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Some Recent Developments in Complex Differential Geometry

We survey here some recent results in the theory of complex manifolds obtained by the methods of global differential geometry. These results stemmed from attempts to generalize to the higher-dimensional case the uniformization theorem for Riemann surfaces. A simply connected Riemann surface must be either the Euclidean plane, the Riemann sphere, or the open unit disc in the Euclidean plane. In the higher-dimensional case there is no such simple classification of simply connected complex manifolds, even with the imposition of very strong curvature conditions. A slight perturbation of the open unit ball in C^n for $n > 1$ will in general give us a new complex structure (not biholomorphic to the ball) [9], where the Bergman metric is still complete and has sectional curvature very close to that of the ball [16]. We will discuss the various results in higher dimensions for the parabolic, elliptic, and hyperbolic cases, generalizing those for Riemann surfaces. At the end we mention some recent results on K3 surfaces.

§ 1. Parabolic case

The problem concerns the characterization of C^n as a Kähler manifold with sufficiently fast decay of curvature. The first result of this kind was obtained by Siu and Yau [55] in response to a question posed by Greene and Wu [18]: A simply connected complete Kähler manifold of complex dimension $n \geqslant 2$ with $0 \geqslant$ sectional curvature $\geqslant -A/(1+r^{2+\varepsilon})$ (where A, $\varepsilon > 0$ and r is the distance from a fixed point) is biholomorphic to C^n. The method is to use the L^2 estimates of $\bar\partial$ to produce n holomorphic functions of nearly linear growth which are local coordinates at one point. The map from the manifold to C^n defined by these n functions is a local biholomorphism, because the growth of the Jacobian n-form is related to the growth of the volume of its zero-set. Moreover, the map is proper, because the manifold is Stein and the power series obtained by expanding any global holomorphic function in terms of these n functions at any point converges uniformly on any compact subset of the manifold. The curvature condition is used to get suitable weight functions for the L^2 estimates and to get sup norm estimates for holomorphic functions and n-forms with weighted L^2 bounds. Later Greene and Wu [19] made some quantitative improvements on the assumption of the decay order of the curvature.

Recently Mok, Siu and Yau [36] showed that under the above assumption the manifold is actually biholomorphically isometric to C^n. Moreover, if the curvature condition is replaced by $|\text{sectional curvature}| \leqslant A_\varepsilon/(1+r^{2+\varepsilon})$ (where A_ε is a positive number depending on ε) and the exponential map at the fixed point is a diffeomorphism, then the manifold is still biholomorphic to C^n. In the case of positive curvature of fast decay, if the scalar curvature is $\leqslant A/(1+r^{2+\varepsilon})$ and the volume of the geodesic ball centered at the fixed point of radius s is $\geqslant Cs^{2n}$ (C being a positive constant), then the manifold is biholomorphically isometric to C^n when the sectional curvature is non-negative or when the bisectional curvature is non-negative and the manifold is Stein. These two isometry results are obtained as follows. One solves the equation $\partial\bar\partial u = \text{Ricci}$ either by the L^2 estimates of $\bar\partial$ or by solving first $\Delta u = \text{scalar curvature}$ and applying the Bochner technique to $\Delta\,|\partial\bar\partial u - \text{Ricci}|^2$ and then one proves that u must be constant.

The isometry results in Mok, Siu and Yau [36] prompted the conjecture for similar results in Riemannian geometry. Such results in Riemannian geometry were afterwards obtained by Greene and Wu [20] and also independently by Gromov [21].

An open problem about the characterization of C^n is whether a complete non-compact Kähler manifold with positive sectional curvature is biholomorphic to C^n. The case $n = 2$ under some mild additional conditions was recently proved by Mok [35] together with some other partial results. Greene and Wu [17] showed that such a manifold is Stein. (Under the weaker assumption of positive bisectional curvature such a manifold is conjectured to be Stein. Though it is easy to produce global holomorphic functions separating points and giving local coordinates, the conjecture remains unproved.)

