A

NEW METHOD

OF

SOLVING EQUATIONS

WITH EASE AND EXPEDITION;

BY WHICH

THE TRUE VALUE OF THE UNKNOWN QUANTITY IS FOUND WITHOUT PREVIOUS REDUCTION.

WITH

A SUPPLEMENT,

CONTAINING

TWO OTHER METHODS OF SOLVING EQUATIONS, DERIVED FROM THE SAME PRINCIPLE.

By THEOPHILUS HOLDRED.

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PREFACE.

THIS method of extracting the roots of Equations is perfect, and expeditious: and since Logarithms do not apply to mixed Equations, I presume the method cannot fail of being acceptable to the lovers of Mathematics.

Sir ISAAC NEWTON, Dr. HALLEY, Mr. RAPHSON, Mr. WARD, and other eminent Mathematicians, have applied themselves particularly to this subject: but not being able to discover an easy and perfect method, they abandoned the pursuit; contenting themselves with approximations only.

The method which has (by some) been called Mr. HARRIOT'S Universal Method, is so extremely tedious, on account of the repeated involutions, as to exhaust all human patience.

My first discovery of this method was made when I was about twenty years of age, now forty years ago. Regretting that the method of extracting the Cube Root should be so troublesome, I at last thought of forming a canon for finding the square of the root as it became increased, with the addition of every new figure, as they became known, considering it as a binomial root; the first member being the known part of the root, and the other member the newly-discovered figure. This canon being formed, and multiplied by 3, it was easy to perceive that the preceding divisor made a great part thereof: the first member of the preceding divisor being singly taken; the second member being multiplied by the newly-discovered figure, and then doubled, with the treble square of the said newly-discovered figure, made up the canon.

I then considered, that in extracting the square root, the double of the known part of the root is taken for the divisor; to which if the double of the newly-discovered figure be added, the sum will be the double of the increased root for a new divisor. By this means, I saw that the method would apply to mixed equations, and that the sums of the different powers of the increased root are had out of the sums of the respective powers of the preceding part of the root.

I then applied the same principle to the biquadrate root with equal success; and perceiving that the coefficients were the beginning of the figurate numbers, I concluded that it must be universal, which (by means of Sir Isaac Newton's Binomial Theorem) I soon found to be the case.

There may be various ways of writing down the numbers in an operation: my chief aim has been to be clearly understood; for which purpose the Algebraic Symbols are used throughout the work, with the exception of the Supplement.

In June 1818, I submitted to Mr. P. Nicholson, a Manuscript of this Tract, which met with his entire approbation;

and he gave me the following recommendation in writing, which I inserted in the Prospectus.

"*Mr. Theophilus Holdred has submitted to my opinion a small Tract on the Resolution of Equations of all Degrees. I have perused it with care, and have found it to be a most ingenious method: for notwithstanding the numerous attempts and industry of the most eminent Mathematicians, from the time of Cardan to the present, to discover an easy, direct, and correct process of extracting roots, this method eluded their research, and has been reserved to raise the fame of an individual hitherto unknown in the mathematical world.*

"No. 12, London Street, Fitzroy Square,
October 7th, 1818.

PETER NICHOLSON."

Mr. Nicholson recommended a different notation, with some other alterations of little importance; and though I did not perceive any advantage in it, I yielded to his advice, and the Manuscript was written over again: this, on account of my profession, and through indisposition, was not completed before December.

Mr. Nicholson discovered another way of demonstrating my rule, which he requested me to add to my Treatise, by way of Supplement; but finding he was communicating my method to every Mathematician he knew, I became much dissatisfied with his conduct; and discovering the improvement which I have inserted as a Supplement, I considered it far better than that intended by Mr. Nicholson. I therefore resolved to prefer my own; which I have never communicated to him.

After the flattering testimony which Mr. Nicholson gave of the advantages of my method, is it not surprising that he should have published a TREATISE ON ALGEBRA, obviously written for the purpose of introducing that method? My Subscribers well know how long my Manuscript has been completed, as well as the unavoidable causes which procrastinated its publication: let them read Mr. Nicholson's recommendation, and then draw their own conclusions.

The discovery which is the subject of the Supplement was made in the following manner. A Subscriber to this Work sent to me an equation to solve, which I did accordingly, and went to him with two of the roots: but in extracting one of them, I began on too small a piece of paper; and in transcribing it (on another piece of paper, to carry the root further), I committed an error, which made all the following work erroneous. My friend instantly shewed me, that after the sixth figure in decimal parts, I differed from the author whence he had taken the equation: at which I diligently sought to find where the error had taken place. When satisfied on that subject, I next considered by what means I might have proved the root with less trouble to myself. I soon thought of the method with which the Supplement begins. I next considered, that to know how many figures of the root might be depended on, it was necessary to form (at least) the first member of the imperfect divisor. The next consideration was, Whether this could be done from the numbers that had been produced in proving the root: it did not require much consideration to perceive, that not only the first member could, but every member of the divisor could; and perceiving the figurate numbers to be run back one order, I considered this an improvement.

To run the figurate numbers back further, I considered would be of service, for equations above the sixth power; and though I found several ways of doing this, yet, to do it by so simple a method that I could satisfactorily present to the Public, occupied my attention for some weeks.

This last method I conceive to be the perfection of the whole; as in it there is no occasion for the arithmetical equivalents of negative numbers; and it is only necessary to write down the proper signs. It is admirable for its simplicity; as any child who can multiply and divide may perform it with ease, the number of multiplications being greatly reduced.

T. HOLDRED.

No. 2, Denzel Street, Clare Market,
June 1, 1820.

SECTION I.

DEFINITIONS.

THE capital letters *A, B, C,* &c. are used for a rank of numbers; and these letters preceded by a small figure at bottom, denote a rank according to the value of the figure by which it is preceded; thus, ${}_2A$, ${}_2B$, ${}_2C$, &c. signifies the second rank; ${}_3A$, ${}_3B$, ${}_3C$, &c. signifies the third rank; and so on. Those numbers which I call quotient figures are signified by the small letters *a, b, c, d,* &c.

When a negative number is to be added to an affirmative number, it is, in effect, Subtraction; and it is very obvious, that when there are but two numbers, the one positive and the other negative, it will be an easy matter; but if there are three numbers, or more, the operation will become less simple (if done in the usual manner), as there must be both adding and subtracting, and a line added to the work; therefore I have taken the arithmetical complement of negative numbers, which is by subtracting the number from such a number as consists of as many noughts or ciphers as the negative number consists of figures, preceded by 1; to the remainder must be prefixed 1, with a dash over it, which $\bar{1}$, must be considered as negative.

Suppose the number were -6438, then, by subtracting it from 10000, the remainder is 3562, which is the arithmetical complement; but to make it equivalent to the fore-mentioned number, a negative $\bar{1}$ must be prefixed; so that this is only another expression for the same value; that is, $\bar{1}3562, = -6438$. By this form of writing a negative number,

it

it consists of two parts; a negative part, and a positive part. In adding such a number, there is no difference to that of adding any other number, until we come to the negative $\bar{1}$, which must be subtracted.

In subtracting numbers so expressed, a negative $\bar{1}$ in the subtrahend may be considered as a positive 1 in the minuend; and a negative $\bar{1}$ in the minuend may be considered as a positive 1 in the subtrahend; therefore, in the former case, add the 1 to the figure which stands above; and in the latter case, add the 1 to the figure which stands below, and then the subtraction will be as in other cases.

When it may be necessary to multiply such a number, the positive part must be multiplied in the usual manner; but whatever figure may require to be carried from this part, must be taken from the product of the negative part; thus, if the number $\bar{1}85$ were to be multiplied by 3, the product would be $\bar{1}55$. When the negative part of the product shall be any other figure than 1, it will be more convenient to take that from 10, and prefix the remainder to the positive part, and to that remainder prefix the negative $\bar{1}$.

Suppose it were required to multiply $\bar{1}3562$ by 3, the product would be $\bar{2}0686$; in this case I would recommend to take the $\bar{2}$ from 10, and write the difference 8 in its place, to which prefix the negative $\bar{1}$, and thereby the same number is expressed in another manner, for $\bar{2}0686 = \bar{1}80686$. Let a number so expressed be called *The arithmetical equivalent.*

The arithmetical complement of a number may be taken from the left to the right, by subtracting each figure from 9, except the last, which must be subtracted from 10.

If the figure which immediately follows the negative $\bar{1}$ should be 9, it may be omitted, and the $\bar{1}$ put in its place; thus, $\bar{1}97 \doteq \bar{1}7$, &c. If a number so expressed should require to be divided by a single figure,

subtract

subtract 1 from the divisor, and prefix the remainder to the affirmative part; then divide by the proposed divisor, and to the quotient prefix the negative $\bar{1}$; this is on the principle of adding an equal number to both negative and affirmative parts, so as to make the negative part commeasurable by the divisor. Thus the number $\bar{1}80686$ divided by 3, the quotient is $\bar{1}93562 = \bar{1}3562$.

When a colon immediately follows a number, and the symbol of multiplication immediately follows that colon, read, *Which multiplied by*.

SECT. II.

OF THE RULE IN GENERAL.

For a general equation, write $x^n + hx^{n-1} + ix^{n-2} + kx^{n-3} + lx^{n-4} + \&c. = N$. The unknown root sought is represented by x; the exponent n must be a positive whole number; and the rest of the letters may be either affirmative, or negative, something or nothing, but are all understood to be known.

Let r be put for an assumed root; then for a canon raise r to all the powers, &c. of x in the given equation; writing them one under another; and write A for the sum; thus,

$$\begin{array}{c} r^n \\ hr^{n-1} \\ ir^{n-2} \\ kr^{n-3} \\ lr^{n-4} \\ \&c. \\ \hline A \end{array}$$

Next multiply each of those members by the exponent of r in that member, and take 1 from the exponent, and call the sum B; thus,

r^n	nr^{n-1}
hr^{n-1}	$h(n-1)r^{n-2}$
ir^{n-2}	$i(n-2)r^{n-3}$
kr^{n-3}	$k(n-3)r^{n-4}$
lr^{n-4}	$l(n-4)r^{n-5}$
&c.	&c.
A	B

Then multiply each of those by the exponent of r in that member; divide by 2, and take 1 from the exponent, and call the sum C; thus,

r^n	nr^{n-1}	$\frac{n(n-1)}{2}r^{n-2}$
hr^{n-1}	$h(n-1)r^{n-2}$	$h\frac{(n-1)(n-2)}{2}r^{n-3}$
ir^{n-2}	$i(n-2)r^{n-3}$	$i\frac{(n-2)(n-3)}{2}r^{n-4}$
kr^{n-3}	$k(n-3)r^{n-4}$	$k\frac{(n-3)(n-4)}{2}r^{n-5}$
lr^{n-4}	$l(n-4)r^{n-5}$	$l\frac{(n-4)(n-5)}{2}r^{n-6}$
&c.	&c.	&c.
A	B	C

Then multiply each of those members by the exponent of r in that member; divide by 3, and take 1 from the exponent, and call the sum D; thus,

r^n

r^n	nr^{n-1}	$\frac{n(n-1)}{2}r^{n-2}$	$\frac{n(n-1)}{2}\frac{(n-2)}{3}r^{n-3}$
hr^{n-1}	$h(n-1)r^{n-2}$	$h\frac{(n-1)(n-2)}{2}r^{n-3}$	$h\frac{(n-1)(n-2)}{2}\frac{(n-3)}{3}r^{n-4}$
ir^{n-2}	$i(n-2)r^{n-3}$	$i\frac{(n-2)(n-3)}{2}r^{n-4}$	$i\frac{(n-2)(n-3)}{2}\frac{(n-4)}{3}r^{n-5}$
kr^{n-3}	$k(n-3)r^{n-4}$	$k\frac{(n-3)(n-4)}{2}r^{n-5}$	$k\frac{(n-3)(n-4)}{2}\frac{(n-5)}{3}r^{n-6}$
lr^{n-4}	$l(n-4)r^{n-5}$	$l\frac{(n-4)(n-5)}{2}r^{n-6}$	$l\frac{(n-4)(n-5)}{2}\frac{(n-6)}{3}r^{n-7}$
&c.	&c.	&c.	&c.
A	B	C	D

Proceed in this manner until the exponent of r shall be $=0$, or until there shall be as many terms as necessary, increasing the divisor by 1 each time.

When A shall be taken from the given number N, the remainder $N-A$ is the resolvend, and $B+C+D+$ &c. is *An imperfect divisor*, by which the next figure of the root may be discovered, which when found I call a; then $B+Ca+Da^2+$ &c. I call *The true divisor*; multiply this divisor by a, the product $Ba+Ca^2+Da^3+$ &c. will be the subtrahend, which being taken from the resolvend, leaves the new resolvend.

