Market liquidity and its incorporation into risk management

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The excessively optimistic assessment of market liquidity, i.e. *the belief that transactions can be settled at current prices without any notable delays or transaction costs, may be a serious threat to financial stability –the near failure of the LTCM hedge fund in 1998 was a case in point. Admittedly, the financial community today appears to have a better grasp of the risks arising from liquidity illusion. The fact nonetheless remains that current risk management tools, particularly the most common Value at Risk (VaR) measures, do not capture this complex component of market risk satisfactorily. In fact, standard VaR calculations do not take specific account of the risk to which a portfolio is exposed at the time it is liquidated.*

This article aims to explore the different aspects of liquidity risk and provide signposts to methods for incorporating this risk into existing risk control tools. We first examine "normal" or average liquidity risk, which corresponds to the costs of liquidating or hedging a position in tranquil periods, then illiquidity risk that arises in crisis periods and results in the market's inability to absorb order flows without violent price adjustments. Two separate methodologies, which must nonetheless be combined in a comprehensive approach, are required to analyse these two situations. In the first case we seek to assess the frictions that emerge in imperfect markets by using bid-ask spread measures and by analysing the negative impact on prices resulting from the liquidation of a sizeable portfolio. In the case of extreme risk, we assess the potential consequences of occurrences that are rare, fundamentally uncertain and systemically important.

In each case, we suggest and describe a number of techniques that aim to incorporate these elements into the risk measurement and management systems used by private market participants, while underscoring the obstacles to application given the frequent unavailability of the data required. We show that these techniques are relevant because they provide a more cautious and more realistic assessment of financial institutions' exposure to risk.

Lastly, it is in market participants' own interest for central banks and supervisory bodies to have at their disposal the information required to construct indicators for monitoring market liquidity or conducting sufficiently comprehensive stress tests in order to assess the financial system's resilience to liquidity shocks, while taking into account all the externalities that market participants do not individually consider.

The increased marketability of financial instruments and transferability of risks has been one of the major features of the modernisation of financial systems over the last twenty years. Bank balance sheets have been transformed for both assets and liabilities, with traditional bank intermediation being supplanted by market operations. Structured finance and collateralised transactions have developed rapidly, as have methods for making credit risk marketable. In the current financial environment, a considerable part of balance sheet management and structured financing is thus founded on the assumption that markets for underlying assets will regularly fulfil their transfer function, thanks to the constant presence of counterparties. However, market liquidity, *i.e.* the ability to settle transactions at current prices and at all times with no notable transaction costs, is never totally ensured. In fact, one of the most pernicious threats to market liquidity is the very illusion of its continuity. This illusion means that market participants overestimate their ability to unwind transactions or hedge their positions smoothly and rapidly to meet requirements in unforeseen circumstances, which could lead them to take excessive risks *ex ante*.

Market liquidity is at the heart of central banks' financial stability concerns because it is a precondition for market efficiency and also because its sudden disappearance from markets may degenerate into a systemic crisis, as evidenced by the turbulence of summer and autumn 1998, which resulted in the abrupt drying up of liquidity on the bond markets.

An extensive body of the literature dealing with the microstructure of financial markets has been devoted to identifying the determinants of liquidity and modelling liquidity. Similarly, numerous studies on financial crises show that the shortage of liquidity is an element that is always present in times of major crises (currency crises, banking crises, bursting of speculative bubbles, payment system gridlock, etc.). However this risk, a component of market risk that is difficult to capture, is still not sufficiently accounted for in risk measurement and management tools. This could be harmful because it is precisely the failure of internal control systems that has often been the culprit in past episodes of financial turmoil such as the virtual collapse of the LTCM hedge fund in 1998.[1]

The purpose of this article is to present a number of techniques that aim to assess the risks incurred in the event of a decline in market liquidity and to examine to what extent these techniques may contribute to improving risk control in financial institutions and also help the relevant authorities to enhance their analyses and assessments of financial stability. We will examine beforehand the exact nature of market liquidity and the causes of its imperfection and chronic inconstancy.

1| NATURE OF MARKET LIQUIDITY AND RISK OF ILLIQUIDITY

1|1 Imperfect market liquidity and the importance of transaction costs

Liquidity may be defined as a range of characteristics rather than a one-dimensional attribute of assets and of the markets on which they are traded. It is also a relative concept, as the more liquid the asset, the more it is easily traded for liquidity "*par excellence*": money, *i.e.* at low cost, at short notice and with no risk of a notable change in price. A perfectly liquid market would therefore guarantee a single bid/ask price at all times and irrespective of the quantities being traded. Financial markets, even those deemed the most liquid, conform less than perfectly to this ideal configuration. Liquidity risk is therefore the risk of not being able to immediately liquidate or hedge a position at current market prices. This *market liquidity* risk is different from *balance sheet liquidity risk*, which is the inability to raise liquid funds by offloading assets *or borrowing*. It results from the fact that markets are not perfect at all times and in all segments (atomicity of participants, free entry

1 See CGFS (1999), Jorion (P.) (2000).

and exit at no cost, transparent information). The degree of liquidity of a market is traditionally assessed on the basis of three essential criteria:

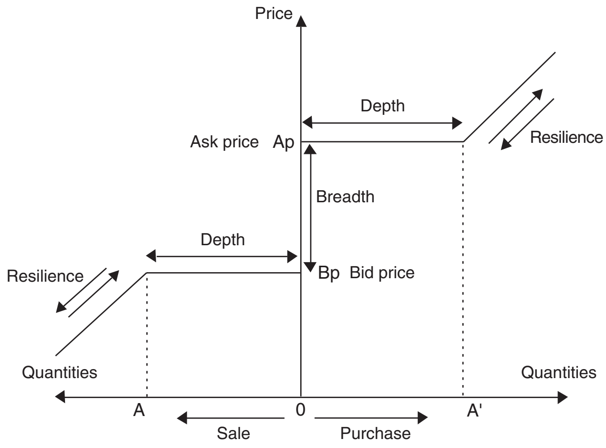
- the tightness of the bid-ask spread, which measures the cost of a reversal of position at short notice for a standard amount,

- market depth, which corresponds to the volume of transactions that may be immediately executed without slippage of best limit prices,

- market resilience, *i.e.* the speed with which prices revert to their equilibrium level following a random shock in the transaction flow.