There is another kind of characterization of C^n. It is by the existence of an exhaustion function satisfying a certain Monge–Ampère equation. This characterization works not only for C^n but also for bounded balls in C^n. So it is also a result in the hyperbolic case. Suppose M is an n-dimensional complex manifold with a strongly plurisubharmonic exhaustion function u: $M \to [0, r^2)$ $(0 < r \leqslant \infty)$ such that $(\partial\bar{\partial}\log u)^n = 0$, $\sqrt{-1}\,\partial\bar{\partial}\log u \geqslant 0$, and $(\partial\bar{\partial}\log u)^{n-1} \neq 0$. Stoll [59] proved that M with the Kähler metric whose potential is u is biholomorphically isometric to the ball of radius r in C^n with the metric induced from the Euclidean metric of C^n. Alternative proofs and some generalizations were given by Burns [7]. Another alternative proof was given by P.-M. Wong [65]. The idea is to consider complex curves in M whose tangent spaces are the annihilators of $\partial\bar{\partial}\log u$ and to show that the space of all such curves is P_{n-1}. One then establishes the isometry by using the tautological line bundle of P_{n-1}.

§ 2. Elliptic case

The problem concerns the curvature characterization of the complex projective space or, more generally, irreducible compact Hermitian symmetric manifolds. Frankel [14] conjectured that a compact n-dimensional Kähler manifold with positive sectional curvature is biholomorphic to P_n. He and Andreotti [14] gave a proof for $n = 2$. Mabuchi [33] proved it for $n = 3$. Mori [37], using the methods of algebraic geometry of characteristic $p > 0$, proved the general case with the weakened assumption that the tangent bundle is ample. Siu and Yau [56] gave a differential-geometric proof for the general case with the assumption that the bisectional curvature is positive.

The differential-geometric proof of the Frankel conjecture makes use of harmonic maps. The energy $E(f)$ of a map f: $N \to M$ between two Riemannian manifolds is the L^2 norm over N of its differential df. The map f is harmonic if it is critical for the energy functional. In particular, it is harmonic if it minimizes energy among all maps in its homotopy class. Hartshorne and Kobayashi and Ochiai [28, 29] observed that a compact Kähler manifold M with positive bisectional curvature is biholomorphic to P_n if a generator of $H^2(M, Z)$ can be represented by a rational curve. On the other hand, by the result of Sachs and Uhlenbeck [44], one can represent in this case a generator of $H^2(M, Z)$ by a finite sum of harmonic maps f_i: $P_1 \to M$ so that $\sum E(f_i)$ is minimum among maps in the homotopy class of $\sum f_i$. By using the second variation of the energy functional, one concludes from the positivity of the bisectional curvature that each f_i is either holomorphic or antiholomorphic (see also [47]). There can be only one f_i, otherwise we have one holomorphic f_i and one antiholomorphic f_j and, by the positivity of the tangent bundle of M, the images of f_i and

f_j can be holomorphically deformed until they touch at some point and one can then modify f_i+f_j near the point of contact to produce a map in the homotopy class of f_i+f_j with smaller energy. Thus a generator of $H^2(M, \mathbf{Z})$ is represented by a rational curve.

The above method was refined in Siu [49] to yield the following curvature characterization of the complex hyperquadric. A compact Kähler manifold M of complex dimension $\geqslant 3$ with non-negative bisectional curvature is biholomorphic to either the complex projective space or the complex hyperquadric if at every point there exists no tangent subspace V of complex dimension 2 such that the bisectional curvature in the direction of any two vectors always vanishes when one of them is in V.

The general problem is to determine whether a compact Kähler manifold with non-negative sectional (or bisectional) curvature and positive definite Ricci curvature is necessarily biholomorphic to a compact Hermitian symmetric manifold. Moreover, if it is biholomorphic to an irreducible compact Hermitian symmetric manifold of rank at least two, one wants to know whether the Kähler metric with non-negative sectional (or bisectional) curvature is necessarily an invariant metric.

In Siu [52] the following weak partial result on the above problem was obtained by using the Bochner–Kodaira technique and holonomy groups. Let M be a compact Kähler manifold whose cotangent bundle with the induced metric is seminegative in the sense of Nakano [41]. Assume that at some point of M it is not possible to decompose the tangent space of M into two non-zero orthogonal direct summands such that the bisectional curvature in the direction of two vectors with one in each summand must vanish. Then either M is an irreducible Hermitian symmetric manifold with respect to the given Kähler metric or its cohomology ring with real coefficients is isomorphic to that of the complex projective space of the same dimension. This partial result is unsatisfactory, because seminegativity in the sense of Nakano is a curvature operator condition.

Recently Bando [4] proved that a three-dimensional compact Kähler manifold of non-negative bisectional curvature whose Ricci curvature is semipositive everywhere and positive definite somewhere is biholomorphic to the complex hyperquadric or a product of complex projective spaces. His proof makes use of the methods of Siu and Yau [56], Siu [49], the classification of compact Kähler surfaces of non-negative bisectional curvature by Howard and Smyth [25], the work of Howard, Smyth and Wu [26] and Wu [67] on the splitting of Kähler manifolds of non-negative bisectional curvature, and Hamilton's method [22] of solving evolution equations to increase the positivity of curvature.

