to some result identifiable with such a law.

'I soon found, by combining (A.6) and (A.7), according to the rules of my calculus, the expression

$$\phi = \frac{1}{2}(3\nu S.\nu\lambda\lambda' - \nu^2 V.\lambda\lambda'); \tag{A.8}$$

after having written, for conciseness,

$$\lambda = d\rho, \lambda' = d\rho', \nu = \Delta\rho^{-1}, \tag{A.9}$$

making also

$$V.\lambda\lambda' = \mu, \tag{A.10}$$

we have this somewhat shortened expression,

$$\phi = \frac{1}{2}(3\nu S.\nu\mu - \nu^2\mu); \tag{A.11}$$

or (by the rules of this calculus),

$$\phi = \nu(S + \frac{1}{2}V).\mu\nu. \tag{A.12}$$

Up to this stage, I assure you that no recollection of anything about terrestrial magnetism (respecting which I know very little) had in *any* degree consciously influenced my transformations. So mush had those things been out of my head that I was obliged to ask my son Archy whether in fact the Dip was greater than the Latitude, or the Latitude than the Dip. But Having seen that equation (A.12) gave

$$\frac{V}{S}.\nu^{-1}\phi = \frac{1}{2}\frac{V}{S}.\mu\nu, \tag{A.13}$$

I perceived that (by The principles of my *Lectures*)

$$\tan\mu\nu = 2\tan\nu\phi; \tag{A.14}$$

and this not only *suggested* things about dip, &c., though very vaguely remembered, but led me to see that, for a first approximation, I was to assimilate the joining line $\nu^{-1} = \Delta\rho$, to the *vertical;* and to compare the line μ, perpendicular to the elements of the current, to the *magnetic axis* of the earth; and finally to consider my conjectured *line of directive force*, ϕ, as being physical represented by the *dipping needle*. At least it is so in *direction*, even when *plane* is taken into account; but I thought it *very likely* (such was my faith in the quaternions), that the formula (A.12) for ϕ would be found to represent also the *intensity* of terrestrial magnetism; to the same order of approximation, I mean, as the formula,

$$\tan(\text{dip}) = 2\tan(\text{mag. lat.}).$$

By Taking the *tensor* of ϕ, using (A.11) rather than (A.12), I inferred this magnetic *intensity*[1] to be proportional to

$$\sqrt{1 + 3\sin l^2};$$

because the formula (A.11) gives (on my hypothesis, *intensity* =)

$$T\phi = \frac{1}{2}T\nu^2 T\mu\{1 + 3(SU.\nu\mu)^2\}^{\frac{1}{2}}. \tag{A.15}$$

(I use here the symbols T and U to which I am accustomed).

' (Recapitulation. – $f = SQ$ = Ampère's *attractive force* between the elements $d\rho$, $d\rho'$, of a current, with intensities each = 1, and separated by the interval $\Delta\rho$; $\phi = VQ = \nu(S + \frac{1}{2}V)\mu\nu$, when $\nu = \Delta\rho^{-1}$, $\mu = V.d\rho d\rho'$; and $\phi =$ a (new?) sort of *directive force*, which seems to be nearly represented by the *dipping needle*, if ν be treated as the vertical at the place, and ν as the magnetic axis of the earth).

'When I had got so far I conjured my son Archy, after his returning from our parish church – (I am ashamed to say that I had got *caught* in this investigation early, and did not observe the hours, till it was too late for me to shave and walk) – to hunt out all the books in this house bearing any relation to terrestrial magnetism, and to search among them whether any law of intensity, of the form

$$\text{const} \times \sqrt{1 + 3(\text{sin.mag.lat})^2}, \tag{A.16}$$

was recognized. He had to perform duties of hospitality to a party from this neighborhood, and could not immediately find any information for me of the

[1]It was *partly* through a *foresight* of some *such* use as this that I ventured to introduce the word "Tensor," in my writings.