Make $r+a={}_2r$; then if ${}_2r$ were written instead of r in the preceding canons, we should get the values of ${}_2A$, ${}_2B$, ${}_2C$, ${}_2D$, &c.; but there is no need of those tedious involutions, for those values may be had by the following easy canons;

${}_2A=$

$$
\begin{aligned}
{}_2A &= A + Ba + Ca^2 + Da^3 + \&c.\\
{}_2B &= B + 2Ca + 3Da^2 + 4Ea^3 + \&c.\\
{}_2C &= C + 3Da + 6Ea^2 + 10Fa^3 + \&c.\\
{}_2D &= D + 4Ea + 10Fa^2 + 20Ga^3 + \&c.\\
\&c. &= \&c.
\end{aligned}
$$

Therefore the new resolvend is $N - {}_2A$, and the new imperfect divisor is ${}_2B + {}_2C + {}_2D + \&c.$

If the given equation be a cubic equation, then we have $n=3$, and the fore-mentioned general equation becomes $N = x^3 + hx^2 + ix$.

Example 1.

Make $N = 247661856$, $h = 12$, and $i = 30$, then the equation becomes $x^3 + 12x^2 + 30x = 247661856$; make $r = 600$, and we shall have,

$r^3 = 216000000$	$3r^2 = 1080000$	$3r = 1800$
$hr^2 = \quad 4320000$	$2hr = \quad 14400$	$h = \quad 12$
$ir = \quad 18000$	$i = \quad 30$	
$A = 220338000$	$B = 1094430$	$C = 1812$

The last line is the sum of the three lines above; A is the subtrahend; and B and C are the terms of the divisor, with which proceed in the following manner;

Write down the given number N, and under it the subtrahend A; which subtract, and call the remainder the resolvend; thus,

$$
\begin{aligned}
N &= 247661856\\
A &= 220338000\\
\hline
&\quad\ 27323856 \text{ resolvend.}
\end{aligned}
$$

Then

Then write the number denominated B on the left-hand side of the resolvend, at a little distance; and write the number denominated $C = 1812$, further to the left, but one figure lower down the page; thus,

$$
\begin{array}{lrr}
 & & N = 247661856 \\
 & & A = 220338000 \\
 & B = 1094430\,) & 27323856 \text{ resolvend.} \\
C = 1812 & &
\end{array}
$$

Then seek how often $1094430 = B$ can be had out of the resolvend; with this precaution, that when $1812 = C$ shall be multiplied by the quotient figure and added to B, the sum shall still bring out the same quotient figure; in this case it is found to be 2, i. e. 20, the 2 being in the place of tens; then multiply $1812 = C$ thereby, and write the product under the number denominated B, and the work will stand thus,

$$
\begin{array}{lrr}
 & & N = 247661856\,(\,r = 600 \\
 & & A = 220338000 \\
 & B = 1094430\,) & 27323856\,(\,a = 20 \\
C = 1812 : \times 20 = & 36240 &
\end{array}
$$

Then write the square of the quotient figure under the last-mentioned product; and add those three numbers into one sum, which call *The true divisor*; then multiply the true divisor by the quotient figure $a = 20$, and write the product right against it, under the resolvend; and the work will stand thus,

$N =$

$$N = 247661856 \left(r = 600 \right.$$
$$A = 220338000$$

$$B = 1094430 \quad) \qquad 27323856 \left(a = 20 \right.$$
$$C = 1812 : \times 20 = \quad 36240 = Ca$$
$$400 = a^2$$

True divisor 1131070 : × 20 = 22621400 subtrahend.

Next draw a line under the last-mentioned number, and subtract it from the resolvend. Then add the three numbers next below the number which I call B into one sum, doubling the square of the quotient figure, while you add it. To prevent the possibility of a mistake, put a star against each of those numbers, and two against that which is to be doubled; which done, the work will stand thus,

$$N = 247661856 \left(r = 600 \right.$$
$$A = 220338000$$

$$B = 1094430 \quad) \qquad 27323856 \left(a = 20 \right.$$
$$C = 1812 : \times 20 = {}^{*} \quad 36240 = Ca$$
$$** \quad 400 = a^2$$

True divisor *1131070 : × 20 = 22621400 subtrahend

$${}_2B = 1168110 \qquad 4702456$$

By this means the value of ${}_2B$ is had by addition only.

Multiply the quotient figure ($a = 20$) by 3, and write the product under the number denominated C; add them together, and write the sum one line lower down the page than ${}_2B$; and the work will stand thus,

$N =$

$$\begin{array}{lll}
 & & N = 247661856 \;(r = 600 \\
 & & A = 220338000 \\
 & B = 1094430\,) & \quad 27323856 \;(a = 20 \\
C = 1812 : \times 20 = {}^{*} & 36240 = Ca & \\
3a = \quad 60 & {}^{**} \quad 400 = a^{2} & \\
\text{divisor} & {}^{*}\,1131070 : \times 20 = & 22621400 \text{ subtrahend} \\
 & {}_{2}B = \quad 1168110\,) & \quad 4702456 \text{ resolvend} \\
{}_{2}C = 1872 & &
\end{array}$$

Seek how often $1168110 = {}_{2}B$ can be had out of the resolvend; it is found to be 4 times; then multiply $1872 = {}_{2}C$ thereby, and write the product under the number $1168110 = {}_{2}B$, and under that write the square of the quotient figure; then draw a line under, and add those three numbers together into one sum, which multiply by 4 (the quotient figure), and write the product under the resolvend for a subtrahend, which subtract, and nothing remains; therefore 624 is a root of the equation; and the whole work will stand thus,

$$\begin{array}{lll}
 & & N = 247661856 \;(r = 600 \\
 & & A = 220338000 \\
 & B = 1094430\,) & \quad 27323856 \;(a = \; 20 \\
C = 1812 : \times \; 20 = {}^{*} & 36240 = Ca & \\
3a = \quad 60 & {}^{**} \quad 400 = a^{2} & \\
\text{divisor} & {}^{*}\,1131070 : \times 20 = & 22621400 \text{ subtrahend} \\
 & {}_{2}B = 1168110\,) & \quad 4702456 \;(b = 4 \\
{}_{2}C = 1872 : \times \; 4 \; = & 7488 = {}_{2}Cb & \\
 & 16 = b^{2} & \\
 & 1175614 : \times 4 = & \quad 4702456 \text{ subtrahend} \\
 & & \qquad 0
\end{array}$$

Example 2.

Let it be required to extract the cube root out of this number,

159220088.

By comparing this with the general cubic equation $N = x^3 + hx^2 + ix$, we have $N = 159220088$, $h = 0$, and $i = 0$; the first figure of the root is 5; therefore $r = 500$; then, by the general rule,

$r^3 = 125000000$	$3r^2 = 750000$	$3r = 1500$
$hr^2 = 0$	$2hr = 0$	$h = 0$
$ir = 0$	$i = 0$	
$A = 125000000$	$B = 750000$	$C = 1500$

$N = 159220088$ ($r = 500$
$A = 125000000$

$B = 750000$) 34220088 resolvend ($a = 40$
$C = 1500 : \times 40 =$ * 60000 $= Ca$
$3a =$ 120 * *1600 $= a^2$

$B + Ca + a^2 =$ divisor * 811600 : $\times 40 =$ 32464000 subtrahend

$B + 2Ca + 3a^2 = {}_2B = 874800$) 1756088 resolvend ($b = 2$
$C + 3a = {}_2C = 1620 : \times 2 =$ 3240 $= {}_2Cb$
4 $= b^2$

${}_2B + {}_2Cb + b^2 =$ divisor 878044 : $\times 2 =$ 1756088 subtrahend

0

$r = 500$
$a = 40$
$b = 2$

$r + a + b = x = 542$

Example

Example 3.

Let it be required to extract the cube root of this number,

665224304197.

The first figure of the root will be found, by trial, to be 8; therefore $8000 = r$; then, by the general rule,

$$r^3 = 512000000000 = A : 3r^2 = 192000000 = B : 3r = 24000 = C$$

$$B + 2Ca + 3a^2 = {}_2B \qquad C + 3a = {}_2C$$

$${}_2B + 2\,{}_2Cb + 3b^2 = {}_3B \qquad {}_2C + 3b = {}_3C$$

$$N = 665224304197 \; (8000 = r$$

$$A = 512000000000$$

	$B =$	$192000000\,)$		$153224304197 \; (\; 700 = a$
$C = 24000 : \times 700$	$=$	$*\,16800000 = Ca$		
$3a = 2100$		$*\,*\;490000 = a^2$		
$\delta = B + Ca + a^2$	$=$	$*\,209290000 : \times 700 =$		146503000000 subtrahend
${}_2B = \delta + Ca + 2a^2$	$=$	$227070000\,)$		$6721304197 \; (20 = b$
${}_2C = 26100 : \times 20$	$=$	$*\;522000 = {}_2Cb$		
$3b = 60$		$*\,*\;400 = b^2$		
${}_2\delta = {}_2B + {}_2Cb + b^2$	$=$	$*227592400 : \times 20$	$=$	4551848000 subtrahend
${}_3B = {}_2\delta + {}_2Cb + 2b^2 =$		$228115200\,)$		$2169456197 \; (9 = c$
${}_3C = 26160 : \times 9$	$=$	$235440 = {}_3Cc$		
		$81 = c^2$		
${}_3\delta = {}_3B + {}_3Cc + c^2$	$=$	$228350721 : \times 9$	$=$	2055156489 subtrahend
				114299708 remainder

For brevity, I have written δ for divisor.

Here

Here being a remainder, proves there is no true cube root to the above number. When the root shall be required in decimal parts, it will be convenient to determine what number of figures you would have in the answer at first; and use no more figures of the divisors and resolvends than what are likely to affect the root in the intended number of figures; but more of that further on.

Example 4.

Let it be required to find the root of this equation,

$$x^3-6438x=104785688.$$

By comparing this with the general equation, we have $N=104785688$, $h=0$, $i=-6438$; as this is a negative number, rather write the arithmetical equivalent; thus, $i=\bar{1}3562$.

I intend, in this Example, to shew that the estimate root r may be taken greater than the truth; in which case the unknown part being negative, the first resolvend becomes $N-A=-By+Cy^2-y^3$, putting y for the unknown part of the root; then, by changing the signs, it becomes $A-N=By-Cy^2+y^3$. That the signs of every other term is to be changed, is all that is to be understood here, for sometimes the quantities are negative from the nature of the equation. The imperfect divisor is $B-C$; and the quotient figures are to be taken as affirmative in multiplying, although to be taken from the estimate root r.

Make $r=500$

$r^3=125000000$	$3r^2=750000$	$3r=1500=C$, properly.
$ir=\bar{1}6781000$	$i=\bar{1}3562$	$\therefore -C=\bar{1}8500$, but rather write
$A=121781000$	$B=743562$	$C=\bar{1}8500$, for the convenience of the operation.

$A=$

$$A = 121781\,00 \;\big(500 = r$$
$$N = 104785688$$

$$B = 743562\,\big) \qquad 16995312 \;\big(20 \;= a$$
$$C = \bar{1}8500 : \times 20 = {}^{*}\bar{1}70000 = Ca$$
$$3a = \quad 60 \qquad {}^{**} \quad 400 = a^2$$

$$\delta = B + Ca + a^2 = {}^{*}713962 : \times 20 = \; 14279240 \text{ subtrahend}$$

$${}_2B = \delta + Ca + 2a^2 = \; 684762\,\big) \qquad 2716072 \;\big(4 \;= b$$
$${}_2C = C + 3a = \bar{1}8560 : \times 4 \; = \; \bar{1}4240 = {}_2Cb$$
$$16 = b^2$$

$${}_2\delta = {}_2B + {}_2Cb + b^2 = \; 679018 : \times 4 \; = \; 2716072 \text{ subtrahend}$$

$$0$$

$$500 = r$$
$$24 = a + b$$
$$476 = x$$

Example 5.

Let it be required to extract the biquadrate root of 99757432336

The first figure of the root is 5, therefore $500 = r$, then $r^4 = 62500000000 = A$, $4r^3 = 500000000 = B$, $6r^2 = 1500000 = C$, $4r = 2000 = D$.

$N =$

$N = 99757432336\ (500 = r$

$A = 62500000000$

$B = 500000000\ )\quad 37257432336\ (60 = a$

$C = 1500000 : \times 60 = 90000000 = Ca$

$D = 2000,\quad 3Da = 360000\quad 7200000 = Da^2$

$Da = 120000\quad 6a^2 = 21600\quad 216000 = a^3$

$4a = 240$

divisor $597416000 : \times 60 = 35844960000$ subtrahend

$a^2 = 3600$

${}_2B = B + 2Ca + 3Da^2 + 4a^3 = 702464000\ )\quad 1412472336\ (2 = b$

${}_2C = C + 3Da + 6a^2 = 1881600 : \times 2 = 3763200 = {}_2Cb$

$8960 = {}_2Db^2$

${}_2D = D + 4a = 2240$

$8 = b^3$

${}_2Db = 4480$

divisor $706236168 : \times 2 = 1412472336$ subtrahend

0

$500 = r$

$60 = a$

$2 = b$

$562 = x$

In finding the number denominated ${}_2B$, I have made no use of the divisor, as in the cubic equation, not seeing any advantage in it; but have done it by the universal rule delivered in the beginning.

