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Taxonomy of Universal and Other Classes

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1. The problem. I was attracted to mathematics by its generality, its ability to give information where apparently total chaos prevails, rather than by its ability to give much concrete and exact information where we a priori know a great deal. So, not surprisingly, the following represents a theme which has been central in my mathematical interests since starting my thesis. (We give "universal classes" as an example, as the definition is "logic-free" (see Definition).)

1.1. *The first problem: The taxonomy = classification problem for universal classes.* Find the main dividing lines in the family of universal classes; each line is significant in the sense that all classes in one side "enjoy" common properties witnessing "simplicity," "analyzability," and those of the other side have common properties witnessing complications, unanalyzability; we define

1.2. *Universal classes.* (1) Examples are the class of groups, the class of rings, any variety, and the class of locally finite groups.

(2) Generally, let τ denote a vocabulary = set of function symbols and predicates (= relation symbols) each with an assigned arity, $(n(F), n(R))$; a τ-structure M is a nonempty set $|M|$ (its universe), and interpretations of any function symbol $F \in \tau$ and relation symbol $R \in \tau$ is an $n(F)$-place function from $|M|$ to $|M|$, and $n(R)$-place relation on $|M|$, respectively.

(3) A universal class K is a class of τ-structures for some $\tau = \tau(K)$ such that $M \in K$ *iff* every finitely generated substructure of M belongs to K.

I also love, in mathematics, that there is no argument (at least usually) about whether or not one solves a problem. It suffices to find the correct solution; being untalented in convincing people is no serious hindrance. Hence I like problems that are precise, preferably with a yes/no answer, and I believe that usually the way to treat more elusive problems is by choosing the right test question. So we shall specify our problem below.

1.3. *The second problem: The structure/nonstructure problem.* (1) Describe for some (e.g., universal classes) K a structure theory (see below) and prove for the other classes (in the family) nonstructure theorems; that is, demonstrate

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the impossibility of a structure theory, construction of many and/or complicated structures in K.

(2) A *structure theory* for K is a theory that gives, for each $M \in K$, a complete set of invariants; i.e., each invariant should depend only on the isomorphism type of M, and if M_1, M_2 have the same invariants, then they are isomorphic.

Of course, we do not want the invariants to be too complicated (e.g., the isomorphism type of M) (and, preferably, derivation of the invariant from the structure and vice versa are explicit constructions). We shall not deal with this, but it would not change the theory much. See [**Sh86**] for more.

What objects should we use as invariants for structures of cardinality λ? In the prototype of structure theorems, the celebrated Steinitz theorem, for algebraically closed fields of a fixed characteristic, a cardinality (= transcendence dimension) is used. Certainly if K_i has a structure theory for $i \in I$, then so does $\sum_{i\in I} K_i \stackrel{\mathrm{df}}{=} \{\sum_{i\in I} M_i : M_i \in K_i \text{ for } i \in I\}$ (with any reasonable definition of $\sum_i M_i$, such that $\sum_{i\in I} K_i$ is a class of the right kind). Also, if K has a structure theory so does $\sum K \stackrel{\mathrm{df}}{=} \{\sum_{j\in J} M_j$: for some set J, with $M_j \in K\}$. So we have to admit λ-values of kind (α, χ) as invariant for structures of cardinality λ, where

1.4. *Definition.* (1) A λ-value of kind (α, χ) (λ, χ cardinals, α an ordinal) is defined by induction on α:

$\alpha = 0$. A λ-value of kind (α, χ) is a cardinal $\leq \lambda$.

$\alpha = \beta + 1$. A λ-value of kind (α, χ) is a λ-value of kind (β, χ) *or* sequence of length χ, each entry a function from $\{x : x$ a λ-value of kind $(\beta, \chi)\}$ into the set of cardinals $\leq \lambda$.

α limit. A λ-value of kind (α, χ) is a λ-value of kind (β, χ) for some $\beta < \alpha$.

(2) We say an invariant of kind $(\beta, 1)$ is of depth β.

(3) An invariant of kind (α, χ), for K, is a function which gives, for each $M \in K$ of cardinality λ, a λ-value of kind (χ, α), and which depends on M only up to isomorphism.