kind required. I went roaming about the house; hunted out Biot's *Physique*, 3rd vol.; Turner's *Chemistry;* Kane's; a book of Hassenfratz; and several others, without being able to find anything bearing on the point. At last I lit on Christie's *Report on the Magnetism of the Earth*, embodied in the Report of the B. A. for 1833, in which, at page 121, I found the formulœ,

$$\tan\delta = 2\tan\lambda,$$

$$I = \frac{m}{2}\sqrt{3\sin^2\lambda + 1};$$

deduced, it is true, from "the hypothesis of two magnetic poles not far removed from the centre of the earth," and very far inferior in accuracy to what is given by the Gaussian constants; but yet having a encouraging analogy to my own theoretical results, especially as my investigation (it must always be remembered) relates only to *two linear elements*.

'On the whole, I think that it is not without some just cause that I propose to call the quaternion

$$Q = -\frac{1}{2}(\frac{d\rho d\rho'}{\Delta\rho^2} + 3V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}),$$

the "Electro-Magnetic Quaternion," of, or resulting from, the *two linear elements*, $d\rho$ and $d\rho'$, separated by the interval $\Delta\rho$.

'You see that the notion of "current" is quite eliminated here, though the mere *word* has been retained. The $d\rho$ and $d\rho'$ may be *directed tensions* or elementary *axes of rotation*, or anything else which answers to a *directed and linear element* in space. I must confess that I am strongly tempted to believe, that a *differential action*, represented by the vector ϕ of my quaternion Q, exists in nature; but if experiments shall overthrow this opinion, it will still have been proven that my Calculus furnishes an *organ of* expression, adapted to very complex phenomena."

A.2 Letter to Augustus De Morgan, 27 May 1854

'Observatory, May 27, 1854.

'Though I let a letter go off this morning, after causing a copy to be kept, which letter had been written two days ago, I feel much less sanguine than when writing it, about its having any *physical value at present;* even if, with modifications to be indicated by facts, it shall ever come to have any.

'The *scalar*, SQ, of the quaternion

$$Q = -\frac{1}{2}\left(\frac{d\rho d\rho'}{\Delta\rho^2} + 3V\frac{d\rho}{\Delta\rho}V\frac{d\rho'}{\Delta\rho}\right), \tag{A.17}$$

undoubtedly represent Ampère's law of the attractive or repulsive action between two linear and directed distance $\Delta\rho$. That is a mere mathematical fact of calculation. But you know that *any vector*, whatever, suppose κ, may be added to a quaternion Q, without changing its scalar part:

$$S(Q+\kappa) = SQ, \text{if } S\kappa = 0. \tag{A.18}$$

My choice of the form (A.17) must therefore be admitted to be eminently conjectural, *even if* the principle to which I still cling, that *some* quaternion form Q, of a simple kind, exists, which by its scalar part SQ expresses *one* mode of physical action, and by its vector part VQ expresses *another* connected and connate mode of force or influence in nature.

'On the whole, without having perceived any mathematical error in what I lately wrote, I *withdraw* the epithet "Electro-Magnetic," as assuming too mush, on the *physical* side, for my recent quaternion Q. But though I cannot hope that your own avocations, and researches, have allowed you as yet to catch much more than the *spirit* of my calculus (what a splendid quaternionist you would become, if you ever really set about it!), I think that this one short expression of mine for Q contains at once, by its scalar part the Ampèrian function

$$f = r^{-2} ds ds' (\sin\theta \sin\theta' \cos\omega - \frac{1}{2} \cos\theta \cos\theta'),$$

and by its vector part the three other laws: – 1st, of a certain resultant axis ϕ being *in the same plane* with the connecting line $\Delta\rho$ and the common perpendicular to the elements $d\rho$, $d\rho'$; 2ed, of the *tangent* of a certain *dip* (δ) being *twice* the tangent of a certain *latitude* (λ); and 3rd, of the *intensity* varying as $\sqrt{1 + 3\sin\lambda^2}$ when the elements are otherwise given.