The first aspect is a direct measure of transaction costs (excluding other operational costs such as brokerage commissions and clearing and settlement fees). The last two indicate the market's ability to absorb significant volumes without adverse effects on prices. The rest of the article will focus mainly on market breadth and depth insofar as it will pay more attention to the costs of immediacy than to how long it takes prices to return to equilibrium (see Chart 1).

Chart 1
Aspects of market liquidity



Note : The bid price Bp and the ask price Ap are defined for the standard amounts OA and OA'. The Bp-Ap spread represents the "breadth" of the market. The amounts OA and OA' are those that may be traded without price slippage: they reflect market "depth". Beyond points A and A', one sees the negative impact of large-value transactions on the execution price. Resilience refers to the time aspect of liquidity and indicates how quickly prices adjust to their equilibrium value following a shock in transaction flow.

The bid price is the highest price that the market maker is willing to pay at a given time to acquire a specific amount of assets. Symmetrically, the ask price is the lowest price at which the market maker is willing to sell a given amount of assets. The gap between the bid price and the ask price (the bid-ask spread) compensates the market maker for the immediacy of execution that it offers to its counterparties. The spread measures the cost of a sell/buy or buy/sell sequence over a short period (two-way transaction); only the half-spread should therefore be attributed to a single transaction (sale or purchase) if one considers that the mid-price is the one that should be paid in a perfectly liquid market. The tightness of the spread depends, *inter alia*, on the costs of processing orders from market makers, the size and volatility of accumulated order flows as well as the degree of information asymmetry between market makers and initiators of transactions (the market maker is exposed to the risk of dealing with investors that have private information regarding the real value of the asset). In a quote-driven market, the quoted spread[2] corresponds to the difference between the best bid price and the best ask price offered by market makers, whilst in an order-driven market, what is important is the difference between the best limit order book prices.[3] However, the spread quoted in the markets is not generally an exact reflection of transaction costs (for a buy/sell sequence) because certain transactions may be traded not at the bid or the ask price but at prices located within this spread, or even outside this spread, even for standard amounts.[4] In addition, the spread is a measure of the liquidity available at a given time. With a view to risk measurement and management, it is therefore important to take account of its variability over time.

In particular, the spread is quoted for limited amounts and it normally tends to widen in the presence of massive order flows, which is what the concept of depth refers to. In the case of a sale,

2 *In general, the quoted spread is expressed as a ratio of the mid-price. It is then called the relative (quoted) spread.*
3 *These two types of market organisation differ with respect to the price setting methods and the way in which liquidity is ensured. On an order-driven market, liquidity is created by matching orders in a central order book. On a price-driven market, liquidity is created through the actions of intermediaries (market makers) who guarantee investors a bid price and an ask price for a minimum amount.*
4 *One may therefore calculate the relative* effective spread*, corresponding to the absolute difference between the price at which a transaction was performed and the midpoint (the difference is expressed as a ratio of the midpoint).*

the negative impact of order flows on the bid price comes from the decline in the market aggregate demand curve depending on the amounts offered, *i.e.* its imperfect elasticity. The latter may be attributed, in particular, to the information asymmetry between market participants. A substantial flow of sell orders for an asset is in fact likely to arouse the suspicion that initiators of transactions have privileged information on the quality of this asset, and lead potential buyers to demand a price discount in exchange (the drop in the price may in fact lead to the total disappearance of the market[5]). Execution risk, *i.e.* the possibility of errors and delays in the settlement of large-value orders, may also prompt such market reactions. Price sensitivity to block transactions is often estimated using Kyle's λ (1985) in the following econometric equation:[6]

$$\Delta P_t = \alpha + \lambda NVOL_t + \varepsilon_t \qquad (1)$$

where price changes ΔP_t are a linear function of the net volume of trades ($NVOL_t$: difference between the amount of buy orders and sell orders in period t), with ε_t representing a random error. The λ coefficient assesses the market's ability to absorb large-value transactions: the higher the value, the smaller the market's absorption capacity.

Ultimately, to realistically assess portfolios' risk exposure it is necessary to take into account the transaction costs incurred during liquidation. Transaction costs cover the costs of resorting to the market for the allocation of resources and transfer of property rights, *i.e.* the execution costs referred to above (spread, impact on prices and other operational costs), and also the possible opportunity cost if transactions have had to be deferred (forced relinquishment of the benefits of immediacy).[7]

These liquidation costs are relatively predictable in tranquil periods: the corresponding risk appears to be fairly manageable at these times. Nevertheless, tranquil periods may mask the development of financial vulnerabilities and liquidity is a fragile feature of markets. Therefore, liquidation costs are much less predictable in the presence of market stress.

1|2 Fragility of liquidity: co-ordination problems and crises

Market liquidity depends essentially on the presence of a sufficient number of counterparties and their willingness to trade. The latter depends on investors' expectations regarding price developments and also their risk aversion at a given time, as well as the information available (*e.g.* on issuers' creditworthiness). A "good equilibrium" of regular liquidity therefore presupposes heterogeneous expectations and behaviour, ensuring the execution of orders irrespective of the transaction direction. Some analysts thus emphasise the contribution highly leveraged institutions operating under few regulations make to market making and liquidity. Similarly, it is rightly believed that liquidity is cumulative in nature and that, for example, the opening up of markets to new participants is likely to strengthen the positive externalities produced by a broader investor base. However, this must not lead us to ignore the precariousness of the collective valuation that –at any given time– characterises financial markets and even those that are deemed the most robust, for it is subject to sudden swings in opinion.

The perception of guaranteed liquidity has in the past therefore led a number of financial players to take excessively risky positions. C. Borio (2004) shows, in an anatomy of financial crises, that these crises are typically preceded by phases of excessive confidence in which risk exposure is heightened, fuelled by leverage. A hint of doubt creeping into market operators' minds is all it then takes to radically change the market configuration and trigger a liquidity crisis.