For this problem, if the Kähler metric is assumed to be Einstein, then the result of Berger [5] and Gray [15] tells us that a Kähler–Einstein metric on a compact complex manifold is locally symmetric if its sectional curvature is non-negative. It is unknown in dimension at least four whether in this result one can replace sectional curvature by bisectional curvature.

§ 3. Hyperbolic case

The problem is to produce non-constant bounded holomorphic functions on a simply connected complete Kähler manifold whose sectional (or bisectional) curvature is bounded from above by a negative number, with the goal of embedding it into a bounded domain. Very little is known about this problem. Wu [66] observed that in the case of negative sectional curvature the manifold is Stein. It is an open problem whether the manifold is Stein in the case of negative bisectional curvature. It is even unknown whether such a manifold is necessarily non-compact. Recently B. Wong [64] constructed a simply connected compact complex surface with ample cotangent bundle. This would be a negative answer to the last question about non-compactness if the ampleness of the cotangent bundle could be replaced by the negativity of the bisectional curvature of some Kähler metric. On the other hand, in the corresponding situation in Riemannian geometry, Anderson [1] and Sullivan [61] recently succeeded in constructing non-constant bounded harmonic functions on any simply connected complete Riemannian manifold whose sectional curvature is bounded from above by a negative number. (Earlier Prat [*C.R. Acad. Sci. Paris* **284** (1977), pp. 687–690] did the real 2-dimensional case under additional conditions.) Later a very simple construction was given by Schoen [46].

Even when one confines oneself to the special case of manifolds which are the universal covers of compact Kähler manifolds with negative sectional (or bisectional) curvature, one still does not know how to construct non-constant bounded holomorphic functions. For a long time, for lack of examples, it was believed that every compact Kähler manifold with negative sectional curvature is covered by the ball. This belief was reinforced by the result of B. Wong [63] characterizing balls as smooth bounded (strongly pseudoconvex) domains admitting compact quotients and by Yang's result [68] that a bidisc cannot cover a compact Kähler surface with negative bisectional curvature. Finally an example was constructed by Mostow and Siu [40] of a compact Kähler surface with negative sectional curvature not covered by the ball. Actually its curvature is even strongly negative in the sense of [48]. The manifold is constructed by using an almost discrete subgroup of the automorphism group of the ball generated by three complex reflections [39] and the metric is obtained by piecing together the Bergman metrics of the ball and a domain ramified over the ball along a complex line. For this construction one can also use the almost discrete subgroup constructed from the extension by Deligne and Mostow [11] of a method of Picard [43].

It is not known whether requiring the Kähler metric to be Einstein would force the universal cover of a negatively curved compact Kähler manifold to be the ball. A partial result in the surface case was obtained by Siu and Yang [54]. This question is related to the problem of obtaining conditions under which the Kähler–Einstein metric constructed by Yau [69] has negative sectional curvature.

Compact Kähler manifolds of complex dimension at least two with suitable negative curvature conditions enjoy the property of strong rigidity. In the Riemannian case Mostow [38] obtained the following result on strong rigidity. Two compact locally symmetric Riemannian manifolds of non-positive sectional curvature are isometric (up to normalizing constants) if they are of the same homotopy type and they do not admit closed geodesic submanifolds of real dimension $\leqslant 2$ which are locally direct factors. In the complex case one has a stronger kind of strong rigidity. A compact Kähler manifold is said to be strongly rigid if any compact Kähler manifold of the same homotopy type is either biholomorphic or antibiholomorphic to it. Siu [48] proved that a compact Kähler manifold of complex dimension at least two is strongly rigid if its Kähler metric makes its cotangent bundle positive in the sense of Nakano (or if the weaker condition of strongly negative curvature in the sense of [48] is satisfied). The idea of the proof is as follows. Let M be the compact Kähler manifold with strongly negative curavture and let N be a compact Kähler manifold of the same homotopy type as M. By Eells and Sampson [13], there exists a harmonic map $f\colon N \to M$ which is a homotopy equivalence. By applying the Bochner–Kodaira technique to $\bar\partial f$, one obtains the vanishing of a term involving $\bar\partial f$, ∂f, and the curvature of M. The curvature condition on M forces the vanishing of $\bar\partial f$ or ∂f at points where the rank of df over $\boldsymbol{R}$ is at least 4. The proof can be regarded as a quasilinear analog of the vanishing theorem of Kodaira for negative bundles in which $\bar\partial f$ takes the place of a harmonic form.