Now if we do not bound α, even for $\chi = 1$, any structure of cardinality λ can be coded up to isomorphism; also note that for any χ, α there is a β such that every λ-value of kind (χ, α) can be coded naturally by a λ-value of depth β (i.e., kind $(\beta, 1)$). So we are led to

1.5. *First thesis*: A class K has a structure theory iff for some β there are invariants of depth β for K which determine each $M \in K$ up to isomorphism.

I think that the part of the thesis that says that such invariants give a structure theory is very strong. First of all

1.6. *Claim.* If a class K has a structure theory according to Thesis 1.5, by an invariant of kind (α, χ), then for every cardinal $\aleph_\gamma$, $I(\aleph_\gamma, K) \leq \beth_\alpha(|\gamma| + \chi)$ (see below), so if the Generalized Continuum Hypothesis (= G.C.H.) holds, then $\beth(\aleph_\gamma, K) \leq (\chi + |\gamma|)^{+\beta}$ where

1.7. *Definition.* (1) $|\gamma|$ is the cardinality of the ordinal γ.

(2) $I(\aleph_\gamma, K)$ is the number of structures from K of cardinality $\aleph_\gamma$, up to isomorphism.

(3) The G.C.H. says that $2^{\aleph_\gamma} = \aleph_{\gamma+1}$ for every γ.

(4) $(\aleph_\alpha)^{+\beta} = \aleph_{\alpha+\beta}$, $\beth_\alpha(\lambda)$ is defined by induction on α as $\aleph_0 + \sum_{\beta<\alpha} 2^{\beth_\beta(\lambda)}$. (Note that $\beth_\alpha(\chi + |\gamma|)$ may be $> 2^{\aleph_\gamma}$ even for $\alpha = 2$, $\chi = 1$.)

Because knowing the number of members of K in one cardinality is a natural and important problem, $I(\lambda, K)$ is an important function; if its values are small this signifies that the class K is simple. For me, the really important thing is Thesis 1.8 below (and not the detailed computation as $I(\lambda, K)$).

1.8. *Second thesis*: *The (first) main dividing line = the main gap.* For a "nice" family of classes (like the family of universal classes), the dividing line "is there α such that for every $\gamma, I(\aleph_\gamma, K) \leq \beth_\alpha(|\gamma|)$" is a good one, i.e.,

(a) It coincides with "having a structure theory according to 1.5";

(b) Every class in the "complicated side" has strong evidence for nonstructure: a jump in the lower bound for $I(\lambda, K)$—i.e., among the possible functions $I(\lambda, K)$ there is a large gap:

(1) $I(\lambda, K) = 2^\lambda$ for large enough λ which is the maximum value when $\lambda \geq |\tau(K)|$, or at least

(2) $I(\lambda, K) > \lambda$ for large enough class of cardinals λ. We much prefer that the nonstructure proof be carried out in ZFC alone, but note that in order to show that we cannot prove a structure theorem, the consistency of nonstructure is enough.

(c) We believe that if we succeed in solving (b), we will have developed extended taxonomy having many tools to deal with, and we will know much on each side of those dividing lines; i.e., we suggest 1.8(b) as the test question.

Note that we do not claim that only dividing lines are interesting: the class of rings is a very interesting subclass of the class of structures with two-place functions, although its complement is not interesting at all. On the other hand, dividing lines, in addition to having intrinsic interest, help in proving theorems by cases.

Implicit is

1.9. *Thesis.* (d) In understanding a class you should look at a large enough cardinality in order to iron out singularities.

Note that, e.g., theories having unique countable model can have many complicated uncountable ones. Note that a priori the answer to the following question is not clear.

1.10. *Question.* Is there a reasonably general family of classes for which Thesis 1.8 can be confirmed?

We shall return to this question later.

2. Background. Why do we speak on universal classes? Now in model theory the primary family of classes is the family $\{\mathrm{Mod}(T): T$ a countable first-order theory$\}$, where

2.1. *Definition.* $\mathrm{Mod}(T)$ is the class of models of T (and we write T instead of $\mathrm{Mod}(T)$).