The number $Da = 120000$, as also $a^2 = 3600$, are written down for the convenience of multiplying them; they are both multiplied by $a = 60$, and the products written under the number denominated B, as also Ca, to be added together, the sum being the true divisor. Da is also multiplied by 3, and a^2 is multiplied by 6, and written under C, to be added together, their sum being ${}_2C$.

SECT. III.

DEMONSTRATION IN THE CUBIC EQUATION.

LET the equation be $x^3+hx^2+ix=N$. Here I must request my reader to be particular in observing the quantities placed one under another, to be added together; under which the capital letters are written, as the sums thereof. Those quantities being known numbers to be added together, a single letter is the fittest to represent the sum; and where I mention an equation which gives the value of any of those capital letters, those columns should be referred to, in order to perceive their agreement.

Make $r+y=x$; raise this value of x to all the powers of x in the given equation, and multiply by the respective coefficients, and we get the following equations;

$$\begin{array}{rrcl} & x^3 = & r^3+3r^2y+3ry^2+y^3 \\ & hx^2 = & hr^2+2hry+hy^2 \\ & ix = & ir+iy \\ \hline \text{The sum} & N = & A+By+Cy^2+y^3 \end{array}$$

A, B, & C, being functions of the estimate root r, and the known coefficients of the given equation; and y the remaining unknown part of the root x. Transpose A, and we get $N-A=By+Cy^2+y^3$; this is the first resolvend, therefore $B+C$ is the first imperfect divisor. Then put a for the second figure of the root, and make ${}_2r=r+a$; raise this

this equation to all the powers of x in the given equation, and multiply by the respective coefficients, and we get these following equations;

$$
\begin{aligned}
{}_2r^3 &= r^3 + 3r^2a + 3ra^2 + a^3 \\
h_2r^2 &= hr^2 + 2hra + ha^2 \\
i_2r &= ir + ia \\
\hline
\text{The sum } {}_2A &= A + Ba + Ca^2 + a^3
\end{aligned}
$$

As ${}_2r$ is the sum of r and a, therefore ${}_2A$ is a function of r, a, and the known coefficients of the given equation, h, and i.

The first member on the other side of the equation, viz. A, was taken from the given number N, which left the first resolvend $N-A$; and now take the remaining part, viz. $Ba + Ca^2 + a^3$, there will remain $N - {}_2A$, for the new resolvend.

Now put u for the remaining part of the root, we have $x = {}_2r + u$; this equation being raised to all the powers of x in the given equation, and multiplied by the respective coefficients, we get these following equations,

$$
\begin{aligned}
x^3 &= {}_2r^3 + 3{}_2r^2u + 3{}_2ru^2 + u^3 \\
hx^2 &= h_2r^2 + 2h_2ru + hu^2 \\
ix &= i_2r + iu \\
\hline
\text{The sum } N &= {}_2A + {}_2Bu + {}_2Cu^2 + u^3
\end{aligned}
$$

A, ${}_2B$, and ${}_2C$, are functions of r, a, and the known coefficients of the given equation; and u is the remaining unknown part of the root. Now, by transposition, we obtain this equation,

$$N - {}_2A = {}_2Bu + {}_2Cu^2 + u^3;$$

this

This is the new resolvend, the same as was had by taking the subtrahend from the first resolvend; therefore ${}_2B+{}_2C$ is the new imperfect divisor.

The next thing to be done is, to raise the canons for finding the values of ${}_2B$, and ${}_2C$, without involution; thus;

Because ${}_2B=3{}_2r^2+2h{}_2r+i$, and ${}_2r=r+a$, raise this equation to all the powers of ${}_2r$ in the equation which gives the value of ${}_2B$, and multiply by the respective coefficients, and the following equations are had,

$$\begin{array}{rrl} & 3{}_2r^2= & 3r^2+6ra+3a^2 \\ & 2h{}_2r= & 2hr+2ha \\ & i= & i \\ \hline \text{The sum} & {}_2B= & B+2Ca+3a^2 \end{array}$$

And because ${}_2C=3{}_2r+h$, therefore, by proceeding in the like manner, we get these following equations;

$$\begin{array}{rrl} & 3{}_2r= & 3r+3a \\ & h= & h \\ \hline \text{The sum} & {}_2C= & C+3a \end{array}$$

SECT. IV.

DEMONSTRATION OF THE RULE IN THE BIQUADRATIC EQUATION.

LET the equation $x^4+hx^3+ix^2+kx=N$ be proposed. Make $x=r+y$; raise this equation to all the powers of x in the given equation, and multiply by the respective coefficients; and we get the following equations;

$$\begin{array}{rl} x^4 = & r^4 + 4r^3y + 6r^2y^2+4ry^3+y^4 \\ hx^3= & hr^3+3hr^2y+3hry^2+hy^3 \\ ix^2= & ir^2+2iry+iy^2 \\ kx = & kr + ky \\ \hline \text{The sum}\quad N= & A + By + Cy^2+Dy^3+y^4 \end{array}$$

A, B, C, and D, being functions of the estimate root r, and the known coefficients of the given equation; and y the remaining unknown part of the root x; then, by transposing A, we get the following equation,

$$N-A=By+Cy^2+Dy^3+y^4$$

This is the first resolvend, therefore $B+C+D$ is the first imperfect divisor. Then put a for the next figure of the root, and make ${}_2r=r+a$; raise this equation to all the powers of x in the given equation, and multiply by the respective coefficients, and we get these following equations;

${}_2r^4=$

$$
\begin{aligned}
{}_2r^4 &= r^4 + 4r^3a + 6r^2a^2 + 4ra^3 + a^4 \\
h\,{}_2r^3 &= hr^3 + 3hr^2a + 3hra^2 + ha^3 \\
i\,{}_2r^2 &= ir^2 + 2ira + ia^2 \\
k\,{}_2r &= kr + ka \\
\hline
\text{The sum } {}_2A &= A + Ba + Ca^2 + Da^3 + a^4
\end{aligned}
$$

As ${}_2r$ is the sum of r and a, therefore ${}_2A$ is a function of r, a, and the known coefficients of the given equation.

The first member on the other side of the equation, viz. A, was taken from the given number N, which left the first resolvend $N-A$; and now take the remaining part, viz. $Ba+Ca^2+Da^3+a^4$, there will remain $N-{}_2A$ for the new resolvend.

Now put u for the remaining part of the root, and we shall have $x={}_2r+u$; this equation being raised to all the powers of x in the given equation, and multiplied by the respective coefficients, we get these following equations,

$$
\begin{aligned}
x^4 &= {}_2r^4 + 4\,{}_2r^3u + 6\,{}_2r^2u^2 + 4\,{}_2ru^3 + u^4 \\
hx^3 &= h\,{}_2r^3 + 3h\,{}_2r^2u + 3h\,{}_2ru^2 + hu^3 \\
ix^2 &= i\,{}_2r^2 + 2i\,{}_2ru + iu^2 \\
kx &= k\,{}_2r + ku \\
\hline
\text{The sum } N &= {}_2A + {}_2Bu + {}_2Cu^2 + {}_2Du^3 + u^4
\end{aligned}
$$

${}_2A$, ${}_2B$, ${}_2C$, and ${}_2D$, are functions of r, a, and the known coefficients of the given equation; and u is the remaining unknown part of the root. Now, by transposition, we obtain this equation,

$$N-{}_2A={}_2Bu+{}_2Cu^2+{}_2Du^3+u^4$$

This is the new resolvend, the same as was had by taking the subtrahend

trahend

trahend from the first resolvend; therefore ${}_2B+{}_2C+{}_2D$ is the new imperfect divisor.

The next thing to be done, is to raise the canons for finding the values of B, ${}_2C$, and ${}_2D$, without involution; thus,

Because ${}_2B=4{}_2r^3+3h{}_2r^2+2i{}_2r+k$, and ${}_2r=r+a$; raise this value of ${}_2r$ to all the powers of ${}_2r$ in the equation which gives the value of ${}_2B$, and multiply by the respective coefficients, and the following equations are had,

$$\begin{aligned} 4{}_2r^3 &= 4r^3+12r^2a+12ra^2+4a^3 \\ 3h{}_2r^2 &= 3hr^2+6hra+3ha^2 \\ 2i{}_2r &= 2ir+2ia \\ k &= k \\ \hline \text{The sum } {}_2B &= B+2Ca+3Da^2+4a^3 \end{aligned}$$

Because ${}_2C=6{}_2r^2+3h{}_2r+i$; therefore raise the equation ${}_2r=r+a$ to all the powers of ${}_2r$ in the preceding equation, which gives the value of ${}_2C$; and multiply by the respective coefficients, and we get the following equations,

$$\begin{aligned} 6{}_2r^2 &= 6r^2+12ra+6a^2 \\ 3h{}_2r &= 3hr+3ha \\ i &= i \\ \hline \text{The sum } {}_2C &= C+3Da+6a^2 \end{aligned}$$

And because ${}_2D=4{}_2r+h$, doing in like manner, we get these following equations,

$$\begin{aligned} 4{}_2r &= 4r+4a \\ h &= h \\ \hline \text{The sum } {}_2D &= D+4a \end{aligned}$$

Exam.

Example 6.

Let the equation $x^4+5x^3+7x^2+3x=91672020000$ be proposed.

By comparing this with the general equation, we have $h=5$, $i=7$, and $k=3$.

Make $r=500$

$r^4=62500000000$	$4r^3=500000000$	$6r^2=1500000$	$4r=2000$
$hr^3=625000000$	$3hr^2=3750000$	$3hr=7500$	$h=5$
$ir^2=1750000$	$2ir=7000$	$i=7$	$D=2005$
$kr=1500$	$k=3$	$C=1507507$	$_2D=D+4a$
$A=63126751500$	$B=503757003$	$_2C=C+3Da+6a^2$	
	$^2B=B+2Ca+3Da^2+4a^3$		

$N=91672020000\ (500=r$

$A=63126751500$

$B=503757003\ )\quad 28545268500\ (40=a$

$C=1507507:\times 40=\quad 60300280=Ca$

$D=2005,\ 3Da=\ 240600 \qquad 3208000=Da^2$

$Da=80200\quad 4a=\ 160\quad 6a^2=\ 9600 \qquad 64000=\ a^3$

$a^2=\ 1600$

Divisor $567329283:\times 40=22693171320$ subtrahend

$_2B=634237563 \qquad 5852097180\ (9=b$

$_2C=1757707:\times\ 9=\ 15819363={}_2Cb$

$_2D=2165 \qquad 175365={}_2Db^2$

$_2Db=19485 \qquad 729=\ b^3$

$a^2=\ 81$

Divisor $650233020:\times\ 9=\ 5852097180$ subtrahend

0

SECT. V.

PRELIMINARY OBSERVATIONS TO THE GENERAL DEMONSTRATION.

Before I treat this subject in general terms, it will be convenient to explain a new notation, whereby the product of a rank of numbers in arithmetical progression is very briefly expressed; which is in this manner. Write the first term of the series; and like an exponent of a power, write the number of terms, draw a downright line to the right-hand side of that exponent, and write the common difference to the right of that; thus; $n^{2|1} = n\ (n+1) : n^{3|1} = n\ (n+1)\ (n+2)$, &c.; but when the series shall be decreasing, a small line is drawn over the number, which signifies the common difference; thus,

$$n^{2|\bar{1}} = n\ (n-1), \text{ and } n^{3|\bar{1}} = n\ (n-1)\ (n-2), \text{ \&c.}$$

These are known by the name of *factorial*. Sometimes the first term, though a simple quantity, is written like a compound quantity, thus, $(n+0)$, or $(n-0)$, according as the series shall be ascending or descending.

A factorial may be divided into two factorial factors, by dividing the exponent into two parts; making one of those parts the exponent of the base or first term (for the first factorial factor), and the other part the exponent of the other factorial factor, whose base is had by adding the exponent of the first factorial factor to the base: as in these,

$$n^{5|1} = n^{2|1} \times (n+2)^{3|1} = n\ (n+1) \times (n+2)\ (n+3)\ (n+4)$$

If the series shall be decreasing, subtract the exponent of the first factorial factor from the base, and the remainder will be the base of the other factorial factor; thus, $n^{5|\bar{1}} = n^{2|\bar{1}} \times (n-2)^{3|\bar{1}} = n\ (n-1) \times (n-2)\ (n-3)\ (n-4)$.

When

When the common difference shall be any other number than 1, the exponent must be multiplied thereby, and the product added to, or subtracted from the base, for the other factorial factor.