Liquidity crisis is illiquidity risk that has reached its paroxysm. It may be defined as the market's inability

5 *This phenomenon was highlighted and illustrated by Akerlof in 1970 for the second-hand car market. In the same vein, Genotte and Leland (1990) consider the 1987 stock market crash to be a result of problems of information asymmetry between market participants.*
6 *This conventional econometric specification postulates the stationarity of* ΔP_t *and* $NVOL_t$ *or the existence of a cointegration relation between these variables.*
7 *The latter may be calculated as the difference between the value of unexecuted orders at date* t *and that corresponding to the actual liquidation date* T *(based on the mid-price at* T*), see Perold (1988).*

to absorb order flows without provoking violent price adjustments that are unrelated to fundamental value. It is characterised by the sudden widening of the bid-ask spread, or even the total disappearance of buy (or sell) flows and the inability to trade. It often leads to an increase in short-term volatility as well as the slump of the primary markets. It therefore contains the seeds of serious systemic upheaval. The risk of having to face this type of event is that of booking major losses insofar as it is necessary to unwind positions in order to settle liabilities or meet hedging requirements.

This risk is difficult to capture because it refers back to the paradoxical nature of liquidity clearly outlined by Keynes in 1936: the liquidity of a financial asset simply does not exist for the financial community as a whole. In other words, an asset can only remain liquid if its liquidity is not put to the test by all investors simultaneously. The drying up of liquidity is in fact the consequence of the unanimous co-ordination of market players at a "bad equilibrium", with all players wishing to exit the market at the same time. Obviously, this type of occurrence can hardly be forecast: it may be triggered by a simple swing in the collective opinion, as shown, for instance, by the collapse, and then virtual disappearance of the Perps (Perpetual floating-rate notes) market in 1987.[8] In addition, the collapse of liquidity generates incentives that rationally induce economic agents to behave in ways that worsen the market situation as a whole. This negative chain of events is amply described in the literature on banking crises leading to rushes on bank deposits.[9] It occurs in exactly the same way in market crises involving flights to liquidity. It is rational to try to act first, *i.e.* to seek to exit as soon as distrust sets in, even if this behaviour only serves to speed up the evaporation of liquidity:[10] when it spreads, the fear of being short of liquidity is self-fulfilling. Naturally, a large financial institution (such as the main market makers on certain highly concentrated OTC derivative markets) may paralyse the market by pulling out. The difficulties faced by smaller but very active and highly leveraged participants, such as hedge funds (which may account for one-third or even half of daily transactions on the New York Stock Exchange and the London Stock Exchange) may cause the same types of problems. When they liquidate their assets in times of financial distress, their actions are likely to alarm other market participants and thus trigger "predatory trading",[11] *i.e.* selling in anticipation, which withdraws liquidity instead of providing it at the appropriate time.

Ultimately, liquidity crises reflect the dysfunction in the price regulation mechanism: instead of restoring the balance, price movements exacerbate pro-cyclical behaviour.[12]

Extreme liquidity risk is not a sum of minor independent risks, but rather systemic risk that leads to a major break in the usual statistical relationships between risk factors. It is, admittedly, a rare risk, but one that is inherent to liberalised financial systems where phases of excessive optimism alternate with sharp market decline. More generally, successive financial innovations reveal an increase in the revocability and conditionality of commitments to meet investors' demand for marketability-liquidity. While they serve to increase microeconomic efficiency, these facilities for pulling out of commitments –notional in times of stress– may give rise to the sometimes pathological spread of the preference for liquidity. With such specific risks, preventive measures rather than risky forecasts must be encouraged. Improving risk assessment tools is one essential aspect.

2| INCORPORATING LIQUIDITY RISK INTO RISK MANAGEMENT TOOLS

Value at Risk (VaR) is an estimate of the maximum potential loss that may be incurred on a position at a given time horizon and level of confidence. Though in recent years it has become a reference

8 *"Perps" were perpetual notes, whose coupons were linked to an interest rate reference (usually the Libor) and periodically redefined (every three months or every six months). Created in 1984, Perps were very popular, especially because the authorities allowed banks that issued them to include them in their regulatory capital under certain conditions and also because they enjoyed tax benefits. With higher yields than those offered on the interbank market, Perps attracted a large number of investors, in particular Japanese banks. In December 1986, there was a sudden drop in Perps prices. According to Fernando and Herring (2003), rumours and concerns surrounding a possible change in the Perps accounting regime were sufficient to precipitate the collapse of this market.*

9 *See Diamond (D.), Dybvig (P.) (1983).*

10 *See Bernardo (A.), Welch (I.) (2004).*

11 *See Brunnermeier and Perdersen (2005).*

12 *See Cohen and Shin (2003).*

for market risk management —and this is why we are focusing on it in this chapter— the VaR model does not satisfactorily capture liquidity risk, which is an integral component of market risk. In calculating VaR, it is assumed that the positions concerned can be liquidated or hedged within a fixed and fairly short timeframe (in general one day or ten days), that the liquidation of positions will have no impact on the market and that the bid-ask spread will remain stable irrespective of the size of the position. The price referred to is often the mid-price or the last known market price. However, as we have seen, the quoted market price cannot be used as a basis for valuating a portfolio that is to be sold on a less than perfectly liquid market: in practice, account must be taken of its orderly liquidation value or even its distress liquidation value. The standard VaR model is not a reliable guide because it neglects the risk to which a portfolio is exposed during its liquidation.

It is nonetheless possible to adjust VaR measures so as to incorporate trading execution costs into risk assessment. For the sake of simplicity, we will examine the liquidity of a single asset, as the measurement of the liquidity risk of a portfolio of assets leads us to non-specific considerations on the correlation of risk factors.

There are indeed *ad hoc* techniques for re-evaluating VaR by artificially increasing the volatility of positions deemed illiquid, or by lengthening the time horizon used for calculating VaR. However, because they propose arbitrary adjustments, these techniques do not deal directly with the issue of liquidity. The principle of adjustments that we present here consists essentially in re-calculating the distribution of asset returns by using not the market value but the liquidation value in "normal" times or in times of stress.