Of course, more complicated classes—uncountable theories, theories in infinitary logics or ones with generalized quantifiers on the one hand, and universal theories and even varieties (= equational theories)—are also interesting. I think this approach is right, but here there is no need to justify it.

Let us return to what is for me prehistory (you can ignore notions unknown to you). Generally, around 1960, research in mathematical logic became deeper and more complicated mathematically. At that time, the aim of much of the research in model theory was advancement toward the solution of the Los Conjecture, which was as follows.

LOS CONJECTURE. *If T is (first-order) countable, then if T is categorical in λ for some $\lambda > \aleph_0$ (i.e., $I(\lambda, T) = 1$), then this holds for every $\lambda > \aleph_0$.*

Certainly, in wanting to know something about $I(\lambda, T)$, this was a very good problem to start with: it was foolish to consider far-reaching conjectures like 1.8(b), considering the knowledge available. Those investigations culminated in the positive solution of the Los Conjecture by Morley in [**Mo**], which used many of the tools developed previously—in particular, the works of Vaught and Ehrenfeucht-Moslowski. Morley's theorem is considered by many (including myself) to be one of the main achievements of mathematical logic during the sixties.

For quite some time, little happened. This certainly has its reasons—among them, that Morley and Keisler, at least, thought that the (model) theory of first-order theories was finished, or essentially finished (they told me so in 1969). However, since then the field has increased in popularity in model theory, as witnessed by the research book [**Sh78**], and numerous articles, as well as the (largely) expository books Pillay [**Pi**], Lascar [**La**], Poizat [**Po**], and Baldwin [**Ba**], and lately some conferences dedicated to it. On early history see [**Sh74**]; this article overlaps with [**Sh85**] (which speaks on countable first-order T) and Baldwin, introduction to [**Ba1**] (which deals generally with the theory).

The papers of most researchers in the field, however, reflect a very different outlook—a "fine structure" one—wanting to know much (or everything) about what we already have considerable knowledge about or investigating families of classes which have some structure theory and/or tendency to be (relatively) more concrete. Illustrious examples are the Baldwin-Lachlan theorem (on first-order countable T, categorical in $\aleph_1$ but not $\aleph_0$) and the works of Zilber and Cherlin, Harrington and Lachlan (on T categorical in $\aleph_0$, $\aleph_1$, or totally transcendental T categorical on $\aleph_0$). I hope Lachlan and Peretyatkin, in this volume, will do justice to some of this.

Note that there is no real conflict: solutions of Problem 1.1 give, and are intended to give, instances for fine structure investigation with considerable tools to start with. Here I shall continue to present my personal outlook. When I started in 1967, I was interested in Problem 1.1, introducing the stability and superstability as dividing lines, and in [**Sh71**, p. 283, (13)] (also in [**Sh74**]) I conjectured what the functions $I(\lambda, T)$ for λ large enough, T countable first-order, should be like; in particular, the "main gap" of 1.7(b) was of interest.

The first class for which this was confirmed (with a minor correction) was for the family of $\{M: M$ an $\aleph_\varepsilon$-saturated model of $T\}$: T first-order (announced in [**Sh74**], see [**Sh83**; **Sh83a**; **Sh77**, Chapter X, Example 3.3]). but I felt this was cheating, as I invented $\aleph_\varepsilon$-saturativity. The second one was for universal (first-order) theories T (see [**Sh86**]); i.e., the set of models of T, T consisting of formulas of the form $\forall x_1, \dots, x_n \ \bigvee_i \bigwedge_j \phi_{i,j}$, $\phi_{i,j}$ atomic or negation of atomic. We will give more details on this result.