It is also necessary to say some little about Figurate Numbers. Let m, and n, represent absolute whole numbers, then $\frac{n^{m-1|1}}{1^{m-1|1}}$ will represent the nth term of the mth order. Now expound m, and n, by the numbers 1, 2, 3, 4, &c. thus;

First order . . . $\frac{1^0}{1^0}, \frac{2^0}{1^0}, \frac{3^0}{1^0}, \frac{4^0}{1^0}$, &c.

Second order . . . $\frac{1^1}{1^1}, \frac{2^1}{1^1}, \frac{3^1}{1^1}, \frac{4^1}{1^1}$, &c.

Third order . . . $\frac{1^{2|1}}{1^{2|1}}, \frac{2^{2|1}}{1^{2|1}}, \frac{3^{2|1}}{1^{2|1}}, \frac{4^{2|1}}{1^{2|1}}$, &c.

Fourth order . . . $\frac{1^{3|1}}{1^{3|1}}, \frac{2^{3|1}}{1^{3|1}}, \frac{3^{3|1}}{1^{3|1}}, \frac{4^{3|1}}{1^{3|1}}$, &c.

&c. &c.

When m shall be $=1$, then $m-1=0$; and when 0 shall be the exponent, the power will be 1, whatever the root be; so that the first order is a rank of units; the second order is the natural scale of numbers; or, in other terms, a rank of numbers in arithmetical progression, the first number and common difference being 1. I have not written the first and second orders like Factorials, but like Powers, as they contain but one term, and therefore become Powers. The Numbers come out in the following manner;

First order . . .	1,	1,	1,	1,	&c.
Second order . . .	1,	2,	3,	4,	&c.
Third order . . .	1,	3,	6,	10,	&c.
Fourth order . . .	1,	4,	10,	20,	&c.

&c.

Any

Any term in any order, is the sum of all the terms, of the next preceding order, up to that place.

A rank of figurate numbers may be signified by a negative increment or common difference, as in these;

$$\frac{(n-0)^{m-1|\bar{1}}}{1^{m-1|1}},\ \frac{(n-1)^{m-1|\bar{1}}}{1^{m-1|1}},\ \frac{(n-2)^{m-1|\bar{1}}}{1^{m-1|1}},\ \frac{(n-3)^{m-1|\bar{1}}}{1^{m-1|1}},\ \&c.$$

in which m is the number of the order, and $(n-0,)$ $(n-1)$, $(n-2)$, $(n-3)$, &c. are the numbers of the respective terms; and therefore the series goes backwards; for, put $r=n-3$, then $r+1=n-2$, $r+2=n-1$, and $r+3=n-0$.

SECT. VI.

THE GENERAL DEMONSTRATION.

There is a series given by Sir Isaac Newton, whereby any power of a binomial root may be expressed in general terms; thus,

Let n denote the exponent of the power; then

$$(r+y)^n = r^n + \frac{n}{1}r^{n-1}y + \frac{n^{2|\bar{1}}}{1^{2|1}}r^{n-2}y^2 + \frac{n^{3|\bar{1}}}{1^{3|1}}r^{n-3}y^3 + \frac{n^{4|\bar{1}}}{1^{4|1}}r^{n-4}y^4 + \frac{n^{5|\bar{1}}}{1^{5|1}}r^{n-5}y^5 + \&c.$$

and the nth power of $(r-y)$ will be expressed in the very same manner, only the signs must be negative where the odd powers of y are involved.

Let it be required to extract the root of this equation,

$$x^n + hx^{n-1} + ix^{n-2} + kx^{n-3} + lx^{n-4} + \&c. = N;$$

Put $x=r+y$; this equation being raised to all the powers of x in the given equation, and multiplied by the respective coefficients, will give the following equations;

$x^n =$

$$x^n = r^n + \frac{n}{1} r^{n-1} y + \frac{n^{2|1}}{1^{2|1}} r^{n-2} y^2 + \frac{n^{3|1}}{1^{3|1}} r^{n-3} y^3 + \frac{n^{4|1}}{1^{4|1}} r^{n-4} y^4 + \&c.$$

$$hx^{n-1} = hr^{n-1} + \frac{n-1}{1} hr^{n-2} y + \frac{(n-1)^{2|1}}{1^{2|1}} hr^{n-3} y^2 + \frac{(n-1)^{3|1}}{1^{3|1}} hr^{n-4} y^3 + \frac{(n-1)^{4|1}}{1^{4|1}} hr^{n-5} y^4 + \&c.$$

$$ix^{n-2} = ir^{n-2} + \frac{n-2}{1} ir^{n-3} y + \frac{(n-2)^{2|1}}{1^{2|1}} ir^{n-4} y^2 + \frac{(n-2)^{3|1}}{1^{3|1}} ir^{n-5} y^3 + \frac{(n-2)^{4|1}}{1^{4|1}} ir^{n-6} y^4 + \&c.$$

$$kx^{n-3} = kr^{n-3} + \frac{n-3}{1} kr^{n-4} y + \frac{(n-3)^{2|1}}{1^{2|1}} kr^{n-5} y^2 + \frac{(n-3)^{3|1}}{1^{3|1}} kr^{n-6} y^3 + \frac{(n-3)^{4|1}}{1^{4|1}} kr^{n-7} y^4 + \&c.$$

$$lx^{n-4} = lr^{n-4} + \frac{n-4}{1} lr^{n-5} y + \frac{(n-4)^{2|1}}{1^{2|1}} lr^{n-6} y^2 + \frac{(n-4)^{3|1}}{1^{3|1}} lr^{n-7} y^3 + \frac{(n-4)^{4|1}}{1^{4|1}} lr^{n-8} y^4 + \&c.$$

&c. &c.

The sum

$$N = A + By + Cy^2 + Dy^3 + Ey^4 + \&c.$$

This last line, being the sum of all those above (by supposition); then transpose A, and we get $N - A = By + Cy^2 + Dy^3 + Ey^4 +$ &c. This is the first resolvend; therefore $B + C + D + E +$ &c. is the first imperfect divisor. Put a for the second figure of the root, and make ${}_2r = r + a$; then, by a process similar to what was done in the cubic and biquadratic equations, the subtrahend may be formed.

Now suppose a to be known, ${}_2r$ becomes the known part of the root; and by putting u for the remaining unknown part of the root, we have $x = {}_2r + u$. Raise this equation to all the powers of x in the given equation, and multiply by the respective coefficients, and we get these following equations:

$$x^n = {}_2r^n + \frac{n-0}{1}\,{}_2r^{n-1}u + \frac{(n-0)^{2|\overline{1}}}{1^{2|1}}\,{}_2r^{n-2}u^2 + \frac{(n-0)^{3|\overline{1}}}{1^{3|1}}\,{}_2r^{n-3}u^3 + \frac{(n-0)^{4|\overline{1}}}{1^{4|1}}\,{}_2r^{n-4}u^4 + \text{\&c.}$$

$$hx^{n-1} = h\,{}_2r^{n-1} + \frac{n-1}{1}h\,{}_2r^{n-2}u + \frac{(n-1)^{2|\overline{1}}}{1^{2|1}}h\,{}_2r^{n-3}u^2 + \frac{(n-1)^{3|\overline{1}}}{1^{3|1}}h\,{}_2r^{n-4}u^3 + \frac{(n-1)^{4|\overline{1}}}{1^{4|1}}h\,{}_2r^{n-5}u^4 + \text{\&c.}$$

$$ix^{n-2} = i\,{}_2r^{n-2} + \frac{n-2}{1}i\,{}_2r^{n-3}u + \frac{(n-2)^{2|\overline{1}}}{1^{2|1}}i\,{}_2r^{n-4}u^2 + \frac{(n-2)^{3|\overline{1}}}{1^{3|1}}i\,{}_2r^{n-5}u^3 + \frac{(n-2)^{4|\overline{1}}}{1^{4|1}}i\,{}_2r^{n-6}u^4 + \text{\&c.}$$

$$kx^{n-3} = k\,{}_2r^{n-3} + \frac{n-3}{1}k\,{}_2r^{n-4}u + \frac{(n-3)^{2|\overline{1}}}{1^{2|1}}k\,{}_2r^{n-5}u^2 + \frac{(n-3)^{3|\overline{1}}}{1^{3|1}}k\,{}_2r^{n-6}u^3 + \frac{(n-3)^{4|\overline{1}}}{1^{4|1}}k\,{}_2r^{n-7}u^4 + \text{\&c.}$$

$$lx^{n-4} = l\,{}_2r^{n-4} + \frac{n-4}{1}l\,{}_2r^{n-5}u + \frac{(n-4)^{2|\overline{1}}}{1^{2|1}}l\,{}_2r^{n-6}u^2 + \frac{(n-4)^{3|\overline{1}}}{1^{3|1}}l\,{}_2r^{n-7}u^3 + \frac{(n-4)^{4|\overline{1}}}{1^{4|1}}l\,{}_2r^{n-8}u^4 + \text{\&c.}$$

&c. &c.

The sum

$$N = {}_2A + {}_2Bu + {}_2Cu^2 + {}_2Du^3 + {}_2Eu^4 + \text{\&c.}$$

${}_2A$, ${}_2B$, ${}_2C$, ${}_2D$, and ${}_2E$, &c. being functions of r, a, and the known coefficients of the given equation, and u, the remaining unknown part of the root. Now by transposition we obtain this equation;

$$N - {}_2A = {}_2Bu + {}_2Cu^2 + {}_2Du^3 + {}_2Eu^4 +, \text{\&c.}$$

This is the new resolvend; therefore ${}_2B + {}_2C + {}_2D + {}_2E +$, &c. is the new imperfect divisor.

The next thing to be done is, to raise the canons for finding the values of ${}_2B$, ${}_2C$, ${}_2D$, &c. without involution, thus;

$$\text{Because } {}_2B = \frac{n-0}{1}\,{}_2r^{n-1} + \frac{n-1}{1}h\,{}_2r^{n-2} + \frac{n-2}{1}i\,{}_2r^{n-3} + \frac{n-3}{1}k\,{}_2r^{n-4} + \frac{n-4}{1}l\,{}_2r^{n-5} + \text{\&c.}$$

and because ${}_2r = r + a$, raise this value of ${}_2r$ to all the powers of ${}_2r$ in the equation which gives the value of ${}_2B$, and multiply by the respective coefficients, and we get these following equations:

$$\frac{n-0}{1}\,{}_2r^{n-1} = \frac{n-0}{1}r^{n-1} + \frac{n-0}{1}\times\frac{n-1}{1}r^{n-2}a + \frac{n-0}{1}\times\frac{(n-1)^{2|\overline{1}}}{1^{2|1}}r^{n-3}a^2 + \frac{n-0}{1}\times\frac{(n-1)^{3|\overline{1}}}{1^{3|1}}r^{n-4}a^3 + \&c.$$

$$\frac{n-1}{1}h\,{}_2r^{n-2} = \frac{n-1}{1}hr^{n-2} + \frac{n-1}{1}\times\frac{n-2}{1}hr^{n-3}a + \frac{n-1}{1}\times\frac{(n-2)^{2|\overline{1}}}{1^{2|1}}hr^{n-4}a^2 + \frac{n-1}{1}\times\frac{(n-2)^{3|\overline{1}}}{1^{3|1}}hr^{n-5}a^3 + \&c.$$

$$\frac{n-2}{1}i\,{}_2r^{n-3} = \frac{n-2}{1}ir^{n-3} + \frac{n-2}{1}\times\frac{n-3}{1}ir^{n-4}a + \frac{n-2}{1}\times\frac{(n-3)^{2|\overline{1}}}{1^{2|1}}ir^{n-5}a^2 + \frac{n-2}{1}\times\frac{(n-3)^{3|\overline{1}}}{1^{3|1}}ir^{n-6}a^3 + \&c.$$

$$\frac{n-3}{1}k\,{}_2r^{n-4} = \frac{n-3}{1}kr^{n-4} + \frac{n-3}{1}\times\frac{n-4}{1}kr^{n-5}a + \frac{n-3}{1}\times\frac{(n-4)^{2|\overline{1}}}{1^{2|1}}kr^{n-6}a^2 + \frac{n-3}{1}\times\frac{(n-4)^{3|\overline{1}}}{1^{3|1}}kr^{n-7}a^3 + \&c.$$

$$\frac{n-4}{1}l\,{}_2r^{n-5} = \frac{n-4}{1}lr^{n-5} + \frac{n-4}{1}\times\frac{n-5}{1}lr^{n-6}a + \frac{n-4}{1}\times\frac{(n-5)^{2|\overline{1}}}{1^{2|1}}lr^{n-7}a^2 + \frac{n-4}{1}\times\frac{(n-5)^{3|\overline{1}}}{1^{3|1}}lr^{n-8}a^3 + \&c.$$

&c. &c.