2|1 Taking "normal" liquidity risk into account

"Normal" liquidity risk is a relatively foreseeable average risk. It includes an "exogenous" component that corresponds to the average transaction costs set by the market and an "endogenous" component[13] that corresponds to the impact on prices of the liquidation of a position in an excessively tight market and that therefore applies to orders that are large enough to move market prices. The first component may be incorporated in an adjusted VaR model by using various bid-ask spread measures, while endogenous liquidity risk may be factored in by using measures of the elasticity of prices to volumes (impact measures).

Using bid-ask spread measures

The value realised from reselling assets is generally not equivalent to their theoretical market price because a liquidity cost, which is represented by the half-spread, puts a strain on the sales price.

It is therefore expedient, when examining historical bid-ask price series, to estimate the (exogenous) liquidity cost that may affect the market value of a position. A simple method would consist in adding the observed average of the half-spread to conventional VaR. This method does not however take account of the variability of the half-spread over time. This is why it would be wiser to try to extract from historical data series information on the statistical distribution of bid-ask prices, and in particular on their volatility.

Standard VaR calculations presuppose using a distribution of returns. The distribution may be determined using three types of methods (parametric, historical simulation or Monte Carlo simulation).[14] The same approaches may be applied to the distribution of bid-ask spreads with the aim of calculating the most unfavourable half-spread for a given time horizon and confidence threshold.The highest exogenous liquidity cost is thus obtained. It is added to the standard VaR defined for the same time horizon and the same confidence threshold.

Assessing the overall risk of an asset by simply summing up its price risk –which is reflected by the standard VaR model– and the exogenous liquidity cost amounts to assuming that these two components are perfectly correlated (*i.e.* that high variability of the mid-price is associated with

13 *This distinction is made by Bangia* et al. *(1999).*
14 *See Lévy-Rueff (G.), (2005).*

Box 1

Calculation of VaR adjusted for exogenous liquidity according to the parametric method

The parametric method for calculating VaR, applied to an asset, is based on the assumption of the normal distribution of returns of this asset. Let μ *be the expected value of the distribution and* σ *its standard deviation.*

The lowest return expected at date t*, at a confidence threshold of 99%, is:*

$$R_t^* = \ln (P^*_t / P_t) = \mu - 2{,}33\,\sigma \qquad (2)$$

where P_t *is the asset price (midpoint) at* t *and* P^*_t *the worst price expected at a confidence threshold of 99%. The coefficient 2.33 is the normal-law quantile at the confidence threshold of 99%.*

VaR at date t*, measuring the highest potential loss at a confidence threshold of 99% is by definition equal to* $P_t - P^*_t$ *so that using (2):*

$$VaR = P_t\,(1 - e^{\mu - 2{,}33\,\sigma}) \qquad (3)$$

If we assume that the relative bid-ask spread is also taken from a normal distribution, the worst relative spread at the threshold of 99% is:

$$\mu' + 2{,}33\,\sigma'$$

where μ' *is the expected relative spread and* σ' *its standard deviation. The exogenous liquidity cost (ELC) at a one-day horizon is the worst half-spread at the threshold of 99%:*

$$ELC = \tfrac{1}{2}\,P_t\,(\mu' + 2{,}33\,\sigma') \qquad (4)$$

*Lastly, liquidity adjusted VaR (*L-VaR*) is:*

$$L\text{-}VaR = VaR + ELC = P_t\,(1 - e^{\mu - 2{,}33\,\sigma}) + P_t/2\,(\mu' + 2{,}33\,\sigma') \qquad (5)$$

the high variability of the bid-ask spread itself). This assumption may on occasion lead to the overestimation of the risk.

Several studies show that such a correction leads, in some cases, to significantly boosting the amount of VaR. Bangia *et al.* (1999) estimate for example that, in May 1997, liquidity risk accounted for over 17% of the market risk of a long position on USD/Thai Baht and for only 1.5% for positions on USD/Yen, the Yen/USD market being very liquid. Using a similar methodology for the stock market (CAC 40), Le Saout (2002) shows that the liquidity component may be substantial (up to 52%) for certain securities. Looking at the Indian bond market, Roy (2005) finds that liquidity risk accounts for some 16% of the total risk on little traded securities.

The drawback of this methodology, however, is that it requires large samples of daily or even intra-day trading data, which are not always available.

Another option is to use the Roll (1984) measure, which seeks to provide an estimate of the implied spread using only observed market price series, under a number of assumptions (see Box 2).

This indicator may of course come in for criticism due to the underlying simplifying assumptions. Stoll (1989) considers that it is liable to underestimate the effective spread because it neglects the effects of information asymmetry when evaluating transaction costs.[15] Nevertheless, the Roll measure often provides relevant information on markets' liquidity situation.[16] Chart 2 shows how close Roll's coefficient

15 *See Huang and Stoll (1997) and Stoll (2000) for a review of bid-ask spread modelling and its breakdown into three types of factors: asymmetric information effects, inventory effects and order processing costs.*

16 *See Lesmond (2005), Upper (2000).*

Box 2

The Roll measure as a proxy for the bid-ask spread

Let an asset whose observed price p_t *at* t *is assumed to reflect the (unobservable) fundamental value* m_t *of the asset, adjusted by a fixed transaction cost between dates* t-2 *and* t*, be noted as* c *(half-spread):*

$$p_t = m_t + c\, q_t \qquad (6)$$

with qt = +1 *in the case of a purchase and* -1 *in the case of a sale.*

The fundamental value m_t *of the asset is assumed to follow a constant variance random walk (as dictated by public announcements). The expected value of* Δp_t *is therefore zero:* $E(\Delta p_t) = 0$*. If it is assumed that buy and sell order flows are equiprobable at time* t *and that they are serially independent and do not depend on random public announcements, it may be shown that:*

$$E(\Delta p_t . \Delta p_{t-1}) = \mathrm{Cov}\,(\Delta p_t, \Delta p_{t-1}) = -\, c^2 \qquad (7)$$

Therefore, the estimated half-spread c *at date* t *is:*

$$c = \sqrt{-\,\mathrm{Cov}(\Delta P_t\,, \Delta P_{t-1})} \qquad \text{si Cov}\,(\Delta p_t, \Delta p_{t-1}) \leq 0 \qquad (8)$$