2.2. THEOREM. *For every universal (first-order) T, exactly one of the following occurs:*

(A) *For every $\lambda > |T|$, $I(\lambda, T) = 2^\lambda$, and there are other signs of complicatedness (see* [**Sh85**]*).*

(B) *Still $I(\lambda, T) = 2^\lambda$ for every $\lambda \geq |T| + \aleph_1$, but*

(*) *for every model M of T of cardinality λ, there is $\langle M_\eta : \eta \in I\rangle$ such that:*

(i) *I is a set of finite sequences of ordinals $< \lambda$, nonempty, closed under initial segments.*

(ii) *M_η is a submodel of M of cardinality $\leq |T|$, and if ν is an inital segment of η then M_ν is a submodel of M_λ.*

(iii) *M is freely generated by $\bigcup_{\eta \in I} M_\eta$, which means:*

- (α) *the closure of $\bigcup_{\eta \in I} |M_\eta|$ (union of universes) under the functions of M is $|M|$;*
- (β) *if $\eta \in I$ has length $n+1$, $\nu = \eta \restriction n$, then for every finite sequence $\bar{c}$ from M_η : $\oplus$ for every finite set Φ of quantifier-free formulas with parameters from $\bigcup\{|M_\rho| : \rho \in I$, η not an initial segment of $\rho\}$ which $\bar{c}$ satisfies, there is a finite sequence $\bar{c}'$ from M_ν satisfying all formulas in Φ.*

(C) *The condition (*) from (B) holds, moreover, for some ordinal* $\mathrm{Dp}(T)$, *which is countable if T is countable, and of cardinality $\leq 2^{|T|}$ generally; for every M there is $\langle M_\eta : \eta \in I\rangle$ as in (*) with I of depth $\langle \mathrm{Dp}(T)$: i.e., there is a function $d: I \to \alpha$ such that if ν is a proper initial segment of η then $d(\nu) > d(\eta)$.*

So for α large enough, $I(\aleph_\alpha, T) \leq I_{\mathrm{Dp}(T)}(|\alpha|)$ (in fact, we get very detailed information on $I(\lambda, T)$).

Note that the first dividing line is between (A)+(B) and (C). But a second dividing line between (A) and (B)+(C) is worthwhile; see [**Sh85**] on this, but we shall not deal with it here.

This looks to be a reasonable answer to Question 1.10—a general family where we have a solution; but, having a model theoretic background, I was not satisfied until the solution for countable first-order T (see [**Sh85**, **Sh87**]). Though I thought 1.8 was the point, I felt it was my duty not to avoid the relevant problem which was the legacy of the previous generation—the Morley Conjecture. Having thus answered Question 1.10 fully, we have

2.3. *Question.* Is the theory a theory on first-order classes, or is there really a collection of such theories which may even need a general framework?

There were some advances, some changing of the family of classes, some changing of the questions. In Baldwin-Shelah [**BaSh8**] (see also Shelah [**Sh86a, Sh85a**]), the complexity of class $K = \text{Mod}(T)$, T first-order, in the monadic logic (finitary or infinitary) was investigated (relying on [**Sh78**]).

Quite complete classification (in the relevant sense) was obtained. The results explain why the quite large body of works on monadic logic (e.g., works of Buchi, Rabin, Shelah, and Gurevich (see [**Gu**])) concentrate on linear orders and trees, and discover some neglected cases; also, more general quantifiers were dealt with. This also indicates that the classification in [**Sh78**] is relevant to a reasonably wide spectrum of problems not thought of in the first place. On classifying all quantifiers see [**Sh86x**].

We may work with classes K of two sorted structures and ask how much a $M \in K$ can be described up to isomorphism over the first sort. The prototype of such problems is the structure of a vector space V over a field F, letting both vary; so one cardinal invariant, the dimension, suffices.

So nonstructure will mean that for each (or at least arbitrarily large) cardinal λ, there is a structure M_0 so that for many and complicated $N \in K$, N restricted to the first sort is M_0. It seems that for $K = \text{Mod}(T)$, T countable first-order, there is a complete answer *provided that* we accept indepenent results for the nonstructure side. We say "it seems," as some parts are not worked out, others need considerable expansion; see [**PiSh8**], [**Sh86**], and the notes [**Sh85b**]. However, that is enough to show that there is such a theory.

The situation is similar for universal classes [**Sh87a**].

Grossberg and Hart [**GH**] have been proving the main gap, etc., for the family of excellent classes. Excellent classes were introduced in [**Sh83c**] (in the following context: if $\psi \in L_{\omega_1,\omega}$ has an uncountable model and, for no $n > 0$, has many nonisomorphic models (essentially $2^{\aleph_n}$), then $\text{Mod}(\psi)$ is the union of few excellent classes).