'The analogy to magnetism is perhaps very vague – indeed I suspect it to be so; but if *one* small quaternion can *mean so much*, may not something be hoped from some future shake of Lord Burleigh's head? (*Vide* Sheridan's *Critic*).

'I have, however, the common sense (sometimes) to admit that no ingenuity of speculation can dispense with an appeal to *facts*. And what I am at present extremely curios to know is whether facts are decidedly against what seems to me a natural supposition, that *two rectangular and rectilinear (and non-intersecting) conducting wires*, though *not attracting nor repelling each other*, may have some *tendency to assume parallel positions:*[2] or may be the occasion of *some other* DIRECTIVE FORCE arising, *before attraction* or repulsion begins. Oersted's discovery (I have met Oersted, and he has given me some very pretty and rather poetical German papers) seems to be *almost* a proof that this is so; but my wish is to eliminate, if possible, *magnetism*, at first, as an eminently *complex* phenomenon.'

'OBSERVATORY, June 1, 1854.

'P.S. – This morning's post brings me a letter from Lloyd, influenced no doubt by our old friendship, but containing a far greater degree of encouragement than I had expected to receive, as to what I had called the "Electro-Magnetic Quaternion."

'Hope to write soon on something else.'

[2] I have since seen this rotatory tendency *does* follow from Ampère's *attraction*, though not for the elements at the very extremities of the common perpendicular; and on expressing and integrating the moments of rotation have got results which I have the satisfaction of finding to be coincident with his (see especially his page 86). In short, this little speculation, which as *such* I abandon for the present, has led me to learn more of precise nature of his beautiful theory, in what may be called a *few hours*, than I had done in my *life* before.

Appendix B

Maxwell letters

The following is letters from J. C. Maxwell. Taken from *The scientific letters and papers of James Clerk Maxwell* edited by P. M. Harman.

B.1 Letter to Peter Guthrie Tait, 28 August 1879

[Glenlair]

Headstone in search of a new sensation

While meditating as is my wont on a Saturday afternoon on the enjoyments and employments which might serve to occupy one or two of the aeonian ætherial phases of existence to which I am looking forward, I began to be painfully conscious of the essentially finite variety of sensations which can be elicited by the combined action of a finite number of nerves, whether these nerves are of protoplasmic or eschatoplasmic structure. When all the changes have been rung in the triple bob major of experience, must the same chime be repeated with intolerable iteration through the dreary eternities of paradoxical existence?

The horror of a somewhat similar consideration had as I well know driven the late J. S. Mill to the very verge of despair till he discovered a remedy for his woes in the perusal of Wordsworths Poems.

But it was not to Wordsworth that my mind now turned, but to the noble Viscount the founder of the inductive philosophy and to the Roman city whence he was proud to draw his title, consecrated as it is to the memory of the Protomartyr of Britain.

Might not I, too, under the invocation of the holy ALBAN become inspired with some germinating idea, some age-making notion by which I might burst the shell of circumstance and hatch myself something for which we have not even a name, freed for ever from the sickening round of possible activities and exulting in life every action of which would be a practical refutation of the arithmetic of this present world.

Hastily turning the page on which I had recorded these meditations, I noticed just opposite the name of the saint another name which I did not recollect having written. Here it is - ИAꓭ⅃A.

Here then was the indication, impressed by the saint himself, of the way out of all my troubles. But what could the symbol mean? I had heard that the harp from which Heman or Ethan drew those modulations from the plaintive to the triumphant which modern musée with its fetters of tonality may ignore but can never equal - I had heard that this harp be found, nor yet the lordly music which has not been able to come down through the illimitable years.

Here I was interrupted by a visitor from Dresden who had come all the way with his Erkenntniß Theorie under his arm showing that space must have 3 dimensions and that theres not a villain living in all Denmark but he's an arrant knave. Peruse his last epistle and see whether he could be transformed from a blower of his own trumpet into a Nabladist.

I have been so seedy that I could not read anything however profound without going to sleep over it.

$\frac{dp}{dt}$