*This measure is defined only if the co-variance between successive price changes is negative. To obtain a time series of Roll coefficients, one must therefore neutralise the observations for which the co-variance is positive. Intuitively, the negative co-variance is due to the fact that there cannot be two successive price increases (decreases) in the absence of public announcements (*i.e. *in the absence of changes in fundamental value) and if the spread is fixed; transactions therefore occur at the quoted bid or ask price.*

(calculated for each day between 27 October 2005 and 21 February 2006 on the basis of intra-day data) is to the half-spread actually quoted for a French share.[17]

The interest of using this measure as a tool for managing liquidity risk does not appear to have been considered hitherto. This is despite the fact that, in the absence of detailed databases, it can provide an indicative value of the bid-ask spread and be used to calculate an adjustment factor for VaR, for example as part of a historical simulation (see Box 3). This method for determining VaR adjusted for liquidity cost is recommended for its simplicity, because it does not require the formulation of assumptions on the probability distribution of returns or on that of Roll coefficients. Table 1 in fact shows that the latter cannot be considered here to be normally distributed, as the asymmetry coefficient (skewness) is not zero and the kurtosis is (slightly) above 3. VaR calculations adjusted according to the historical simulation method will therefore consist in using the historical distribution of returns and the historical distribution of Roll coefficients over the same period to estimate the distribution of possible losses –including liquidity cost– on a current position.

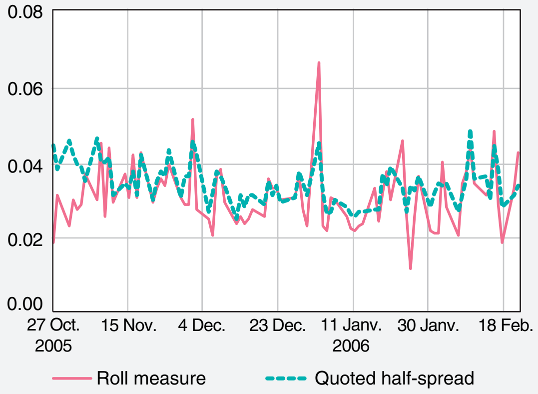
Estimates of the bid-ask spread nonetheless indicate the cost of immediacy for limited value transactions, corresponding to market depth. Prices can therefore be considered as exogenous. However, the higher the transaction amount, the more important it is to take account of the risk that a negative price movement could result from these transactions (endogenous liquidity risk).

[17] *We randomly selected the Essilor share. Identical calculations were carried out for all the CAC 40 companies, which, in general, revealed that the Roll coefficients were extremely close to the quoted half-spreads.*

Box 3

Calculating adjusted VaR using the Roll measure as part of a historical simulation

Chart 2
Roll measure and bid-ask spread of the Essilor share
(Eur)



Source: Bloomberg; Banque de France calculations

Here, we have a sample of daily Roll coefficients over four months only. For the sake of homogeneity, the series of daily returns on this share over the same period is used to calculate VaR adjusted for exogenous liquidity cost.

We apply the historical simulation method, i.e. *we use the observed distribution of daily returns as well as that of Roll coefficients between 27 October 2005 and 21 February 2006.*

At the threshold of 99%, the most negative rate of price change R* *is around –2.2% and the highest Roll coefficient EUR 0.067. As the mid-price* (P_t) *of the share was EUR 73.45 on 22 February 2006, the following results are obtained (for one share).*

Table 1

Features of the empirical distribution of Roll coefficients	
Average	0.031216
Median	0.030148
Minimum	0.011899
Maximum	0.067064
Standard deviation	0.008471
Skewness	1.130643
Kurtosis	3.038863

Table 2

Calculations of VaR adjusted for the Essilor share at a one-day horizon and at a threshold of 99%	
Lowest expected price $P^* = P_t\ e^{R^*}$	71.852
VaR = $P_t - P^*$	1.598
Exogenous liquidity cost (ELC)	0.067
Adjusted VaR LVaR = VaR + ELC	1.665
Share of ELC in adjusted VaR	4.02%

In this case, the exogenous liquidity risk (ELC) accounts for a fairly small proportion of the market risk of the Essilor share. This security therefore appears to have good liquidity on the calculation day (tightness of the spread). This evaluation however fails to cover the endogenous component of liquidity risk. Besides, the same type of calculation for other securities, outside the CAC 40 in particular, could yield significantly less favourable results.

Using impact measures

Endogenous liquidity risk is the risk that the actual price of a transaction may be significantly different from the price quoted just before the transaction was performed. This is also known as slippage. It is therefore important to determine the relation that may exist between the size of an order and the price slippage that this order may cause, *i.e.* define an impact measure (function).[18]

One may therefore seek to estimate a coefficient representing market depth, such as Kyle's λ or variants of it.[19] It must be noted that, in equation (1), the coefficient λ relates to sales as well as purchases, which means that the impact of

18 *It must be pointed out that these impact measures may be high (in other words, asset prices may vary considerably even though turnover is low) without being influenced by market depth or liquidity: for example when a public announcement leads to a price revision. Nonetheless, for long series, it is rare for major price adjustments to occur without corresponding transaction flows.*

19 *Kyle's coefficient is a frequently used indicator in studies on market microstructure. Pastor and Stambaugh's measure (2003) is close to this, the idea being that the reversal in the sign of returns following the trading of large volumes is more pronounced in less liquid markets.*

purchases and sales is assumed to be symmetrical. This simplifying assumption may be removed by estimating, for example, an econometric equation of the following type:

$$\Delta P_t = \alpha + \lambda_1 VOL_{P,t} + \lambda_2 VOL_{S,t} + \varepsilon_t \qquad (9)$$

where $VOL_{P,t}$ and $VOL_{S,t}$ respectively represent the volume of buy orders and the volume of sell orders over the period t, with λ_1 being the coefficient of the impact of purchases and λ_2 that of the impact of sales. In formulae (1) or (9), the term ΔP_t corresponds to the absolute (or relative) change in the mid-price after a transaction.[20]

If one sticks to equation (1), estimating coefficient λ at a given time requires the use of a sample of intra-day data on bid-ask prices, actual transaction prices and trading volumes. The procedure is as follows: based on intra-day data, to each transaction is attributed the midpoint price that precedes it by at least five seconds, allowing time for quotation changes. For the net volume of orders (*NVOL*), orders are classified between those initiated by buyers and those initiated by sellers. Following Lee and Ready (1991), transactions performed at a price above the midpoint are classified as "buy" transactions and those conducted at a price below the midpoint are classified as "sell" transactions.