This method of proof can be used to obtain the holomorphicity of harmonic maps and strong rigidity in more general cases [52]. Let M be a compact Kähler manifold whose cotangent bundle with the induced metric is non-negative in the sense of Nakano. Suppose for some positive integer p the bundle of $(p, 0)$-forms on M with the induced metric is positive in the sense of Nakano and suppose at every point of M it is not possible to find two non-zero complex tangent subspaces V, W such that the sum of the complex dimensions of V and W exceeds p and the bisectional curvature in the direction of one vector of V and one vector of W always vanishes. Then any harmonic map f from a compact Kähler manifold to M must be either holomorphic or antiholomorphic if the rank of the differential df of f over $\boldsymbol{R}$ is at least $2p+1$ at some point. In particular, M is strongly rigid if the complex dimension of M exceeds p. As a consequence, one concludes that a compact quotient of an irreducible bounded symmetric domain of complex dimension at least two is strongly rigid [48, 50, 52]. For a compact quotient of an irreducible bounded symmetric domain Siu [50, 52] and Zhong [70], by using the results of Calabi and Vesentini [10] and Borel [6], computed the smallest p for which the above conditions hold.

The above result on the holomorphicity of harmonic maps enables one to construct complex submanifolds in a compact Kähler manifold with suitable negative curvature conditions and also it leads to results on the relationship between the Kuranishi deformation space [30] and the Douady deformation space [12] for a submanifold of a manifold with suitable negative curvature conditions (see Kalka [27] and Siu [52]).

It is possible to obtain strong rigidity results in the case of non-compact manifolds of finite volume (Siu and Yau [58]). Morevoer, Siu and Yau [57] proved that a complete Kähler manifold of finite volume whose sectional curvature is bounded between two negative numbers can be compactified by adding a finite number of points so that the compactification can be blown up at the added points to become a projective algebraic variety. This generalizes the result on the corresponding locally symmetric cases by Satake [45] and Baily and Borel [3] (see also Andreotti and Grauert [2]).

§ 4. K3 surfaces

One of the most interesting applications of the methods of global differential geometry to the theory of complex manifolds are the results recently obtained on K3 surfaces. A K3 surface is a simply connected compact complex surface with trivial canonical line bundle. Let X be a smooth manifold diffeomorphic to one (and hence every) K3 surface. Let V be $H^2(X, \boldsymbol{Z})$ endowed with the quadratic form defined by the cup product and let Ω be the set of all real 2-planes in $V \otimes \boldsymbol{R}$ on which the quadratic form is positive definite. A marking on a K3 surface M is an isomorphism between $H^2(M, \boldsymbol{Z})$ and V compatible with the cup product. The period of a marked K3 surface is the element of Ω spanned by the elements of $H^2(M, \boldsymbol{R})$ corresponding to the real and imaginary parts of a non-zero holomorphic 2-form on M via the de Rham isomorphism.

There are two important questions concerning K3 surfaces. One is whether the period map for marked K3 surfaces is surjective, i.e. whether every element of Ω is a period of some marked K3 surface. Another is whether every K3 surface is Kähler. These questions were recently answered in the affirmative by using Yau's proof [69] of the Calabi conjecture (Todorov [62], Looijenga [31], Siu [51, 53]). Prior to Yau's proof of the Calabi conjecture, Hitchin [24] observed that from a Kähler–Einstein metric on a K3 surface one can construct a family of complex structures parametrized by the Riemann sphere. The surjectivity proof depends on this construction of Hitchin. The Kähler property of K3 surfaces follows from a refined version of the surjectivity result, the Torelli theorem for algebraic or Kähler K3 surfaces [8, 32, 42], and the existence of a real closed 2-form with positive $(1, 1)$-component on any K3 surface obtained by Sullivan's trick [60] of using the separation theorem for locally convex linear topological spaces. The simpler surjectivity proofs given in [31, 51] are mainly technical streamlining of the original surjectivity proof of Todorov [62].

The Kähler property of K3 surfaces completes the proof of the conjecture of Kodaira that every compact complex surface with even first Betti number must be Kähler. The other cases were already verified by Miyaoka [34]. An alternative proof of Miyaoka's result was recently given by Harvey and Lawson [23] as an application of their newly obtained intrinsic characterization of Kähler manifolds.

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Invited 45-Minute Addresses in Sections