The sum

$${}_2B = B + 2Ca + 3Da^2 + 4Ea^3$$

By comparing these columns with those where the same capital letters are written in the preceding pages for the sums of the coefficients of the respective powers of y, they will be found to be the multiples thereof, agreeing with the numbers here attached to them.

$$\text{Because } {}_2C = \frac{(n-0)^{2|\overline{1}}}{1^{2|1}}\,{}_2r^{n-2} + \frac{(n-1)^{2|\overline{1}}}{1^{2|1}}h\,{}_2r^{n-3} + \frac{(n-2)^{2|\overline{1}}}{1^{2|1}}i\,{}_2r^{n-4} + \frac{(n-3)^{2|\overline{1}}}{1^{2|1}}k\,{}_2r^{n-5} + \&c.$$

therefore raise the equation ${}_2r = r + a$ to all the powers of ${}_2r$ in the preceding equation, which gives the value of ${}_2C$; and multiply by the respective coefficients, and we get the following equations:

$$\frac{(n-0)^{2|\bar{1}}}{1^{2|1}}\,{}_2r^{n-2} = \frac{(n-0)^{2|\bar{1}}}{1^{2|1}}\,r^{n-2} + \frac{(n-0)^{2|\bar{1}}}{1^{2|1}} \times \frac{n-2}{1}\,r^{n-3}a + \frac{(n-0)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-2)^{2|\bar{1}}}{1^{2|1}}\,r^{n-4}a^2 + \frac{(n-0)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-2)^{3|\bar{1}}}{1^{3|1}}\,r^{n-5}a^3 + \&c.$$

$$\frac{(n-1)^{2|\bar{1}}}{1^{2|1}}\,h_2r^{n-3} = \frac{(n-1)^{2|\bar{1}}}{1^{2|1}}\,hr^{n-3} + \frac{(n-1)^{2|\bar{1}}}{1^{2|1}} \times \frac{n-3}{1}\,hr^{n-4}a + \frac{(n-1)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-3)^{2|\bar{1}}}{1^{2|1}}\,hr^{n-5}a^2 + \frac{(n-1)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-3)^{3|\bar{1}}}{1^{3|1}}\,hr^{n-6}a^3 + \&c.$$

$$\frac{(n-2)^{2|\bar{1}}}{1^{2|1}}\,i_2r^{n-4} = \frac{(n-2)^{2|\bar{1}}}{1^{2|1}}\,ir^{n-4} + \frac{(n-2)^{2|\bar{1}}}{1^{2|1}} \times \frac{n-4}{1}\,ir^{n-5}a + \frac{(n-2)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-4)^{2|\bar{1}}}{1^{2|1}}\,ir^{n-6}a^2 + \frac{(n-2)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-4)^{3|\bar{1}}}{1^{3|1}}\,ir^{n-7}a^3 + \&c.$$

$$\frac{(n-3)^{2|\bar{1}}}{1^{2|1}}\,k_2r^{n-5} = \frac{(n-3)^{2|\bar{1}}}{1^{2|1}}\,kr^{n-5} + \frac{(n-3)^{2|\bar{1}}}{1^{2|1}} \times \frac{n-5}{1}\,kr^{n-6}a + \frac{(n-3)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-5)^{2|\bar{1}}}{1^{2|1}}\,kr^{n-7}a^2 + \frac{(n-3)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-5)^{3|\bar{1}}}{1^{3|1}}\,kr^{n-8}a^3 + \&c.$$

$$\frac{(n-4)^{2|\bar{1}}}{1^{2|1}}\,l_2r^{n-6} = \frac{(n-4)^{2|\bar{1}}}{1^{2|1}}\,lr^{n-6} + \frac{(n-4)^{2|\bar{1}}}{1^{2|1}} \times \frac{n-6}{1}\,lr^{n-7}a + \frac{(n-4)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-6)^{2|\bar{1}}}{1^{2|1}}\,lr^{n-8}a^2 + \frac{(n-4)^{2|\bar{1}}}{1^{2|1}} \times \frac{(n-6)^{3|\bar{1}}}{1^{3|1}}\,lr^{n-9}a^3 + \&c.$$

The sum

$${}_2C = C + \frac{2^{2|1}}{1^{2|1}}Da + \frac{3^{2|1}}{1^{2|1}}Ea^2 + \frac{4^{2|1}}{1^{2|1}}Fa^3 + \&c.$$

By comparing the preceding with those equations at page 25, where the capital letters are put for the sums of the coefficients of the respective powers of y, it will be found that the column of coefficients under which C is written, both there and here, agree exactly, and that the others differ only in the factorial part thereof: the numerators are equal, though there they are given in the form of one factorial: but here they are broken into two factorial factors; but the denominators are not equal; being here in two factorials, the base (or first term) being 1 in both these. When two fractional quantities have the same numerator, but different denominators, the greatest is that which has the least denominator, as in these $\frac{a}{3b}$, $\frac{a}{b}$, the ratio they bear to each other is to be had by dividing the greatest denominator by the least denominator: also if the denominators should not be measurable, the one by the other, the fractional quotient by such division will still be the ratio, as in these, $\frac{a}{bc}$, $\frac{a}{cd}$, by dividing the denominator of the latter by the denominator of the former, the quotient is $\frac{d}{b}$, which is the ratio; for $\frac{d}{b} \times \frac{a}{cd} = \frac{a}{bc}$; so it is with the factorial coefficients now treating of. When two factorials have the same base (or first term) the greatest may be divided by the least, by adding the least exponent to the base, and taking the sum for the base of the quotient, and subtract the exponent of the divisor from the exponent of the dividend, and the difference is the exponent of the quotient. Thus in the preceding there was $1^{3|1}$ to be divided by $(1^{2|1} \times 1)$; $1^{4|1}$ by $(1^{2|1} \times 1^{2|1})$; $1^{5|1}$ by $(1^{2|1} \times 1^{3|1})$, &c. By the preceding rule the division is performed, dividing first by the last of the two factors, and writing the other, viz. $1^{2|1}$, under the quotient, as a denominator.

$$\text{Because } {}_2D = \frac{(n-0)^{3|\overline{1}}}{1^{3|1}}\,{}_2r^{n-3} + \frac{(n-1)^{3|\overline{1}}}{1^{3|1}}\,h\,{}_2r^{n-4} + \frac{(n-2)^{3|\overline{1}}}{1^{3|1}}\,i\,{}_2r^{n-5} + \frac{(n-3)^{3|\overline{1}}}{1^{3|1}}\,k\,{}_2r^{n-6} + \frac{(n-4)^{3|\overline{1}}}{1^{3|1}}\,l\,{}_2r^{n-7} + \text{\&c.}$$

therefore raise the equation ${}_2r = r + a$ to all the powers of ${}_2r$ in the preceding equation, which gives the value of ${}_2D$; and multiply by the respective coefficients, and we get the following equations:

$$\frac{(n-0)^{3|\overline{1}}}{1^{3|1}}\,{}_2r^{n-3} = \frac{(n-0)^{3|\overline{1}}}{1^{3|1}}\,r^{n-3} + \frac{(n-0)^{3|\overline{1}}}{1^{3|1}} \times \frac{n-3}{1}\,r^{n-4}a + \frac{(n-0)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-3)^{2|\overline{1}}}{1^{2|1}}\,r^{n-5}a^2 + \frac{(n-0)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-3)^{3|\overline{1}}}{1^{3|1}}\,r^{n-6}a^3 + \text{\&c.}$$

$$\frac{(n-1)^{3|\overline{1}}}{1^{3|1}}\,h\,{}_2r^{n-4} = \frac{(n-1)^{3|\overline{1}}}{1^{3|1}}\,hr^{n-4} + \frac{(n-1)^{3|\overline{1}}}{1^{3|1}} \times \frac{n-4}{1}\,hr^{n-5}a + \frac{(n-1)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-4)^{2|\overline{1}}}{1^{2|1}}\,hr^{n-6}a^2 + \frac{(n-1)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-4)^{3|\overline{1}}}{1^{3|1}}\,hr^{n-7}a^3 + \text{\&c.}$$

$$\frac{(n-2)^{3|\overline{1}}}{1^{3|1}}\,i\,{}_2r^{n-5} = \frac{(n-2)^{3|\overline{1}}}{1^{3|1}}\,ir^{n-5} + \frac{(n-2)^{3|\overline{1}}}{1^{3|1}} \times \frac{n-5}{1}\,ir^{n-6}a + \frac{(n-2)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-5)^{2|\overline{1}}}{1^{2|1}}\,ir^{n-7}a^2 + \frac{(n-2)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-5)^{3|\overline{1}}}{1^{3|1}}\,ir^{n-8}a^3 + \text{\&c.}$$

$$\frac{(n-3)^{3|\overline{1}}}{1^{3|1}}\,k\,{}_2r^{n-6} = \frac{(n-3)^{3|\overline{1}}}{1^{3|1}}\,kr^{n-6} + \frac{(n-3)^{3|\overline{1}}}{1^{3|1}} \times \frac{n-6}{1}\,kr^{n-7}a + \frac{(n-3)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-6)^{2|\overline{1}}}{1^{2|1}}\,kr^{n-8}a^2 + \frac{(n-3)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-6)^{3|\overline{1}}}{1^{3|1}}\,kr^{n-9}a^3 + \text{\&c.}$$

$$\frac{(n-4)^{3|\overline{1}}}{1^{3|1}}\,l\,{}_2r^{n-7} = \frac{(n-4)^{3|\overline{1}}}{1^{3|1}}\,lr^{n-7} + \frac{(n-4)^{3|\overline{1}}}{1^{3|1}} \times \frac{n-7}{1}\,lr^{n-8}a + \frac{(n-4)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-7)^{2|\overline{1}}}{1^{2|1}}\,lr^{n-9}a^2 + \frac{(n-4)^{3|\overline{1}}}{1^{3|1}} \times \frac{(n-7)^{3|\overline{1}}}{1^{3|1}}\,lr^{n-10}a^3 + \text{\&c.}$$

&c. &c.

The sum

$${}_2D = D + \frac{2^{3|1}}{1^{3|1}}Ea + \frac{3^{3|1}}{1^{3|1}}Fa^2 + \frac{4^{3|1}}{1^{3|1}}Ga^3 + \text{\&c.}$$

The figurate coefficients are derived like the former.

In like manner, because $${}_2E = \frac{(n-0)^{4|\overline{1}}}{1^{4|1}}\,{}_2r^{n-4} + \frac{(n-1)^{4|\overline{1}}}{1^{4|1}} h\,{}_2r^{n-5} + \frac{(n-2)^{4|\overline{1}}}{1^{4|1}} i\,{}_2r^{n-6} + \frac{(n-3)^{4|\overline{1}}}{1^{4|1}} k\,{}_2r^{n-7} + \frac{(n-4)^{4|\overline{1}}}{1^{4|1}} l\,{}_2r^{n-8} + \text{\&c.}$$

therefore raise the equation ${}_2r = r + a$ to all the powers of ${}_2r$ in the preceding equation, which gives the value of ${}_2E$; and multiply by the respective coefficients, and we get the following equations:

$$\frac{(n-0)^{4|\overline{1}}}{1^{4|1}}\,{}_2r^{n-4} = \frac{(n-0)^{4|\overline{1}}}{1^{4|1}} r^{n-4} + \frac{(n-0)^{4|\overline{1}}}{1^{4|1}} \times \frac{n-4}{1} r^{n-5}\, a + \frac{(n-0)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-4)^{2|\overline{1}}}{1^{2|1}} r^{n-6}\, a^2 + \frac{(n-0)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-4)^{3|\overline{1}}}{1^{3|1}} r^{n-7}\, a^3 + \text{\&c.}$$

$$\frac{(n-1)^{4|\overline{1}}}{1^{4|1}} h\,{}_2r^{n-5} = \frac{(n-1)^{4|\overline{1}}}{1^{4|1}} h r^{n-5} + \frac{(n-1)^{4|\overline{1}}}{1^{4|1}} \times \frac{n-5}{1} h r^{n-6} a + \frac{(n-1)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-5)^{2|\overline{1}}}{1^{2|1}} h r^{n-7} a^2 + \frac{(n-1)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-5)^{3|\overline{1}}}{1^{3|1}} h r^{n-8} a^3 + \text{\&c.}$$

$$\frac{(n-2)^{4|\overline{1}}}{1^{4|1}} i\,{}_2r^{n-6} = \frac{(n-2)^{4|\overline{1}}}{1^{4|1}} i r^{n-6} + \frac{(n-2)^{4|\overline{1}}}{1^{4|1}} \times \frac{n-6}{1} i r^{n-7} a + \frac{(n-2)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-6)^{2|\overline{1}}}{1^{2|1}} i r^{n-8} a^2 + \frac{(n-2)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-6)^{3|\overline{1}}}{1^{3|1}} i r^{n-9} a^3 + \text{\&c.}$$

$$\frac{(n-3)^{4|\overline{1}}}{1^{4|1}} k\,{}_2r^{n-7} = \frac{(n-3)^{4|\overline{1}}}{1^{4|1}} k r^{n-7} + \frac{(n-3)^{4|\overline{1}}}{1^{4|1}} \times \frac{n-7}{1} k r^{n-8} a + \frac{(n-3)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-7)^{2|\overline{1}}}{1^{2|1}} k r^{n-9} a^2 + \frac{(n-3)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-7)^{3|\overline{1}}}{1^{3|1}} k r^{n-10} a^3 + \text{\&c.}$$

$$\frac{(n-4)^{4|\overline{1}}}{1^{4|1}} l\,{}_2r^{n-8} = \frac{(n-4)^{4|\overline{1}}}{1^{4|1}} l r^{n-8} + \frac{(n-4)^{4|\overline{1}}}{1^{4|1}} \times \frac{n-8}{1} l r^{n-9} a + \frac{(n-4)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-8)^{2|\overline{1}}}{1^{2|1}} l r^{n-10} a^2 + \frac{(n-4)^{4|\overline{1}}}{1^{4|1}} \times \frac{(n-8)^{3|\overline{1}}}{1^{3|1}} l r^{n-11} a^3 + \text{\&c.}$$

&c. &c.