Re-estimating equation (9) by linear regression at each date t using these data thus provides a time series of impact coefficients. Lastly, a corrective factor for VaR that takes account of endogenous liquidity risk must be deduced from the estimated coefficients. To do so, an historical simulation may be carried out. In the case of a sale, a discount determined by the coefficient λ of the day is subtracted from the mid-price. Both components of liquidity risk (endogenous and exogenous) may in fact be incorporated into the calculation in the process.

This methodology is also quite demanding in terms of data availability. A strategy that is easier to implement might be to start directly from restrictive (pessimistic) assumptions on the price-elasticity of demand and supply, in order to estimate the potential costs arising from the liquidation of a portfolio in a shallow market.

Assuming firstly, market demand that is quite inelastic in the short term, so that daily transaction value is assumed to be fixed (equal to that observed on the calculation day), and secondly, inelastic supply (meaning that the seller accepts any price imposed by market demand, provided that its order is fully executed) leads us to form fairly pessimistic expectations of market liquidity: the absorption of an excess supply of securities leads solely to a drop in the average ask price (absence of quantity effect, *i.e.* of adjustment of the quantity demanded to supply). Therefore, if V is the daily trading value of a certain category of securities (assumed to be fixed), N the number of stocks traded daily (assumed to be known), P the average price of the stock before the transaction ($P = V/N$), the excess supply ΔN pushes the average price from P to $P' = V / N + \Delta N$. The rate of change in the price resulting from the liquidation of ΔN stocks is therefore:

$$\Delta P/P = -\Delta N/N \qquad (10)$$

Equation (10) thus makes it possible to adjust the historical returns observed for an impact factor $\lambda' = \frac{\Delta P/P}{\Delta N/N}$ linked to the size of the portfolio to be liquidated, and to calculate adjusted VaR using a historical simulation. This approach is doubly interesting. First because it is based on conservative assumptions, which are appropriate in a risk-control perspective, and second because it uses data available for a large number of stock markets (average daily transaction value and average number of stocks traded daily). D. Cosandey (2001) illustrates this approach by considering the liquidation of equity portfolios.

The same concern about data availability may justify recourse to more general indicators, such as the Amihud (2002) illiquidity ratio, which may be calculated on the basis of daily transaction prices

20 *It is possible to select changes in actual transaction prices as a dependent variable, but it may be preferable to use the mid-price because it can change in the absence of transactions, unlike the transaction price. Using the transaction price could lead to underestimating the impact of a sale. Besides, equation (9), like equation (1), assumes the stationarity of variables at play or the existence of a cointegration relation between these variables.*

and volumes. For a given asset class, the average illiquidity ratio in the month t is defined by:

$$\gamma_t = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{d,t}|}{v_{d,t}}$$

with $r_{d,t}$ being the return on the asset $v_{d,t}$ the volume of the asset traded (in currency units) on day d, during the month t, and D_t the number of observations in the course of the month t. To obtain a daily assessment, the following expression is considered:

$$\gamma_d = \frac{|\ln P_d - \ln P_{d-1}|}{v_d}$$

where P_d is the price observed on day d, and v_d the turnover on day d. The γ_d coefficient may be considered as the equivalent of Kyle's λ estimated at low frequency. As a general indicator, it applies symmetrically to purchases and sales. No distinction is made here between fundamental factors and liquidity problems in price changes. In a study of the US stock market, Hasbrouck (2005) however finds that the Amihud ratio is the best indicator based on daily data of the impact of volumes on prices. The γ_d coefficient may therefore replace Kyle's λ in the calculation of adjusted VaR.

It must here again be emphasised that liquidity risk estimates are often hampered by a lack of data: for example, the trading volumes required to calculate the Amihud ratio are rarely available on the foreign exchange markets. In addition, there is a lack of directly usable market data for certain little traded derivatives. Pricing these instruments therefore means using valuation models and the issue of liquidity risk assessment therefore becomes part of the larger problem of model risk (see below).

Moreover, a more in-depth analysis of impact functions would in fact require more sophisticated modelling than that derived from Kyle's model. Thus, the dynamic properties of prices would be better captured by introducing lagged variables into the impact function. Similarly, the linear specification of the impact function may be called into question, as the liquidity premium (or discount) demanded by the market for the execution of orders is probably not strictly proportional to their size.[21] In addition, we have hitherto considered a stable relationship between turnover and price changes. Another methodological approach would be to consider that price elasticity is random in nature due to the impact of unpredictable factors, other than order size, on price changes. Some recent theoretical studies thus attempt to directly model the reaction function of the market to volumes as a stochastic process,[22] but the possibilities of practical implementation with respect to risk management remain uncertain.

Lastly, we made the implicit assumption of a block and instantaneous liquidation of a position to measure its incidence on prices. However, the market impact differs depending on the duration of the liquidation. An element that is decisive for the impact on prices is in fact the *liquidation rate*: the aggressive liquidation of a position may provoke a powerful negative reaction from the market. Conversely, chopping transactions up into small-sized orders that are presented on the market progressively may, in theory, significantly reduce liquidity costs. Nonetheless, slower liquidation generates higher opportunity costs and exposes the position to price volatility (price risk) over a protracted period. There is therefore a trade-off between the risk of price change and endogenous liquidity risk, with the optimal liquidation strategy making it possible to partially control the impact on prices by choosing the amount to be liquidated periodically or the time frame for liquidation.[23] The assumption of immediate liquidation is in fact justified in crisis situations when investors cannot choose their liquidation strategy, but are subject to panic (see below).

The spread and elasticity measures suggested, based on past observations, are supposed to provide relevant information for "average" situations and fall smoothly within the framework of the VaR model. They therefore appear to be useful for the management of normal liquidity risk.