3. Outside interactions. Of course, the theory answered almost all relevant problems of the model theorist of the sixties and some which do not a priori look connected (like investigating Keisler order on first-order theories).

Note that the conclusion applies also to universal algebra. In particular, Theorem 2.2 gives the possible function $I(\lambda, K)$ for K a variety, except that we do not know if all values of the parameters really appear (mainly whether the depth of the theory can be infinite; also there are some problems for small cardinals). It seemed that though they have interest in the problem, universal algebraists have not learned the material we mention.

We have been interested in this theory for its own sake; and applications were sought in order to convince the "heathen." However, we sincerely believe that it should help in investigating specific classes.

Note, however, that there is an asymmetry between the structure and nonstructure side. You can deal with a structure theory for a specific class without having any formal definition of what a structure theory is. But we need one (or some alternatives) for showing that no structure theory exists.

Note that having a theory as we desire makes it natural and profitable to investigate for specific classes where they are on this classification. This line of research starts with Macintyre [**Mc**], dealing with first-order theories of fields (via [**Mo**]). For field and division rings this was continued in Cherlin [**Ch**] and Cherlin and Shelah [**ChSh**], thus giving the following

Conclusion. If T is a first-order theory, Mod(T) a class of infinite fields (or division rings), not all algebraically closed fields, then T is not superstable; hence Mod(T) has many and complicated models and modules.

There is extensive literature on first-order theories of rings, modules and groups. A very successful case is the theory T^p_{dcf} of differentially closed fields. Robinson, relying on Seidenberg [**Se**], proved that T^0_{dcf} is first-order. Blum [**B**] gave concrete axioms for it, and proved it totally transcendental. She deduced (relying on Morley's work) that over every differential field F of characteristic 0, there is a prime differentially closed field over F (extending F) (i.e., one embeddable in every differentially closed field extending F). We deduce, by a theorem in classification theory (see [**Sh78**, Chapter IV, §4]), the uniqueness of the prime differentially closed field over F. Wood [**W**], relying again on algebraic work, proved T^p_{dcf} is first-order, but not totally transcendental. Independently, Shelah [**Sh**] and Wood [**W1**] proved the existence of prime differentially closed fields over any differential field. Shelah [**Sh73**] proved that the theory is stable; so by [**Sh78**, Chapter IV, §5], the prime differentially closed field above is unique. But by [**Sh73**], T^p_{dcf} ($p > 0$) is not superstable, hence there is no structure theory for Mod(T^p_{dcf}).

The existing theory on Mod(ψ), ψ a sentence in infinitary logic ($L_{\omega_1,\omega}$), is used in Mekler and Shelah [**MSh**] to prove (when $V = L$) that for every variety either $L_{\infty,\omega}$-freeness implies freeness *or* there are λ-free not free ones for every λ (continuing work of Eklof and Mekler [**EM**]).

In Grossberg and Shelah [**GSh**] a problem of Fuchs and Salce (see [**FS**]) on the possibility of a structure theory of torsion divisible modules over a uniserial ring is answered (using a general theorem proved there). This theorem can also be used to deduce directly an older result from [**Sh74a**] (solving a problem of Fuchs [**F**]) that there are many complicated separable reduced abelian p-groups in every $\lambda > \aleph_0$.

Quite naturally, in many cases the theorems have not applied directly; rather, the proofs or the method apply. We have tried to adapt the theorems to general use in [**Sh83b**], the applications there being constructions of Boolean algebras which are complicated in various ways (e.g., have no automorphisms or one-to-one endomorphisms, are complete and/or satisfy the CCC).

Another attempt to adapt the theorems for applications is [**Sh85c**], which has been of use in several instances in representing rings as endomorphism groups of abelian groups, in works of Corner, Gobel, and the author.

We generally believe that the method should be useful in constructing structures in specific classes which are "complicated," e.g., have no "nontrivial" automorphism or endomorphism or are indecomposable, etc.

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