The sum

$${}_2E = E + \frac{2^{4|1}}{1^{4|1}} F a + \frac{3^{4|1}}{1^{4|1}} G a^2 + \frac{4^{4|1}}{1^{4|1}} H a^3 + \text{\&c.}$$

These equations, which give the values of ${}_2B$, ${}_2C$, ${}_2D$, &c. will be found to agree with those delivered in the second section.

Example 7.

Let it be required to find one of the roots of this equation;

$$x^3 - 7035x^2 + 15262754x = 10000730880;$$

by comparing this with the general equation we have $h = -7035$, $i = 15262754$, and $N = 10000730880$. Make $r = 2000$, for $-7035 = h$, take the arithmetical equivalent $\bar{1}2965$,

$r^3 =$ 8000000000	$3r^2 =$ 12000000	$3r =$ 6000	
$hr^2 = \bar{1}71860000000$	$2hr = \bar{1}71860000$	$h = \bar{1}2965$	
$ir =$ 30525508000	$i =$ 15262754		$D = 1$
$A =$ 10385508000	$B = \bar{1}99122754$	$C = \bar{1}8965$	
	or $B =$ $\bar{1}122754$		

A, being greater than N, the resolvend becomes negative; but the divisor being so too, the quotient will be affirmative; therefore it will be more convenient to change the signs; by which they become,

$B = 877246$,	$C = 1035$	$D = -1 = \bar{1}$
${}_2B = B + 2Ca + 3Da^2$	${}_2C = C + 3Da$	${}_2D = D$
${}_3B = {}_2B + 2{}_2Cb + 3{}_2Db^2$	${}_3C = {}_2C + 3{}_2Db$	

$$A = 10385508000$$
$$N = 10000730880$$

$B = 877246)$ $\quad$ $384777120 \,(a = 300$

$C = 1035 : \times 300 = {}^{*}310500 = Ca$

$3\,Da = \bar{1}100$ $\quad$ ${}^{**}\bar{1}10000 = Da^2$

divisor ${}^{*}\,1097746 : \times 300 = 329323800$ subtrahend

${}_2B = 1228246)$ $\quad$ $55453320 \,(b = 40$

${}_2C = 135 : \times 40 = {}^{*}\ 5400 = {}_2Cb$

$3\,Db = \bar{1}880$ $\quad$ ${}^{**}\bar{1}8400 = Db^2$

divisor ${}^{*}1232046 : \times 40 = 49281840$ subtrahend

${}_3B = 1234246)$ $\quad$ $6171480 \,(c = 5$

${}_3C = 15 : \times 5 = 75 = {}_3Cc$

$\bar{1}75 = Dc^2$

divisor $1234296 : \times 5 = 6171480$ subtrahend

0

$$r = 2000$$
$$a = 300$$
$$b = 40$$
$$c = 5$$
$$x = 2345$$

Example 8.

Let it be required to find the roots of this equation, $x^3 - 3x = -1$; make $r = 1$, then $A = -2 = \bar{1}8$, $B = 0$, $C = 3$; as there is no rational root to this equation, I shall make use of the contraction before hinted at.

$N = -1 = \bar{1}9$ (1,53208888623

$A = -2 = \bar{1}8$

$B = 0,$ 1,000 ($a = ,5$

$C = 3 : \times a =$ . . . . * . 1,5

$3a = 1,5$

* * ,25 $= a^2$

divisor 1,75 : $\times a =$,875 subtrahend

${}_2B =$ 3,75 125000 ($b = ,03$

${}_2C = 4,5 : \times b =$ . . . * . ,135

$3b =$,09

* * . . . 9 $= b^2$

divisor * 3,8859 : $\times b =$ 116577 subtrahend

${}_3B = 4,0227$ 8423000 ($c = ,002$

${}_3C = 4,59 : \times c =$ . . * . . . 918

$3c =$,006

* * 4 $= c^2$

divisor * 4,031884 : $\times c =$ 8063768 subtrahend

${}_4B = 4,041072$ 359232000 ($d = 0$

${}_4C = 4,596$ 00 } $\times 0 =$ 0 { nothing to subtract

$3d =$ 00 00

${}_5B = 4,04107200$ 359232000 ($e = ,00008$

${}_5C = 4,5960 : \times e =$ . . * . . . 36768 +

divisor * 4,04143968 : $\times e =$ 323315174 +, subtrahend

${}_6B =$ 4,0418073 + 35916826 ($f = ,000008$

${}_5C \times f =$ * . . . . 367 +

divisor * 4,0418440 : $\times f =$ 32334752 +, subtrahend

${}_7B =$ 4,041880 + 3582074 ($g = ,0000008$

${}_5C \times g =$ . . . . . . 3 +

4,041883 : $\times g =$ 3233506 +, subtrahend

${}_8B =$ 4,04188 +. 348568 (8623

323350 +

25218

24251 +

967

808

159

121

38

As B is $=0$, it may seem difficult at first, how to find the value of a; but we get over that by considering the subtrahend to be $Ba+Ca^2+a^3$; now because $B=0$, it becomes Ca^2+a^3, which being divided by C, the quotient will be but a little more than a^2; divide $1{,}00=$ the resolvend by $3=C$, and the quotient will be $,33+$ the square root of which is less than $,6$ and more than $,5$, therefore 5 must be the value of a.

After four figures of decimal parts are found, I annex no more ciphers to the resolvends, but abridge the work, by striking off a figure from every succeeding divisor; by putting a dot under the figure, which is only to be regarded for its increase in multiplication; and two figures are struck off from $_5C$ each time, in the like manner. After the last figure is so struck off, it becomes plain division: the reason of all which will plainly appear, by considering the value of each respective quotient figure.

Example 9.

For example sake, let r be taken greater than the truth in the preceding equation. Make $r=2$; then, by the general rule,

$r^3=8$	$3r^2=12$	$3r=6$
$-3r=\bar{1}4$	$-3=\bar{1}7$	
$A=2$	$B=9$	$C=6$

$$N=-1$$
$$A=2$$
$$-3 \text{ resolvend.}$$

As the resolvend is negative, and the divisor affirmative, the quotient will be negative; but the trouble of multiplying by a negative number is so much more than multiplying by an affirmative number, on account of changing the signs at every multiplication; therefore it will be more convenient to change the signs of every other term of the divisor; thus the first resolvend is,

$$-3 = N - A = -By + Cy^2 - y^3$$

As the unknown part of the root y is negative, those terms must be negative where the odd powers thereof are involved; change the signs, and it becomes,

$$3 = A - N = By - Cy^2 + y^3;$$

therefore make C negative, or rather take the arithmetical equivalent.

Thus,

$$A = 2.\left(r = 2{,}00000000000\right.$$
$$-N = 1.\ \ -\ {,}46791111376$$
$$1{,}53208888624 \text{ the root nearly}$$

	$B =$	$9,$	$3{,}000\left(a = {,}4\right.$
$C = \bar{1}4,\ \times a =$		$*\bar{1}7{,}6$	
$3a = 1{,}2$		$**\ 16 = a^2$	
	divisor	$6{,}76\ :\times a =$	$2{,}704$ subtrahend
	$_2B =$	$4{,}68$	$29600\left(b = {,}06\right.$
$_2C = \bar{1}5{,}2\ :\times b =$		$*\ \bar{1}{,}712$	
$3b = 18$		$**\ 36 = b^2$	
	divisor	$4{,}3956\ :\times b =$	263736 subtrahend
	$_3B =$	$4{,}1148$	$32264000\left(c = {,}007\right.$
$_3C = \bar{1}5{,}38\ :\times c =$		$*\ \bar{1}6766$	
$3c = 21$		$**\ 49 = c^2$	
	divisor	$*4{,}082509\ :\times c =$	28577563 subtrahend
	$_4B =$	$4{,}050267$	$3686437000\left(d = {,}0009\right.$
$_4C = \bar{1}5{,}401 :\times d =$		$*\ \bar{1}58609$	
$3d = 27$		$**\ 81 = d^2$	
	divisor	$*4{,}04612871 \times d =$	3641515839 subtrahend
	$_5B =$	$4{,}04199123$	$44921161\left(e = {,}00001\right.$
$_5C = \bar{1}5{,}4037 :\times e =$		$*\ \bar{1}5403\ +$	
	divisor	$*4{,}04194526 :\times e =$	$40419452 +$, subtrahend
	$_6B =$	$4{,}0418992\ +$	$4501709\left(f = {,}000001\right.$
	$_5C \times f =$	$*\ \bar{1}54\ +$	
	divisor	$*4{,}0418946\ :\times f =$	$4041894 +$, subtrahend
	$_7B =$	$4{,}041890\ +$	$459815\left(11376\right.$
		.....	404189
			55626
			40418
			15208
			12125
			3083
			2828
			255
			242
			13

It may be perceived by the preceding, that if too great a quotient figure were anywhere taken, so that the subtrahend should be greater than the resolvend; the resolvend might be taken from the subtrahend; and by changing the signs of every other term of the divisor, you may proceed as before; but the following quotient figures must have the contrary sign to the preceding, when you add them to the estimate root.

By making $r = 0{,}3$ the lesser root might have been had, by a process like the former of these two examples. But rather let the equation be lowered by division to a quadratic; to do which, let b be put for the known root, to lessen the trouble in division; thus,

$$\begin{array}{l} x - b = 0 \,\big)\, x^3 - 3x + 1 = 0 \,\big(\, x^2 + bx + b^2 - 3 = 0 \\ \qquad\qquad x^3 - bx^2 \\ \hline \qquad\qquad\quad bx^2 - 3x + 1 \\ \qquad\qquad\quad bx^2 - b^2x \\ \hline \qquad\qquad\qquad \left.\begin{array}{r} b^2 \\ -3 \end{array}\right\} x + 1 \\ \qquad\qquad\qquad \left.\begin{array}{r} b^2 \\ -3 \end{array}\right\} x - b^3 + 3b \\ \hline \qquad\qquad\qquad\qquad\quad b^3 - 3b + 1 = 0 \end{array}$$

That this last remainder is equal to nothing, is evident from this consideration; that b being one of the values of x, if x were written in the stead of b, it would become the original equation.

Example 10.

Now to solve this quadratic equation by the general rule, transpose the known part of the equation, and it becomes $x^2 - bx = 3 - b^2$; which turned into numbers, is,

$$x^2 - 1{,}5320888862x = 0{,}65270364466$$

Make $r=0{,}3$ and we shall have

$$A=0{,}54962666587 \qquad B=2{,}1320888862 \qquad C=1.$$

The value of C in this instance is merely nominal; for as 1 does not multiply, it is only writing the quotient figure in the place thereof; therefore B is the only term of the imperfect divisor; the true divisor is $B+a$, and the new imperfect divisor is $B+2a$, so that the quotient figure is only to be added in forming the true divisor, and added again for the new divisor. If x^2 had had a coefficient, that coefficient would have been the value of C.

		$N=,65270364466$	(,3472963553
		$A=,54962666587$	
$B=2{,}1320888862$		10307697879	
$a=$ 4			
divisor 2,1720888862	$:\times a=$	8688355544	subtrahend
${}_2B=2{,}212088886+$		1619342335	
$b=$ 7			
divisor 2,219088886	$:\times b=$	1553362220	subtrahend
${}_3B=2{,}22608888+$		65980115	
$c=$ 2			
divisor 2,22628888	$:\times c=$	44525777	subtrahend
${}_4B=2{,}2264888+$		21454338	
$d=$ 9			
divisor 2,2265788	$:\times d=$	20039209	subtrahend
${}_5B=2{,}226668+$		1415129	
$e=$ 6			
divisor 2,226674	$:\times e=$	1336004	subtrahend
${}_6B=2{,}22668+$		79125	(3553
		66800	
		12325	
		11133	
		1192	
		1113	
		79	
		66	
		13	

I shall now take two equations, deduced from the properties of a circle.