However, market liquidity may undergo violent upheavals. The impact of *collective* liquidation in times of stress is not covered by measures of endogenous

21 *See Almgren and Chriss (2003).*
22 *See Jarrow and Protter (2005), Çetin* et al. *(2004).*
23 *On optimal liquidation models, see Bertsimas and Lo (1998), Almgren and Chriss (2000), Berkowitz (2000), Subramanian and Jarrow (2001).*

risk –which concern the liquidation of individual positions– and even less so by the measures of exogenous risk considered above. This underlines the difficulty for market participants to protect themselves from a risk that is systemic in nature.

2|2 Capturing extreme liquidity risk

A general liquidity crisis is an extreme event: it occurs infrequently but its consequences may be very costly. The singularity of such events calls for appropriate risk management tools. The extreme value theory and stress tests may help to prevent these types of events. More generally, the fundamental uncertainty surrounding market liquidity, in particular in periods of turmoil, calls for a precautionary approach and the consideration of the model risk that affects the various indicators of liquidity risk.

EXTREME VALUE THEORY AND MANAGEMENT OF LIQUIDITY CRISIS RISK

Beyond the limitations stemming from the fact that they do not cover transaction costs in normal times, conventional VaR calculation methods appear to be poorly adapted to the capture of liquidity crises, which are extreme events. The historical simulation is in fact liable to be based on a sample of data that is too narrow to take account of the worst eventualities, while the parametric method is based on the use of normal distributions that tend to underestimate the distribution tails of losses.[24] Nonetheless, it is in fact these tails that must be properly assessed in financial risk management.

The extreme value theory provides theoretical results on the limit (asymptotic) probability distribution of extreme losses and it does so without having to formulate strong assumptions on the form of the underlying distribution of returns. The principle consists in extrapolating the behaviour of possible extreme values based on the sample of the greatest losses actually observed.

From a practical perspective, *i.e.* for the calculation of the parameters of the asymptotic distribution, we therefore do not use a complete series of returns and losses, but only the series of maximum losses (the sharpest drops in prices). Two approaches are used to construct this sub-sample:

• the block maxima method, which consists in breaking the historical observation period into data "blocks" of equal length (*e.g.* monthly, quarterly or half-yearly blocks) which are divided into n time periods, and using only the upper bound of each block. When n becomes very large, the maxima follow a "generalised extreme value distribution".

• The peaks over threshold method, which is based on the analysis of losses exceeding a high threshold u to be defined. When u becomes very large, the losses exceeding the value u follow a "generalised Pareto distribution".

Each of these methods therefore shows a limit probability distribution whose parameters are estimated using the sub-sample of extreme values. This distribution may then be used to assess exceptional risks.[25]

To apply extreme value theory to liquidity crisis risk one must therefore, ideally, have long series of indicators on average liquidity risk. For example, where one is fortunate enough to have a significant sample of bid-ask spreads for a financial product, one must extract from this series the widest spreads in order to determine their distribution and calculate the corresponding VaR adjusted for extreme liquidity cost. This also holds for indicators of endogenous risk.[26]

Another way of proceeding, which is justified by the interdependence of risks during major crises –in particular between price risk and liquidity risk– would be to directly calculate an extreme VaR, while considering that the market data that were used to construct this VaR model provide information on the risks of price changes as well as the risks of loss arising from the evaporation of liquidity. While this

24 *This is why even CVaR (Conditional VaR) or Expected Shortfall, which corresponds to the average amount of losses exceeding a predefined amount of VaR and which therefore concerns possible extreme losses, is liable to underestimate the real distribution of extreme losses so long as it is calculated –like the VaR on which it is based– on the basis of a normal distribution.*

25 *On extreme value theory, see Embrechts* et al. *(1997).*

26 *It must be noted that the extreme value method is a* semi-parametric *method for assessing risk, for it is based on the explicit formulation of a probability distribution. Some studies use a more empirical approach, by determining a corrective factor for non-normality based on the historical distribution of the spread. See Bangia* et al. *(1999) and Le Saout (2002).*

method is simpler, it is probably less precise because it does not model the determinants of liquidity.

Extreme value theory is an attractive approach for assessing extreme illiquidity risks that is grounded on clear foundations. Nevertheless, as an asymptotic theory, it requires sufficiently long observation series, which could pose problems, especially when the markets are not very mature. It is also difficult to implement in its multivariate version when too many risk factors are taken into account. This highlights the limits of a strictly statistical approach to liquidity crisis risk.

Liquidity stress tests

The difficulty of applying extreme value theory in the face of the complex chain of events that is set in train during crises may lead to a preference for other approaches based on simulation, *i.e.* stress tests. The Basel Committee moreover acknowledges stress tests to be an integral element of the internal control of market risks.[27]

Stress tests encompass various practices (scenarios, sensitivity analyses) that aim to assess the consequences of infrequent but plausible large-scale shocks on the value of a portfolio. Stress tests therefore presuppose first, the definition of the relevant market movements and second, the quantification of their impact on the value of the portfolio. Following the Asian crisis of 1997 and the events of autumn 1998, the large financial institutions improved their ability to conduct such tests.[28]

From the perspective of extreme liquidity risk, stress tests make it possible to assess the implications of certain phenomena that characterise crisis episodes: unusual increase in the correlation of risks, atypical rise in volatility or even the inability to trade on certain market segments. They therefore enable the detection of complex risks that are difficult to capture using a probabilistic approach.

Liquidity crisis is not only an extreme event, it is also a collective phenomenon that develops and fuels itself in an endogenous manner on the spread of liquidity fears. It is therefore crucial to examine the factors that may lead to such one-way market movements. One of these factors is the practice of stop-loss rules, *i.e.* the mechanical triggering of a sell order when the market price overshoots a predefined threshold, another is dynamic hedging, which leads net issuers of put options to liquidate the underlying securities for hedging purposes when the prices of these securities drop sharply and a large majority of option holders seek to exercise their options. These waves of mechanical selling on bearish markets are instrumental in the spread of liquidity problems.

More generally, an institution's crises scenarios must, as much as possible, incorporate the fragility of its entire balance sheet, *i.e.* the possible mismatches between illiquid investments on the assets side and precarious sources of finance on the liabilities side. For example, the difficulties of meeting margin calls on collateralised loans or uncertainty surrounding borrowing capacity may herald more major difficulties when they lead to the unwinding of positions under stress.

An especially edifying stress test for a large institution would be to envisage the consequences of the simultaneous withdrawal of its counterparties from the market. This was in fact the path taken by JP Morgan and Deutsche Bank in the aftermath of the near bankruptcy of LTCM. Aware of the potential dangers arising from the high concentration of certain markets (interest rate options and credit derivatives), the two banks developed "dealer exit stress tests" aimed at estimating the risks of a sudden drying up of market liquidity generated by one or several of their counterparties.[29] In this regard, it is important for institutions to consider liquidity conditions from a *systemic* angle by paying particular attention not only to the possible impacts of the concentration of the positions of their counterparties and other market participants, but also to their own impact on the markets when they account for a substantial fraction of these markets.

Besides the problems relating to the relevance of the scenarios chosen, the implementation costs and the low frequency of stress tests, one of the essential difficulties of stress tests remains endowing the scenarios envisaged with a degree of plausibility. This intrinsic limitation of risk measurement,

27 *See the 1996 Amendment to the Capital Accord to incorporate market risks.*
28 *See CGFS (2000) and (2001).*
29 *See Jeffery (2003).*

particularly for very rare and systemically important events, highlights the importance of experience and good judgement in the analysis of the potential scale of liquidity risk.

MODEL UNCERTAINTY AND THE PRECAUTIONARY APPROACH

Indeed, in liquidity risk management, the problem of *measurement* is closely linked to that of *decision-making* under uncertainty. Faced with the uncertainty that surrounds future liquidity conditions, and consequently measures of illiquidity risk, *forecasting* appears in fact to be a delicate or even insurmountable task, especially for major phases of stress. This in no way signifies that all efforts of quantification must be abandoned. It instead requires the adoption of a *precautionary* attitude in the face of the inaccuracy of the risk indicators.

One must therefore acknowledge the shortcomings that at times result from the lack of information and the complexity of the interactions, and appreciate that the models designed to assess risks may be subject to uncertainty. This is particularly clear for the valuation of little traded derivatives, whose pricing is based exclusively on the construction of models.[30] Two main types of errors may, in general, compromise a model's reliability: errors concerning parameters and errors of specification. In any case, the most harmful attitude in practice would be to place one's trust in biased indicators tending towards the underestimation of liquidity problems.

Taking into account the risk linked to model uncertainty and the precautionary principle therefore leads to the consideration, around a reference model, of a set of possible alternative models and the choice in the risk assessment process, of the model that yields the greatest losses. This conservative strategy, which it seems appropriate to use when it is not possible to rely on conventional statistical inference procedures, refers back to a type of uncertainty that cannot be perfectly measured in terms of probabilities and which affects the liquidation value of an asset in certain critical situations. This strategy may in fact be transposed into mathematical terms and yield quantified operational results.

In the probabilistic context of the extreme values theory, it has been suggested that the calculation of an extreme VaR model encompassing market risk and liquidity risk may be a way of factoring in extreme liquidity risk. Similarly, the determination of a sufficiently robust adjusted VaR measure could play this role in a context of strong uncertainty. In this regard, Rey *et al.* (2004) provide an interesting illustration. The authors develop a calculation of VaR by linking each possible loss not to a point probability, but to an interval of probabilities, whose width is dependent on a parameter that represents perceived uncertainty. By setting the value of this parameter at a level that is sufficiently high, in accordance with the precautionary principle, the aim is to capture the highest liquidity risk within certain bounds of plausibility. Recognising the incomplete nature of the information available and the limited capacity to process this information should in fact be an integral part of all liquidity risk control strategies.

30 *See Cont (2004).*

The international financial community currently appears to have a better grasp of the risks associated with liquidity illusion. A working group made up of large private financial institutions[31] recently focused on the initiatives that need to be taken by private players in their risk management procedures in order to bolster the stability and efficiency of the financial system. The report resulting from this work especially underlines the need for financial institutions to strengthen their capacity to assess the possible threat of crowded trades, which means that they need to re-appraise their usual risk measures and enhance their stress tests.

To this end, this article has attempted to clarify the different aspects of illiquidity risk and explore a few avenues for incorporating this risk into the existing risk control tools. Two main conclusions may be drawn.

- *There is a productive convergence between research on financial market microstructure and risk management practices. This convergence should make it possible to construct and track relevant indicators of market liquidity risk.*

- *It is essential to make a distinction between normal times and times of stress. This distinction calls not only for the development of separate methodologies, of which we have provided a few examples, but also for the combination of these different methods in a comprehensive risk approach.*

Overall, the construction of appropriate models that help financial institutions to better evaluate their risk exposure must be encouraged as an element of market self-discipline. However, liquidity risk management cannot of course be reduced to the search for quantitative indicators: experience and good judgement are crucial in this area, particularly for coping with periods of stress when market behaviour sometimes diverges from conventional models.

In addition, it must be borne in mind that from a macroeconomic point of view, market liquidity is the product of externalities generated by all market participants, which agree to act as counterparties in sale/purchase transactions and by so doing perform a market making function. Even though financial institutions may be aware of the collective nature of liquidity, there is little likelihood of them being able to make the most of this: first because their information on the scale and overlapping of risks is necessarily limited; second because they have no incentive, from the point of view of individual rationality, to shore up market liquidity when it is floundering. It is therefore a major stake and clearly a matter of concern for public authorities.

Communication between markets and authorities is at the heart of the prevention of generalised illiquidity risk. This is why it is necessary not only to encourage information transparency between market participants (for example between hedge funds and their counterparties) so that they can build appropriate tools for managing liquidity risk, but also for supervisors to have better means of assessing such risks at the financial system level, and transfer, if need be, the results of this assessment to the markets. From this point of view, the performance of aggregate stress tests could assist in the quantitative assessment of the financial system's resilience to liquidity shocks. These stress tests should incorporate participants' size and level of leverage, identify their sources of finance and more generally take account of the multiple interdependence networks between counterparties. Similarly, the development of regular indicators for monitoring market liquidity would be useful to public authorities in the carrying out of their financial stability tasks.

31 *The Counterparty Risk Policy Management Group II, chaired by Corrigan (E. G.). See the group's report, "Toward greater financial stability: A private sector perspective" published in July 2005.*

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