It is demonstrable; if x be the chord of an arc, and N be the chord of three times that arc, the radius $=1$, it will be $3x-x^3=N$.

Also, if x be the chord of an arc, and N be the chord of five times that arc, the radius $=1$, it will be $x^5-5x^3+5x=N$.

Also, if x be the chord of an arc, and N be the chord of fifteen times that arc, the radius $=1$, it will be

$$15x-140x^3+378x^5-450x^7+275x^9-90x^{11}+15x^{13}-x^{15}=N.$$

This is easily deduced from the two preceding equations.

The first of these equations; if N be made $=1$, and then all the signs changed, it will become the equation last solved.

Example 11.

In the equation $x^5-5x^3+5x=N$, make $N=1=$ the chord of 60 degrees.

and by writing the arithmetical equivalent for the negative coefficient, the equation becomes $x^5+\bar{1}5x^3+5x=1$; make $r=0,2$

$r^5=0,00032$	$5r^4=0,008$	$10r^3=\ 0,08$	$10r^2=\ 0,4$	$5r=1;$
$\bar{1}5r^3=0,\bar{1}6000$	$\bar{1}85r^2=\bar{1},400$	$\bar{1}85r=\bar{1}7,00$	$\bar{1}95=\bar{1}5,0$	$E=1,$
$5r=1,00000$	$5=5,000$	$C=\bar{1}7,08$	$D=\bar{1}5,4$	
$A=0,96032$	$B=4,408$			

$N=1,0000000000\,(0,2090569265$, the chord of 12 degrees.
$A=\ \ 96032$
$396800000\ (a=,009$

$E=1,$
$Ea=\ ,009$
$Ea^2=\ ,000081$

$D=\bar{1}5,4$
$Da=\ ,\bar{1}586$

$B\ =4,408$
$C=\bar{1}7,08:a\ =\ .\ .\ .\ .\ .\ \bar{1}7372$
$Da^2=\ .\ .\ .\ \bar{1}6274$
$Ea^3=\ .\ .\ .\ .\ .\ .\ .\ 729$
$a^4=\ .\ .\ .\ .\ .\ .\ .\ .\ 6\ +$
divisor . . . . $4,381348135:\times a=$ $394321332+$ subtrahend
$_2B=4,3543251$ $2478668\ (b=,00005$
$_2C=\bar{1}6,956:\times b=$ * $\bar{1}8478$
divisor . . . *$4,3541729\ :\ \times b=$ $2177086+$ subtrahend
$_3B\ =4,35402$ $301582\ (c=,000006$
$_2Cc=\ \ \bar{1}8$
divisor . . . . $4,35400\ \ :\ \ \times c=$ $261240+$ subtrahend
. . . . .

$40342\ (9265$
39186
1156
871
285
261
24
21
3

$B=4,408$
$2Ca=.,\bar{1}4744$
$3Da^2=.,.\bar{1}88822$
$4Ea^3=\ .\ .\ .\ .\ .\ ,2916$
$5a^4=\ .\ .\ .\ .\ .\ .\ .\ ,32$
$_2B=4,3543251|48$

$C=\bar{1}7,08$
$3\,Da=\ .\bar{1},8758$
$6\,Ea^2=\ .\ .\ .\ .\ .\ .\ 486$
$_2C=\bar{1}6,956|286$

EXAMPLE 12.

In the equation $15x - 140x^3 + 378x^5 - 450x^7 + 275x^9 - 90x^{11} + 15x^{13} - x^{15} = N$, make $N = 1$ = the chord of 60 degrees, and write the arithmetical equivalents for the negative coefficients, and the equation becomes $15x + \bar{1}860x^3 + 378x^5 + \bar{1}550x^7 + 275x^9 + \bar{1}10x^{11} + 15x^{13} + \bar{1}x^{15} = 1$, make $r = ,06$; from which we get these following numbers; $A = 0,8700526758$, $B = 13,51234785$, $C = \bar{1}75,609159$, $D = \bar{1}873,4050$, $E = 110,02$, $F = 344$. These are as many figures as are likely to effect the root to the tenth decimal place.

$N = 1,0000000000$ (,0697989934 = the chord of 4°.
$A = 0,8700526758$
1299473242 ($a = ,009$

$F = 344$,
$Fa = ..3,09$
$Fa^2 = ..\ ,0278$
$Fa^3 = ..\ ,000250$

$Da = \bar{1}8,860645$
$E = 110,02$
$Ea = \ ,9902$
$Ea^2 = \ ,008912$

$D = \bar{1}873,4050$
$4Ea = ..3,96 +$
$10Fa^2 = ...,27$

${}_2D = \bar{1}877,6$
${}_2Db = \ ,\bar{1}143$

$C = \bar{1}75,609159$: $\times a =$
$3Da = \bar{1}6,58193 +$
$6Ea^2 = ...5346$
$10Fa^3 = ...\ 250$

${}_2C = \bar{1}72,2470$: $\times b =$
$3{}_2Db = \bar{1},743$

${}_3C = \bar{1}71,99$: $\times c =$

$B = 13,51234785$
$\times a = ..\bar{1},78048243 +$
$\bar{1}8974580 = Da^2$
$8021 = Ea^3$
$225 = Fa^4$
divisor $13,28265854$: $\times a = 1195439269$ subtrahend
104033973 ($b = ,0007$
${}_2B = 13,0428822$
$\times b = *\ \bar{1}805729$
$*\ *\ \bar{1}400 = {}_2Db^2$
divisor $*13,0233951$: $\times b = 91163766$ subtrahend
12870207 ($c = ,00009$
${}_3B = 13,003848$
$\times c = *\ \bar{1}7479$
divisor $*13,001327$: $\times c = 11701194$ subtrahend
1169013 ($d = ,000008$
${}_4B = 12,99880$
$*\ \bar{1}77$
divisor $*12,99857$: $\times d = 1039886$ subtrahend
129127 (9934
116984
12143
11698
445
389
56
52
4
${}_5B = 12,9983$
....

$B = 13,51234785$
$2Ca = \bar{1},56096486$
$3Da^2 = \ ,\bar{1}6923740$
$4Ea^3 = ...\ 32084$
$5Fa^4 = ...\ 1125$
${}_2B = 13,0428822|0$

The following method of extracting the simple cube root has lately been communicated to me, by the inventor Mr. W. Hall, a native of Roxburghshire in Scotland; and it is at the request of this venerable and most worthy character that I publish it.

Let the number be 665224304197, whose cube-root is required.

MR. HALL'S METHOD.

665224304197 (8729

$8^3 = 512$

153224 resolvend

6400 : × 3 = divisor . . . . 19200 } × 7
560 : × 3 : + 7^2 = ... 1729 }

560 — 146503 subtrahend

7^2 = . 49 — 6721304 resolvend

756900 : × 3 = ... 2270700 } × 2
1740 : × 3 : + 2^2 = ... 5224 }

1740 — 4551848 subtrahend

2^2 = . . 4 — 2169456197 resolvend

76038400 : × 3 = . . . 228115200 } × 9
78480 : × 3 : + 9^2 = ... 235521 }

2055156489 subtrahend

114299708 remainder.

After the first resolvend is formed, the square of the known part of the root is written to the left, with two ciphers annexed; which being multiplied by 3, the product is written under the resolvend, which Mr. Hall calls the divisor; next, the root is multiplied by the quotient figure, with a cipher annexed, and written down under the square of the foreknown part of the root; this number being multiplied by 3, and the square of the quotient figure added in, which is done mentally, and then written under the divisor, the sum of those two numbers is multiplied by the quotient figure, and the product written underneath for a subtrahend, which is subtracted from the resolvend, and the next period of figures is brought down for a new resolvend. The number which is written under the square of the root, is written again; and under that the square of the quotient figure. Those four numbers are added into one sum; which is the square of the increased root, to which two ciphers are annexed, and then multiplied by 3, for a new divisor; and proceed as before.

In contemplating Mr. Hall's Method, I have projected the following method for extracting the cube root, which applies to Cubic Adfected Equations also.

After the first resolvend is formed, write the treble root underneath, in the usual manner, and the treble square at the left end of the resolvend, but at a considerable distance, with two ciphers annexed. When the quotient figure shall be fixed upon, annex it to the treble root, then multiply that compound number by the said quotient figure, and write the product under the treble square; then add it thereto; and write the sum underneath, which will be the true divisor; then multiply the true divisor by the quotient figure, and write the product under the resolvend, for a subtrahend, which subtract from the resolvend, and bring down the next period of figures to the remainder for a new resolvend. Then write the square of the quotient figure under the divisor; then add those three numbers (viz. the square of the quotient figure, the true divisor, and the number which stands immediately above the true divisor) into one sum; which sum will be the treble square of the increased root; with which proceed as before.

I here take the same number as in the last example.

		665224304197 (8729
	$8^3 = 512$	
$80^2 \times 3 =$	19200	153224 resolvend
	1729	247
true divisor	20929 : $\times 7 =$	146503 subtrahend
$7^2 =$	. . 49	6721304 resolvend
	2270700	2612
	5224	
true divisor	2275924 : $\times 2 =$	4551848 subtrahend
$2^2 =$	 4	2169456197 resolvend
	228115200	26169
	235521	
true divisor	228350721 : $\times 9 =$	2055156489 subtrahend
		114299708 remainder.

A

SUPPLEMENT

CONTAINING

AN IMPORTANT IMPROVEMENT IN EQUATIONS,

DISCOVERED BY THE AUTHOR

SINCE THE COMMENCEMENT OF THIS WORK.

THE best way of proving the root of an equation, is by first multiplying it by the coefficient of the first term, if the first term should have any coefficient; if not, it will be only to write down the root, and then add the coefficient of the second term thereto; then multiply that sum by the root, and add the coefficient of the third term thereto; then multiply that sum by the root, and so on, till you come to the last term, the coefficient of which being added, and the sum multiplied by the root, the last product will be the given number, if it be the true root; if not, the several terms of the divisor (for continuing the root further) may be easily formed from the numbers so produced. This may be advantageously done with the assumed root from the beginning.

Let the equation $gx^5 + hx^4 + ix^3 + kx^2 + lx = N$, be proposed; write r for an assumed root. I here make use of some Greek letters, for the convenience of references, and proceed thus;

$$\alpha = gr$$
$$\alpha + h = \beta = gr + h$$
$$\beta r = \gamma = gr^2 + hr$$
$$\gamma + i = \delta = gr^2 + hr + i$$
$$\delta r = \epsilon = gr^3 + hr^2 + ir$$
$$\epsilon + k = \zeta = gr^3 + hr^2 + ir + k$$
$$\zeta r = \eta = gr^4 + hr^3 + ir^2 + kr$$
$$\eta + l = \vartheta = gr^4 + hr^3 + ir^2 + kr + l$$
$$\vartheta r = A = gr^5 + hr^4 + ir^3 + kr^2 + lr$$

By a process like this, the value of A, which is the subtrahend, is obtained; and the values of B, C, D, and E, are obtained by the following canons;

$$B = \vartheta + \eta + \epsilon r + \gamma r^2 + \alpha r^3$$
$$C = \zeta + 2\epsilon + 3\gamma r + 4\alpha r^2$$
$$D = \delta + 3\gamma + 6\alpha r$$
$$E = \beta + 4\alpha$$
$$F = g$$

To work an operation in numbers, the best way that I know is, to write the coefficients in a line, one after another, leaving room enough between for all the figures that may be necessary, as is here done with the letters

g   *h*   *i*   *k*   *l*

If the first term of the equation should have no coefficient, then 1 must be taken for the coefficient, that is $g = 1$; then multiply it by r, and write the product under the coefficient of the second term (h); add it thereto, and write the sum underneath; then multiply that sum by r, and write the product under the coefficient of the third term (i);

add it thereto, and write the sum underneath; and so on. The last sum, which will be under the coefficient of the single power of x, being multiplied by r, will give the subtrahend; and the numbers will stand as the symbols do here;—

g	h	i	k	l
	$\alpha = gr$	$\gamma = gr^2 + hr$	$\varepsilon = gr^3 + hr^2 + ir$	$\eta = gr^4 + hr^3 + ir^2 + kr$
	$\beta = gr + h$	$\delta = gr^2 + hr + i$	$\zeta = gr^3 + hr^2 + ir + k$	$\vartheta = gr^4 + hr^3 + ir^2 + kr + l$

The last of these quantities, being multiplied by the assumed root, gives the subtrahend; which being subtracted from the given number, leaves the resolvend; and the imperfect divisor is to be formed by the preceding canons, which may be in columns, under the preceding work; in the following manner—

g	h	i	k	l
	$\alpha = gr$	$\gamma = gr^2 + hr$	$\varepsilon = gr^3 + hr^2 + ir$	$\eta = gr^4 + hr^3 + ir^2 + kr$
	$\beta = gr + h$	$\delta = gr^2 + hr + i$	$\zeta = gr^3 + hr^2 + ir + k$	$\vartheta = gr^4 + hr^3 + ir^2 + kr + l$
	$4\alpha = 4gr$	$3\gamma = 3gr^2 + 3hr$	$2\varepsilon = 2gr^3 + 2hr^2 + 2ir$	$\eta = gr^4 + hr^3 + ir^2 + kr$
		$6\alpha r = 6gr^2$	$3\gamma r = 3gr^3 + 3hr^2$	$\varepsilon r = gr^4 + hr^3 + ir^2$
			$4\alpha r^2 = 4gr^3$	$\gamma r^2 = gr^4 + hr^3$
				$\alpha r^3 = gr^4$
$F = g$	$E = 5gr + h$	$D = 10gr^2 + 4hr + i$	$C = 10gr^3 + 6hr^2 + 3ir + k$	$B = 5gr^4 + 4hr^3 + 3ir^2 + 2kr + l$

The values of B, C, D, E, and F, found by the rules delivered in the second Section, would agree with the preceding; therefore it requires no further demonstration.

There is room sufficient in the blank spaces for all the extra work; such as αr, αr^2, γr, &c. so that the work may be all together.

But for high equations, such as is the 12th Example of this Tract, where the abridged method is used from the beginning, there is a method which answers better, by which B is formed from the numbers which are produced in forming the subtrahend in the same manner as the subtrahend is formed from the coefficients of the given equation. In like manner, C is formed out of the numbers produced in forming B; and D is formed from the numbers produced in forming C; and so on, in the following manner. Suppose the subtrahend formed as before directed, transcribe the first product (gr), and write it down under the number next below it, viz. $gr + h$; draw a line under, and add it thereto, and write the sum ($2gr + h$) underneath; then multiply it by r, and write the product ($2gr^2 + hr$) in the next column under $gr^2 + hr + i$; draw a line under, and add it thereto; and write the sum ($3gr^2 + 2hr + i$) underneath; multiply it by r, and write the product ($3gr^3 + 2hr^2 + ir$) in the next column under $gr^3 + hr^2 + ir + k$; draw a line under, and add it thereto; and write the sum ($4gr^3 + 3hr^2 + 2ir + k$) underneath; multiply it by r, and write the product ($4gr^4 + 3hr^3 + 2ir^2 + kr$) in the next column, under $gr^4 + hr^3 + ir^2 + kr + l$; draw a line under, and add it thereto; and the sum will be the value of B, and the numbers will stand as the symbols do here.

g	h	i	k	l
	gr	$gr^2 + hr$	$gr^3 + hr^2 + ir$	$gr^4 + hr^3 + ir^2 + kr$
	$gr + h$	$gr^2 + hr + i$	$gr^3 + hr^2 + ir + k$	$gr^4 + hr^3 + ir^2 + kr + l$
	gr	$2gr^2 + hr$	$3gr^3 + 2hr^2 + ir$	$4gr^4 + 3hr^3 + 2ir^2 + kr$
	$2gr + h$	$3gr^2 + 2hr + i$	$4gr^3 + 3hr^2 + 2ir + k$	$5gr^4 + 4hr^3 + 3ir^2 + 2kr + l = B$

Next, bring down the first product gr, and write it under $2gr + h$; draw a line under, and add those two numbers together, and write the sum ($3gr + h$) underneath; multiply it by r, and write the product in the next column, under $3gr^2 + 2hr + i$; draw a line under, and add it thereto; and write the sum ($6gr^2 + 3hr + i$) underneath; then multiply it by r, and write the product in the next column, under

$4\,gr^3+3hr^2+2ir+k$; draw a line under, and add it thereto. The sum will be the value of C.

Again, bring down gr, and write it below $3gr+h$; draw a line under, and add those two numbers together, and write the sum underneath; then multiply the sum by r, and write the product $(4gr^2+hr)$ in the next column under $6gr^2+3hr+i$; draw a line under, and add it thereto. The sum will be the value of D.

Again, bring down the number gr, and write it below $4gr+h$; draw a line under, and add it thereto; and the sum is the value of E; and the whole work will stand in the following manner.

g	h	i	k	l
	gr	gr^2+hr	gr^3+hr^2+ir	$gr^4+hr^3+ir^2+kr$
	$gr+h$	gr^2+hr+i	gr^3+hr^2+ir+k	$gr^4+hr^3+ir^2+kr+l$
	gr	$2gr^2+hr$	$3gr^3+2hr^2+ir$	$4gr^4+3hr^3+2ir^2+kr$
	$2gr+h$	$3gr^2+2hr+i$	$4gr^3+3hr^2+2ir+k$	$5gr^4+4hr^3+3ir^2+2kr+l=B$
	gr	$3gr^2+hr$	$6gr^3+3hr^2+ir$	
	$3gr+h$	$6gr^2+3hr+i$	$10gr^3+6hr^2+3ir+k=C$	
	gr	$4gr^2+hr$		
	$4gr+h$	$10gr^2+4hr+i=D$		
	gr			
	$5gr+h=E$			

I shall now shew the application of this rule, in finding the first imperfect divisor for solving this equation—

$$15x-140x^3+378x^5-450x^7+275x^9-90x^{11}+15x^{13}-x^{15}=1;$$

which is the question solved in the Twelfth Example.

The value of r is found by dividing (1), the number on the known side of the equation, by (15), the coefficient of the single power of x; and taking the first significant figure only, that is $r = ,06$. The root being only required to the tenth decimal place, there will be no occasion to regard the highest power in the equation; the value of r being so small, it is plain, when raised to the fifteenth power, there will be more than fifteen ciphers between the decimal point and the first significant figure. Therefore it will be sufficient to begin with the coefficient of the ninth power of x; writing all the coefficients thence to the coefficient of the single power of x, as before directed; not omitting to write ciphers for the coefficients of those terms which are wanting; thus—

$$275 \;\Big|\; 00,0 \;\Big|\; \bar{1}550,00 \;\Big|\; 0,0000 \;\Big|\; 378,000000 \;\Big|\; 0,00000000 \;\Big|$$

$$\bar{1}860,0000000000 \;\Big|\; 0,0000000000 \;\Big|\; 15,0000000000$$

Multiply 275, the coefficient of the ninth power, by $,06 = r$, and write the product 16,5 under the coefficient of the eighth power, which being nothing, there is nothing to add, (but for the sake of order, it will be well to draw a line under, and write it again underneath, as the sum of the product and coefficient,) which multiply again by $,06 = r$, and write the product under the coefficient of the seventh power; and so on, as before directed.

The multiplier having two places in decimal parts, every product will have two places in decimal parts more than the multiplicand; therefore after there shall be ten places of decimal parts, omit two figures in every succeeding product, to have the value of A, to ten figures, being all that is necessary.

The next figure of the root being expected to be in the third place of decimals, every multiplication thereby will add three to the number of places of decimal parts; but because it will be necessary to regard the increase which arises from the multiplication of the next figure to the right, it will be proper to count off but two figures; that is, the value of B must be had to eight places in decimals; the value of C, to six; the value of D, to four; the value of E, to two; and the value of F may be without decimal parts; the value of G may be dispensed with.

The value of B is had by eight multiplications, and eight additions; the value of C, by seven multiplications, and seven additions; the value of D, by six multiplications, and six additions; the value of E, by five multiplications, and five additions; and the value of F, by four multiplications, and four additions. In all these operations, the only multiplier is ,06; and the first multiplication in each operation is, ,06 × 275. This multiplication may be dispensed with (after the first), by transcribing the first product, as has been shewn in the literal performance. The following is an example of the numeral operation.

275	0,00	$\bar{1}$550,0000	0,000000	378,00000000	0,0000000000	$\bar{1}$860,0000000000	0,0000000000	15,0000000000
	16,50	,9900	$\bar{1}$73,059400	$\bar{1}$8,38356400	22,5830138400	1,3549808304	$\bar{1}$1,6812988498	$\bar{1}$,5008779309
	16,50	$\bar{1}$550,9900	$\bar{1}$73,059400	376,38356400	22,5830138400	$\bar{1}$861,3549808304	$\bar{1}$1,6812988498	14,5008779309 : × ,06 = ,8700526758 | 5 = *A*
	16,50	1,9800	$\bar{1}$73,178200	$\bar{1}$6,77425600	22,38946920	2,69834898	$\bar{1}$1,84319978	$\bar{1}$,01146992
	33,00	$\bar{1}$552,9700	$\bar{1}$46,237600	373,15782000	44,97248304	$\bar{1}$864,05332981	$\bar{1}$83,52449863	13,51234785 = *B*
	16,50	2,9700	$\bar{1}$73,356400	$\bar{1}$5,175640	22,100008	4,024349	$\bar{1}$2,084661	
	49,50	$\bar{1}$555,9400	$\bar{1}$19,594000	368,333460	67,072491	$\bar{1}$868,077678	$\bar{1}$75,609159 = *C*	
	16,50	3,9600	$\bar{1}$73,5940	$\bar{1}$3,5913	21,7155	5,3273		
	66,00	$\bar{1}$559,9000	$\bar{1}$893,1880	361,9247	88,7879	$\bar{1}$873,4049 = *D*		
	16,50	4,95	$\bar{1}$73,89	$\bar{1}$2,02	21,24			
	82,50	$\bar{1}$564,85	$\bar{1}$867,07	353,94	110,02 = *E*			
	16,50	6,	$\bar{1}$74,	$\bar{1}$1,				
	99,0	$\bar{1}$570,	$\bar{1}$841,	344, = *F*				

There is (at every step) an equation between the resolvend and the sum of the several terms of the divisor; each term of the divisor being supposed to be multiplied by the respective powers of the unknown part of the root; and therefore the method shewn here, of forming the divisor from the coefficients of the given equation, may be used in forming every new divisor from the preceding divisor, the new quotient figure being the common multiplier instead of r. Thus, when the subtrahend is taken from the given number, $N - A = By + Cy^2 + Dy^3 + Ey^4 +$ &c. is the equation; in which $B, C, D, E,$ &c. are the coefficients, y the unknown root, and $N - A$ the known side of the equation.

If the coefficient of the single power of x should be much greater than the given number N, and not less than any of the other coefficients in the equation; if then the given number N be divided by the coefficient of the single power of x, the quotient will be nearly equal to one of the roots of the equation.

This rule will often be of service, in fixing on the value of r.

As there are never more than two numbers to be added together, in this method, I have made no use of arithmetical equivalents; it being only necessary to attend to the signs.

Let it be required to find (by the last improved method) one of the roots of this equation, $x^4 - 80x^3 + 1998x^2 - 14937x = -5000$. By dividing -5000 (the known side of the equation) by -14937 (the coefficient of the single power of x), the first figure of one of the roots (viz. $+$ Q,3) is obtained. Write down the coefficients of the given equation, as before directed, and proceed thus—

+ 1	− 80,0	+ 1998,00	− 14937,000		− 5000,0000(0,3
	+ ,3	− 23,91	+ 592,227		
	− 79,7	+ 1974,09	− 14344,773	: × ,3 =	− 4303,4319 subtrahend
	+ ,3	− 23,82	+ 585,081		
	− 79,4	+ 1950,27			
	+ ,3	− 23,73			
	− 79,1				
	+ ,3				
+ 1	− 78,80	+ 1926,5400	− 13759,692000)		− 696,56810000(,05
	+ ,05	− 3,9375	+ 96,130125		
	− 78,75	+ 1922,6025	− 13663,561875	: × ,05 =	− 683,17809375 subtrahend
	+ ,05	− 3,9350	+ 95,933375		
	− 78,70	+ 1918,6675			
	+ ,05	− 3,9325			
	− 78,65				
	+ ,05				
*	− 78,60	+ 1914,7350	− 13567,628500)		− 13,39000625(,0009
		− ,0707	+ 1,723198		
		+ 1914,6643	− 13565,905302	: × 0009 =	− 12,20931477 subtrahend
		− ,07	+ 1,7231		
		+ 1914,59			
		+ 1914,	− 13564,1822)		− 1,18069148(,00008
			+ ,1531		
			− 13564,0291	: × ,00008 =	− 1,08512233 subtrahend
			+ ,153		
		+ 1914,	− 13563,876)		− ,09556915(,000007
			+ 13		
			− 13563,863	: × ,000007 =	− ,09494704 subtrahend
			+ 1		
			− 13563,85)	. . . . .	− ,00062211(,0000000458
					54255
					7956
					6782
					1174
					1085
					89

* Here I desist annexing more ciphers, and make use of the abridgment; in which I put dots under those figures which are only to be regarded for the increase of the next figures by multiplication.

The root is 0,3509870458 +

THE END.

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